Exercise 1

Connected to a one phase rectifier (figure 1), the rms value of the grid voltage is $V_s = 120\text{V}$ at a frequency $f = 50\text{Hz}$ with a inductance $L_s = 1\text{mH}$.

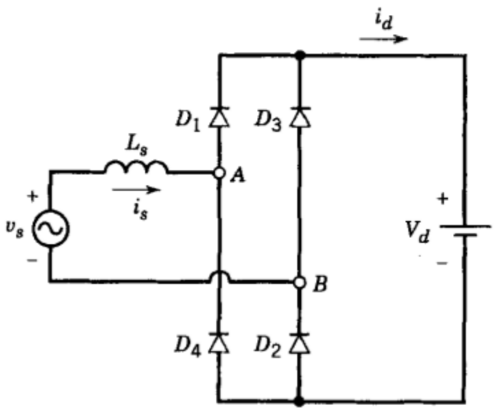

Figure 1: Rectifier with a constant DC-side voltage.

a) Obtain the current i_d of the DC-side and its peak value when the voltage over the DC-side is $V_d = 150\text{V}$.

b) The average value of the current $i_d \approx 10,5\text{A}$. Using the figure 2, define the value of V_d, the power factor PF, the displacement power factor DPF, the crest factor and the ratio I_s/I_d. What is the current i_d peak value?

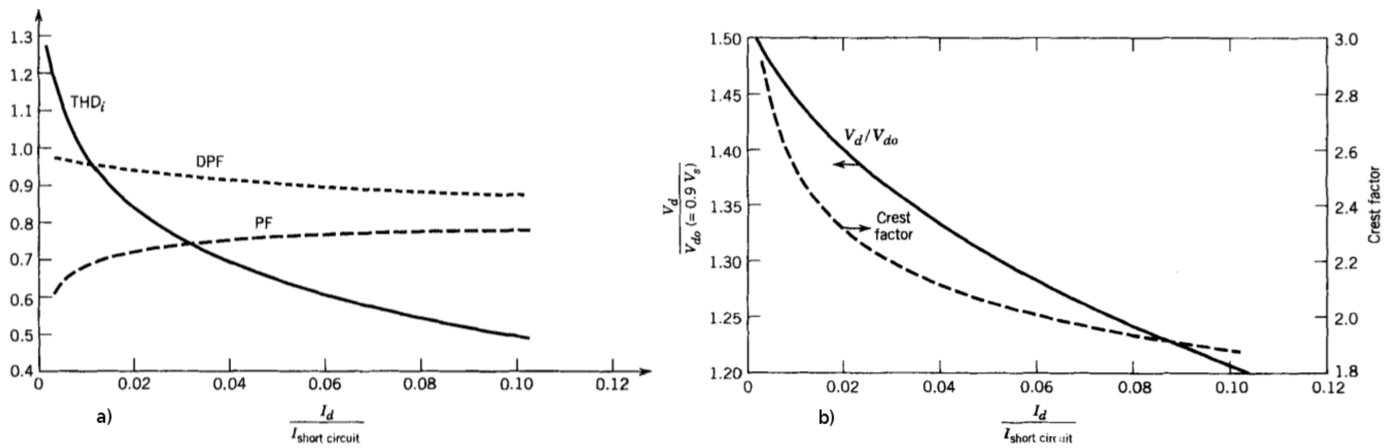

Figure 2: a)Total harmonic distortion, DPF, and PF. b)Normalized V_d and the crest factor

Solution

Part a

Using the figure 3, the angle θ_b from which the bridge starts conducting is obtained from

$$v_s(\omega t = \theta) = \sqrt{2}V_s \sin(\theta) = V_d \Rightarrow \theta_b = \arcsin\frac{V_d}{\sqrt{2}V_s} \approx 1,08 \quad \text{or} \quad \theta_p \approx \pi - 1,08 \approx 2,057 \quad (1)$$

These angles correspond to the times $t_b = \theta_b/\omega \approx 3,44$ms and $t_p \approx 6,55$ms.

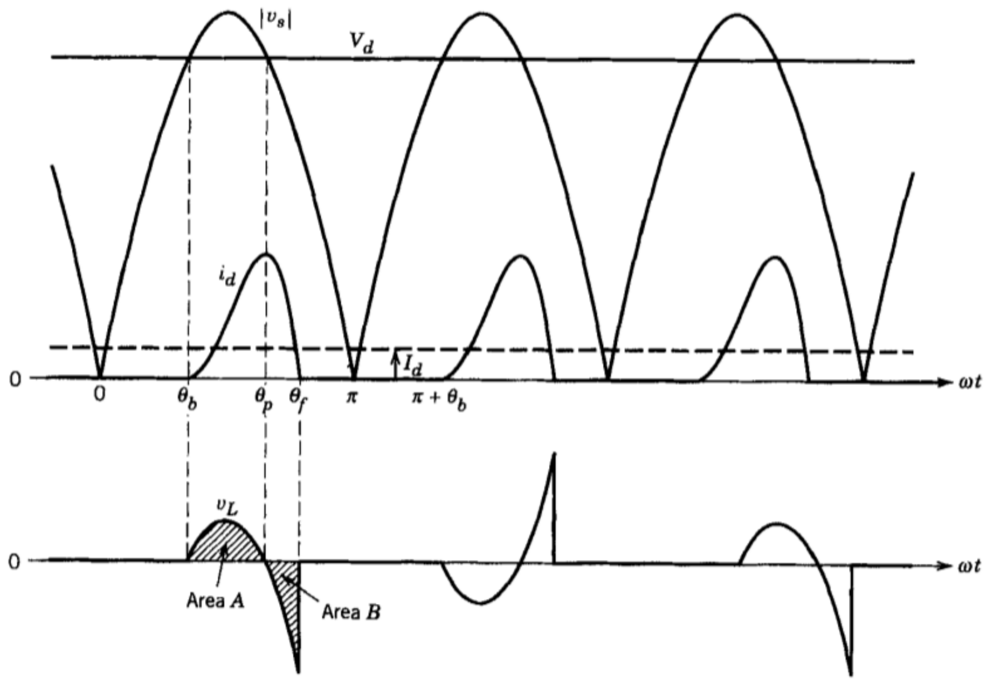

Figure 3: Waveforms of the rectifier.

After angle θ_p, the voltage of the grid is under the output voltage V_d and the voltage over the inductance v_L becomes negative and the current i_d starts to drop. The voltage over the inductance is

$$v_L(t) = \sqrt{2}V_s\text{sin}(\omega t) - V_d = Ls\frac{di_d(t)}{dt} \tag{2}$$

from which we get the current

$$i_d(t) = \frac{1}{L_s}\int_{t_b}^{t} v_L(t)dt = \frac{1}{L_s}\int_{t_b}^{t} (\sqrt{2}V_s\text{sin}(\omega t) - V_d)dt \tag{3}$$

$$i_d(t) = \frac{\sqrt{2}V_s}{\omega L_s}(\cos(\omega t_b) - \cos(\omega t)) - \frac{V_d}{L_s}(t - t_b), \qquad \text{when } t > t_b \tag{4}$$

The peak value of the current $i_d(t)$ is obtain at $t = t_p$ and we obtain from equation 4: $i_{d,peak} \approx 40,54$A.
The angle θ_f, when the current $i_d(t)$ goes to zero, is obtained from the equation 4 equal to zero and we get

$$i_d(t) = \frac{\sqrt{2}V_s}{\omega L_s}(\cos(\omega t_b) - \cos(\omega t_f)) - \frac{V_d}{L_s}(tf - t_b) = 0 \tag{5}$$

which becomes

$$\frac{\sqrt{2}V_s}{V_d}\text{cos}\theta_f + \theta_f = \frac{\sqrt{2}V_s}{V_d}\theta_b + \theta_b \tag{6}$$

This kind of equation cannot be solve easily. We can do with "trial and error" or with a software like Matlab and we get $\theta_f = 2,556$ with $t_f = 8,14$ms.

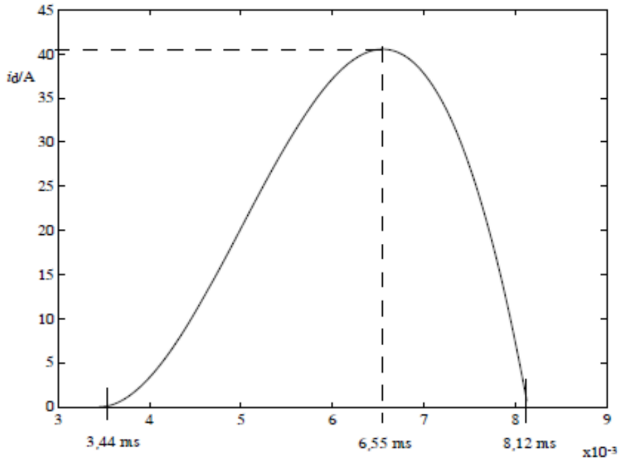

Figure 4: Waveforms of the current in the DC-side $i_d(t)$.

Part b

To use the figure 2, we need to calculate $I_d/I_{\text{short circuit}}$. The short circuit current is given by

$$I_{\text{short circuit}} = \frac{V_s}{\omega L_s} \tag{7}$$

and we get

$$\frac{I_d}{I_{\text{short circuit}}} = \frac{I_d \omega L_s}{V_s} \approx 0,0275 \tag{8}$$

And from the figure 2, we get PF$= 0,72$, DPF$= 0,93$ and $V_d/V_{d0} = 1,375$, the crest factor$=$ $2,25$ where V_{d0} is the DC-voltage V_d when the inductance $L_s = 0$. When $L_s = 0$ we have

$$V_{d0} = \frac{1}{T/2}\int_0^{T/2} \sqrt{2}V_s \sin(\omega t)dt = \frac{2}{\pi}\sqrt{2}V_s \approx 0,9V_s \tag{9}$$

And we obtain $V_d = 1,375 \times 0,9 \times V_s \approx 148,5$V. Which is close to exercise value.
The crest factor is defined as

$$\text{crest factor} = \frac{I_{s,peak}}{I_s} = \frac{I_{d,peak}}{I_s} \approx 2,25 \tag{10}$$

and we obtain $I_s = 18,01$A. And finally $\dfrac{I_s}{I_d} = 1,71$A.

Exercise 2

In the figure 6 is represented different factors of the three phase diode full-bridge of figure 5.

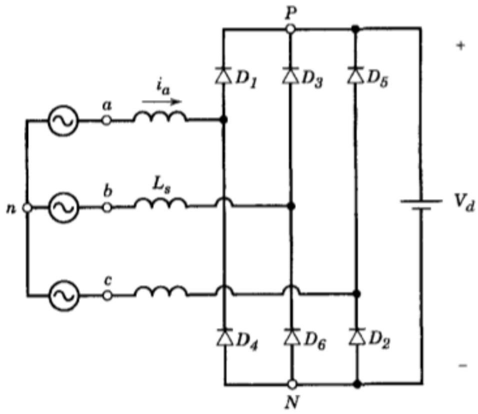

Figure 5: 3-phase rectifier with a constant DC-side voltage.

In a extreme case, the value of the inductance approaches infinity which makes the output current i_d flat and the maximal power factor is PF$= 0,955$. Calculate in the same case, the maximal power factor for a single-phase diode full-bridge (figure 1)?

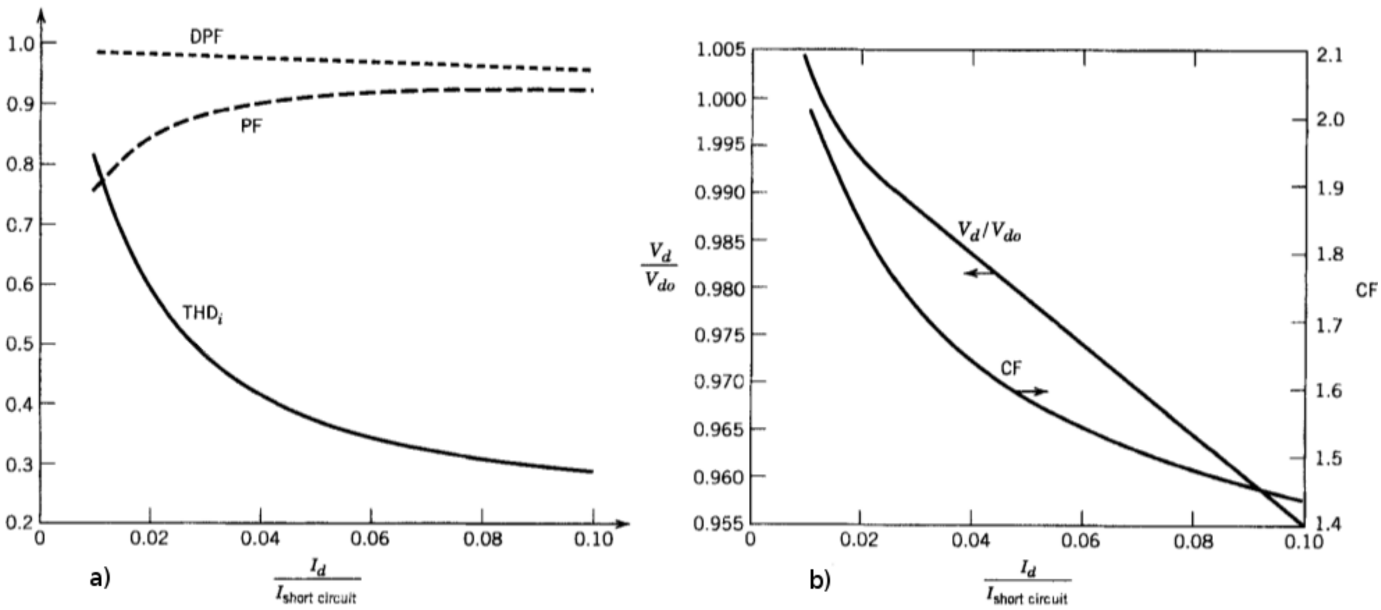

Figure 6: a)Total harmonic distortion, DPF, and PF. b)Normalized V_d, and crest factor

Solution

When the current id is considered to be "flat" (figure 7), the current is form with positive and negative π-large pulses of amplitude I_d.

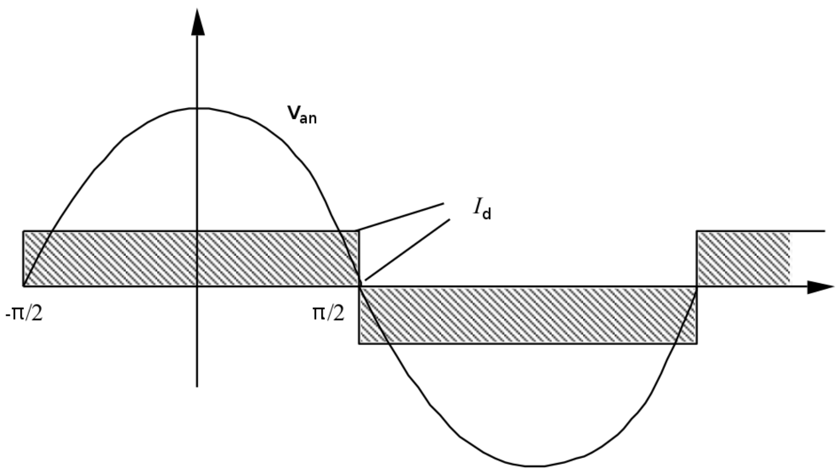

Figure 7: The waveform of the current of one phase i_a.

The current i_s fundamental $\hat{i}_{s1}$ is obtain from the Fourier components and is given by

$$\hat{i}_{s1} = \frac{2}{\pi}\int_{-\pi/2}^{\pi/2} I_d \cos x dx = \frac{4I_d}{\pi} \tag{11}$$

The rms-value of the current of the grid is I_d. From the figure 7, it is easy to see that the current is in phase with the voltage if the network. The displacement power factor is then 1. The power factor is

$$\mathrm{PF} = \frac{P}{S} = \frac{V_{s_1} I_{s1}\cos\phi_1}{V_s I_s} = \frac{I_{s1}}{I_s}\cos\phi_1 = \frac{I_{s1}}{I_d} = \frac{4}{\sqrt{2}\pi} \approx 0,9 \tag{12}$$

It is smaller than with a three phase diode full-bridge.
We can also calculate that the displacement power factor is 1 like in the figure 7. The average value of the DC-side voltage is

$$V_d = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \sqrt{2} V_s \cos x dx = \frac{2\sqrt{2}V_s}{\pi} \tag{13}$$

The power flows only through the grid fundamental. With the harmonic currents the average of the power is zero. They only add distortion. When the bridge is considered as lossless we can write

$$P = V_s I_s \cos\phi_1 = V_d I_d \tag{14}$$

$$P = V_s \frac{4I_d}{\sqrt{2}\pi}\cos\phi_1 = \frac{2\sqrt{2}V_s}{\pi} I_d \Rightarrow \cos\phi_1 = 1 \tag{15}$$

Exercise 3

In a forward-converter, the inductance of the output filter is $L = 20,7\mu$H and its current minimal, maximal and rms values are $i_{L,min} = 5,16$A, $i_{L,max} \approx 14,84$A and $I_{L,rms} = 10,38$A. The switching frequency is $f_s = 100$kHz. The inductance L is built with a double E-core where the maximal magnetic flux density is $B_{max} = 0,3$T. It is Assumed that the surface of the core is $A_c = a^2$ and the dimensions for the winding are $l_w = 0,5a$ and $h_w = a$. The surface of the airgap is $A_g = 1,2A_c$ and the copper fill factor $k_{Cu} = 0,7$.

a) Calculate the surface of the core A_c when the current density is $J = 2\text{A.mm}^{-2}$.

b) Considering the temperature, calculate the maximal allowed value of the current density J when

$$J = 450AP^{-0,125}\text{A.cm}^{-2} \text{ and } AP = A_w A_c \tag{16}$$

c) Obtain the number of turns N and the airgap length g.

Solution

Part a

To obtain the core surface area A_c, we need to find a which is obtain from

$$L_{max} I_{L,max} I_{L,rms} = (k_{Cu} J B_{max}) A_w A_c = (k_{Cu} J B_{max}) 0,5a^2 a^2 \tag{17}$$

which becomes

$$a = \left(\frac{L_{max} I_{L,max} I_{rms}}{0,5 k_{Cu} J B_{max}}\right)^{\frac{1}{4}} \approx 11,1\text{mm} \Rightarrow A_c = a^2 \approx 123,2\text{mm}^2 \tag{18}$$

Part b

To obtain the J, we must first calculate the value of AP. Using equation 17, we get

$$AP = A_w A_c = \frac{L_{max} I_{L,max} I_{L,rms} \times 10^4}{k_{Cu} J B_{max}} = \frac{L_{max} I_{L,max} I_{L,rms} \times 10^4}{k_{Cu} 450 AP^{-0,125} B_{max}} \tag{19}$$

which gives

$$AP = \left(\frac{L_{max} I_{L,max} I_{L,rms} \times 10^4}{450 k_{Cu} B_{max}}\right)^{\frac{1}{0,875}} \approx 0,288\text{cm}^4 \tag{20}$$

with that result, we get $J \approx 525,7\text{A.cm}^{-2} = 5,257\text{A.mm}^{-2}$ and $a = 8,71\text{mm}$

Part c

To calculate the number of turns

$$N = A_w \frac{k_{Cu} J}{I_{L,rms}} = 0,5a^2 \frac{k_{Cu} J}{I_{L,rms}} \approx 13,45 \approx 13 \tag{21}$$

The airgap is obtain from

$$g = \frac{N' I_{L,max}}{2H_{g,max}} = \frac{\mu_0 A_g N' I_{L,max}}{2B_{max} A_c} \approx 0,485 \tag{22}$$

Exercise 4

In a forward-converter the voltage over the filtering capacitor is $210 \leq V_d \leq 325\text{V}$. It is varying according to changes in the load and the changes in the voltage of the grid. The output voltage is $V_o = 5\text{V}$ and the output power is $15 \leq P_o \leq 50\text{W}$.
The transformer is built from ferrite material 3C8 with a core of model ETD 34. Its volume is $V_e = 7640\text{mm}^3$, the effective area $A_e = 97,1\text{mm}^2$ and the smallest surface area of the core is $A_{c,min} = 86,6\text{mm}^2$. The switching frequency is $f_s = 50\text{kHz}$ and the demagnetization winding number of turns is $N_3 = N_1$ with N_1 the primary winding number of turns. The copper fill factor is $k_{Cu} = 0,6$.

a) Calculate the switching conduction time ratio $D_{e,max}$ and the winding ratio a when for the switching transistor has a reduced conducting of 1μs. The conducting time is shortened as followed

$$D_{e,max} = D_{max} - aI_of_s \times 1,2 \times 10^{-9} \qquad \text{with } a = \frac{N_1}{N_2} \tag{23}$$

b) The saturation flux density is $B_s > 0,32\text{T}$ at $100°$. Calculate the primary and secondary winding turns when the transformer has a small airgap such as the remanence flux density $B_r \approx 0\text{T}$.

c) What is the rms value of the maximal current allowed in the secondary when $A_w = 90\text{mm}^2$ when taking the temperature in account knowing that

$$J = 450AP^{-0,125}\text{A.cm}^{-2} \text{ and } AP = A_wA_e \tag{24}$$

d) Calculate the losses in the core when the hysteresis and eddy current losses in the material at the frequencies 10kHz to 100kHz are respectively

$$P_h \approx 16,7 \times f_s^{1,3}\Delta B_c^{2,5}V_e \;\text{ and }\; P_e \approx 0,8 \times f_s^2\Delta B_c^2\frac{A_eV_e}{\rho} \qquad \text{with } \rho = 0,4\Omega\text{m} \tag{25}$$

e) Calculate the length of the airgap needed when the inductance of the primary is $L = 10\text{mH}$.

Solution

Part a

To have the magnetization returning to zero before the next cycle, the switch has to not conduct for a long enough period of time. In the exercise 2 of session 8, in a ideal case the maximal duty cycle was

$$D_{max} = \frac{1}{1 + \frac{N_3}{N_1}} = \frac{1}{2} \tag{26}$$

The temperature and the value of the current influence the transistor properties. Because of these uncertainties, 1μs is remove the the conducting of the transistor (5% of the switching period T_s). We obtain

$$D_{max} = \frac{1}{2} - \frac{1 \times 10^{-6}}{T_s} = 0,45 \tag{27}$$

The conducting time has also to be reduced according to the given equation 24. For that, the needed winding ratio is given by

$$\frac{V_o}{V_d} = \frac{N_2}{N_1} D \Rightarrow a = \frac{N_1}{N_2} = D\frac{V_d}{V_o} \tag{28}$$

The winding ratio is chosen in order that in the worst case, i.e. minimal input voltage with maximal output voltage, the desired output voltage is obtained. Because the output voltage U_o is constant, we get from equation 25

$$a = D_{e,max}\frac{V_{d,min}}{V_o} \tag{29}$$

and using equation 23, we obtain

$$D_{e,max} = D_{max} - D_{e,max}\frac{V_{d,min}}{V_o} I_o f_s \times 1,2 \times 10^{-9} \tag{30}$$

$$D_{e,mac} = \frac{D_{max}}{1 + \frac{V_{d,min}}{V_o} I_o f_s \times 1,2 \times 10^{-9}} \approx 0,439 \tag{31}$$

and the winding ratio is then

$$a = D_{e,max}\frac{V_{d,min}}{V_o} \approx 18,44 \tag{32}$$

Part b

Before the calculation of the winding number of turns, the flux density of the core must be determined. The first forth of magnetizing curve is usually only used for forward converters.

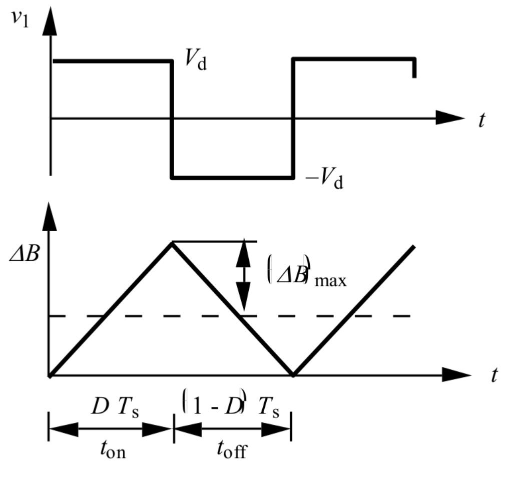

Figure 8: Waveforms of a forward converter: v_1 voltage over the primary and ΔB the variation of the flux density

When the flux density variation is exceptionally determined from the average value, and the remanence is assumed to be zero for the airgap we can write

$$(\Delta B)_{max} < \frac{B_{sat}}{2} \tag{33}$$

It has to be noticed that the input voltage is varying and that the core cannot saturate even with the largest voltages. The flux density is reduced according to the ratio of the voltage and we obtain

$$B_{ac} = \frac{B_s}{2}\frac{V_{d,min}}{V_{d,max}} = \frac{B_s}{2\alpha} = \frac{0.32}{2}\frac{210}{325} \approx 0,103\text{T} \tag{34}$$

In the manufacturer's tables, for instance Philips, the factor α is often $1,72$. In the exercise, the input voltage has a smaller variation $\alpha = 325/210 \approx 1,55$.
The number of turns in the primary is calculated with the induced voltage. When the switch is conducting the flux changes from zero to its maximal value $\phi_{max} = B_{max}A_{c,min} = 2A_{c,min}B_{ac}$ and we get

$$v_1 = N_1\frac{d\phi}{dt} \Rightarrow N_1 = v_1\frac{dt}{d\phi} = v_1\frac{\Delta t}{\Delta\phi} = \frac{V_1 D T_s}{2A_{c,min}B_{ac}} \tag{35}$$

The number of turns can be calculate with the minimum value of V_d because when B_{ac} was defined, the variation in V_d was taken in account.

$$N_1 = \frac{V_{d,min}D_{e,max}T_s}{2A_{c,min}B_{ac}} \approx 106,5 \Rightarrow N_1 = 107 \tag{36}$$

and

$$N_2 = \frac{N_1}{a} \approx 5,773 \Rightarrow N_2 = 6 \tag{37}$$

The results of the number of turns are rounded up because it reduces the flux density such as the flux density does not go over the maximal value allowed.

Part c

From the current density (equation 24) given in the exercise we $J \approx 457,7\text{A.cm}^{-2}$. The rms value of the maximal current in the secondary is found using the equation

$$I_{2,max} = \frac{k_{Cu}JA_w}{2N_2} \approx 20,59\text{A} \tag{38}$$

The current $I_{2,max}$ will probably not be exceeded because the output current is $I_o = 50/5 = 10\text{A}$. It should be checked when the inductance of the output filter is defined.

Part d

Before calculating the losses in the core, the flux density must be calculated with the winding turn calculated in the previous question.

$$B_{ac} = \frac{V_{o,max}T_s}{2A_{c,min}N_2} \approx 96,23\text{mT} \tag{39}$$

This flux density corresponds to the maximal it can reach because it is obtained with the minimal surface area $A_{c,min}$. The losses are usually calculated with the average flux density which is obtain from

$$B_{e,ac} = \frac{B_{ac}A_{c,min}}{A_e} \approx 85,82\text{mT} \tag{40}$$

The losses are obtain by summing the hysteresis and eddy currents losses. The equations are given in the exercise and we obtain

$$P_h \approx 16,7 \times f_s^{1,3} \Delta B_c^{2,5} V_e = 16,7 \times f_s^{1,3} B_{e,ac}^{2,5} V_e \approx 0,354\text{W} \tag{41}$$

and

$$P_e \approx 0,8 \times f_s^2 \Delta B_c^2 \frac{A_e V_e}{\rho} = 0,8 \times f_s^2 B_{e,ac}^2 \frac{A_e V_e}{\rho} \approx 0,027\text{W} \tag{42}$$

So, the total losses in the core are $P_h + P_e \approx 0,381\text{W}$.

Part e

The magnetic flux in a toroid is given by

$$\Phi = BA = \mu HA = \mu \frac{N_i}{L} = \Lambda N_i = \Lambda F_m \tag{43}$$

The flux in a an inductor is

$$\Psi = N\Phi = Li \Rightarrow L = N^2 \Lambda = \frac{N^2}{R} \qquad \text{when } R = \frac{l}{\mu A} \tag{44}$$

where R is the reluctance. From equation 43 and 44, we get

$$L = \frac{N^2 \mu A}{l} = \frac{N^2 \mu_r \mu_0 A}{l} \tag{45}$$

The reluctance of the magnetic circuit R must be also calculated. In this question, the magnetic circuit has an airgap. When the airgap is small, its surface area A_g can be assumed to be approximately equal to the surface area of the core A_c. In this case, the reluctance is

$$R = R_m + R_g = \frac{l_e}{\mu A_e} + \frac{l_g}{\mu_0 A_g} = \frac{1}{\mu_0 A_g} \left(l_g + \frac{l_e}{\mu_r} \right) \tag{46}$$

The equation 44 becomes

$$L = \frac{N^2}{R} = \frac{N^2 \mu_0 A_e}{\frac{l_e}{\mu_r} + l_g} = \frac{N^2 \mu_0 \mu_r A_e}{l_e + \mu_r l_g} = \frac{N^2 \mu_0 \mu_r A_e}{l_e \left(1 + \mu_r \frac{l_g}{l_e} \right)} \tag{47}$$

From which we obtain the airgap length

$$l_g = \frac{N^2 \mu_0 A_e}{L} - \frac{l_e}{\mu_r} \approx 0,1\text{mm} \tag{48}$$