Exercise 1

Obtain the output inductance of a forward converter which allows in steady-state a ripple of maximal peak-to-peak value of 0, 8% of the of the output voltage. In the output is a single capacitor of 15mF whose equivalent series resistance (ESR) is 37mΩ and the absolute value of the series inductance (ESZ) is 20mΩ at the switching frequency $f_s = 50$kHz. The variation of the DC-side voltage is $210 \leq V_d \leq 325$V, the maximal switching ratio is $D_{e,max} = 0,439$. The maximal current at the output is $I_{o,max} = 20$A and its power is $15 \leq P_o \leq 50$W with a output voltage $V_o = 5$V. When the load current changes by 50% (changes in the load), the variation in its voltage can only be of maximum 400mV and the voltage has to return to the reference value in maximum $t_{TR} = 400\mu$s.

Solution

Let us first check the steady-state while assuming that the capacitor is ideal, i.e. the leakage resistance is infinite. The output voltage variation can be calculated the same way as the voltage variation in a buck converter because a forward converter is a buck converter with a transformer. When the switch is not conducting, the voltage over the choke is

$$v_L = -V_o \qquad \text{when } t_{on} \leq t \leq T_s \tag{1}$$

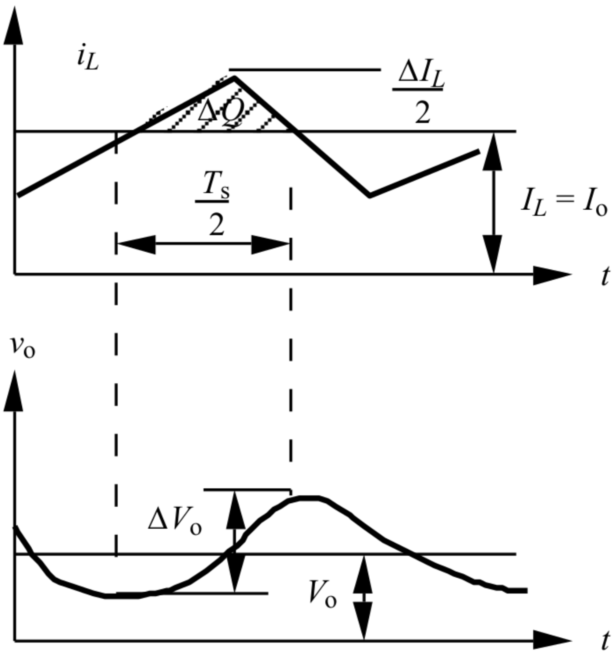

Figure 1: Waveforms of the forward converter.

From the figure 1, and in conduction conduction mode (CCM), the variation of the current flowing in the choke L is

$$\Delta I_L = \frac{V_o}{L} t_{off} = \frac{V_o}{L}(T_s - T_{on}) = \frac{V_o}{L}(1 - D)T_s \tag{2}$$

and for the voltage we have

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{1}{2C}\frac{\Delta I_L}{2}\frac{T_s}{2} = \frac{V_o}{8}\frac{T_s^2}{LC}(1-D) \tag{3}$$

The output voltage variation is maximal when the inductance and the control ratio are minimal. So we obtain

$$L_{min} = \frac{V_o}{8\Delta V_o}\frac{T_s^2}{C}(1-D_{min}) \tag{4}$$

The minimal and applicable value of the control ratio is a function of the ratio of the minimal and maximal value of the input voltage V_s.

$$D_{min} = D_{e,max}\frac{V_{d,min}}{V_{d,max}} = 0,439\frac{210}{325} \approx 0,284 \tag{5}$$

Using the equation 4, we get $L_{min} \approx 0,298\mu$H.

In reality, the situation is not that simple. When the current in the choke changes like previously calculated and when assuming that the output voltage V_o is constant, the current in the capacitor also changes the same way. The average current in the choke is equal to the current in the output I_o and the ac component of the current i_L goes to the capacitor. The current changes in the capacitor give changes, in voltage, of same frequency. The capacitor was assumed to be ideal but in reality, there is also an series inductance. At the frequency $f_s = 50$kHz, the absolute value of its impedance is 20mΩ. The current value should be calculated at the frequency f_s but for the sake of simplicity the values are calculated in a simpler way.

$$\Delta I_L = \frac{V_o}{L}(1-D)T_s = \frac{\Delta V_o}{\text{ESZ}} \tag{6}$$

The inductance of the choke L becomes

$$L_{min} = \frac{V_o}{\Delta V_o}\text{ESZ}(1-D_{min})T_s \approx 35,8\mu\text{H} \tag{7}$$

The inductance is hundred times higher than with the ideal capacitor.

Change in the Load

The stepwise current change in the load of 50% gives a current variation $\Delta I_o = 10$A. The choke in the filter for a current source and its current cannot change suddenly. The variation in the output voltage is

$$\Delta V_o = -\text{ESR}\ \Delta I_o = -370\text{mV} < -400\text{mV} \tag{8}$$

The converter has a close loop that regulates the parameters with the control ration D. When the change in the load happens, the maximal value of the control ratio is D_{TR}. In steady state, before the change in the load, the output voltage is

$$V_o = \frac{N_2}{N_1}DV_d \tag{9}$$

and the output voltage would be after the change

$$V_o = \frac{N_2}{N_1}D_{TR}V_d \tag{10}$$

Since it is assumed that the output voltage is constant, using the equations 9 and 10, the average value of the variation over the choke during the change in the load is

$$\Delta V_L = \frac{N_2}{N_1}(D_{TR} - D)V_d = \frac{V_o}{D}(D_{TR} - D) \tag{11}$$

The time duration of the change of the average value of the current is

$$t_{TR} = \frac{L\Delta I_o}{\Delta V_L} = \frac{L\Delta I_o D}{V_o(D_{TR} - D)} \tag{12}$$

The inductance of the choke

$$L = \frac{t_{TR}V_o(D_{TR} - D)}{\Delta I_o D} \tag{13}$$

The worst case scenario is when the control ratio is close to the maximal value. In the best case the control ration is minimal

$$L = \frac{400 \times 10^{-6} \times 5 \times (0,439 - 0284)}{10 \times 0,284} \approx 109,2\mu\text{H} \tag{14}$$

In most cases, the inductance creates a too large delay. In the average situation, it is good to use the input minimum voltage and average input voltage ratio

$$D_{ave} = D_{e,max}\frac{V_{d,min}}{V_{d,ave}} = 0,439\frac{210}{(210 + 325)/2} \approx 0,344 \tag{15}$$

and the equation 13 becomes

$$L = \frac{t_{TR}V_o(D_{TR} - D_{ave})}{\Delta I_o D_{ave}} \approx 54,76\mu\text{H} \tag{16}$$

Discontinuous conduction mode (DCM)

With a buck converter, we obtained the inductance minimal value that keep the converter working at the limit of DCM and CCM. So, the equation 2 can also be used here. At that limit, the current at the beginning of the cycle is always zero which means that the output current is half the previously calculated. The output minimal power is 15W which gives that the minimal average value of the output current is $I_{LB,min} = 3$A. So, the inductance has to be larger than

$$L_{min} = \frac{1 - D_{min}}{2I_{LB,min}}V_oT_s \approx 11,93\mu\text{H} \tag{17}$$

So, in steady state, the converter would work in CCM for all the input voltage.

Final choice

We got 3 different values for the choke inductance. When the capacitor was taken in account, the inductance was $L_{min} \approx 35,8\mu$H. When a change of 50% happens in the load with a recovery time of 400μs, we obtain $L_{max} \approx 54,76\mu$H. If the circuit work in DCM, we got $L_{min} \approx 11,93\mu$H. The inductance has to be chosen between those bounds. For example, $L = 40\mu$H is a good choice as the recovery time is in average better than the one asked.

Exercise 2

Obtain the switch maximal voltage and current of the forward converter of figure 2 with the values of exercise 1 and $N3 = N_1$.

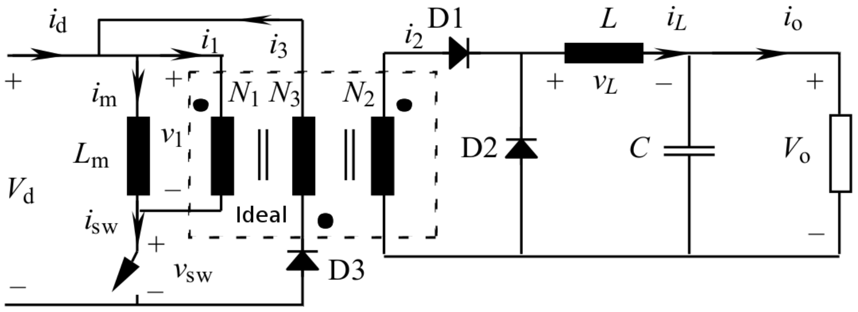

Figure 2: Rectifier with a constant DC-side voltage.

Solution

Current i_{sw}

When the switch is conducting the current is forms from the magnetizing current i_m and the primary current i_1, i.e.

$$i_{sw} = i_m + i_1 = i_m + \frac{N_2}{N_1} i_2 \tag{18}$$

The magnetizing current is given by

$$i_m = \frac{V_d}{L_m} t, \qquad 0 \leq t \leq t_{on} \tag{19}$$

and the maximal value is at $t = t_{on}$

$$i_{m,max} = \frac{V_{d,max}}{L_m} D_{max} T_s \approx 0,285\text{A} \tag{20}$$

When the switch is conducting, the voltage over the secondary is positive and the current i_2 and i_L increase. When the switch is not conducting, the voltage over the choke is $v_L = -V_o$ and its current decreases from the peak values according to

$$i_L = i_{L,\text{peak}} - \frac{V_o}{L_{out}} t, \qquad t \leq t_{on} \leq T_s \tag{21}$$

In the CCM, the average value of the output current is from the minimal and maximal values and we get

$$I_o = I_L = i_{L,min} + \frac{i_{L,\text{peak}} - i_{L,min}}{2} = \frac{i_{L,\text{peak}} + i_{L,min}}{2} \tag{22}$$

The maximal value of the current in the choke is the same as the maximal current in the secondary. The current in the switch is

$$i_{sw,max} = i_{m,max} + \frac{N_2}{N_1} i_{L,\text{peak}} \approx 0,896\text{A} \tag{23}$$

Voltage v_{sw}

When the switch is conducting its voltage is zero. When the switch is not conducting, the energy in the transformer goes through the third winding and diode D3. A voltage in induce in the primary that keeps the magnetization constant. With the winding ratio we get

$$v_1 = -\frac{N_1}{N_3}V_d, \qquad t_{on} \leq t \leq t_{on} + t_m \tag{24}$$

The voltage over the switch is

$$v_{sw} = V_d - v_1 = V_d + \frac{N_1}{N_3}V_d = \frac{N_3 + N_1}{N_3}V_d \tag{25}$$

In this exercise the $N_3 = N_1$ and we get

$$v_{sw,max} = \frac{N_3 + N_1}{N_3}V_{d,max} = 650\text{V} \tag{26}$$

Exercise 3

The transformer of a full-bridge converter is built from ferrite whose magnetizing and core loss curves are represented in the figure 3.

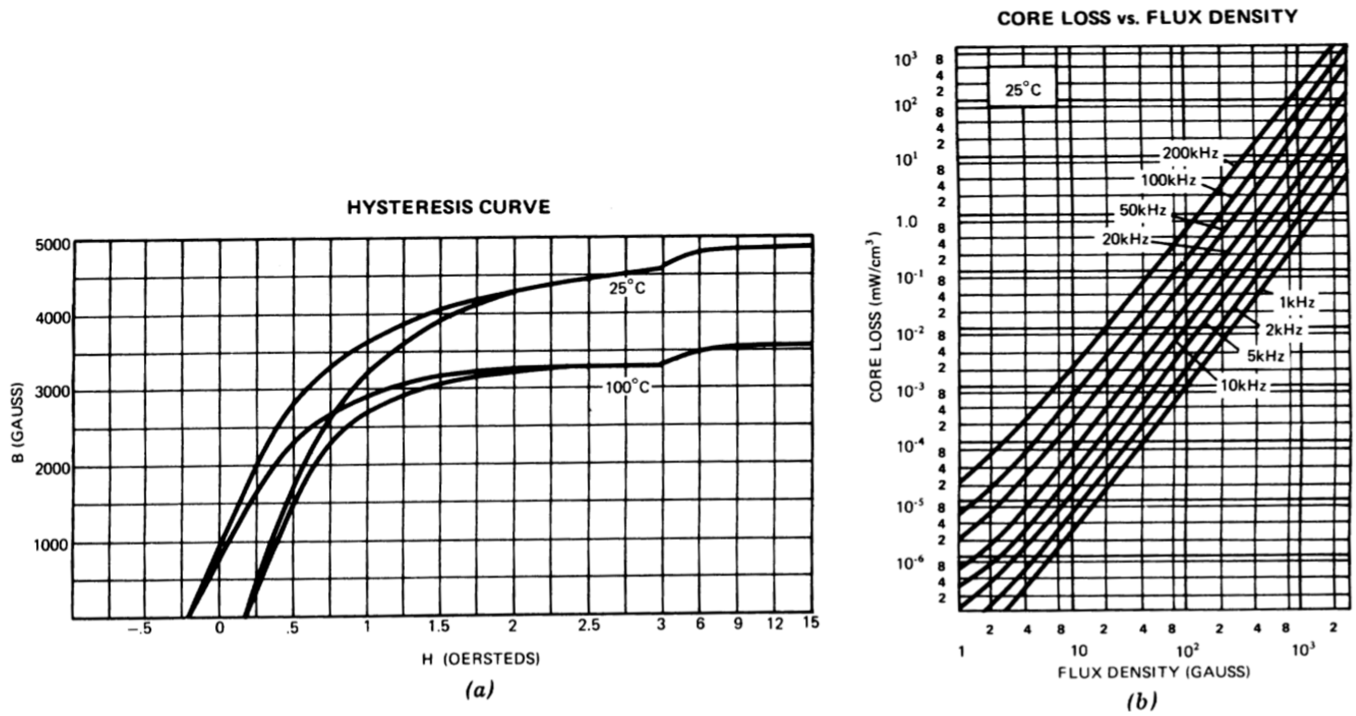

Figure 3: Rectifier with a constant DC-side voltage.

The DC-side voltage $V_d = 170$V, the switching frequency $f_s = 50$kHz, the maximal flux density variation is $\Delta B_{max} = 0,2\text{T}(\text{Wb/m}^2)$, the maximal magnetizing current $I_{L,max} = 0,5$A. The control ratio is $D = 0,5$. Obtain the power loss in the transformer's core when the temperature is 25°C.

Solution

In a full-bridge converter the magnetization is double and $\Delta B_{max} = 0,2$T= 2000Gauss is the high value of the flux density. In the figure 3, we see that at the frequency $f = f_s$ the power loss in the core is 200mW/cm^3. The problem is to resolve the volume of the core $V_m = A_c l_m$.

When the losses due to the resistivity is not taken in account, the input voltage is as high as the flux variation.

$$V_d = \frac{d\Psi}{dt} = N\frac{\phi}{dt} = NA_c\frac{dB}{dt} = NA_c\frac{2\Delta B_{max}}{DT_s} \tag{27}$$

The surface area of the core is

$$A_c = \frac{V_d DT_s}{2N\Delta B_{max}} = \frac{V_d}{4N\Delta B_{max} f_s} \tag{28}$$

The equation still have one unknown variable N. It can be obtained from the magnetization current and the inductance. The magnetizing current peak value is already known and it can also be obtain from the magnetizing inductance L_m.

$$\hat{i}_m = \frac{V_d DT_s}{2L_m} \Rightarrow L_m = \frac{V_d DT_s}{2\hat{i}_m} \tag{29}$$

The magnetizing inductance is obtained from the core surface area and its length.

$$L_m = \frac{N^2}{R_m} = \frac{N^2 \mu A_c}{l_m} \tag{30}$$

Using equations 29 and 30, we get

$$L_m = \frac{V_d DT_s}{2\hat{i}_m} = \frac{N^2 \mu A_c}{l_m} \tag{31}$$

and we obtain

$$l_m = 2\frac{N^2 \mu A_c \hat{i}_m}{V_d DT_s} \tag{32}$$

The volume of the core can be now calculated with equation 28.

$$V_m = A_c l_m = 2\frac{V_d}{4N\Delta B_{max} f_s}\frac{N^2 \mu A_c}{V_d DT_s}\hat{i}_m = NA_c\frac{\mu \hat{i}_m}{2N\Delta B_{max}} \tag{33}$$

and changing A_c again with equation 28, we obtain

$$V_m = \frac{2V_d \mu \hat{i}_m}{(4\Delta B_{max})^2 D f_s} \tag{34}$$

To calculate the volume V_m, we need to obtain the permeability of the ferrite. Its value can be approximated using the ferrite B-H curve on figure 3. The flux density is under $0,2$T (2000 Gauss), which is in the linear zone

$$\mu = \frac{2000[\text{Gauss}]}{0,25 - 0,2[\text{örstedt}]} = \frac{0,2[\text{T}]}{0,45\frac{1}{4\pi}10^3[\text{A/m}]} \approx 5,58 \times 10^{-3}[\text{Vs.(Am)}^{-1}] \tag{35}$$

Using equation 33, the volume of the core is then $V_m \approx 59,34$cm^3.
Using the figure 3, the core loss are

$$P_v = 200[\text{mW/cm}^3] \times 59,34[\text{cm}^3] \approx 11,9\text{W} \tag{36}$$