

### Exercise 1

The output voltage  $U_o = 5V$  is kept constant by adjusting the switch duty ratio  $D$  when the input voltage  $U_d$  is varying between 10V and 40V. The output power is  $P_o \geq 5W$ , and the switching frequency is  $f_s = 50kHz$ . Calculate the value of the inductance  $L$  that keeps the circuit working in the continuous conduction mode (CCM). The current of the choke,  $i_L$ , is considered greater than zero.

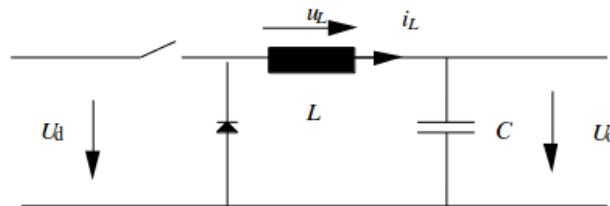


Figure 1: Buck converter.

### Hints

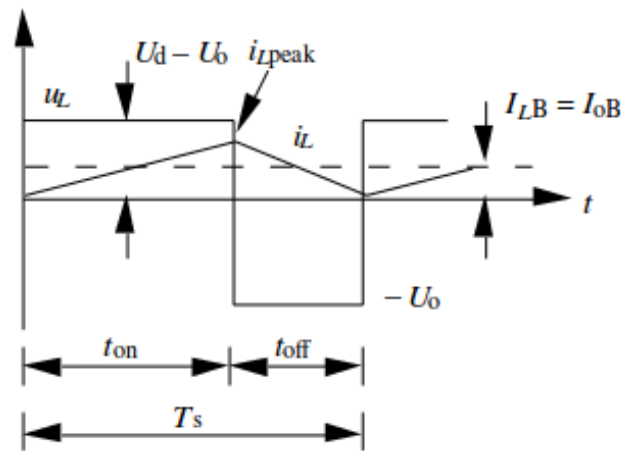


Figure 2: Waveforms of a buck converter at the DCM limit.

The figure 2 shows the waveform of the buck converter when working in the CCM but at the limit of DCM. The current of the choke is given by

$$i_L = \frac{1}{L} \int u_L dt \tag{1}$$

When the switch is conducting

$$i_L = \frac{U_d - U_o}{L} t, \quad 0 < t < t_{on} \tag{2}$$

and the current at the limit of the DCM

$$I_{LB} = I_{oB} = \frac{i_{L,peak}}{2} = \frac{U_d - U_o}{2L} t_{on} \tag{3}$$

where  $t_{on}$  is the time that the switch is conducting.

If the average output current is lower than  $i_{LB}$ , the circuit enters in DCM. The switch duty ratio is given by

$$D = \frac{U_o}{U_d} = \frac{t_{on}}{T_s} \quad (4)$$

Where  $T_s$  is the period of switching. The equation 4 is only true in the CCM and at the limit of the DCM.

## Solution

From the equations 3 and 4, we can rewrite the current in the choke at the limit of DCM

$$I_{LB} = \frac{1-D}{2L} U_o T_s \quad (5)$$

from which we get the inductance

$$L = \frac{1-D}{2I_{LB}} U_o T_s \quad (6)$$

The most demanding case is when the average current  $I_{LB}$  and the switch duty ratio  $D$  are minimal.

$$L_{min} = \frac{1-D_{min}}{2I_{LB,min}} U_o T_s = \frac{1-\frac{5}{40}}{2 \times 1} \times 5 \times 20 \times 10^{-6} \approx 43,75 \mu\text{H} \quad (7)$$

A smaller value for the choke can be found when  $D$  is maximal but the converter would enter the DCM for an input voltage greater than 10V.

## Verification

We can check the average output current at the limit of DCM with the calculated value of the inductance.

$$U_d = 10\text{V} \implies I_{LB} = 0,5714\text{A}$$

$$U_d = 40\text{V} \implies I_{LB} = 1\text{A}$$

For both values of  $U_d$  the current does not exceed 1A which means that the value of the choke is enough.

## Exercise 2

In the previous exercise, the capacitance  $C$  was considered large enough so that the output  $u_o(t)$  was considered as constant. Calculate the voltage the peak-to-peak voltage ripple  $\Delta U_o$  when the capacitance  $C = 470 \mu\text{F}$ .

### Hints

To solve this exercise the following formula is needed

$$\Delta U_o = \frac{\Delta Q}{C} \tag{8}$$

where  $\Delta Q$  [C,A.s] is the variation of charge. Use the geometry of the current  $i_L(t)$  and its average value to find  $\Delta U_o$ .

### Solution

To solve this exercise, it is useful to use the waveforms of the figure 3

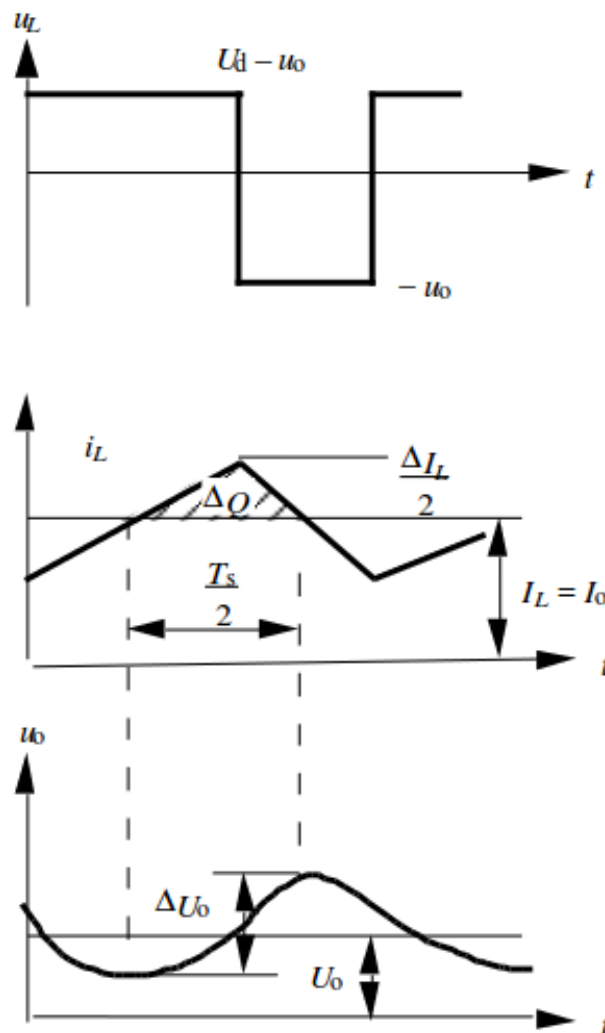


Figure 3: Waveform of a buck converter in CCM.

When the current is over its average value, the capacitor charges according to their difference, and the voltage increases. The increase of the voltage can be calculate with the variation of charge  $\Delta Q$  which depends of the peak-to-peak value of the current. From the figure 3 we get the triangle area which correspond to  $\Delta Q$ .

$$\Delta U_o = \frac{\Delta Q}{C} = \frac{1}{2C} \frac{\Delta I_L T_s}{2} \tag{9}$$

When the switch is not conducting, the voltage over the choke is  $-U_o$ , and the peak-to-peak current is

$$\Delta I_L = \frac{U_o}{L} t_{off} = \frac{U_o}{L} (T_s - t_{on}) = \frac{U_o}{L} (1 - D) T_s \quad (10)$$

Using equations 9 and 10, we obtain

$$\Delta U_o = \frac{T_s^2}{8C} \frac{U_o}{L} (1 - D) \quad (11)$$

Adding in the equation 11 the cut-off frequency

$$f_c = \frac{1}{2\pi\sqrt{LC}} \quad (12)$$

and knowing that the switching frequency  $f_s = 1/T_s$ . We can rewrite the equation 11 as

$$\Delta U_o = U_o \frac{\pi^2}{2} (1 - D) \left( \frac{f_c}{f_s} \right)^2 \quad (13)$$

With the numerical values we get the maximal variation is then

$$\Delta U_{o,max} = \frac{T_s^2}{8C} \frac{U_o}{L} (1 - D_{min}) = U_o \frac{\pi^2}{2} (1 - D_{min}) \left( \frac{f_c}{f_s} \right)^2 \approx 10,64\text{mV} \quad (14)$$

and in percent of the output voltage

$$\frac{\Delta U_o}{U_o} = \frac{\pi^2}{2} (1 - D_{min}) \left( \frac{f_c}{f_s} \right)^2 \approx 0,213\% \quad (15)$$

## Exercise 3

Continuing the same exercise, calculate the peak-to-peak voltage ripple  $\Delta U_o$  when the output power  $P_o$  decreases by 50%. Therefore, the average of the output current is now  $I_o = 0,5\text{A}$  and the converter is working in a discontinuous conduction mode (DCM).

## Hints

In the exercise 1, we calculated the minimum value of the inductance to keep the converter functioning in CCM. In exercise 2, the peak-to-peak output voltage in CCM was calculated. Now, obtain the current in the choke,  $i_L$ , when it decreasing as a function of the time  $\Delta_1$  and the switching duty ratio  $D$  with the new load. The same was as in the previous exercise, when the current  $i_L$  is greater than its average value, the capacitor charges. And from  $\Delta Q$  and the geometry of the signal,  $\Delta U_o$  can be obtained. The average value of the output voltage can be assumed as constant.

**Solution**

In the exercise 1 the current was 1A was the limit of CCM and now it is 0,5A which leads to DCM. The output voltage peak-to-peak ripple is given

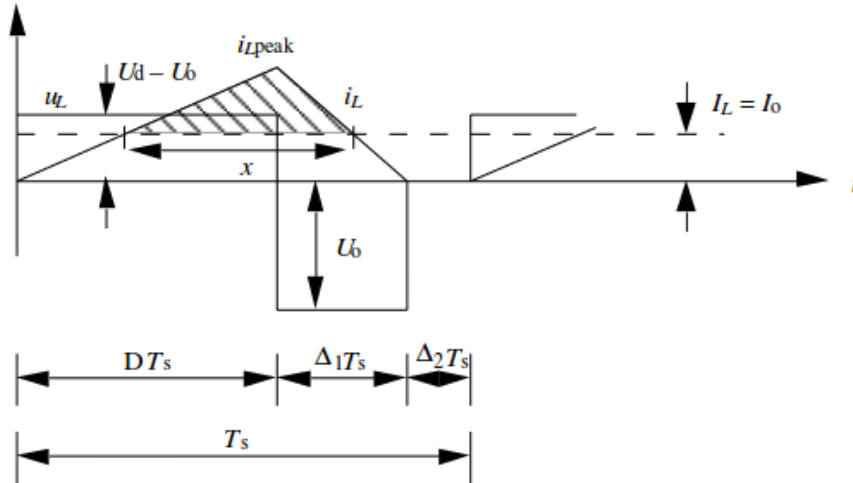


Figure 4: Waveform of a buck converter in DCM.

We first compute the time intervals  $DT_s$  and  $\Delta T_s$ . The integral of the voltage of an inductance over a period is zero. Using  $u_L$  waveform on figure 4, we get

$$(U_d - U_o)DT_s - U_o\Delta_1 T_s + 0 \times \Delta_2 T_s = 0 \tag{16}$$

which leads to

$$\frac{U_o}{U_d} = \frac{D}{D + \Delta_1} \tag{17}$$

Now, using  $i_L$  waveform on figure 4, the average value of the output current is

$$I_o = \frac{i_{L,peak}}{2}(D + \Delta_1) \tag{18}$$

The voltage of an inductance is given by

$$u_L(t) = L \frac{di_L(t)}{dt} \tag{19}$$

Because  $U_L = -U_o$  over the time interval  $\Delta_1 T_s$  the equation 19 can be rewritten as

$$-U_o = L \frac{\Delta i_L}{\Delta_1 T_s} \tag{20}$$

With  $\Delta i_L = 0 - i_{L,peak} = -i_{L,peak}$  and we get

$$i_{L,peak} = \frac{U_o}{L} \Delta_1 T_s \tag{21}$$

Using equations 21 in 18 we obtain

$$I_o = \frac{U_o}{2L} \Delta_1 T_s (D + \Delta_1) \tag{22}$$

and using equation 17, we get

$$I_o = \frac{U_d D \Delta_1 T_s}{2L} \quad (23)$$

and finally the time interval

$$\Delta_1 = \frac{2LI_o}{U_d D T_s} \quad (24)$$

Now we can insert the time interval  $\Delta_1$  into the equation 17 to obtain

$$\frac{U_o}{U_d} = \frac{D}{D + \frac{2LI_o}{U_d D T_s}} \quad (25)$$

and few modifications we get

$$D = \sqrt{\frac{U_o}{U_d(U_d - U_o)} \frac{2LI_o}{T_s}} \quad (26)$$

With the equation 26, the time of conduction can be calculated when the input and output voltages are known. The average output voltage  $U_o$  is considered as constant.

In the first exercise we found that at the limit of the DCM the current

$$I_{LB} = \frac{1-D}{2L} U_o T_s \quad (27)$$

Its value depends of the value of  $D$  and its maximal value is when  $D = 0$ . Therefore,

$$I_{LB,max} = \frac{U_o T_s}{2L} \quad (28)$$

Inserting equation 28 in equation 26, we obtain

$$D = \frac{U_o}{U_d} \sqrt{\frac{\frac{I_o}{I_{L,max}}}{1 - \frac{U_o}{U_d}}} \quad (29)$$

And finally with the values of the exercise we get  $I_{LB,max} = 1,143A$ .

when  $U_d = 10V$   $D \approx 0,4677$  and  $\Delta_1 \approx 0,4677$ .

when  $U_d = 40V$   $D \approx 0,08838$  and  $\Delta_1 \approx 0,6188$ .

In both case,  $D + \Delta_1 < 1$  with mean that the converter enter in DCM ( $D + \Delta_1 + \Delta_2 = 1$ ).

Now, we can calculate the peak-to-peak value of the output voltage  $\Delta U_o$ . During the time interval  $\Delta_2 T_s$  the current is zero and the capacitor give to the load current and the its voltage drops. In the figure 4, the area above the hatched area represents the charge in the capacitor that rise the its voltage. Like in the previous exercise, the variation of charge  $\Delta Q$  can be calculate with the geometry of the waveform. First the "distance"  $x$  has to be determined. The segment  $x$  is situated on the average value of  $I_L = I_o$ . Using Thales' theorem (a.k.a. intercept theorem) we get

$$\frac{x}{(D + \Delta_1)T_s} = \frac{i_{L,peak} - I_o}{i_{L,peak}} \quad (30)$$

son we get

$$x = (D + \Delta_1)T_s \frac{i_{L,peak} - I_o}{i_{L,peak}} \quad (31)$$

using equation 18, we get

$$x = (D + \Delta_1)T_s \left(1 - \frac{D + \Delta_1}{2}\right) \quad (32)$$

Knowing x we can calculate  $\Delta Q$  using simple geometry

$$\Delta Q = \frac{1}{2}x(i_{L,peak} - I_o) = \frac{x}{2}i_{L,peak} \left(1 - \frac{I_o}{i_{L,peak}}\right) \quad (33)$$

using equation 18 again, we get

$$\Delta Q = \frac{1}{2}(D + \Delta_1)T_s i_{L,peak} \left(1 - \frac{D + \Delta_1}{2}\right)^2 \quad (34)$$

Using the equation 19, the peak-to-peak output voltage is

$$\Delta U_o = \frac{\Delta Q}{C} = \frac{1}{2}(D + \Delta_1)\Delta_1 T_s^2 \frac{U_o}{LC} \left(1 - \frac{D + \Delta_1}{2}\right)^2 \quad (35)$$

And finally with the numerical values we get:

when  $U_d = 10\text{V}$   $\Delta U_o = 6,03\text{mV}$

when  $U_d = 40\text{V}$   $\Delta U_o = 8,89\text{mV}$