Exercise 1

The voltages and current in the boost converter of figure 1 are $U_d = 12\text{V}$, $U_o = 24\text{V}$, $I_o = 1\text{A}$ and the value of the components are $L = 150\mu\text{H}$, $C = 470\mu\text{F}$ and the switching frequency is $f_s = 20\text{kHz}$. Calculate the variation in the output voltage ΔU_o.

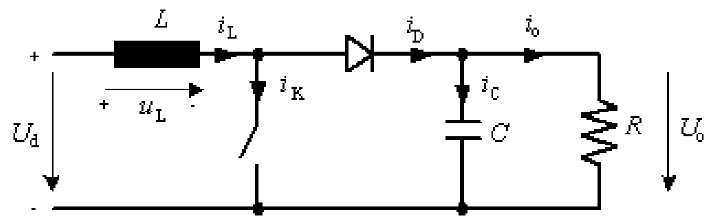

Figure 1: Boost converter.

Solution

In the exercise 3 of the previous session, we got the result that at the limit of the continuous and discontinuous conduction mode we have an average output current of

$$I_{oB} = \frac{T_S U_o}{2L} D(1-D)^2 = 0,5\text{A} \tag{1}$$

In this exercise the average of the output current is $I_o = 1\text{A}$ which means that the circuit is working in CCM.
With the figure 2, we can calculate the value of ΔQ that is need to obtain ΔU_o.

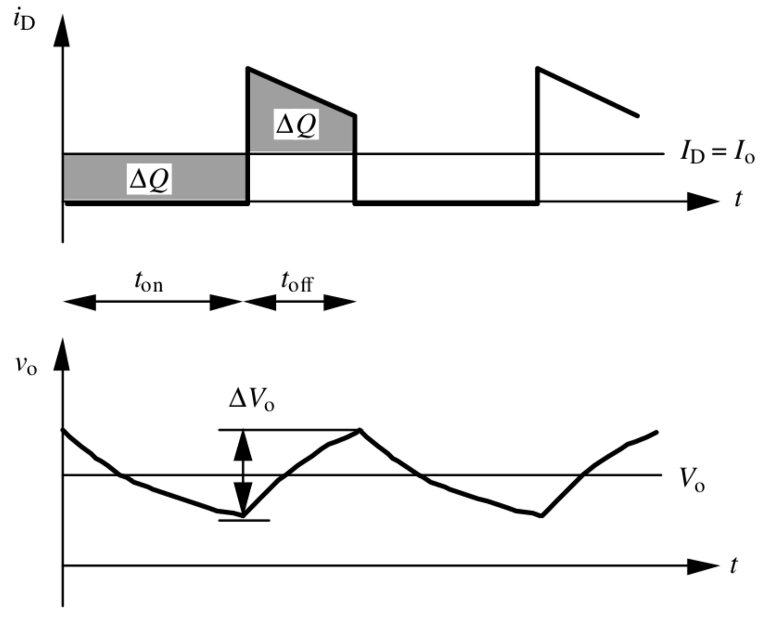

Figure 2: Diode current i_D and output voltage u_o in CCM in a boost converter.

$$\Delta U_o = \frac{\Delta Q}{C} = \frac{I_o D}{f_s C} = \frac{U_o D}{RC f_s} \tag{2}$$

In the CCM, we have the duty ratio being

$$D = 1 - \frac{U_d}{U_o} = 0,5 \tag{3}$$

and we obtain with equation 2, $\Delta U_o \approx 53,2\text{mV}$.
With simulation we can obtain the following waveforms: In the figure 4, the voltage variation

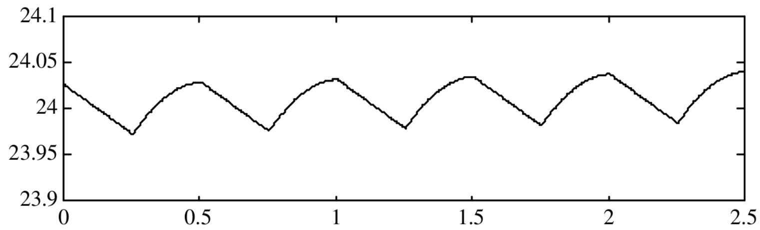

Figure 3: Output voltage u_o as a function of time $\times 0,1\text{ms}$.

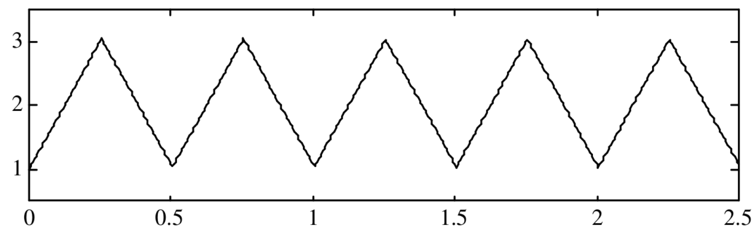

Figure 4: Choke current i_L as a function of time $\times 0,1\text{ms}$.

is $\Delta U_o = 56,16\text{mV}$ which is close to the theoretical value.

Exercise 2

The voltages and current in the boost converter of figure 1 are $U_d = 12\text{V}$, $U_o = 24\text{V}$, $I_o = 1\text{A}$ with a capacitor of $C = 470\mu\text{F}$ and a switching frequency of $f_s = 20\text{kHz}$. Calculate the choke peak current $i_{L,peak}$, which is also the peak value of the switch current $i_{K,peak}$,

a) when the choke value is $L = 150\mu\text{H}$. What happens is the value of the choke is higher?

b) when the choke value is $L = 50\mu\text{H}$.

Calculate the ratio of the peak current of the choke with its average values, i.e. $i_{L,peak}/I_L$, as a function of L_{oB}/L. L_{oB} the minimal value of the choke L to keep the circuit working in the continuous conduction mode (CCM).

Solution

Part a

For $L = 150\mu\text{H}$ we have the exact same case as in the previous exercise. The circuit is working in CCM because the average output current I_o is higher than I_{oB}.

Using the equation 1, we can also find the minimum value of L for which the system remains in CCM. This values is $L \approx 75\mu$H. Under this value, the circuit works in DCM.
In this case, the circuit is in CCM and we can use the figure 5 to calculate the peak current $i_{L,peak}$. The calculation is the same as in the exercise 3 of the previous exercise session. The

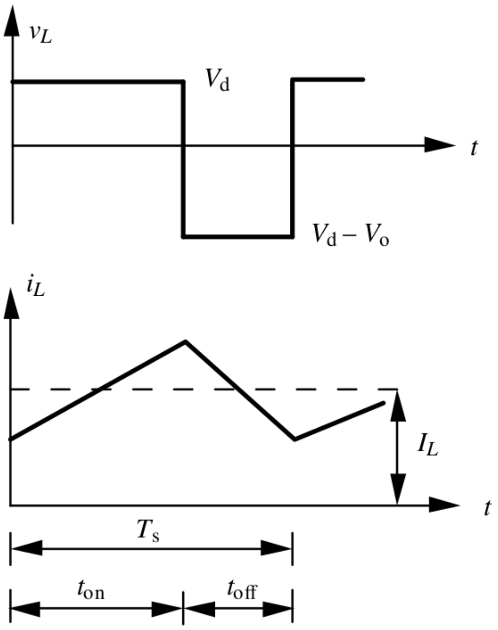

Figure 5: Choke current $i_L(t)$and voltage $u_L(t)$.

peak value of i_L is

$$i_{L,peak} = i_{L,min} + \frac{U_d}{L} DT_s \tag{4}$$

the average value of i_L is

$$I_L = i_{L,min} + \frac{i_{L,peak} - i_{L,min}}{2} = i_{L,min} + \frac{U_d}{2L} DT_s = \frac{I_o}{1-D} \tag{5}$$

Because the circuit does not have any loss we can write $U_d I_L = U_o I_o$ and with equation5, we get that the minimum current in L is

$$i_{L,min} = \frac{I_o}{1-D} - \frac{U_d}{2L} DT_s = \frac{I_o}{1-D} - \frac{U_o(1-D)}{2L} DT_s \tag{6}$$

and its peak value is

$$i_{L,peak} = \frac{I_o}{1-D} + \frac{U_o(1-D)}{2L} DT_s \approx 3\text{A} \tag{7}$$

If the value of the choke is increasing, we get

$$\lim_{L \to \infty} i_{L,peak} = \frac{I_o}{1-D} = 0,5\text{A} \tag{8}$$

The peak value of the current in the choke reduces if the choke value increases.

Part b

Like shown in part a, the minimum value of the choke that keeps the circuit in CCM is 75μH. With a value of 50μH the circuit enters in discontinuous conduction mode (DCM). In

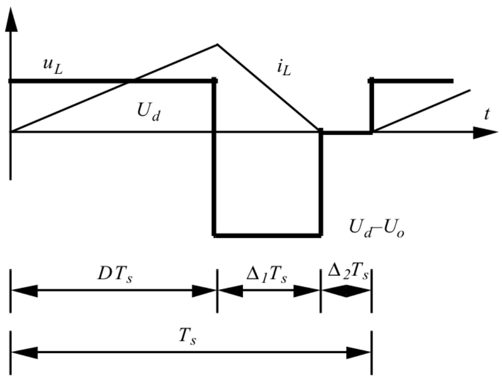

Figure 6: The current and the voltage in the choke, $u_L(t)$ and $i_L(t)$ in DCM.

2. DCM, we can use the figure 6 to compute $i_{L,peak}$ The average voltage over a choke is zero. So, we can write

$$\frac{1}{T_s}\int_0^{T_s} u_L(t)dt = \frac{1}{T_s}\left(U_d DT_s + (U_d - U_o)\Delta_1 T_s + 0\Delta_2 T_s\right) = 0 \tag{9}$$

From the equation 9, we get

$$\frac{U_o}{U_d} = \frac{D+\Delta_1}{\Delta_1} \tag{10}$$

Because there is no loss in the circuit we can write

$$\frac{U_o}{U_d} = \frac{D+\Delta_1}{\Delta_1} = \frac{I_L}{I_o} \tag{11}$$

1. The average current in the choke is

$$I_L = \frac{1}{2T_s}(DT_s + \Delta_1 T_s)i_{L,peak} = \frac{1}{2}(D+\Delta_1)i_{L,peak} \tag{12}$$

The voltage of the choke is

$$u_L(t) = L\frac{di(t)}{dt} \tag{13}$$

Over the time interval DTs the current rise monotonically so we can rewrite the equation 13 as

$$u_L \Delta t = L\Delta i = U_d \Delta t \tag{14}$$

with $\Delta i = i_{L,peak} - 0$ and $\Delta t = DT_s - 0$. We obtain

$$i_{L,peak} = \frac{U_d DTs}{L} \tag{15}$$

Using equation 15 in equation 12, we obtain

$$I_L = \frac{U_d DTs}{2L}(D+\Delta_1) \tag{16}$$

3. and using equation 11, we get the average output current

$$I_o = \frac{U_d DTs}{2L}\Delta_1 \tag{17}$$

The time duration Δ_1 is then

$$\Delta_1 = \frac{2LI_o}{U_d D T_s} \tag{18}$$

Using equation 11 and equation 18, we obtain

$$D = \frac{U_o}{U_d}\Delta_1 - \Delta_1 = \sqrt{\frac{2LI_o}{U_d T_s}\left(\frac{U_o}{U_d} - 1\right)} \approx 0,408 \tag{19}$$

From equation 15, we obtain $i_{L,peak} \approx 4,899$A. The peak current is higher than in part a. The following waveform were obtained with simulation The figure 9 represents the values

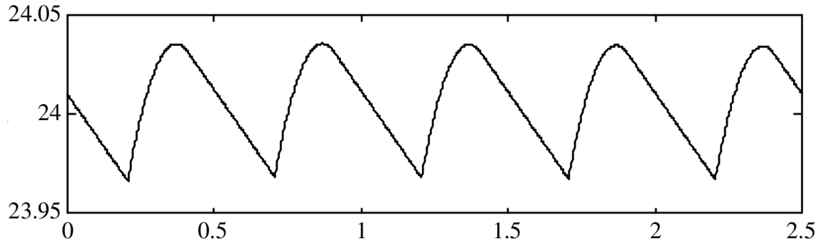

Figure 7: Output voltage u_o as a function of time $\times 0,1$ms.

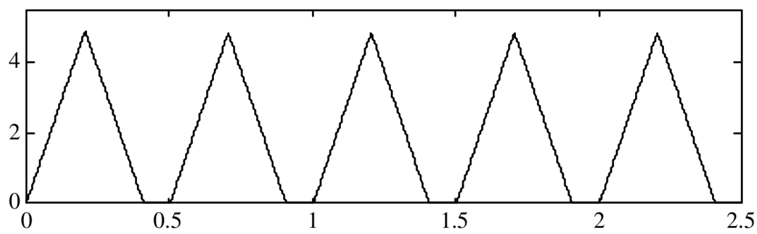

Figure 8: Choke current i_L as a function of time $\times 0,1$ms.

of the current in the choke as a function of its value L. The value $L = 75\mu$H is the limit

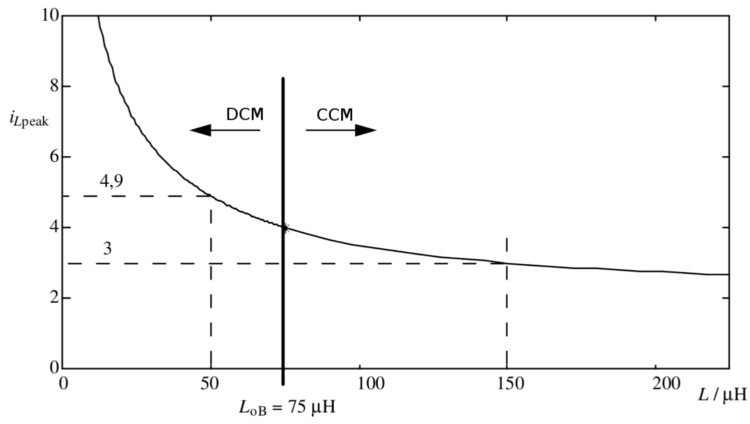

Figure 9: Choke current i_L as a function of its value L when $U_o/U_d = 0,5$

between CCM and DCM. We can see that the peak current in the DCM region the current rises faster than the inductance decreases. As the value of the choke increase the value of the peak current converges to 2A. The use of the boost converter in the DCM region is not practical.

5. In the following question, we must obtain $i_{L,peak}$. It is already calculated in equation 7. With the equation 1 of exercise 1, we obtain,

$$i_{L,peak} = \frac{I_o}{1-D} + \frac{U_o(1-D)}{2L}DT_s = I_d + \frac{L_{oB}}{L}I_d \tag{20}$$

and then

$$\frac{i_{L,peak}}{I_d} = 1 + \frac{L_{oB}}{L} \tag{21}$$

4. Variable I_d in this case is an input current. By combining equations 15 and 19 we obtain,

$$i_{L,peak} = \frac{U_dT_s}{L}\sqrt{\frac{2LI_o}{U_dT_s}\left(\frac{U_o}{U_d}-1\right)} = \sqrt{\frac{2T_sI_oU_d}{L}\left(\frac{U_o-U_d}{U_d}\right)} \tag{22}$$

the minimal inductance to keep the circuit in CCM (from (1)) is,

$$L_{oB} = \frac{T_sU_o}{2I_{oB}}D(1-D)^2 = \frac{T_sU_o}{2I_d}D(1-D) \tag{23}$$

with $D = 1 - U_d/U_o$ because we are at the limit of CCM and DCM. We get

$$L_{oB} = \frac{T_s}{2I_d}\left(1-\frac{U_d}{U_o}\right)U_d \tag{24}$$

inserting the equation 24 in the equation 22, we obtain

$$i_{L,peak} = \sqrt{\frac{2T_sI_o}{L}(U_o-U_d)} = \sqrt{\frac{4I_o}{L}\left(\frac{L_{oB}U_oI_d}{U_d}\right)} \tag{25}$$

Using the fact that the circuit is lossless ($U_oI_o = U_dI_d$) we get

$$\frac{i_{L,peak}}{I_d} = 2\sqrt{\frac{L_{oB}}{L}} \tag{26}$$

The equation 26 is draw on the figure 10.

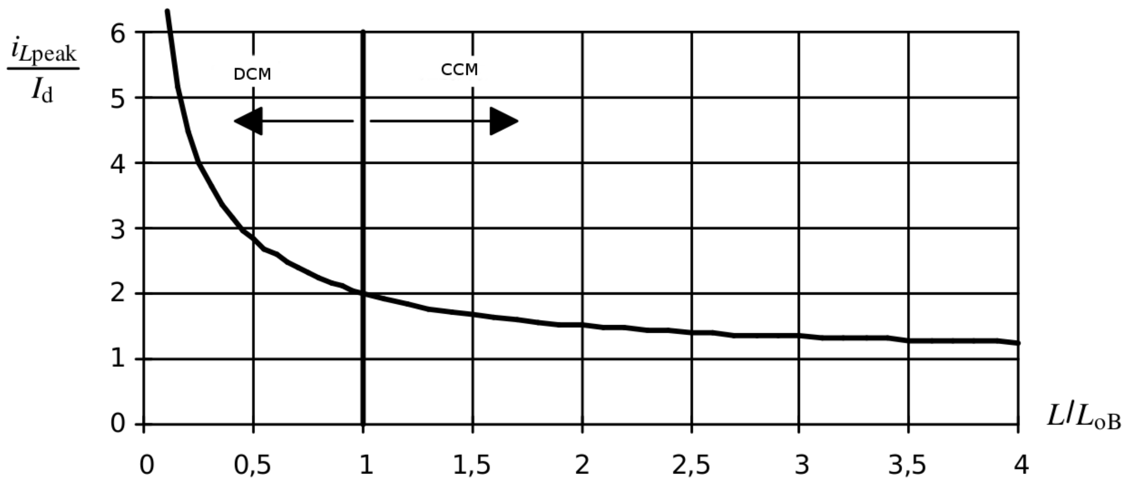

Figure 10: Ratio $i_{L,peak}/I_d$ as a function of the choke value L when $U_o/U_d = 0,5$

Exercise 3

Using figure 1, calculate the rms-value of the current i_D (figure 11) which is also the current in capacitor C, i_C, rms-value. The characteristic of the circuit are:
$8 \leq U_d \leq 16$V, $U_o = 24$V, $I_o = 1$A, the output power $P_o \geq 5$W, $C = 470\mu$F, $L = 427\mu$H, $f_s = 20$kHz and the duty ratio $D = 0,5$.

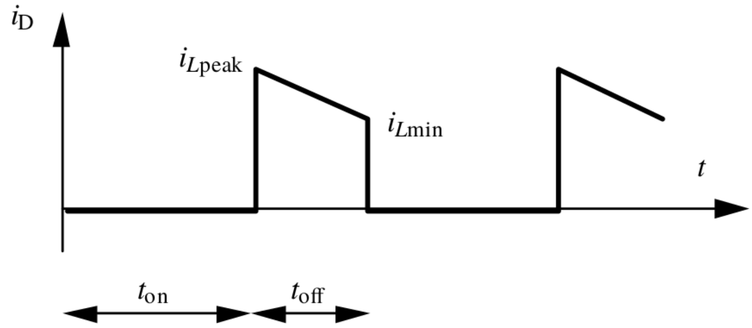

Figure 11: Diode current $i_D(t)$.

Solution

When the switch is conducting the current in the diode is zero. When it is not conducting, the current is passing through the diode and the inductance. Let's calculate first the choke current maximal ($I_{L,peak}$) and "minimal" ($I_{L,min}$) values. In the exercise 2 the peak value $I_{L,peak}$ (equation 7) and the minimum $I_{L,min}$ (equation 6) are already calculated. In the figure 11, we see that the variation of current follow the equation $\Delta i_D = k\Delta t$ where k is a gradient of i_D slope.

$$\Delta i_D = i_{L,min} - i_{L,peak} = kT_s(1 - D) \tag{27}$$

using equations 6 and 7, we get,

$$k = -\frac{U_o D}{L} \tag{28}$$

The form of $i_D(t)$ is therefore,

$$i_D(t) - i_{L,peak} = k(t - DT_s), \tag{29}$$

$$i_D(t) = kt + con, \tag{30}$$

$$con = \frac{U_o D^2 T_s}{L} + i_{L,peak}. \tag{31}$$

The rms-value of id is

$$I_{D,rms} = \sqrt{\frac{1}{T_s}\int_{D_{Ts}}^{Ts} i_D^2(t)dt} = \sqrt{\frac{1}{T_s}\int_{D_{Ts}}^{Ts} (kt + con)^2 dt}, \tag{32}$$

$$= \sqrt{\frac{1}{T_s}\int_{D_{Ts}}^{Ts} \left(k^2t^2 + 2.con.kt + con^2\right)^2 dt}, \tag{33}$$

$$I_{D,rms} = \sqrt{\frac{1}{T_s}\left[\frac{k^2t^3}{3} + con.kt^2 + con^2.t\right]_{D_{Ts}}^{Ts}}, \tag{34}$$

$$I_{D,rms} = \sqrt{\frac{1}{T_s}\left(\frac{k^2}{3}\left(T_s^3 - D^3T_s^3\right) + con.k\left(T_s^2 - D^2T_s^2\right) + con^2\left(T_s - DT_s\right)\right)}. \tag{35}$$

Combining equation 7, 28 and 31, leads to,

$$I_{D,rms} = \sqrt{\frac{U_oD^2}{L^2}T_s^2\left[\frac{(1-D^3)}{3} - D(1-D^2) - \frac{(1-D)(1-D^2)}{2}\right] - \frac{U_oDI_o}{(1-D)L}T_s(1-D^2) + \lambda}, \tag{36}$$

$$\lambda = \left[\frac{U_oD^2T_s}{L} + \frac{I_o}{1-D} + \frac{U_o(1-D)}{2L}DT_s\right]^2(1-D). \tag{37}$$

With the numerical values we get $I_{D,rms} = 1,422$ A, $I_{L,min} = 1.649$ A and $I_{L,peak} = 2.351$ A.