Exercise 1

A boost converter is tested with a resistor $R = 60\Omega$ and the output average current is $I_o = 0,3$A. The other parameters are $U_d = 12$V, $L = 150\mu$H, $C = 470\mu$F and $f_s = 20kHz$. There is no feedback control over the output voltage, i.e. the control ratio D is constant. What happens when the resistor R is removed. It is assumed that the resistor is removed when the switch is conducting.

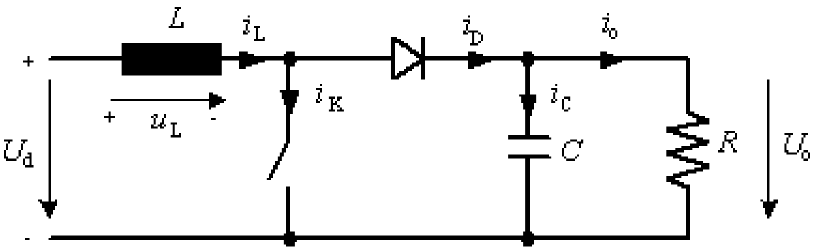

Figure 1: Boost converter.

Solution

The maximal current that keep the circuit in continuous conduction mode (CCM) was obtain in the last exercise of the session 2 with $D = 1/3$. We got

$$I_{oB,max} = \frac{2}{27}\frac{T_sU_o}{L} \approx 0.444 \text{ A} \tag{1}$$

Because the I_o is smaller that $I_{oB,max}$, the circuit can only works in discontinuous conduction mode (DCM). Now, we can calculate the value of D in DCM. In the session 3 exercise 2 we got

$$D = \sqrt{\frac{2LI_o}{U_dT_s}\left(\frac{U_o}{U_d} - 1\right)} \tag{2}$$

where $U_o = RI_o$. Like calculated in the previous exercise session, we got

$$I_{L,peak} = \frac{UdDT_s}{L} \approx 1.0954 \text{ A} \tag{3}$$

and

$$\Delta_1 T_s = \frac{2LI_o}{DU_d} \approx 27.39 \ \mu\text{s} \tag{4}$$

When the switch is conducting, the capacitor is charging from a constant voltage source and when the switch is not conducting anymore the capacitor is discharging into the resistor. When the resistor is removed the capacitor cannot discharge anymore and keeps on charging whenever the switch is conducting.
We can first approximate the rise of the voltage in the capacitor. Let assume that the $\Delta_1 T_s$ calculated in equation 4 remains constant even when the voltage is rising. At the beginning of each conducting time of the switch the voltage in the capacitor is at its maximal value. When the switch is conducting the current rise till $i_{L,peak}$, the voltage in the capacitor remains

constant because there is load anymore and the output current is $I_o = 0$A. The voltage of the capacitor after the first switching period when the charge was removed can be approximated with energy balance. In a capacitor and in a inductance the energy is expressed as

$$E_c = \frac{1}{2}Cu_c^2 \qquad E_L = \frac{1}{2}Li_L^2 \tag{5}$$

Since the capacitor does not discharge anymore, the energy coming from the input voltage source during the instant Δ_1 (switch is not conducting) can be written as

$$E_{c,\Delta_1} = \frac{1}{2}i_{L,peak}U_d\Delta_1 T_s \tag{6}$$

Over a period T_s, we can write the energy being

$$\frac{1}{2}Li_{L,peak}^2 + \frac{1}{2}CU_{co}^2 + \frac{1}{2}i_{L,peak}U_d\Delta_1 T_s = \frac{1}{2}CU_{c1}^2 \tag{7}$$

and we obtain the voltage over C after one switching period when the load R is removed.

$$U_{C1}^2 = U_{C0}^2 + \frac{Li_{L,peak}^2 + U_d i_{L,peak}\Delta_1 T_s}{C} \tag{8}$$

The term in the equation 6 is introducing error in the calculation because $\Delta_1 Ts$ change with the voltage rising.
We can write the voltage over the capacitor after n switching periods and we obtain

$$U_{Cn}^2 = U_{Cn-1}^2 + n\frac{Li_{L,peak}^2 + U_d i_{L,peak}\Delta_1 T_s}{C} \tag{9}$$

Let's calculate the time it would take for the voltage of the capacitor to double when starting with a voltage of $U_{C0} = 18$V. We can obtain the number of switching necessary by using the equation 9 and we get $n = 46$ which gives a time of $nT_s = 2.3$ ms. So, it would takes only 2.3 ms for the voltage to double from a value of 18V.
The tester does not have much time to think of what to do. The voltage rises but its rising "speed" decreases as the inductor takes every period a constant energy and the voltage of the capacitor rises proportionally with the square root of the energy.
$\Delta_1 T_s$ was considered as constant but because the energy taken from the voltage source reduces as the output voltage rises and at the same time $\Delta_1 T_s$ decreases.
A less pessimist analysis can be done by considering the model in the figure 2 The analysis

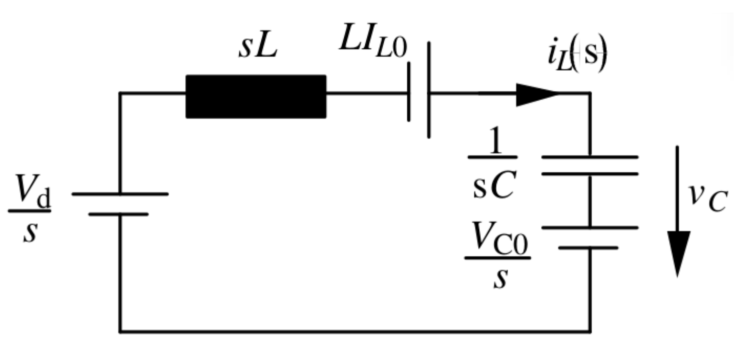

Figure 2: Equivalent circuit when the diode conducts.

can be easily done in the Laplace plane and we get

$$i_L(s) = \frac{\frac{U_d}{s} + \frac{U_{C0}}{s} + LI_{Lo}}{sL + \frac{1}{sC}} \tag{10}$$

which becomes

$$i_L(s) = \frac{U_d - U_{C0}}{L(s^2 + \frac{1}{LC})} + \frac{sI_{Lo}}{s^2 + \frac{1}{LC}} \tag{11}$$

if we set the resonance frequency $\omega_0 = \frac{1}{\sqrt{RC}}$, we get

$$i_L(s) = \frac{U_d - U_{C0}}{L(s^2 + \omega_0^2)} + \frac{sI_{Lo}}{s^2 + \omega_0^2} \tag{12}$$

Transforming the current from the Laplace form to the temporal form we get

$$i_L(t) = I_{Lo}\cos(\omega_0 t) + \frac{U_d - U_{C0}}{Z_o}\sin(\omega_0 t) \qquad 0 \leq t \leq DT_s \tag{13}$$

Let's calculate the time necessary for the current in the inductance to reach zero using the equation 13 when $i_L(t) = 0$A.

$$i_L(t_0) = 0 = I_{Lo}\cos(\omega_0 t_0) + \frac{U_d - U_{C0}}{Z_o}\sin(\omega_0 t_0) \tag{14}$$

and we get

$$t_0 = \frac{1}{\omega_0}\arctan\left(\frac{-Z_0 i_{L,peak}}{U_d - U_{C0}}\right) \approx 27,29\mu s \tag{15}$$

It can be noticed that t_0 is smaller than Δ_1. When the output voltage rises, the current in the inductance decreases faster. The voltage over the capacitor C can be expressed as

$$u_c(s) = \frac{1}{sC}i_L(s) + \frac{U_{C0}}{s} = \frac{U_{C0}}{s} + \frac{U_d - U_{C0}}{sLC(s^2 + \omega_0^2)} + \frac{I_{L0}}{C(s^2 + \omega_0)} \tag{16}$$

With inverse Laplace transform we get

$$u_c(t) = U_{C0} + \frac{U_d - U_{C0}}{LC}(1 - \cos(\omega_0 t)) + \frac{\sqrt{LC}}{C}I_{L0}\sin(\omega_0 t) \tag{17}$$

which becomes

$$u_c(t) = U_d - (U_d - U_0)\cos(\omega_0 t) + Z_0 i_{L,peak}\sin(\omega_0 t) \tag{18}$$

The current in the inductor and in the capacitor are represented in the figure 3. The current becomes negative and the diode stop conducting. The system stays stable for the following switching. Like shown in the current figure, the current decreases linearly to zero.

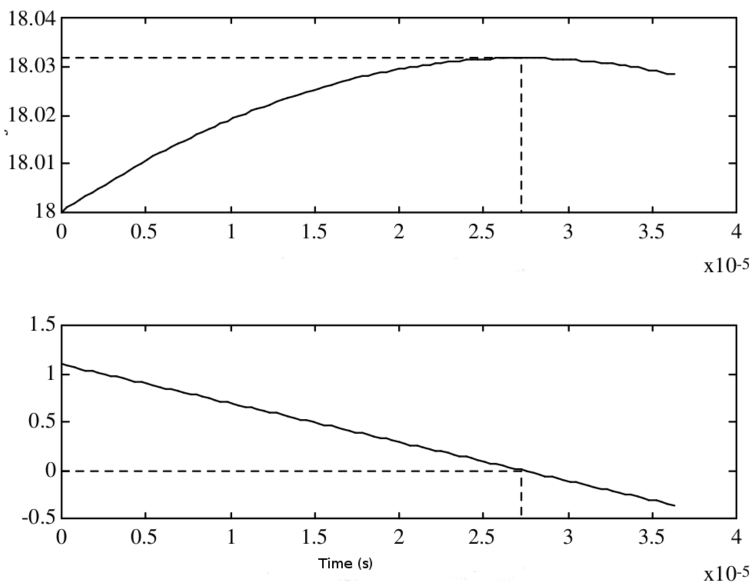

Figure 3: Up, the voltage over the capacitor. Down, current in the inductor.

Despite having calculated the first period after the removing of the resistor, it does not answer entirely the question. The voltage in the capacitor can be calculated as a function of the previous state. For example in the next switching we need to calculate the new time for when the current reaches zero, then the new maximal voltage. An the same procedure for the following states. Unfortunately, no general recursive equation can be found. The solution can be found using a numerical simulation. In the figure 4 is represented the voltage over the capacitor when the load is removed.

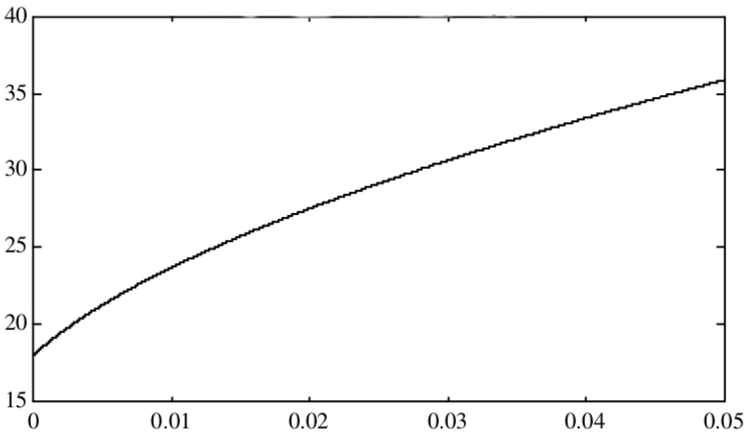

Figure 4: Voltage over the capacitor when the load is removed, $U_c(t) = f(t)$.

The output voltage with $U_{C0} = 18$V, reaches 36V in 0, 05s. The difference from the approximation is small ($\Delta_1 T_s = 0,0423$s).

Exercise 2

Draw output power $P_o(t)$ of the full-bridge with an bipolar and unipolar control when the output voltage $U_d = 40$V, the emf of the motor is $e_a = 24$V and the output current $I_o = 12$A. An inductance $L = 150\mu$H is in series with the load and a switching frequency $f_s = 20$kHz. It can be assumed that the output voltage is constant (large capacitance in the output that is not represented in the figure 5).

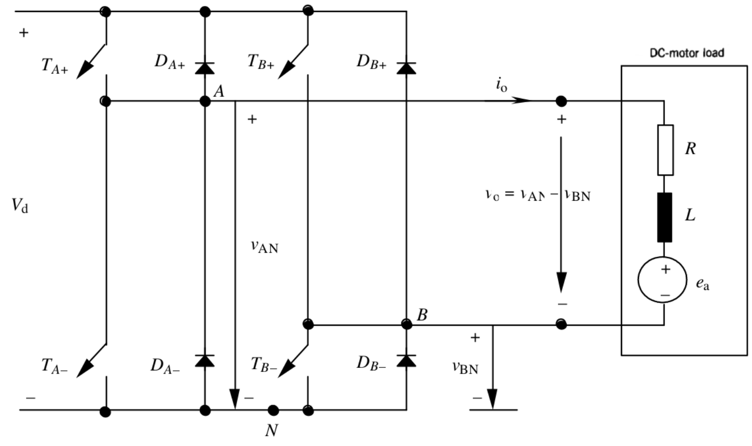

Figure 5: Full-bridge converter.

Solution

Bipolar control

PWM with bipolar voltage switching control uses the switching combinations, (T_{A+}, T_{B-}) with a switching ratio D_1 and (T_{A-}, T_{B+}) with a switching ratio D_2. Only one pair is on at the time. So, if $v_{control} > v_{tri}$, (T_{A+}, T_{B-}) are conducting. Otherwise (T_{A-}, T_{B+}) are conducting.
The average voltage over the inductor is $u_L = V_d - e_a$. When the switches TA_+, TB_- are conducting the current in the inductor increases linearly. we can write

$$i_L(t) = i_{L,min} + \frac{V_d - e_a}{L}t, \qquad \text{when } \frac{T_s}{2} - t_1 < t < T_s + t_1 \tag{19}$$

where the conducting time is

$$t_{on} = T_s + t_1 - \left(\frac{T_s}{2}\right) = 2t_1 + \frac{T_s}{2} \tag{20}$$

The voltage over the DC-motor is

$$V_o = V_{AN} - V_{BN} = D_1 V_d - D_2 V_d = V_d(D_1 - D_2) \tag{21}$$

and because $D_1 = 1 - D_2$ we obtain

$$V_o = V_d(2D_1 - 1) \tag{22}$$

And we get the peak value of the choke

$$i_{L,peak} = i_{L,min} + \frac{V_d - e_a}{L}\left(\frac{T_s}{2} + 2t_1\right) = i_{L,min} + \frac{V_d - e_a}{L} D_1 T_s \tag{23}$$

We can calculate the output current as

$$I_o = \frac{i_{L,peak} + i_{L,min}}{2} = i_{L,peak} - \frac{i_{L,peak} - i_{L,min}}{2} = I_{L,peak} - \frac{V_d - e_a}{2L} D_1 T_s \tag{24}$$

and we get

$$i_{L,peak} = I_o + \frac{V_d - e_a}{2L} D_1 T_s \tag{25}$$

and

$$i_{L,min} = I_o - \frac{V_d - e_a}{2L} D_1 T_s \tag{26}$$

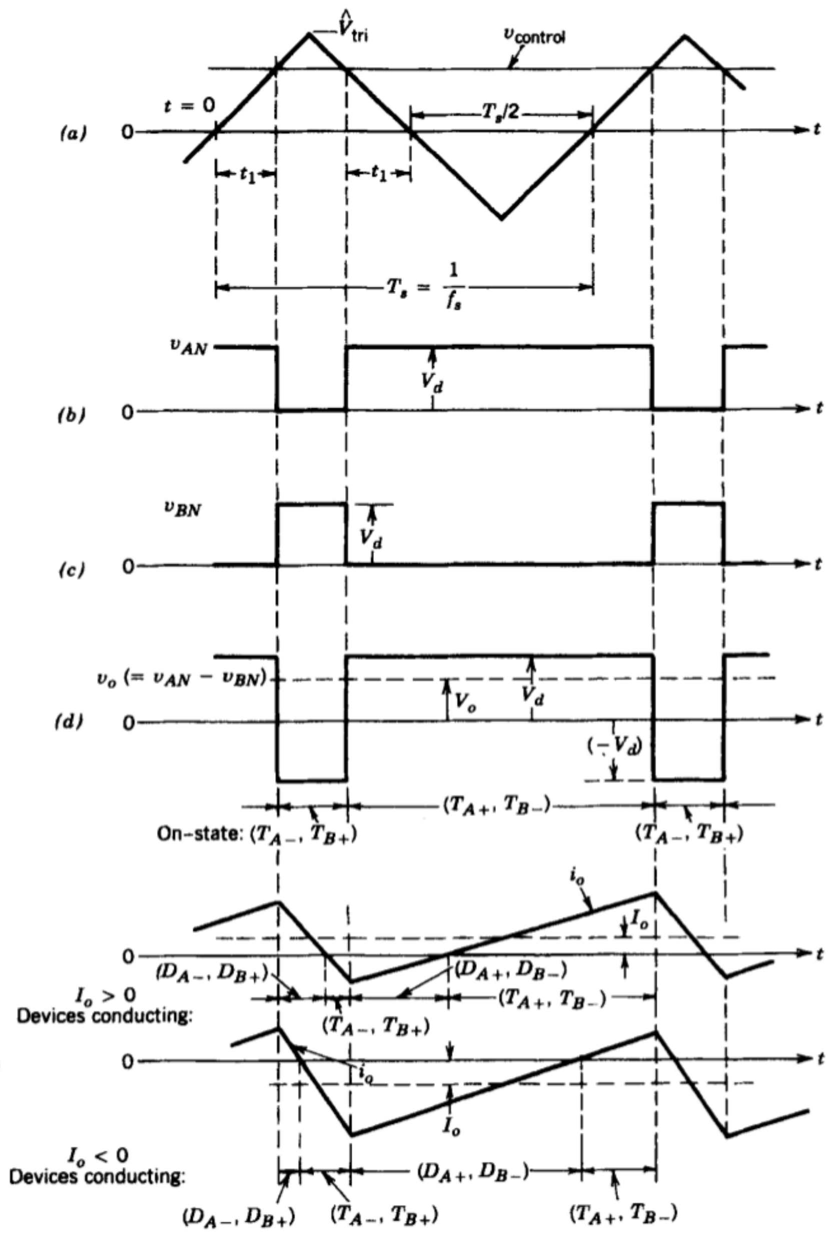

Figure 6: PWM with bipolar voltage switching [1].

The switching ration D_1 is obtain from the equation 22 with $R_a = 0\Omega$ which gives $V_o = e_a$ since the average voltage over an inductance is zero. With the numerical values we get $i_{L,peak} \approx 14,133$A and $i_{L,min} \approx 9,866$A. In the figure 7 is draw the instantaneous power $P_o(t)$ with an inductance of $L = 15\mu$H with a minimum current being negative. In the case of this exercise the current does not go any lower than $i_{L,min} = 9,866$A.This implies that the plot for this exercise are slightly different..

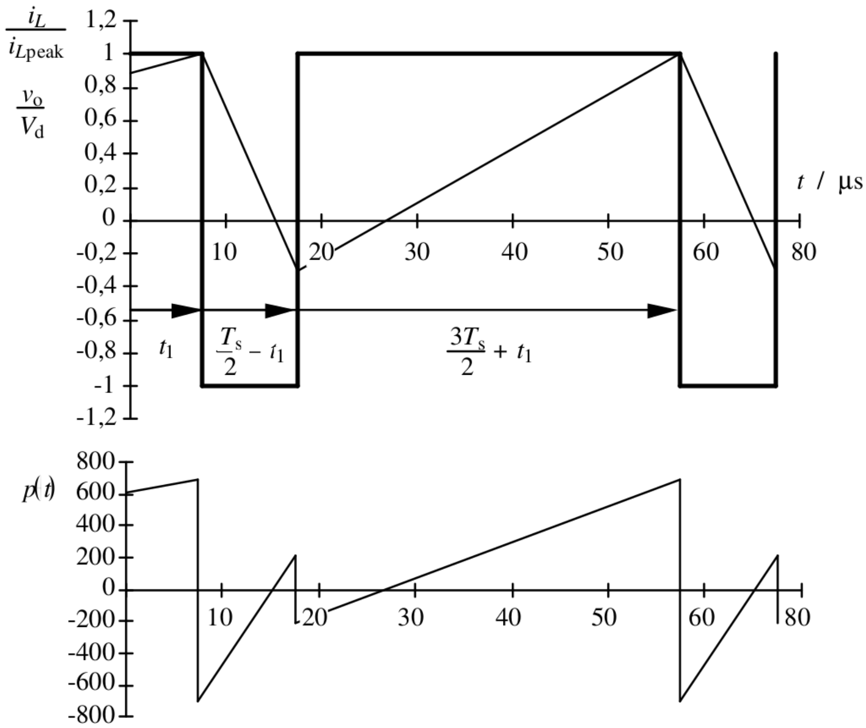

Figure 7: Up, the normalized current in the choke. Down, the instantaneous output power with $L = 15\mu$H

Unipolar control

The unipolar control works the following way:
if $v_{control} > v_{tri}$ T$_{A+}$ is conducting. Otherwise T$_{A-}$ is conducting
if $-v_{control} > v_{tri}$ T$_{B+}$ is conducting. Otherwise T$_{B-}$ is conducting
From the figure 8, we can obtain the current in the choke

$$i_L(t) = i_{L,min} + \frac{V_d - e_a}{L}t, \qquad \text{when } 0 < t < 2t_1 \tag{27}$$

The conduction time is

$$t_{on} = 2t_1 + \frac{T_s}{2} \tag{28}$$

Ans the output voltage is

$$V_o = V_d(2D_1 - 1) \tag{29}$$

and we get

$$t_1 = \frac{t_{on}}{2} - \frac{T_s}{4} = \frac{2D_1 - 1}{4}T_s = \frac{V_o}{V_d}\frac{T_s}{4} \tag{30}$$

From the figure 8 we can also obtain the choke peak current

$$i_{L,peak} = i_{L,min} + \frac{V_d - e_a}{L}2t_1 = i_{L,min} + \frac{V_d - e_a}{L}\frac{2D_1 - 1}{2}T_s \tag{31}$$

and the average output current

$$I_o = \frac{i_{L,peak} + i_{L,min}}{2} = i_{L,peak} - \frac{i_{L,peak} - i_{L,min}}{2} = i_{L,peak} - \frac{V_d - e_a}{L}\frac{2D_1 - 1}{4}T_s \tag{32}$$

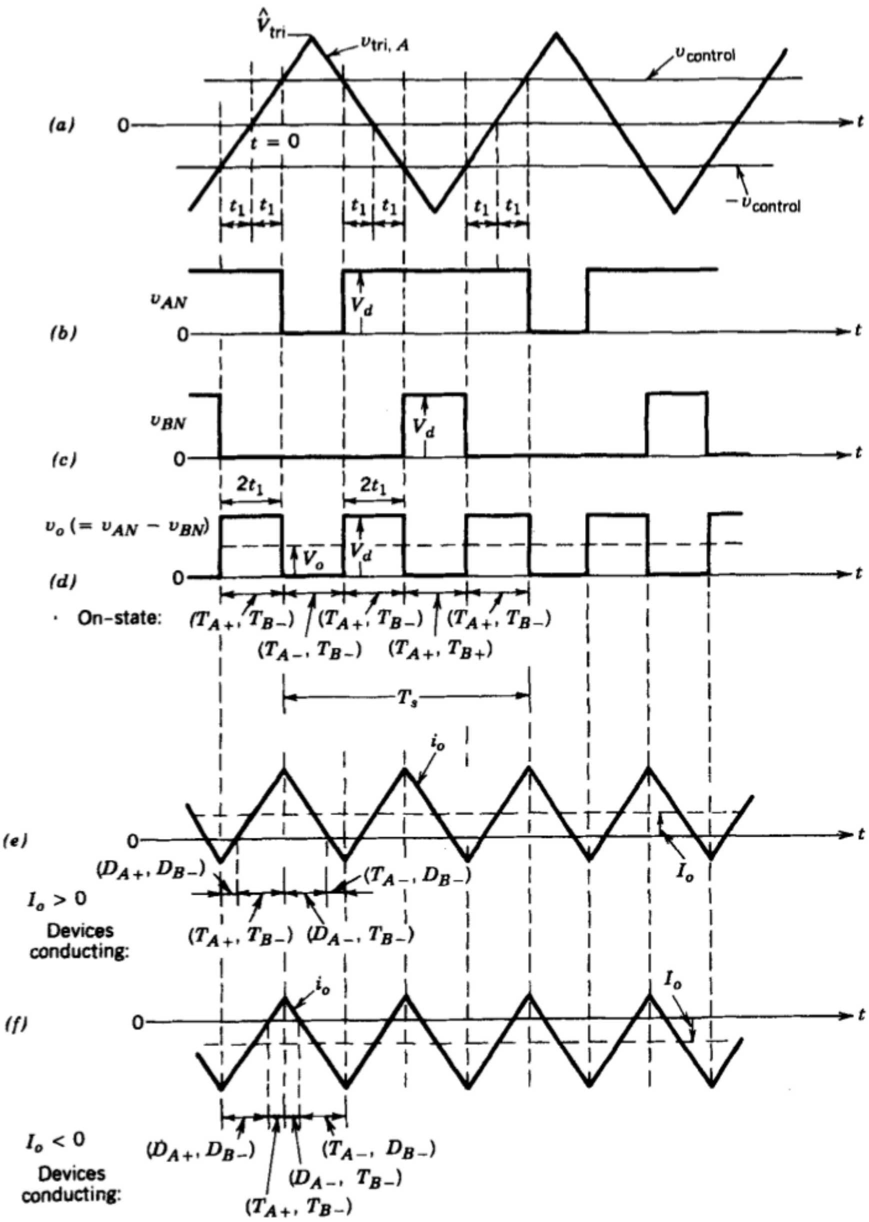

Figure 8: PWM with unipolar voltage switching [1]

and we get

$$i_{L,peak} = I_o + \frac{V_d - e_a}{L}\frac{2D_1 - 1}{4}T_s \tag{33}$$

and

$$i_{L,min} = I_o - \frac{V_d - e_a}{L}\frac{2D_1 - 1}{4}T_s \tag{34}$$

The switching ration D_1 is obtain from the equation 29 with $R_a = 0\Omega$ which gives $V_o = e_a$ since the average voltage over an inductance is zero. With the numerical values we get $i_{L,peak} \approx 12,8$A and $i_{L,min} \approx 11,2$A. In the figure 9 is draw the instantaneous power $P_o(t)$ with an inductance of $L = 15\mu$H with a minimum current being negative. In the case of this exercise the current does not go any lower than $i_{L,min} = 11,2$A. This implies that the plot for this exercise are slightly different.

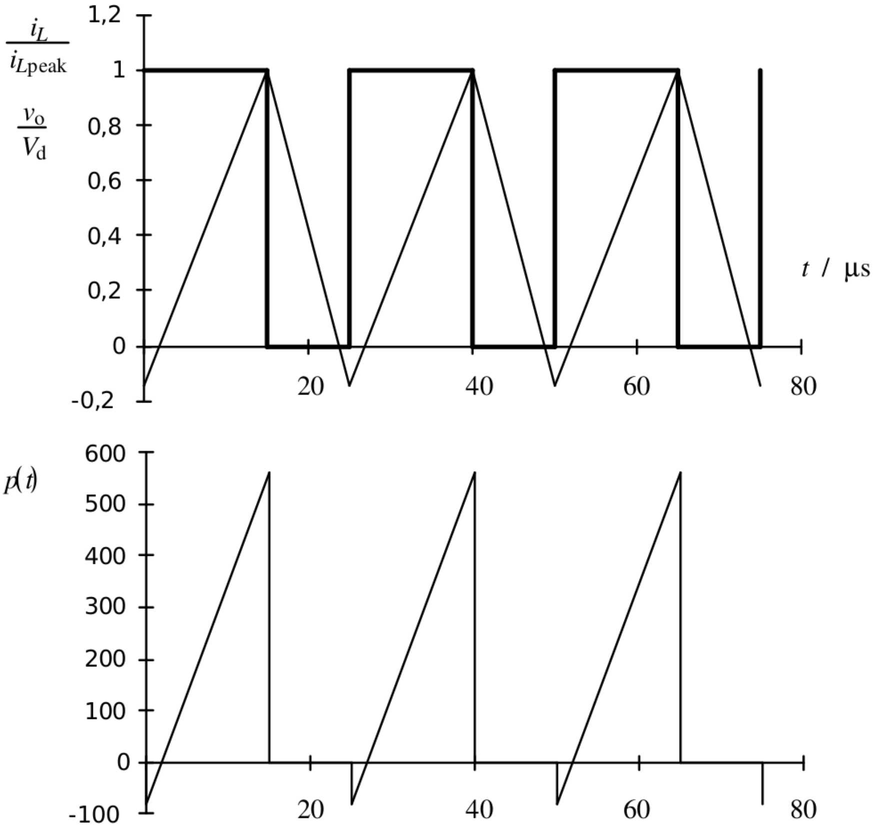

Figure 9: Up, the normalized current in the choke. Down, the instantaneous output power with $L = 15\mu$H.

With the unipolar control the output voltage, current and power vary less that with the bipolar control.

References

[1] N.Mohan, *Power Electronics, Converter Applications, and Design*, 3nd edition, John Wiley & Sons, Inc.,2003, ISBN 978-0-471-22693-2.