Exercise 1

In continuous conduction mode, obtain the power ratio P_o/P_T of the buck, boost, buck-boost, Cùk and full-bridge converter. The ratio can be expressed as

$$\frac{P_o}{P_T} = \frac{U_o I_o}{U_T I_T} \tag{1}$$

where U_T is the maximal voltage over the switch, and I_T the maximal current in the switch. It is assumed that the current in the choke does not have ripple, i.e. $i_L(t) = I_L$ =constant

Solution

In the solution, the ripple in the currents is not taken in account. So, we can use the average currents.

Buck converter

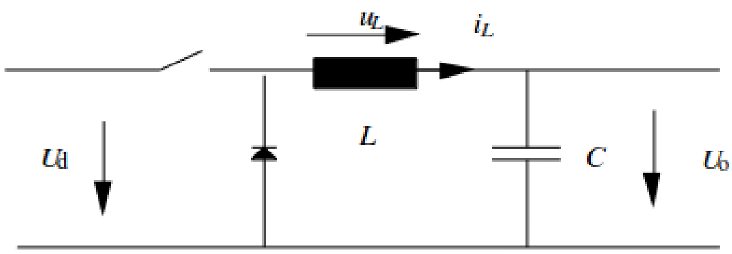

Figure 1: Buck converter.

The duty ration is given by

$$D = \frac{t_{on}}{Ts} = \frac{U_o}{U_d} \tag{2}$$

The current ratio is

$$\frac{I_o}{I_d} = \frac{U_d}{U_o} = \frac{1}{D} \tag{3}$$

The current $I_T = I_o = I_L$ and we get

$$\frac{P_o}{P_T} = \frac{U_o I_o}{U_d I_o} = D \tag{4}$$

Boost converter

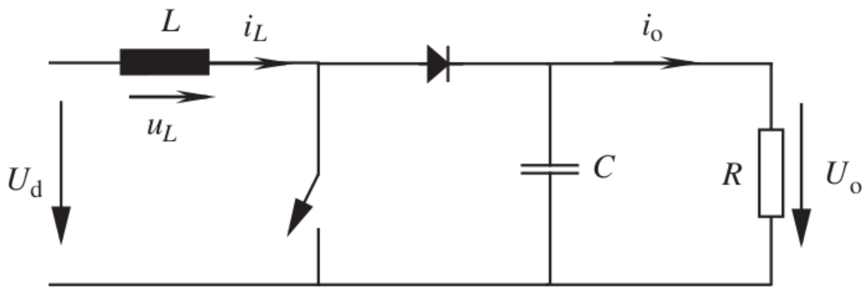

Figure 2: Boost converter.

The output and input voltage ratio is given by

$$\frac{U_o}{U_d} = \frac{1}{1-D} \tag{5}$$

and the current ratio in the

$$\frac{I_o}{I_d} = \frac{U_d}{U_o} = 1 - D \tag{6}$$

The maximal current in the switch is $I_T = I_d = I_L$ and we get

$$\frac{P_o}{P_T} = \frac{U_o I_o}{U_d I_o} = \frac{I_o}{I_d} = 1 - D \tag{7}$$

Buck-Boost converter

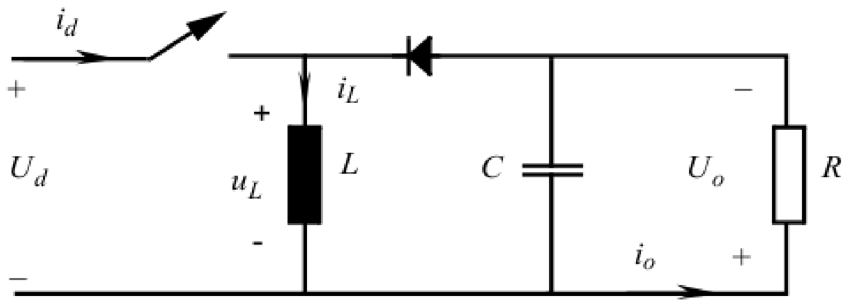

Figure 3: Buck-Boost converter with one switch.

The ration of the output and input voltage is

$$\frac{U_o}{U_d} = \frac{D}{1-D} \tag{8}$$

The current ratio is

$$\frac{I_o}{I_d} = \frac{U_d}{U_o} = \frac{1-D}{D} \tag{9}$$

the maximal current in the switch is $I_T = I_L = I_o + I_d$

$$I_T = \left(1 + \frac{1-D}{D}\right) I_d = \frac{I_d}{D} \tag{10}$$

and we get the power ratio

$$\frac{P_o}{P_T} = \frac{U_o I_o}{(U_o + U_d) I_L} = \frac{1}{1 + \frac{U_d}{U_o}} \frac{I_o}{\frac{I_d}{D}} = \frac{1}{1 + \frac{1-D}{D}} \frac{D(1-D)}{D} = D(1-D) \tag{11}$$

The maximal current of the switch is the same as the current in the inductance. The switch maximal current is not as large as I_d because I_d is the input average current not its maximal value. The maximal voltage is obtained when the switch is open and at that moment the voltage becomes the sum of the output and input voltage.

Cùk converter

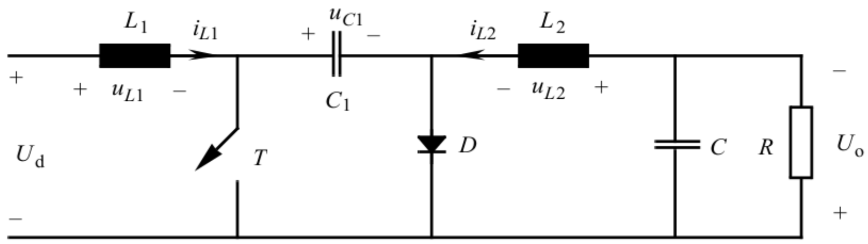

Figure 4: Cùk converter.

The ratio of the voltage is

$$\frac{U_o}{U_d} = \frac{D}{1-D} \tag{12}$$

and the voltage ratio is

$$\frac{I_o}{I_d} = \frac{U_d}{U_o} = \frac{1-D}{D} \tag{13}$$

the maximal current in the switch is $I_T = I_o + I_d$

$$I_T = \left(1 + \frac{1-D}{D}\right) I_d = \frac{I_d}{D} \tag{14}$$

The voltage over the switch is

$$U_T = U_{C1} = U_d + U_o \tag{15}$$

and the power ratio is

$$\frac{P_o}{P_T} = \frac{U_o I_o}{(U_o + U_d) I_L} = \frac{1}{1 + \frac{U_d}{U_o}} \frac{I_o}{\frac{I_d}{D}} = \frac{1}{1 + \frac{1-D}{D}} \frac{D(1-D)}{D} = D(1-D) \tag{16}$$

Full-bridge converter

The ratio of the voltages is

$$\frac{U_o}{U_d} = 2D_1 - 1 \tag{17}$$

and the current ratio is

$$\frac{I_o}{I_d} = \frac{U_d}{U_o} = \frac{1}{2D_1 - 1} \tag{18}$$

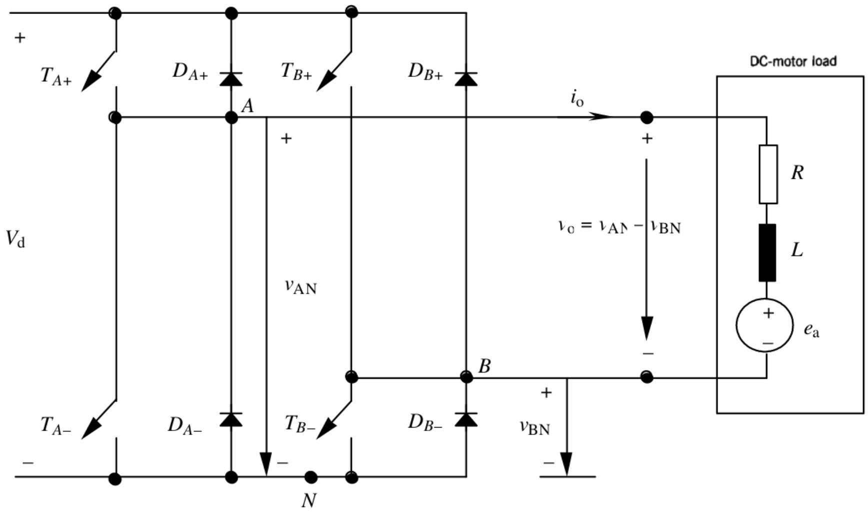

Figure 5: Full-bridge converter.

The thyristor maximal voltage is U_d and the converter has four switches. The power ratio

$$\frac{P_o}{P_T} = \frac{U_o I_o}{4 U_d I_o} = \frac{2D_1 - 1}{4} \tag{19}$$

In the figure 6 is represented the power ratios.

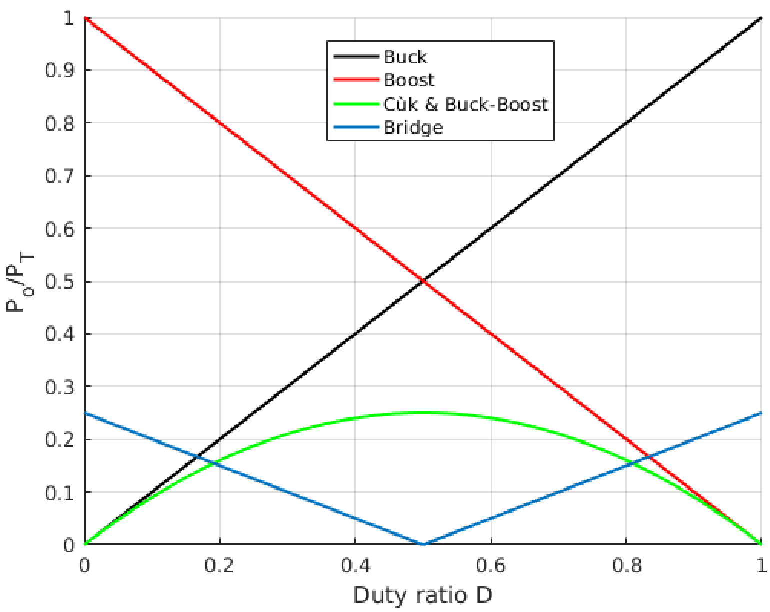

Figure 6: Power ratio of the converters.

Exercise 2

Obtain the current i_L and the voltage v_C in the figure 7. At $t = t_0$ the current in the choke is I_{L0} and the voltage in the capacitor is V_{C0}.

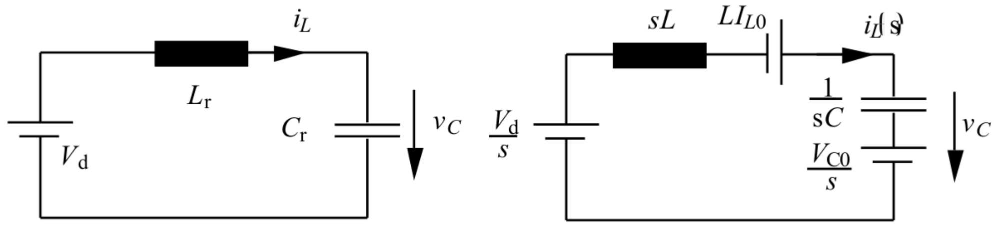

Figure 7: LC circuit and its Laplace equivalent.

The resonance frequency is

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}} \tag{20}$$

and the impedance

$$Z_0 = \sqrt{\frac{L}{C}} \tag{21}$$

Following are the useful Laplace-transformations for this exercise.

$$\frac{s}{s^2 + a^2} \rightarrow \cos(at) \tag{22}$$

$$\frac{a}{s^2 + a^2} \rightarrow \sin(at) \tag{23}$$

$$\frac{a^2}{s(s^2 + a^2)} \rightarrow (1 - \cos(at)) \tag{24}$$

$$\frac{1}{s} \rightarrow 1 \tag{25}$$

Solution

The current in the capacitor is

$$i_c(t) = C\frac{dV_c(t)}{dt} \rightarrow i_c(s) = sCv_c(s) - CV_{C0} \tag{26}$$

the voltage over an inductor is

$$v_L(t) = L\frac{di_L(t)}{dt} \rightarrow v_L(s) = sLi_L(s) - LI_{L0} \tag{27}$$

Using the figure 7 we get

$$\frac{V_d}{s} - \frac{V_{C0}}{s} + LI_{L0} = \left(sL + \frac{1}{sC}\right) i_L(s) \tag{28}$$

$$i_L(s) = \frac{\frac{V_d - V_{C0}}{s} + LI_{L0}}{sL + \frac{1}{sC}} = \frac{V_d - V_{C0}}{L\left(s^2 + \frac{1}{LC}\right)} + \frac{sI_{L0}}{s^2 + \frac{1}{LC}} = \frac{V_d - V_{C0}}{L\left(s^2 + \omega_0^2\right)} + \frac{sI_{L0}}{s^2 + \omega_0^2} \tag{29}$$

and using the Laplace transform inverse we get

$$i_L(t) = I_{L0}\cos(\omega_0 t) + \frac{(V_d - U_{C0})\sqrt{LC}}{L}\sin(\omega_0 t) \tag{30}$$

The voltage over the capacitor is

$$u_c(s) = \frac{V_{C0}}{s} + \frac{1}{sC} i_L(s) = \frac{V_{C0}}{s} + \frac{V_d - V_{C0}}{sLC(s^2 + \omega_0^2)} + \frac{I_{L0}}{C(s^2 + \omega_0^2)} \tag{31}$$

And with the Laplace transform inverse we get

$$u_C(t) = V_{C0} + \frac{V_d - V_{C0}}{\omega_0^2 LC}(1 - \cos(\omega_0 t)) + \frac{\sqrt{LC}}{C} I_{L0} \sin(\omega_0 t) \tag{32}$$

Exercise 3

A symmetrical square-wave is fed to the RLC of the figure 8. The frequency of the signal is $f_s = f_0 = 50$kHz and the amplitude varies between $\pm V_d/2$ with $V_d = 20$V. The quality factor of the circuit is $Q = 7$ and the components are chosen so that the current component at the fundamental frequency f_0 has a peak value of 5A. What is the value of the lowest current harmonic when the resistance R can be about zero at the harmonic frequencies, i.e.when the harmonic number $n > 1$. What happens to the current component when the frequency of the input signal rises to 55kHz?

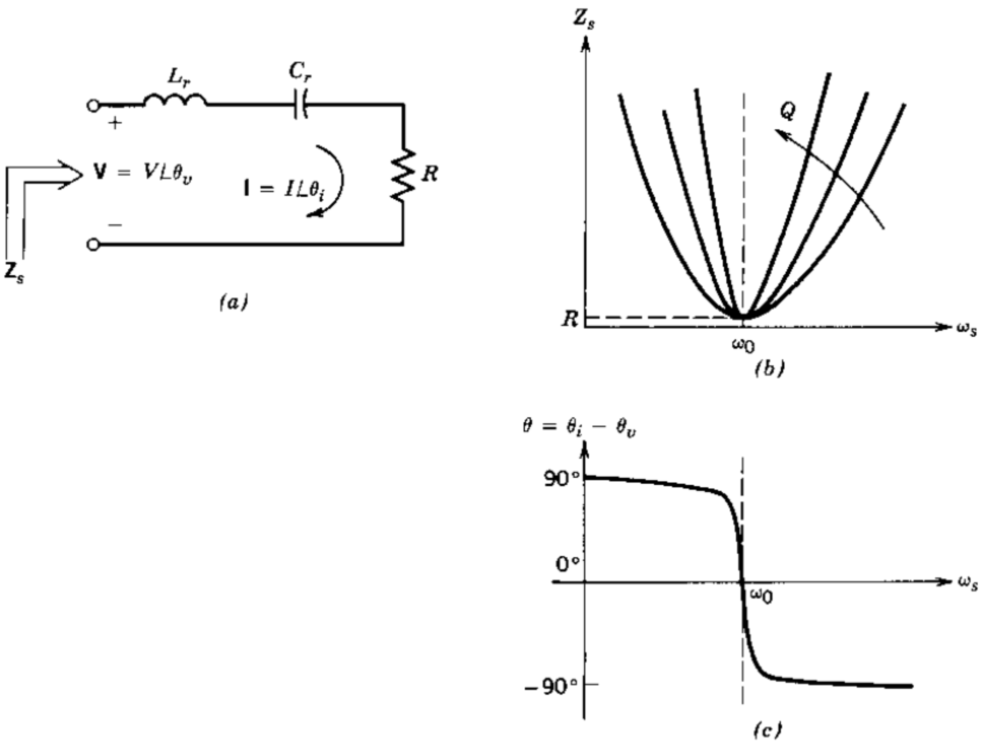

Figure 8: RLC circuit.

The quality factor of the circuit is $Q = \frac{\omega_0 L_r}{R} = \frac{1}{\omega_0 C_r R} = \frac{Z_0}{R}$.

Solution

The input signal is on the figure 9.

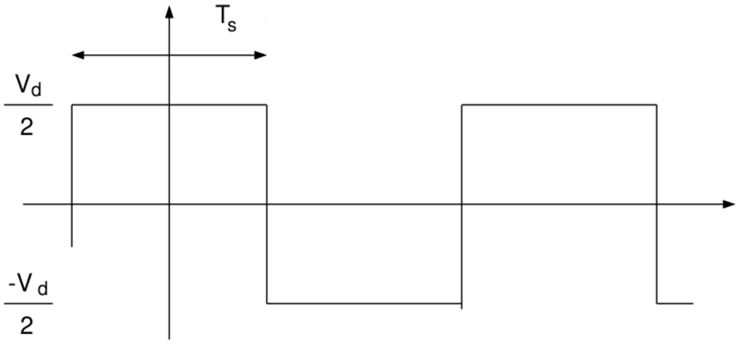

Figure 9: Input signal V

The Fourier-components of the input signal is

$$a_n = \frac{2V_d}{n\pi}\sin\left(\frac{n\pi}{2}\right) \tag{33}$$

with $a_0 = 0$. The impedance of the circuit is

$$Z_n = R + j\left(n\omega L - \frac{1}{n\omega C}\right) \tag{34}$$

At the resonance frequency the imaginary part of the impedance is zero and we get

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}} \tag{35}$$

The components are chosen so that at the resonance frequency ω_0, the fundamental peak value is 5A. Using the equation 33, this current can be written as

$$\hat{i} = \frac{2V_d}{R\pi} \tag{36}$$

and to obtain a peak value of 5A, the resistance is $R \approx 2,55\Omega$. And from the quality factor we get

$$L = \frac{QR}{\omega_0} \approx 56,74\mu\text{H} \tag{37}$$

And with the resonance frequency we get,

$$C = \frac{1}{\omega_0^2 L} \approx 0,178\mu\text{F} \tag{38}$$

The impedance Z_n can be rewritten as

$$Z_n = R + j\frac{n^2\left(\frac{\omega}{\omega_0}\right)^2 - 1}{n\omega C} \tag{39}$$

In the exercise, $R \approx 0$ at the harmonic frequencies. if $n > 1$

$$Z_n \approx j\frac{n^2\left(\frac{\omega}{\omega_0}\right)^2 - 1}{n\omega C} = j\frac{n\left(\frac{\omega}{\omega_0}\right)^2 - \frac{1}{n}}{\omega C} \tag{40}$$

When $\omega = \omega_0$, the impedance is purely resistive and when $n = 3$ the impedance $Z_3 = 47,42j$ whose module is 18,59 times higher than the resonance impedance $2,55\Omega$.
The peak value of the current of the 3rd harmonic is

$$\hat{i} = \frac{\frac{2V_d}{3\pi}}{|Z_3|} \approx 89\text{mA} \tag{41}$$

Despite have a square-wave form signal at the input, the current are quite close to a sinusoidal form. When the circuit quality factor is higher the form of the waveforms becomes more sinusoidal.
When the switching frequency rises to 55kHz, the impedance at the first harmonic is

$$Z_1 = R + j\frac{\left(\frac{\omega}{\omega_0}\right)^2 - 1}{\omega C} \approx 2,55 + 3,43j \tag{42}$$

The peak value of the fundamental is

$$\hat{u}_1 = \frac{2V_d}{\pi} \tag{43}$$

and we get a fundamental peak current of

$$\hat{i}_1 = \frac{2V_d}{\pi|Z_1|} \approx 2,98\text{A} \tag{44}$$

and the power in the resistor is

$$P_{55kHz} = RI^2 = R\left(\frac{\hat{i}_1}{\sqrt{2}}\right)^2 \approx 11,32\text{W} \tag{45}$$

The power in the resistor at the frequency $50kHz$ is

$$P_{50kHz} = RI^2 = R\left(\frac{\hat{i}_1}{\sqrt{2}}\right)^2 \approx 31,87 = 2,82 \times 11,32\text{W} \tag{46}$$