Exercise 1

The Series load resonant (SLR) DC-DC converter of figure 1 works in discontinuous conduction mode (DCM) at a switching frequency ω_s smaller than the resonant frequency $\omega_0 = 1\sqrt{L_r C_r}$ of the LC-circuit. The switching frequency is set to be $\omega_s < \omega_0/2$.

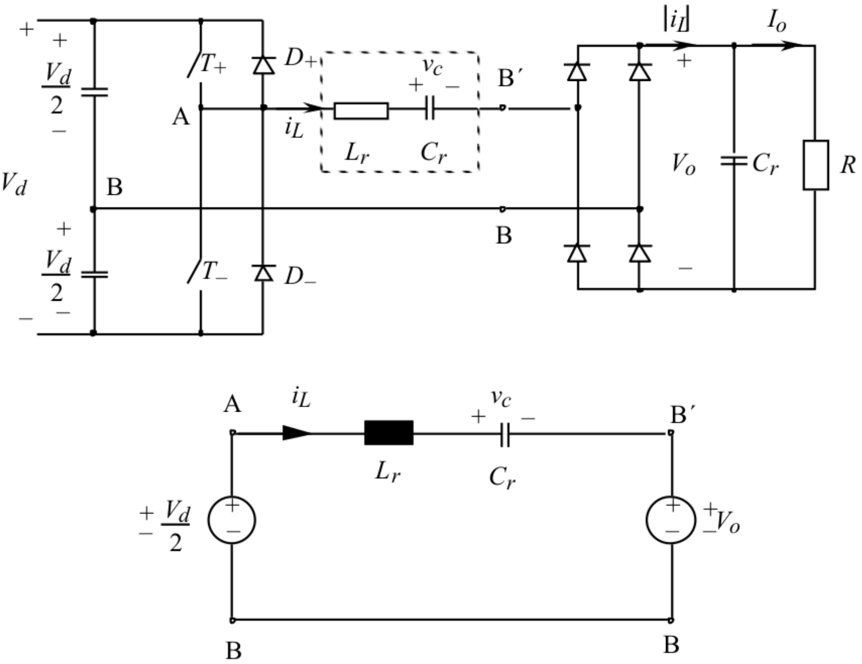

Figure 1: SLR DC-DC converter.

a) Show that the choke maximal current ratio is

$$\frac{I_{L_r,peak}}{I_b} = 1 + 2\frac{V_o}{V_d}, \qquad \text{with } I_b = \frac{V_d}{2Z_0} \text{ and } Z_0 = \sqrt{\frac{L_r}{C_r}} \tag{1}$$

b) Show that the voltage ratio of the peak value over the capacitor C_r is

$$\frac{V_{C_r,peak}}{V_b} = 2 \qquad \text{with } V_b = \frac{V_d}{2} \tag{2}$$

c) Show that the SLR-converter can only work as a voltage step-down.

The instantaneous voltage over C_r and current in the resonant circuit can be expressed as

$$v_{C_r}(t) = V_{AB'} - (V_{AB'} - V_{C0})\cos(\omega_0 t) + Z_0 I_{L0}\sin(\omega_0 t) \tag{3}$$

and

$$i_{L_r}(t) = I_{L0}\cos(\omega_0 t) + \frac{V_{AB'} - V_{C0}}{Z_0}\sin(\omega_0 t) \tag{4}$$

where $V_{AB'}$ is the input voltage of the resonant circuit (figure 2), V_{C0} is the initial voltage over C_r, and I_{L_0} the initial current of L_r.

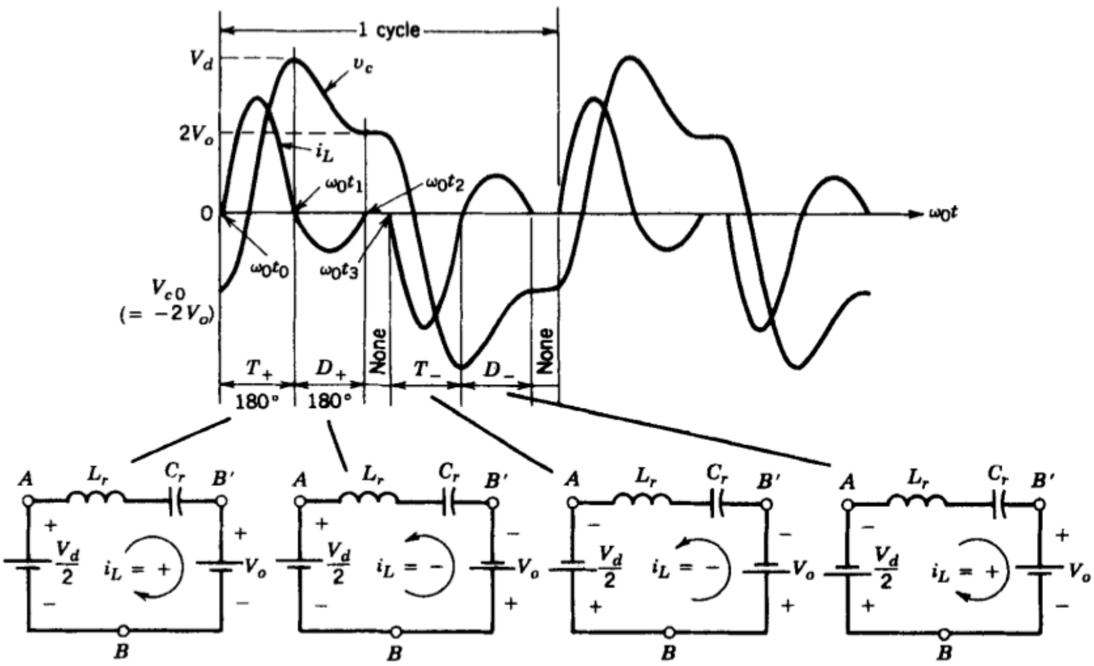

Figure 2: SLR DC-DC converter; discontinuous-conduction mode with $\omega_s < \frac{1}{2}\omega_0$.

Solution

Note: In the DCM mode, the waveforms with $\omega_s < \frac{1}{2}\omega_0$ are represented in figure 2. These waveforms are used to calculate the values. For each 4 states of the circuit the values I_{L0}, V_{C0} and $V_{AB'}$ have to be check from the graph and state-equivalent circuits.

By reading the figure 2, at the beginning, $t = 0$, the switch T_+ is conducting and the current in the choke L_r is $I_{L0} = 0\text{A}$. At that same moment, the voltage over the capacitor C_r is $V_{C0} = -2V_0$. The voltage and current equations 3 and 4 in the LC-circuit can be rewritten by changing $V_{AB'}$ to $\frac{V_d}{2} - V_o$, V_{C0} to $= -2V_o$ and $I_{L0} = 0\text{A}$. We obtain

$$v_{C_r}(t) = \frac{V_d}{2} - V_o - (\frac{V_d}{2} + V_o)\cos(\omega_0 t) \tag{5}$$

and

$$i_{L_r}(t) = \frac{\frac{V_d}{2} + V_o}{Z_0}\sin(\omega_0 t) \tag{6}$$

Part a

From the equation 6, we get the peak value of the current in the choke

$$\hat{i}_{L_r} = \frac{\frac{V_d}{2} + V_o}{Z_0} \tag{7}$$

The current ratio is then

$$\frac{\hat{i}_{L_r}}{I_b} = \frac{\dfrac{\dfrac{V_d}{2} + V_o}{Z_0}}{\dfrac{V_d}{2Z_0}} = 1 + 2\frac{V_o}{V_d} \tag{8}$$

Part b

For this, we check the value of $u_{C_r}(t)$ at $t = 0$ and $t = t_1$. Using equation 5, we have at $t = 0$,

$$v_{C_r}(0) = -2V_o \tag{9}$$

The voltage v_{C_r} is maximal at $t = t_1$, which is when $i_{L_r}(t) = 0$ (figure 2). The equation 6 is equal to zero when $\omega_0 t = 0 + k\pi \ \forall k \in \mathbb{N}$. With the equation 5 at $t = t_1$

$$v_{C_r}(t_1) = v_{C_r,max}\frac{V_d}{2} - V_o + (\frac{V_d}{2} + V_o) = V_d \tag{10}$$

and the ratio is then

$$\frac{v_{C_r,max}}{V_b} = \frac{V_o}{\dfrac{V_o}{2}} = 2 \tag{11}$$

Part c

The current i_{L_r} at $t = t_1$ and after can be expressed as

$$i_{L_r}(t) = \frac{\dfrac{V_d}{2} + V_o - V_{C0}}{Z_0}\sin(\omega_0 t) = \frac{V_o - \dfrac{V_d}{2}}{Z_o}\sin(\omega_0 t) \tag{12}$$

as $V_{AB'} = \frac{V_d}{2} + V_o$ and the initial values $V_{C0} = V_d$ and $I_{L0} = 0\text{A}$ (figure 2).
The current in the choke at an instant $t \geq t_1$ has a lower peak amplitude than in $0 \leq t \leq t_1$. For the circuit to work as a step-down converter, the peak value after of the current t_1 must be negative, i.e.

$$\frac{V_o - \dfrac{V_d}{2}}{Z_0} < 0 \Leftrightarrow V_o < \frac{V_d}{2} \tag{13}$$

The circuit works as a step-down because the output voltage is smaller than the input.

Exercise 2

In the figure, a ZCS (Zero current switch) DC-DC converter is represented.

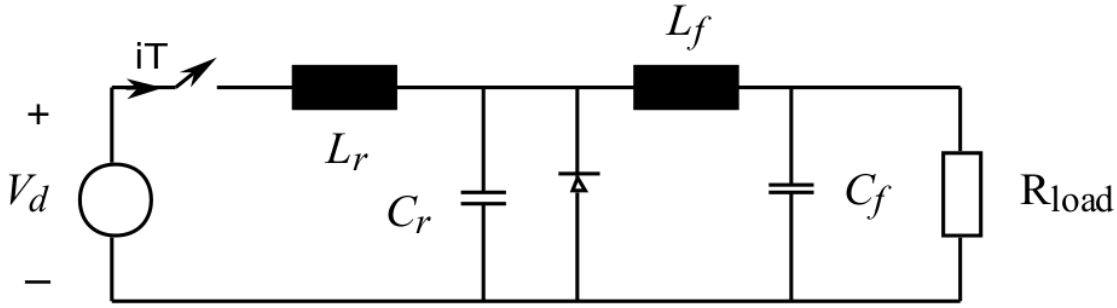

Figure 3: ZCS resonant switch DC-DC converter.

The characteristics of the circuit are as follow:
-the resonance frequency $f_0 = 1\text{MHz}$
-The characteristic impedance $Z_0 = 10\Omega$
-the input voltage is $V_d = 15\text{V}$
-the output voltage is $V_o = 10\text{V}$
-the output power $P_o = 10\text{W}$

Obtain the instantaneous current $i_T(t)$ and voltage $u_{C_r}(t)$ over the capacitor C_r, and the time intervals $t1 - t0, t_1^{'} - t_1, t_1^{''} - t_1, t_2 - t_1, t_3 - t_2, t_3 - t_4$. Calculate the maximal and minimal value of $i_T(t)$ and $u_{C_r}(t)$. What is the switching frequency f_s?

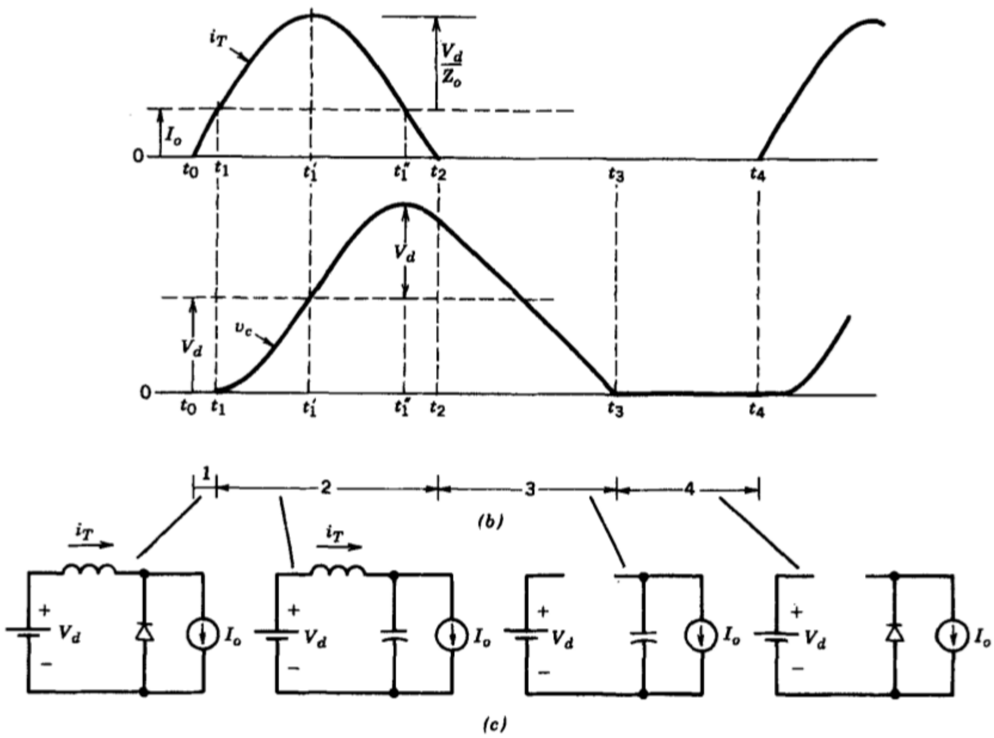

Figure 4: ZCS resonant-switch DC-DC converter waveforms.

The resonant circuit with a parallel load has the following equations:
the current in the choke L_r is

$$i_{L_r}(t) = I_o + (I_{L0} - I_o)\cos(\omega_0 t) + \frac{V_d - V_{C0}}{Z_0}\sin(\omega_0 t) \tag{14}$$

and the voltage over the capacitor C_r

$$v_{C_r}(t) = V_d - (V_d - V_{C0})\cos(\omega_0 t) + Z_0(I_{L0} - I_o)\sin(\omega_0 t) \tag{15}$$

The initial values are V_{C0} for the capacitor C_r and I_{L0} for the inductance L_r.

Solution

Since the characteristic impedance Z_0 and the resonance frequency f_0 are given, we can calculate the value of L_r and C_r.

$$Z_0 = \sqrt{\frac{L}{C}} \Rightarrow \quad \frac{1}{L}Z_0 = \frac{1}{\sqrt{LC}} = \omega_0 \Rightarrow \quad L = \frac{Z_0}{\omega_0} \tag{16}$$

the same way

$$Z_0 = \sqrt{\frac{L}{C}} \Rightarrow \quad CZ_0 = \sqrt{LC} = \frac{1}{\omega_0} \Rightarrow \quad C = \frac{1}{Z_0\omega_0} \tag{17}$$

and their values are $L_r = 1,591\mu$H and $C_r = 16,91$nF.
At $t = 0$ the switch is conducting, the capacitor is at the beginning without voltage as the diode is conducting. The voltage V_d over the choke L_r makes its current rise linearly

$$\frac{di_T(t)}{dt} = \frac{V_d}{L_r} \tag{18}$$

and we get

$$t_1 - t_0 = \frac{L_r}{V_d} \approx 0,106\mu\text{s} \tag{19}$$

As the current i_T increases, the current in the diode decreases till $i_T = I_o$ and the diode stop conducting at the instant $t = t_1$. The current in the resonant circuit increases and the voltage in the capacitor C_r also increases with the current $i_T - I_o$.
To calculate the time interval $t_2 - t_1$, we can use the equations 14 and 15. The initial value at $t = t_1$ are $I_{L0} = I_o$ and $I_{C0} = 0$V and we get

$$i_{L_r}(t) = I_o + \frac{V_d}{Z_0}\sin(\omega_0 t) \tag{20}$$

$$v_{C_r}(t) = V_d - V_d\cos(\omega_0 t) \tag{21}$$

The current is maximal when $\sin(\omega_0 t_1^{'}) = 1$ and we have $t_1^{'} - t_1 = \frac{\pi}{2\omega_0} \approx 0,25\mu$s.
And the maximal current is

$$i_{T,max} = I_o + \frac{V_d}{Z_0} \approx 2,5\text{A} \tag{22}$$

The voltage over the capacitor is a $t = t_1^{'}$

$$v_{C_r}(t_1^{'}) = V_d\left(1 - \cos\left(\frac{\pi}{2}\right)\right) = V_d = 15\text{V} \tag{23}$$

and its maximal value at $\omega_0 t_1^{''} = \pi$ (eq.21) and we get

$$v_{C_r,max} = V_d(1 - \cos(\pi)) = 2V_d = 30\text{V} \tag{24}$$

The time interval $t_1^{''} - t_1 = \pi/\omega_0 \approx 0,5\mu$s.
Using the equation 20 at $t = t_1^{''}$, we obtain $i_{L_r}(t_1^{''}) = I_o$.
The time interval $t_2 - t_1$ can be obtain from equation 20 when it is equal to zero. and we get

$$i_T(t_2) = I_o + \frac{V_d}{Z_0}\sin(\omega_0 t_2) = 0 \tag{25}$$

which gives

$$\sin(\omega_0 t_2) = \arcsin\left(\frac{-I_o Z_0}{V_d}\right) \approx -0,7297 \tag{26}$$

And we obtain $t_2 - t_1 = 0,616\mu$s.
The current in the choke L_r is zero at the instant t_2 and using the equation 21, the voltage of the capacitor is

$$v_{C_r}(t_2) = V_d(1 - \cos(\pi + 0,729)) \approx 26,2\text{V} \tag{27}$$

After that, the current current remains constant and the voltage of the capacitor is decreasing linearly till it becomes zero at $t = t_3$. We can write

$$\Delta t = t3 - t2 = \frac{C}{I_o}\Delta V_{Cr} = \frac{C}{I_o}\Delta V_{Cr}(t_3) - V_{C_r}(t_2) \approx 1,139\mu\text{s} \tag{28}$$

The voltage over the capacitor C_r is also the voltage over the diode and the output voltage. The output filter is tuned to have an average output voltage of V_d. The voltage over the capacitor C_r at different time is

$$v_{C_r}(t) = V_d - V d\cos(\omega_0 t), \qquad t_1 < t_2, \quad t_2 - t_1 = T_1 \approx 0,616\mu\text{s} \tag{29}$$

$$v_c(t) = V_o - \frac{I_o}{C}t = 26,2 - \frac{I_o}{C_r}t, \qquad t_2 < t < t_3, \quad ,t_3 - t_2 = T_2 \approx 0,417\mu\text{s} \tag{30}$$

We calculate the average of $u_{C_r}(t)$ which is equal to V_o and we obtain

$$V_o = \frac{1}{T_s}\left(\int_0^{T_1} V_d(1 - \cos(\omega_0 t))dt + \int_0^{T_2}\left(V_o - \frac{I_o}{C}t\right)dt\right) \tag{31}$$

where the switching period $T_s = t_1 + T_1 + T_2 + T_3$ with $T_3 = t_4 - t_3$. after solving the integral, we obtain

$$Vo = \frac{1}{T_s}\left(V_d T_1 - \frac{1}{\omega_0}\sin(\omega_0 T_1) + V_o T_2 - \frac{I_o}{2C_r}T_2^2\right) \tag{32}$$

and with the numerical values, we get $T_s = 1,63\mu$s and so the switching frequency is $f_s =$ 614kHz. And the last time interval $t_4 - t_3 = T_3 = T_s - T_1 - T_2 - t_1 = 0,490\mu$s.