Exercise 1

A toroidal transformer core is made of ferrite whose magnetization curve is represented in figure 1. In the transformer is built an airgap whose length l_g is a thousandth of the core length l_m

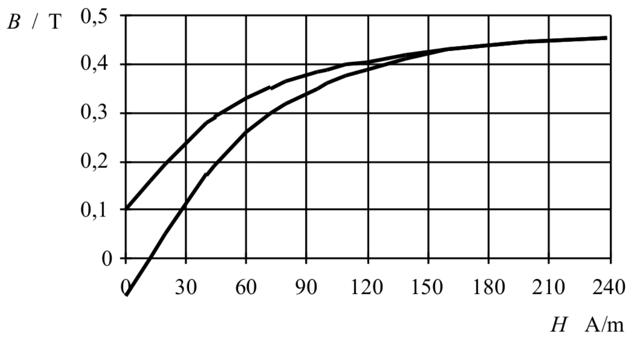

Figure 1: Ferrite magnetization curve.

Draw the B-H curve of the airgaped magnetic circuit and calculate the value of the remanence B_r. The airgap's surface area A_g can be considered as large as the core surface area A_{core}
The remanence is the value of B when $H \to 0$
The vacuum permeability is $\mu_0 = 4\pi \times 10^{-7}$H/m (or Vs.(Am)$^{-1}$)
The flux is

$$\Phi = B_2 A_{core} = B_g A_g \tag{1}$$

Where B_2 is the magnetic flux density of the ferrite core with the airgap.
The flux density is given by

$$B = \mu_m H \tag{2}$$

The magnetic permeability is

$$\mu_m = \mu_r \mu_0 \tag{3}$$

and the magnetomotive force is

$$F_m = H_1 l_m = H_2 l_m + H_g l_g \tag{4}$$

where H_1 is the magnetic field density of the ferrite core without airgap and H_2 with airgap, and H_g the magnetic field density in the airgap.

Solution

Without the airgap and with the Ampère's law over a tore we have

$$\sum_j N_j i_j = F_m = H_1 l_m \qquad \text{and} \qquad B_1 = \mu_m H_1 \tag{5}$$

When a airgap is added the magnetomotive force F_m does not change and we get

$$F_m = H_1 l_m = H_2 l_m + H_g l_g \tag{6}$$

where H_g is the airgap magnetic field and H_2 the magnetic field of the ferrite.
The flux is same in the ferrite and in the airgap. Since it was considered that the airgap surface area is the same as the core surface area, we obtain

$$\Phi = B_2 A_{core} = B_g A_g \tag{7}$$

and

$$B_g = \frac{B_2 A_{core}}{A_g} \approx B_2 = \mu_m H_2 = \mu_0 H_g \tag{8}$$

Let's calculate the magnetic field in the ferrite H_2.

$$F_m = H_1 l_m = H_2 l_m + \frac{\mu_m H_2}{\mu_0} l_g = H_2 l_m \left(1 + \frac{\mu_m l_g}{\mu_0 l_m}\right) \tag{9}$$

and we get

$$H_2 = H_1 \frac{1}{1 + \mu_r \dfrac{l_g}{l_m}} \tag{10}$$

and

$$B_2 = \mu_0 \mu_e H_1 = \mu_0 = \mu_0 \mu_e \frac{B_1}{\mu_m} = \frac{\mu_e}{\mu_r} B_1 \tag{11}$$

with μ_e being the equivalent permeability of the core with an airgap.

$$\mu_e = \frac{\mu_r}{1 + \mu_r \dfrac{l_g}{l_m}} \tag{12}$$

In most case scenario, the airgap is short and ferrite permeability μ_r is high. The equivalent permeability can be approximated as

$$\mu_e = \frac{\mu_r}{1 + \mu_r \dfrac{l_g}{l_m}} = \frac{1}{\dfrac{1}{\mu_r} + \dfrac{l_g}{l_m}} \approx \frac{l_m}{l_g} = 1000 \tag{13}$$

In practice, when measuring the inductances, the magnetic circuit resistance is considered to be zero and we calculate them with only the airgap.
From the equation 11 and the figure 1, we get the figure 2

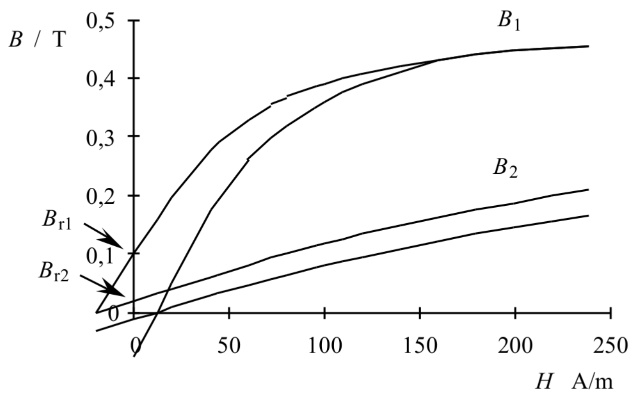

Figure 2: Ferrite magnetization curve with airgap and without.

The remanence of the ferrite with airgap B_{r2} can be obtain from the equation 11 and we get

$$B_{r2} = \frac{\mu_e}{\mu_r} B_{r1} = \frac{B_{r1}}{1 + \mu_r \frac{lg}{l_m}} \approx 0,02\text{T} \tag{14}$$

Where B_{r1} is the remanence of the ferrite without airgap and equal to $0,1$T according to the figure 2. A small airgap linearizes the magnetic circuit and stop efficiently the saturation of the ferrite. This is necessary in the chokes.

Exercise 2

A flyback converter works in the demagnetizing area, i.e. the magnetization of the transformer goes to zero before the next cycle. This region corresponds to discontinuous conduction mode (DCM) where the instantaneous output current goes to zero during the switching period.

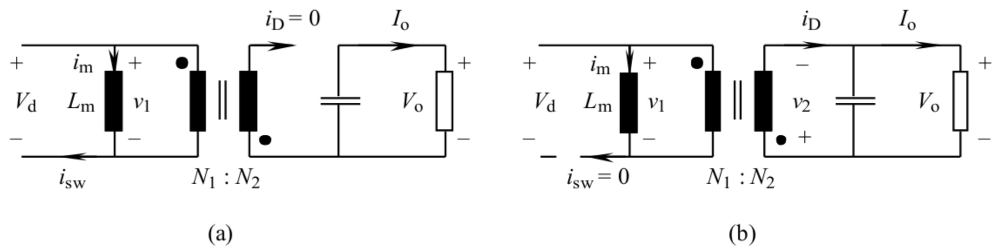

Figure 3: Flyback converter when (a) the switch conducts and (b) the switch does not.

Prove that in DCM the voltage ratio V_o/V_d can be written as

$$\frac{V_o}{V_d} = D\sqrt{\frac{R}{2L_m f_s}} \tag{15}$$

and in Continuous conduction mode (CCM)

$$\frac{V_o}{V_d} = \frac{N_2 D}{N_1 \Delta_1} = \frac{N_2}{N_1}\frac{D}{1-D} \tag{16}$$

where R is the load resistance, f_s the switching frequency, L_m the transformer inductance and D the duty cycle.

Solution

The waveforms of the circuit are given in the figure 4. When the circuit is working in DCM the magnetizing inductance $L_m$and output current i_o are small so that the flux Phi goes to zero before the next cycle. Looking at the figure 3, we can write when the switch is conducting and the diode not conducting that the magnetizing current is

$$i_m(t) = i_{sw}(t) = I_m(0) + \frac{V_d}{L_m}t, \qquad 0 \le t \le t_{on} \tag{17}$$

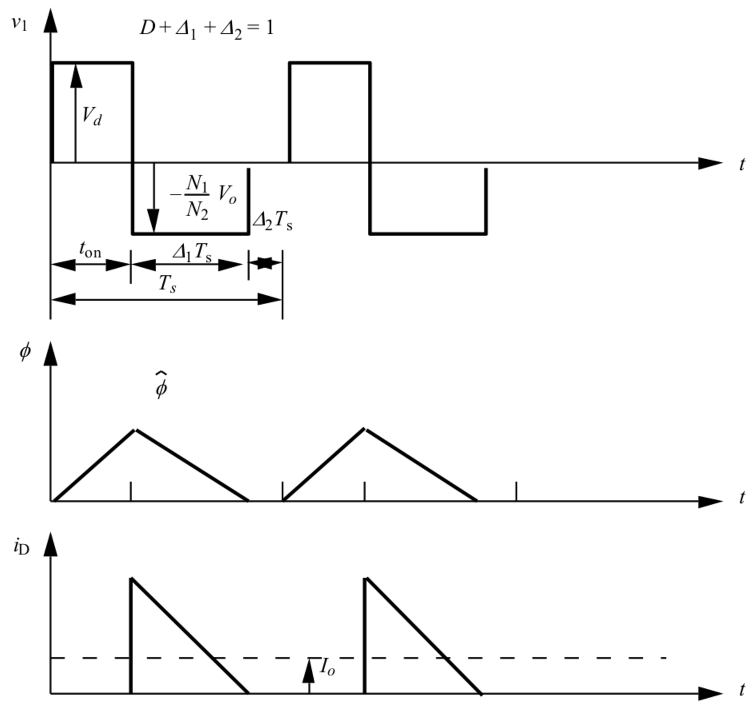

Figure 4: Waveforms of a flyback converter.

When the switch is not conducting, we have

$$v_1 = -\frac{N_1}{N_2}V_o \tag{18}$$

And the magnetizing current is

$$i_m(t) = \hat{I}_m - \frac{N_1}{N_2}\frac{V_o}{L_m}(t - t_{on}) \tag{19}$$

The peak value of $i_m(t)$ is obtain at $t = t_{on}$ and from equation 17 we obtain

$$\hat{I}_m = \frac{V_d}{L_m}t_{on} = \frac{V_d}{L_m}DT_s \tag{20}$$

The current in the diode can be expressed as

$$i_D(t) = \frac{N_1}{N_2}i_m \tag{21}$$

and from equation 20 we obtain the peak current in the diode

$$\hat{I}_D = \frac{N_1}{N_2}\hat{I}_m = \frac{N_1}{N_2}\frac{V_d}{L_m}DT_s \tag{22}$$

From the instant $(D + \Delta_1)T_s$ and forth, the current in the diode is zero and the load current is flowing through the capacitor. The output current I_o is the average value of the current in the diode I_D. Using the figure 4, we obtain the average value of I_D is

$$I_o = \frac{1}{2}\hat{I}_D\Delta_1 T_s\frac{1}{T_s} = \frac{1}{2}\frac{N_1}{N_2}\frac{V_d}{L_m}D\Delta_1 T_s \tag{23}$$

When the switch is not conducting, the magnetizing current is

$$i_m(t) = \frac{V_d}{L_m}DT_s - \frac{N_1}{N_2}\frac{V_o}{L_m}(t - t_{on}) \tag{24}$$

At $t = (D + \Delta_1)T_s$ we have

$$i_m((D + \Delta_1)T_s) = \frac{V_d}{L_m}DT_s - \frac{N_1}{N_2}\frac{V_o}{L_m}(\Delta_1 T_s) = 0 \tag{25}$$

which gives

$$\frac{V_o}{V_d} = \frac{N_2}{N_1}\frac{D}{\Delta_1} \tag{26}$$

The equation 26 remind the equation of a buck-boost converter without the transformer ratio. The time duration Δ_1 can be written as

$$\Delta_1 = \frac{N_2}{N_1}\frac{DV_d}{V_o} \tag{27}$$

And if equation 27 is inserted in the equation 23

$$I_o = \frac{N_1 V_d}{2N_2 L_m}DT_s\frac{N_2}{N_1}\frac{DV_d}{V_o} = \frac{V_o}{R} \tag{28}$$

which leads to

$$\left(\frac{V_o}{V_d}\right)^2 = D^2\frac{RT_s}{2L_m} \tag{29}$$

and finally,

$$\frac{V_o}{V_d} = D\sqrt{\frac{R}{2L_m f_s}} \tag{30}$$

In the CCM we have $\Delta_1 = 1 - D$ and $\Delta_2 = 0$. From the equation 26, we obtain

$$\frac{V_o}{V_d} = \frac{N_2}{N_1}\frac{D}{1 - D} \tag{31}$$

Exercise 3

Using the previous exercise, calculate the maximal value of the magnetizing inductance L_m that keeps the converter working in the demagnetizing area. The numerical values are: $N_1 : N_2 = 1$, $f_s = 200\text{kHz}$, $V_o = 12\text{V}$, $12 \leq V_d \leq 24\text{V}$, the output power $6 \leq P_o \leq 60\text{W}$

Solution

The output current I_o at limit between the CCM and the demagnetizing area is obtain from the previous exercise equation 23 when $\Delta_1 = 1 - D$ and $\Delta_2 = 0$ and we obtain

$$I_{oB} = \frac{N_1 V_d}{2N_2 L_m}D(1 - D)T_s \tag{32}$$

which is the output current limit before entering in demagnetizing area. When the output current goes under I_{oB} the transformer magnetization goes to zero before the next cycle. From equation 31, we can rewrite equation 32 as

$$I_{oB} = \frac{N_1^2(1-D)V_o}{2N_2^2 L_m D} D(1-D)T_s = \left(\frac{N_1}{N_2}\right)^2 \frac{V_o T_s}{2L_m}(1-D)^2 \tag{33}$$

With the numerical values given in the exercise we have $I_o = P_o/V_o \Rightarrow 0,5 \leq I_o \leq 5$A as $6 \leq P_o \leq 60$W. From the equation 33, we obtain the maximal value of the magnetizing inductance required to keep the circuit working in the demagnetizing area.

$$L_m = \left(\frac{N_1}{N_2}\right)^2 \frac{V_o T_s}{2I_{oB}}(1-D)^2 \tag{34}$$

The circuit works in the demagnetizing area when $I_{o,max} < I_{oB,min}$. We need to define the control ratio D_{max} for which the current I_{oB} is minimal. In the CCM, with the equation 31, we obtain the control ratio

$$D = \frac{1}{\frac{N_2}{N_1}\frac{V_d}{V_o} + 1} \tag{35}$$

When the circuit is working in CCM when have $V_o = 12$V. So, the duty ratio $D_{max} = 0,5$ when $V_d = 12$V and $D_{min} = 1/3$ when $V_d = 24$V. From the equation 34, we obtain

$$L_{max} = \left(\frac{N_1}{N_2}\right)^2 \frac{V_o T_s}{2I_{oB,min}}(1-D_{max})^2 \tag{36}$$

with $I_{oB,min} = 5$A and we get $L_{m,max} \approx 1,5\mu$H.
The result can be checked using the equations obtained in the previous exercise. First the duty ratio D from equation 30, we get

$$D = \frac{V_o}{V_d}\sqrt{\frac{2L_m}{RT_s}} = \begin{cases} 0,5 \text{ when } V_d = 12\text{V} \\ 0,25 \text{ when } V_d = 24\text{V} \end{cases} \tag{37}$$

Where $R = V_o/I_o = 12/5\Omega$.
Then the time interval Δ_1 from the equation 23 and we get

$$\Delta_1 = \frac{2I_o N_2 L_m}{N_1 V_d D T_s} \begin{cases} 0,5 \text{ when } V_d = 12\text{V} \Rightarrow \Delta_2 = 0 \\ 0,25 \text{ when } V_d = 24\text{V} \Rightarrow \Delta_2 = 0,25 \end{cases} \tag{38}$$

When $I_o = 5$A and $V_d = 12$V the converter work at the limit of the CCM otherwise it is working in the demagnetizing area.

Shorter Solution

In the limit of CCM and DCM, the equations 15 and 16 are equal.

$$\frac{V_o}{V_d} = D\sqrt{\frac{R}{2L_m f_s}} = \frac{N_2 D}{N_1(1-D)} \tag{39}$$

and we get

$$L_m = \frac{R(1-D)^2}{2f_s} \tag{40}$$

where we use the values $D = 0,5$ and $R = 12/5$ we obtain the same value of $L_m \approx 1,5\mu$H.