General information

The exercise sessions will be held as blackboard sessions, where the participants will present their solutions to the group. As such, the problems should be set up and solved before the session. The focus of the exercises lies on analyzing and discussing the task at hand together with the group: thus, a perfect solution is not required to be awarded points. A point will be awarded for each question, and a person will be chosen to present their solution from the pool.

The paper "*The physics issues that determine inertial confinement fusion target gain and driver requirements: A tutorial*" by M.D. Rosen (Physics of Plasmas, 1999) is a great resource for this set of exercises.

Exercise 1.
Inertial confinement fusion concepts

(a) Explain the difference between direct and in-direct drive in inertial confinement fusion.

(b) Explain the difference between fast and central ignition?

(c) What limits the performance of inertial confinement fusion?

(d) What are the main benefits of compressing the fuel?

(e) How does "rocket equation" relate to ICF?

(f) What are the main subsystems of an inertial confinement fusion power plant?

Exercise 2.
The burn-up ratio Derive the equation for the fraction of fuel burned in inertial confinement fusion:

$$f_B = \frac{\rho R}{\rho R + \beta},$$

where R is the radius of the fuel pellet, $\rho = m_{\mathrm{DT}} n_0$ is the mass density before fusion burn, and $\beta = \frac{8 m_{\mathrm{DT}} C_S}{\langle\sigma v\rangle_{\mathrm{DT}}} \approx 6\,\mathrm{g\,cm^{-2}}$ is the burn parameter at 30 keV (Rosen, 1999). Assume a 50-50 D-T fuel mixture and write the rate of change of the tritium density:

$$\frac{\mathrm{d}n_{\mathrm{T}}}{\mathrm{d}t} = -n_{\mathrm{D}} n_{\mathrm{T}} \langle\sigma v\rangle_{\mathrm{DT}}$$

Set the limits of integration from the start of the fusion reactions $t = 0$ to the confinement time, τ_C. For the confinement time, use the mass-averaged value $\tau_C = R/4C_S$.

Exercise 3.
The Lawson criterion in ICF

(a) In the context of ICF, the Lawson criterion is often expressed in terms of the fuel mass density times the fuel pellet radius, ρR. Assuming a burn-up fraction of $1/3$, calculate the required value of ρR, and then calculate the required value of $n\tau_C$. Compare your result to the reference value for MCF - are we still in the ballpark?

(b) The density of solid DT is approximately $0.23\,\mathrm{g\,cm^{-3}}$. Calculate the required radius of a spherical fuel pellet to obtain a burn up of $1/3$, assuming that the fuel is not compressed.

(c) What is the mass of the resulting D-T fuel pellet?

(d) Calculate a simple estimate for the energy required to heat this fuel pellet uniformly from 0 to 30 keV, i.e. calculate the thermal energy change for the fuel pellet. Neglect any inefficiencies in the the laser system, and any other caveats. The lasers of the National Ignition Facility (NIF) have energies of about 4 MJ, how does this compare to the value you calculated?

(e) How do the results of b), c), and d) change, if the fuel pellet is compressed by a factor of 1000? What kind of conclusion can you draw from this comparison?

Exercise 4.
The gain factor in ICF

(a) The gain factor has been defined as $Q = \textit{energy produced} \,/\, \textit{energy used}$ in previous exercises. Calculate the maximum theoretical gain factor in inertial fusion when the fuel pellet is heated uniformly close to ignition temperatures ($\sim$ 10 keV).

(b) How does a burn-up ratio of 30% affect the gain factor?

(c) How does a combined laser absorption and hydrodynamic efficiency of 10 % affect the gain factor, considering the burn-up ratio as well?

(d) How does considering further losses due to electric conversion and laser efficiency affect the gain factor? Is the resulting gain factor viable for energy production? For a realistic power plant, you should have about $Q = 100$.

(e) If the answer is no, how can the gain factor be increased?