General information

The exercise sessions will be held as blackboard sessions, where the participants will present their solutions to the group. As such, the problems should be set up and solved before the session. The focus of the exercises lies on analyzing and discussing the task at hand together with the group: thus, a perfect solution is not required to be awarded points. A point will be awarded for each question, and a person will be chosen to present their solution from the pool.

The paper "*The physics issues that determine inertial confinement fusion target gain and driver requirements: A tutorial*" by M.D. Rosen (Physics of Plasmas, 1999) is a great resource for this set of exercises.

Exercise 1.
Inertial confinement fusion concepts

(a) Explain the difference between direct and in-direct drive in inertial confinement fusion.

(b) Explain the difference between fast and central ignition?

(c) What limits the performance of inertial confinement fusion?

(d) What are the main benefits of compressing the fuel?

(e) How does "rocket equation" relate to ICF?

(f) What are the main subsystems of an inertial confinement fusion power plant?

Solution 1.

(a) Direct drive: Lasers are used to heat and compress the fuel capsule directly. Indirect drive: Lasers are fired at a hohlraum to emit X-rays which heat and compress the fuel capsule.

(b) In central ignition, the fusion reaction is ignited by a symmetric implosion at the centre of the fuel. In fast ignition, the fusion reaction is ignited by local heating using a short high-intensity laser pulse, typically not at the centre.

(c) The performance of inertial confinement fusion is limited by: the available laser power and pulse length, the ability to focus the compression symmetrically on the fuel capsule, laser-plasma instabilities (reduced laser intensity), hydrodynamic instabilities (Rayleigh-Taylor, Richtmyer-Meshkov).

(d) Benefits of compressing the fuel: 1) The fusion reaction rate scales with the density squared –> minimum fuel mass/laser energy to reach ignition is reduced by orders of magnitude. 2) The kinetic energy of the compressing capsule is converted into heat, contributing to reaching fusion-relevant temperatures.

(e) The rocket equation is applicable to the outer layers of the fuel capsule: when mass is ejected by ablation, conservation of momentum requires that the fuel capsule must compress inwards. The compression velocity can be estimated using the rocket equation.

(f) The subsystems of an ICF power plant:

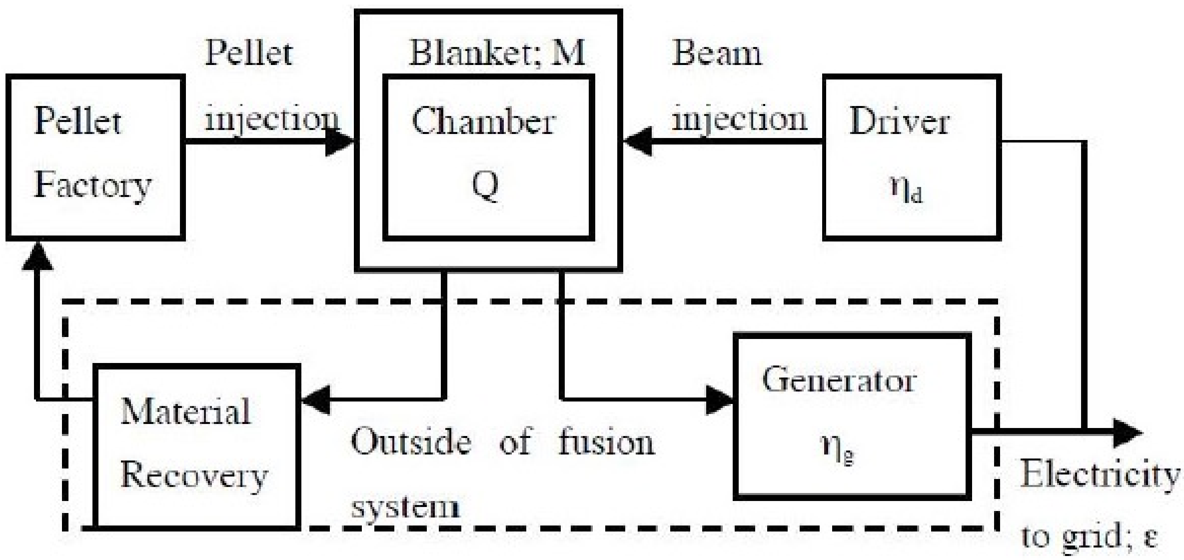

FIG. 10.46. Energy and material flow diagram in a laser fusion power reactor [10.137].

Figure 1: A schematic of an ICF power plant, highlighting the main components

- Driver
  - laser or particle accelerator
- Pellet factory
  - manufactures pellets
  - fill the pellets with D-T fuel
  - send pellets to reactor
  - inject pellets into chamber
- Reaction chamber
  - track the injected pellet (position, direction and velocity - needs to be very accurate!

- target drive beams to the pellet to implode and produce burn (frequency of 10 Hz!)
- thermonuclear emissions captured in the surrounding (blanket) structure, and the energy is converted to heat
- tritium breeding in blanket

- Remainder of plant
  - tritium extracted from the recirculating blanket fluid material and from the reaction chamber exhaust gasses
  - extracted tritium recycled to the pellet factory
  - thermal energy converted into electricity, with a portion being recirculated in the plant

Exercise 2.

The burn-up ratio Derive the equation for the fraction of fuel burned in inertial confinement fusion:

$$f_B = \frac{\rho R}{\rho R + \beta},$$

where R is the radius of the fuel pellet, $\rho = m_{\mathrm{DT}} n_0$ is the mass density before fusion burn, and $\beta = \frac{8 m_{\mathrm{DT}} C_S}{\langle \sigma v \rangle_{\mathrm{DT}}} \approx 6\,\mathrm{g\,cm^{-2}}$ is the burn parameter at 30 keV (Rosen, 1999). Assume a 50-50 D-T fuel mixture and write the rate of change of the tritium density:

$$\frac{\mathrm{d}n_{\mathrm{T}}}{\mathrm{d}t} = -n_{\mathrm{D}} n_{\mathrm{T}} \langle \sigma v \rangle_{\mathrm{DT}}$$

Set the limits of integration from the start of the fusion reactions $t = 0$ to the confinement time, τ_C. For the confinement time, use the mass-averaged value $\tau_C = R/4C_S$.

Solution 2.

Assume a 50-50 fuel mixture, i.e. $n_{\mathrm{D}} = n_{\mathrm{T}} = \frac{1}{2}n$, giving

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -\frac{1}{2} n^2 \langle \sigma v \rangle_{\mathrm{DT}}$$

Separate the variables

$$\frac{\mathrm{d}n}{n^2} = -\frac{1}{2} \langle \sigma v \rangle_{\mathrm{DT}}\,\mathrm{d}t$$

Integrate from n_0 to n and from 0 to τ_C

$$\int_{n_0}^{n} n^{-2}\,\mathrm{d}n = \int_0^{\tau_C} -\frac{1}{2} \langle \sigma v \rangle_{\mathrm{DT}}\,\mathrm{d}t$$

Giving

$$-\left(\frac{1}{n} - \frac{1}{n_0}\right) = -\frac{1}{2} \langle \sigma v \rangle_{\mathrm{DT}} \tau_C$$

Next, we will need the burn fraction, $f_B = 1 - \frac{n}{n_0}$. Substitution gives

$$f_B = \frac{\frac{1}{2}\langle\sigma v\rangle_{\mathrm{DT}}\tau_C}{\frac{1}{2}\langle\sigma v\rangle_{\mathrm{DT}}\tau_C + \frac{1}{n_0}}$$

Finally, using the mass density $\rho = n_0 m_{\mathrm{DT}}$, the mass-averaged confinement time $\tau_C = \frac{R}{4C_s}$ and the definition of the burn parameter β gives, after a number of algebraic steps,

$$f_B = \frac{\rho R}{\rho R + \beta}$$

Exercise 3.
The Lawson criterion in ICF

(a) In the context of ICF, the Lawson criterion is often expressed in terms of the fuel mass density times the fuel pellet radius, ρR. Assuming a burn-up fraction of $1/3$, calculate the required value of ρR, and then calculate the required value of $n\tau_C$. Compare your result to the reference value for MCF - are we still in the ballpark?

(b) The density of solid DT is approximately $0.23\,\mathrm{g\,cm^{-3}}$. Calculate the required radius of a spherical fuel pellet to obtain a burn up of $1/3$, assuming that the fuel is not compressed.

(c) What is the mass of the resulting D-T fuel pellet?

(d) Calculate a simple estimate for the energy required to heat this fuel pellet uniformly from 0 to 30 keV, i.e. calculate the thermal energy change for the fuel pellet. Neglect any inefficiencies in the the laser system, and any other caveats. The lasers of the National Ignition Facility (NIF) have energies of about 4 MJ, how does this compare to the value you calculated?

(e) How do the results of b), c), and d) change, if the fuel pellet is compressed by a factor of 1000? What kind of conclusion can you draw from this comparison?

Solution 3.

(a) Solving ρR from the burn-up relation yields

$$\rho R = \frac{\beta}{\frac{1}{f_B} - 1}$$

A burn-up f_B of $1/3$, with $\beta = 6\,\mathrm{g\,cm^{-2}}$ gives $\rho R = 3\,\mathrm{g\,cm^{-2}}$. To find out the value of $n\tau_C$ this corresponds to, one will need to use $\rho = n_0 m_{\mathrm{DT}}$, $\tau_C = \frac{R}{4C_S}$ and $C_S = \sqrt{2k_B T/m_{\mathrm{DT}}}$. Substitution and rearranging yields

$$n_0\tau_C = \frac{3\,\mathrm{g\,cm^{-2}}}{4\sqrt{2k_B T m_{\mathrm{DT}}}}$$

Or in kilograms and square meters

$$n_0\tau_C = \frac{30\,\mathrm{kg\,m^{-2}}}{4\sqrt{2k_BTm_{\mathrm{DT}}}}$$

Inserting the values $k_BT = 30\,000\,\mathrm{eV} \cdot 1.6 \times 10^{-19}\,\mathrm{J\,eV^{-1}}$ and $m_{\mathrm{DT}} \approx 2.5 \cdot m_{\mathrm{p}} = 2.5 \cdot 1.67 \times 10^{-27}\,\mathrm{kg}$ gives $1.18 \times 10^{21}\,\mathrm{s\,m^{-3}}$. This is approximately a factor of 10 larger than the Lawson criterion value for MCF.

(b) Assuming $\rho R = 3\,\mathrm{g\,cm^{-2}}$ and an initial density of $\rho = 0.23\,\mathrm{g\,cm^{-3}}$ gives $R = 13\,\mathrm{cm}$.

(c) The resulting mass is $M = \rho V = \rho\frac{4}{3}\pi R^3 = 230\,\mathrm{kg\,m^{-3}} \cdot \frac{4}{3}\pi(0.13\,\mathrm{m})^3 \approx 2.11\,\mathrm{kg}$

(d) The number of D/T atoms in the fuel pellet is approximately

$$N = \frac{M}{m_{\mathrm{DT}}} = \frac{2.11\,\mathrm{kg}}{2.5 \cdot 1.67 \times 10^{-27}\,\mathrm{kg}} = 5.05 \times 10^{26}$$

The thermal energy needed to increase the temperature of the fuel pellet from its initial temperature (which, relatively speaking, is close to zero here) to 30 keV is

$$E = 2 \cdot \frac{3}{2}Nk_BT = 3 \cdot 5.05 \times 10^{26} \cdot 30\,000\,\mathrm{eV} \cdot 1.6 \times 10^{-19}\,\mathrm{J\,eV^{-1}} = 7.2 \times 10^{12}\,\mathrm{J},$$

where the factor of 2 accounts for the additional degrees of freedom from the electrons. This is many orders of magnitude more than the lasers at NIF can deliver.

(e) Increasing the density of the pellet by a factor of 1000 reduces the required radius to 0.13 mm, the mass to 2 mg and the required energy to 7.2 MJ, which is the same order of magnitude as the energy delivered by the NIF lasers.

Exercise 4.
The gain factor in ICF

(a) The gain factor has been defined as Q = *energy produced* / *energy used* in previous exercises. Calculate the maximum theoretical gain factor in inertial fusion when the fuel pellet is heated uniformly close to ignition temperatures ($\sim$ 10 keV).

(b) How does a burn-up ratio of 30% affect the gain factor?

(c) How does a combined laser absorption and hydrodynamic efficiency of 10 % affect the gain factor, considering the burn-up ratio as well?

(d) How does considering further losses due to electric conversion and laser efficiency affect the gain factor? Is the resulting gain factor viable for energy production? For a realistic power plant, you should have about $Q = 100$.

(e) If the answer is no, how can the gain factor be increased?

Solution 4.

(a) The energy release in DT fusion is $Q_{\mathrm{DT}} = 17.6\,\mathrm{MeV}$. Heating the two reactants (D,T) and their two electrons to $T = 10\,\mathrm{keV}$ requires

$$E = 4\frac{3}{2}T = 6 \cdot 10\,\mathrm{keV} = 60\,\mathrm{keV}$$

Thus the maximum theoretical gain factor for DT fusion is

$$Q = \frac{17.6\,\mathrm{MeV}}{60\,\mathrm{keV}} \approx 293$$

(b) Assuming the burn-up is 30%, the resulting energy produced, on average, is reduced to 30% of the maximum value, resulting in a gain factor of 88 for DT fusion

(c) Assuming a total efficiency of 10% for the laser absorption and hydrodynamic processes, the resulting gain factor is 8.8.

(d) A realistic driver efficiency is about 10%, and the corresponding electricity conversion efficiency is around 40%, resulting in gain factors below unity. Since realistic power plants require Q>100, uniform ignition is of little use.

(e) The solution is **hot spark ignition**, where only a small part of the fuel is heated to the actual ignition temperatures, and the ignited fuel provides the remaining energy to ignite the pellet.