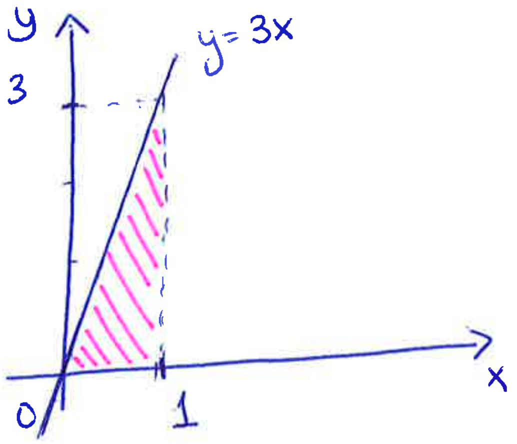
(1)

(a) $\int_0^1 \left(\int_0^{3x} (2x+4y)\,dx \right) dy$ ✗

No, because the variable x from the inside integral can not appear in the limits of integration (and the upper limit is 3x).

(b) $\int_0^1 \left(\int_0^{3x} (2x+4y)\,dy \right) dx$ ✓

Region of integration: $0 \le x \le 1$, $0 \le y \le 3x$



Inside integral:

$$\int_0^{3x} (2x+4y)\,dy = \left[2xy + 2y^2 \right]_0^{3x} =$$

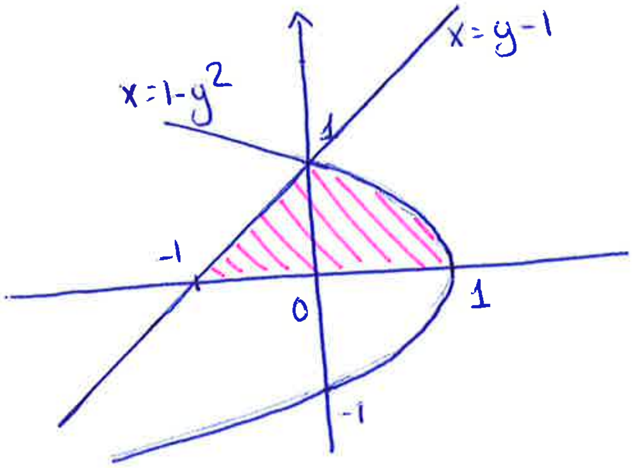
$$= 6x^2 + 18x^2 = 24x^2$$

(c) $\int_0^{3x} \left(\int_0^1 (2x+4y)\,dx \right) dy$ ✗

No, because the variable x from the inside integral can not appear outside the function over which the inside integral is taken.

(d) $\int_0^1 \left(\int_{y-1}^{1-y^2} (2x+4y)\,dx \right) dy$ ✓

Region of integration: $0 \le y \le 1$
$y-1 \le x \le 1-y^2$



Inside integral: $\int_{y-1}^{1-y^2} (2x+4y)\,dx = \left[x^2+4yx\right]_{y-1}^{1-y^2} =$

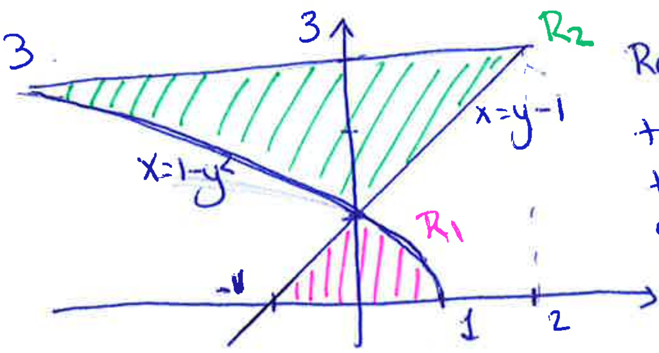
$= (1-y^2)^2 + 4y(1-y^2) - (y-1)^2 - 4y(y-1) =$

$= (1-y^2)^2 - (y-1)^2 + 4y(1-y^2-y+1) =$

$= 1+y^4-2y^2-y^2-1+2y-4y^3-4y^2+8y = y^4-4y^3-7y^2+10y$

(e) $\int_0^3 \left(\int_{y-1}^{1-y^2} (2x+4y)\,dx \right) dy$ ✗

No, because for $1 < y \le 3$ the boundaries in the inside integral do not make sense, since $y-1>0$ but $1-y^2<0$ for $1<y\le 3$.

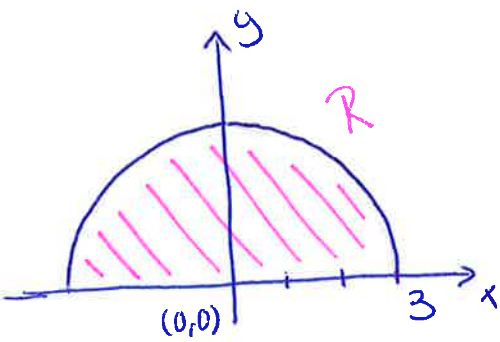


Remark: To make sense, the integral would have to split into an integral over R_1 and an integral over R_2.

(f) $\int_0^{x^3}\left(\int_{y-1}^{1-y^2}(2x+4y)\,dx\right)dy$ X

No, because the variable x from the inside integral can not appear outside the function over which the inside integral is taken.

(2)



Polar coordinates:

$0 \le \theta \le \pi$
$0 \le r \le 3$

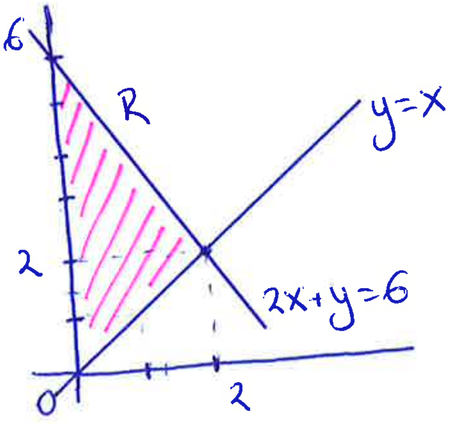
$$\iint_R e^{x^2+y^2}\,dx\,dy = \int_0^{\pi}\left(\int_0^3 e^{r^2} r\,dr\right)d\theta = \int_0^{\pi}\left[\frac{e^{r^2}}{2}\right]_0^3 d\theta =$$

$$= \int_0^{\pi}\frac{1}{2}(e^9-1)\,d\theta = \frac{1}{2}\left[(e^9-1)\theta\right]_0^{\pi} = \frac{\pi}{2}(e^9-1)$$

(3) Center of mass

$R: 0 \le x \le 2$
$x \le y \le 6-2x$

$\rho(x,y) = x^2$



Total mass:

$$M = \iint_R \rho(x,y)\,dx\,dy = \int_0^2\left(\int_x^{6-2x} x^2\,dy\right)dx =$$

$$= \int_0^2 \left[x^2 y\right]_x^{6-2x} dx = \int_0^2 x^2(6-3x)\,dx$$

$$= \int_0^2 (6x^2-3x^3)\,dx = \left[\frac{6x^3}{3} - \frac{3x^4}{4}\right]_0^2 = 2\cdot 2^3 - 3\cdot 2^2 = 4$$

$\bullet\ \bar{x} = M_y / M$

$$M_y = \int_0^2 \left(\int_x^{6-2x} x^3 dy \right) dx = \int_0^2 \left[x^3 y \right]_x^{6-2x} dx = \int_0^2 x^3 (6-3x) dx =$$

$$= \int_0^2 (6x^3 - 3x^4)\, dx = \left[\frac{6x^4}{4} - \frac{3x^5}{5} \right]_0^2 = 3 \cdot 2^3 - \frac{3}{5} \cdot 2^5 = \frac{24}{5}$$

$$\Rightarrow \bar{x} = \frac{24/5}{4} = \frac{6}{5}$$

$\bullet\ \bar{y} = M_x / M$

$$M_x = \int_0^2 \left(\int_x^{6-2x} y x^2 dy \right) dx = \int_0^2 \left[\frac{x^2 y^2}{2} \right]_x^{6-2x} dx =$$

$$= \int_0^2 \frac{x^2}{2} \left((6-2x)^2 - x^2 \right) dx = \int_0^2 \frac{x^2}{2} (36 + 4x^2 - 24x - x^2)\, dx$$

$$= \frac{1}{2} \int_0^2 (3x^4 - 24x^3 + 36x^2)\, dx = \frac{1}{2} \left[\frac{3x^5}{5} - \frac{24x^4}{4} + \frac{36x^3}{3} \right]_0^2 =$$

$$= \frac{1}{2} \left[\frac{3 \cdot 2^5}{5} - \frac{24 \cdot 2^4}{4} + \frac{36 \cdot 2^3}{3} \right] = \frac{48}{5}$$

$$\Rightarrow \bar{y} = \frac{48/5}{4} = \frac{12}{5}$$

The center of mass is $\left(\frac{6}{5}, \frac{12}{5} \right)$