Aalto University
School of Electrical Engineering

Lecture 2: V/Hz-Controlled Induction Motor Drive

ELEC-E8402 Control of Electric Drives and Power Converters

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Learning Outcomes

After this lecture and exercises you will be able to:

- Express the dynamic inverse-Γ model in synchronous coordinates
- Calculate steady-state operating points and draw the corresponding vector diagrams
- Explain the operating principle of V/Hz control

Motor Model

V/Hz Control

Model in Stator Coordinates

- Voltage equations

$$\boldsymbol{u}_\mathrm{s}^\mathrm{s} = R_\mathrm{s}\boldsymbol{i}_\mathrm{s}^\mathrm{s} + \frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{s}^\mathrm{s}}{\mathrm{d}t}$$

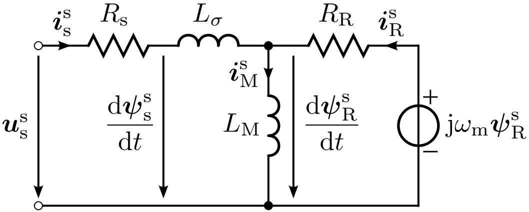
$$\boldsymbol{u}_\mathrm{R}^\mathrm{s} = R_\mathrm{R}\boldsymbol{i}_\mathrm{R}^\mathrm{s} + \frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{R}^\mathrm{s}}{\mathrm{d}t} - \mathrm{j}\omega_\mathrm{m}\boldsymbol{\psi}_\mathrm{R}^\mathrm{s} = 0$$

- Flux linkages

$$\boldsymbol{\psi}_\mathrm{s}^\mathrm{s} = L_\sigma\boldsymbol{i}_\mathrm{s}^\mathrm{s} + \boldsymbol{\psi}_\mathrm{R}^\mathrm{s}$$

$$\boldsymbol{\psi}_\mathrm{R}^\mathrm{s} = L_\mathrm{M}(\boldsymbol{i}_\mathrm{s}^\mathrm{s} + \boldsymbol{i}_\mathrm{R}^\mathrm{s})$$

- Steady state: $\mathrm{d}/\mathrm{d}t = \mathrm{j}\omega_\mathrm{s}$



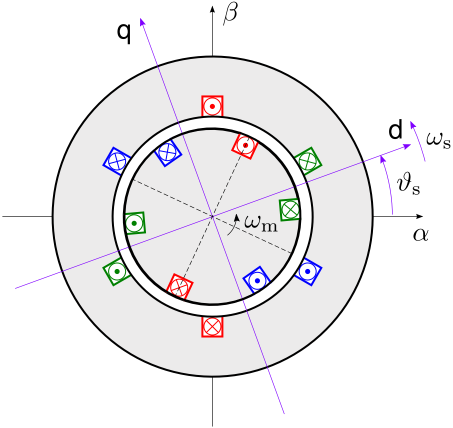
Model in Synchronous Coordinates

- Synchronous (dq) coordinates rotate at the angular speed ω_s
- Coordinate transformation $\boldsymbol{i}_\mathrm{s}^\mathrm{s} = \boldsymbol{i}_\mathrm{s}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{s}}$, where no superscript is used in synchronous coordinates
- Voltage equations become

$$\boldsymbol{u}_\mathrm{s} = R_\mathrm{s}\boldsymbol{i}_\mathrm{s} + \frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{s}}{\mathrm{d}t} + \mathrm{j}\omega_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}$$

$$\boldsymbol{u}_\mathrm{R} = R_\mathrm{R}\boldsymbol{i}_\mathrm{R} + \frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{R}}{\mathrm{d}t} + \mathrm{j}\omega_\mathrm{r}\boldsymbol{\psi}_\mathrm{R} = 0$$

- Angular speed of the coordinate system
  - ω_s with respect to the stator
  - $\omega_\mathrm{r} = \omega_\mathrm{s} - \omega_\mathrm{m}$ with respect to the rotor



Model in Synchronous Coordinates

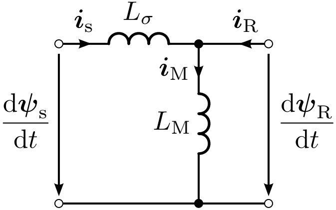
- Voltage equations

$$\boldsymbol{u}_\mathrm{s} = R_\mathrm{s}\boldsymbol{i}_\mathrm{s} + \frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{s}}{\mathrm{d}t} + \mathrm{j}\omega_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}$$
$$\boldsymbol{u}_\mathrm{R} = R_\mathrm{R}\boldsymbol{i}_\mathrm{R} + \frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{R}}{\mathrm{d}t} + \mathrm{j}\omega_\mathrm{r}\boldsymbol{\psi}_\mathrm{R} = 0$$

- Flux linkages

$$\boldsymbol{\psi}_\mathrm{s} = L_\sigma\boldsymbol{i}_\mathrm{s} + \boldsymbol{\psi}_\mathrm{R}$$
$$\boldsymbol{\psi}_\mathrm{R} = L_\mathrm{M}(\boldsymbol{i}_\mathrm{s} + \boldsymbol{i}_\mathrm{R})$$

- Steady state: $\mathrm{d}/\mathrm{d}t = 0$



Power Balance

$$\frac{3}{2}\,\mathrm{Re}\left\{\boldsymbol{u}_\mathrm{s}\boldsymbol{i}_\mathrm{s}^* + \boldsymbol{u}_\mathrm{R}\boldsymbol{i}_\mathrm{R}^*\right\} = \frac{3}{2}R_\mathrm{s}|\boldsymbol{i}_\mathrm{s}|^2 + \frac{3}{2}R_\mathrm{R}|\boldsymbol{i}_\mathrm{R}|^2 + \frac{\mathrm{d}W_\mathrm{f}}{\mathrm{d}t} + \tau_\mathrm{M}\frac{\omega_\mathrm{m}}{n_\mathrm{p}}$$

- Electromagnetic torque

$$\boxed{\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}^*\right\}}$$

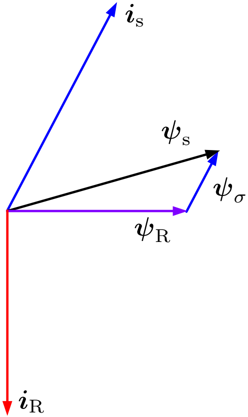
- Rate of change of the magnetic field energy

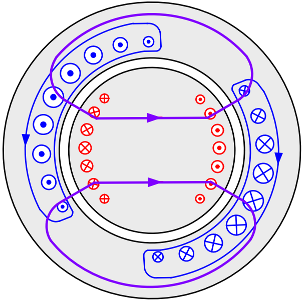
$$\frac{\mathrm{d}W_\mathrm{f}}{\mathrm{d}t} = \frac{3}{2}\,\mathrm{Re}\left\{\boldsymbol{i}_\mathrm{s}^*\frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{s}}{\mathrm{d}t} + \boldsymbol{i}_\mathrm{R}^*\frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{R}}{\mathrm{d}t}\right\} = \frac{3}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}L_\sigma|\boldsymbol{i}_\mathrm{s}|^2 + \frac{1}{2}L_\mathrm{M}|\boldsymbol{i}_\mathrm{M}|^2\right)$$

is zero in the steady state

Vector Diagram: Currents and Flux Linkages

- Airgap and leakage flux paths are sketched
- All vectors are constant in synchronous coordinates in the steady state (but the rotor slips at $-\omega_\mathrm{r}$)





Steady-State Torque

- Torque in the steady state

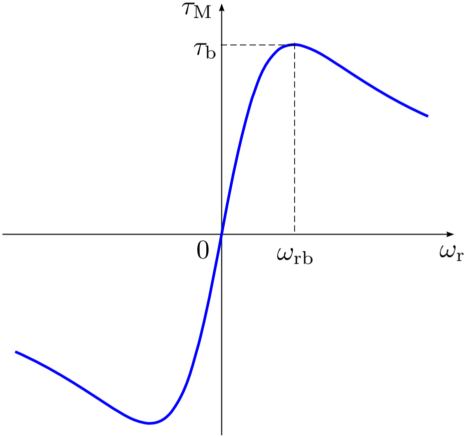
$$\tau_\mathrm{M} = \frac{2\tau_\mathrm{b}}{\omega_\mathrm{r}/\omega_\mathrm{rb} + \omega_\mathrm{rb}/\omega_\mathrm{r}}$$

- Breakdown torque

$$\tau_\mathrm{b} = \frac{3n_\mathrm{p}}{2}\frac{L_\mathrm{M}}{L_\mathrm{M}+L_\sigma}\frac{\psi_\mathrm{s}^2}{2L_\sigma}$$

- Breakdown slip

$$\omega_\mathrm{rb} = \frac{R_\mathrm{R}}{\sigma L_\mathrm{M}} \quad \text{where} \quad \sigma = \frac{L_\sigma}{L_\mathrm{M}+L_\sigma}$$



See the derivation in Exercise 1.

Motor Model

V/Hz Control

Stator Voltage vs. Stator Frequency

- Steady-state stator voltage

$$\boldsymbol{u}_\mathrm{s} = R_\mathrm{s}\boldsymbol{i}_\mathrm{s} + \mathrm{j}\omega_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}$$

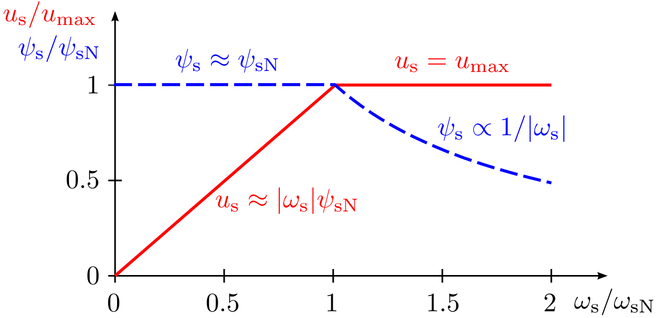
- Approximate magnitude

$$u_\mathrm{s} = |\omega_\mathrm{s}|\psi_\mathrm{s}$$

  where $u_\mathrm{s} = |\boldsymbol{u}_\mathrm{s}|$ and $\psi_\mathrm{s} = |\boldsymbol{\psi}_\mathrm{s}|$

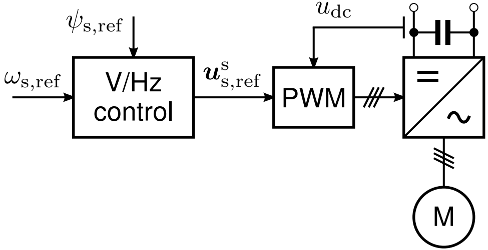
- Maximum voltage is limited

$$u_\mathrm{s} < u_\mathrm{max}$$



Volts-per-Hertz Control (aka Scalar Control)

- Based on the steady-state equations
- Supply frequency $\omega_{\mathrm{s,ref}}$ corresponds to the desired rotor speed
- Some speed error due to the slip (can be partly compensated for)
- Slow, oscillating, or even unstable dynamics[1]
- Torque cannot be controlled
- Current cannot be limited
- For simple applications



$u_{\mathrm{s,ref}} = \omega_{\mathrm{s,ref}}\psi_{\mathrm{s,ref}}$ ($+R_{\mathrm{s}}i_{\mathrm{s}}$ compensation)

$$\vartheta_{\mathrm{s}} = \int \omega_{\mathrm{s,ref}}\mathrm{d}t$$

$$\boldsymbol{u}^{\mathrm{s}}_{\mathrm{s,ref}} = u_{\mathrm{s,ref}}\mathrm{e}^{\mathrm{j}\vartheta_{\mathrm{s}}}$$

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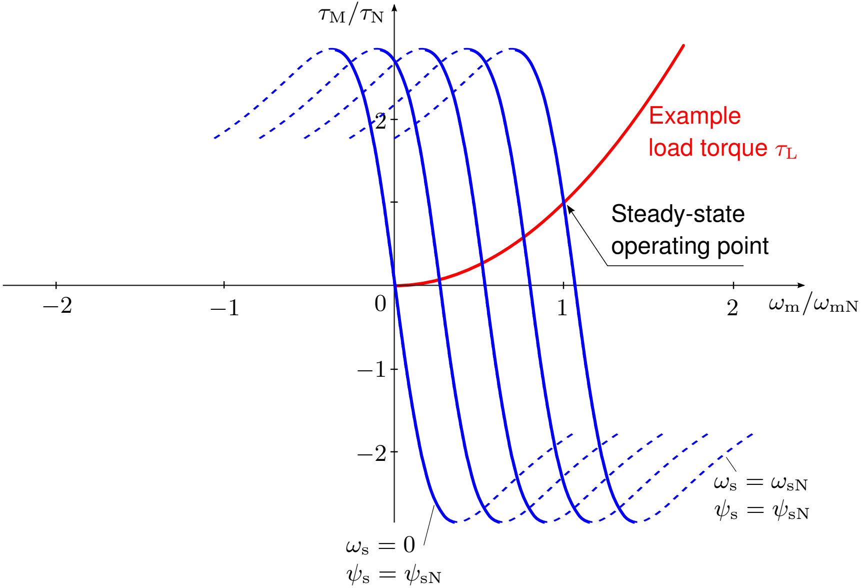
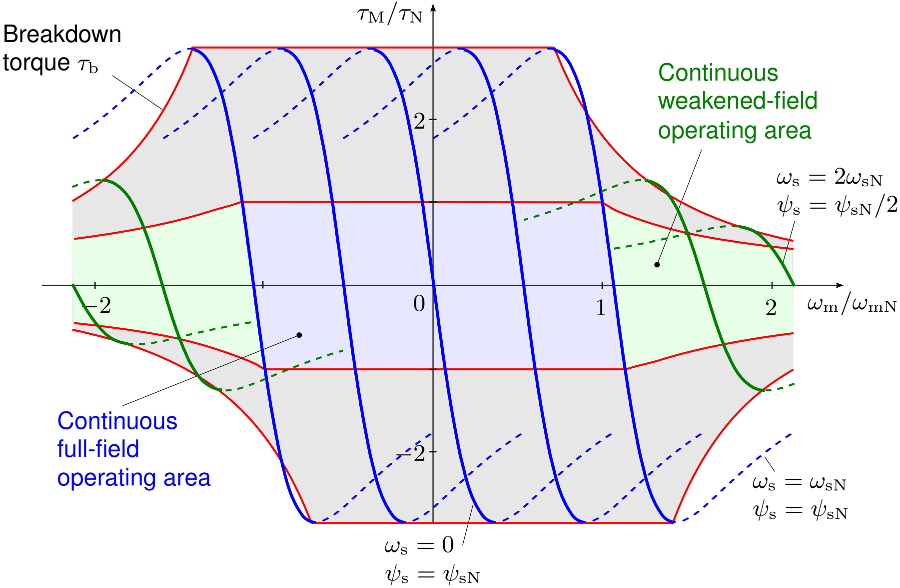
[1] Hinkkanen, Tiitinen, Mölsä, *et al.*, "On the stability of volts-per-hertz control for induction motors," *IEEE J. Emerg. Sel. Topics Power Electron.*, 2021.


τ_M/τ_N
ω_m/ω_{mN}
-2
-1
0
1
2
2
-1
-2
Example load torque τ_L
Steady-state operating point
$\omega_s = \omega_{sN}$
$\psi_s = \psi_{sN}$
$\omega_s = 0$
$\psi_s = \psi_{sN}$


τ_M/τ_N
Breakdown torque τ_b
Continuous weakened-field operating area
$\omega_s = 2\omega_{sN}$
$\psi_s = \psi_{sN}/2$
2
-2
0
1
2
ω_m/ω_{mN}
Continuous full-field operating area
-2
$\omega_s = \omega_{sN}$
$\psi_s = \psi_{sN}$
$\omega_s = 0$
$\psi_s = \psi_{sN}$