Aalto University
School of Electrical
Engineering

Lecture 3: Vector-Controlled Induction Motor Drive

ELEC-E8402 Control of Electric Drives and Power Converters

Marko Hinkkanen

Spring 2023

Learning Outcomes

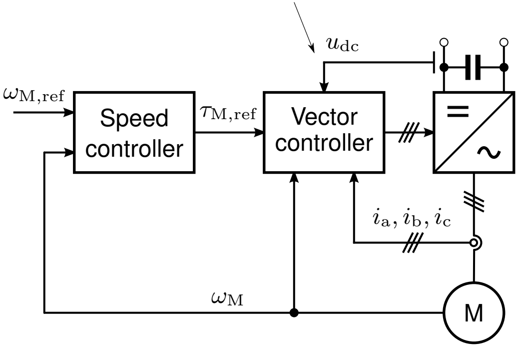
After this lecture and exercises you will be able to:

- Explain the principle of rotor-flux orientation
- Derive the rotor-flux orientation equations (torque, flux dynamics, slip relation) using the inverse-Γ model
- Draw block diagrams for the most typical control schemes and explain them
- Derive the current model and explain its properties

Vector Control Methods

- Based on the dynamic motor model
- Rotor-flux-oriented vector control, direct torque control (DTC)
- Torque can be controlled
- High accuracy and fast dynamics
- Speed measurement can be replaced with speed estimation in most applications

DC-link voltage is typically measured, but this measurement will be omitted in the following block diagrams (or constant u_{dc} is assumed)



State-Space Representation

Principle of Rotor-Flux Orientation

Flux Estimation With the Current Model

Review: Model in Synchronous Coordinates

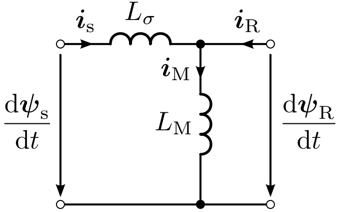
- Voltage equations

$$\boldsymbol{u}_\mathrm{s} = R_\mathrm{s}\boldsymbol{i}_\mathrm{s} + \frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{s}}{\mathrm{d}t} + \mathrm{j}\omega_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}$$
$$\boldsymbol{u}_\mathrm{R} = R_\mathrm{R}\boldsymbol{i}_\mathrm{R} + \frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{R}}{\mathrm{d}t} + \mathrm{j}\omega_\mathrm{r}\boldsymbol{\psi}_\mathrm{R} = 0$$

- Flux linkages

$$\boldsymbol{\psi}_\mathrm{s} = L_\sigma\boldsymbol{i}_\mathrm{s} + \boldsymbol{\psi}_\mathrm{R}$$
$$\boldsymbol{\psi}_\mathrm{R} = L_\mathrm{M}(\boldsymbol{i}_\mathrm{s} + \boldsymbol{i}_\mathrm{R})$$

- Steady state: $\mathrm{d}/\mathrm{d}t = 0$



State-Space Representation

- Stator current $\boldsymbol{i}_\mathrm{s}$ and rotor flux $\boldsymbol{\psi}_\mathrm{R}$ are selected as state variables
- Derivation: rotor current $\boldsymbol{i}_\mathrm{R}$ and stator flux $\boldsymbol{\psi}_\mathrm{s}$ are eliminated from the voltage equations by means of the flux equations

$$L_\sigma \frac{\mathrm{d}\boldsymbol{i}_\mathrm{s}}{\mathrm{d}t} = \boldsymbol{u}_\mathrm{s} - (R_\mathrm{s} + R_\mathrm{R} + \mathrm{j}\omega_\mathrm{s} L_\sigma)\boldsymbol{i}_\mathrm{s} + \left(\frac{R_\mathrm{R}}{L_\mathrm{M}} - \mathrm{j}\omega_\mathrm{m}\right)\boldsymbol{\psi}_\mathrm{R}$$

$$\frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{R}}{\mathrm{d}t} = R_\mathrm{R}\boldsymbol{i}_\mathrm{s} - \left(\frac{R_\mathrm{R}}{L_\mathrm{M}} + \mathrm{j}\omega_\mathrm{r}\right)\boldsymbol{\psi}_\mathrm{R}$$

- Dynamics of the stator current are governed by current control
- Dynamics of the rotor flux are taken into account by rotor-flux orientation

---

Study the derivation of these equations (see the compendium)

Rotor-Flux Dynamics

- Fast closed-loop stator-current controller is used
- Stator current is the input from the point of view of the rotor-flux dynamics
- Rotor equations in synchronous coordinates

$$\frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{R}}{\mathrm{d}t} = -R_\mathrm{R}\boldsymbol{i}_\mathrm{R} - \mathrm{j}\underbrace{(\omega_\mathrm{s} - \omega_\mathrm{m})}_{\omega_\mathrm{r}}\boldsymbol{\psi}_\mathrm{R}$$

$$\boldsymbol{\psi}_\mathrm{R} = L_\mathrm{M}(\boldsymbol{i}_\mathrm{s} + \boldsymbol{i}_\mathrm{R}) \quad \Rightarrow \quad \boldsymbol{i}_\mathrm{R} = \boldsymbol{\psi}_\mathrm{R}/L_\mathrm{M} - \boldsymbol{i}_\mathrm{s}$$

- Rotor current can be eliminated

$$\frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{R}}{\mathrm{d}t} = -\left(\frac{R_\mathrm{R}}{L_\mathrm{M}} + \mathrm{j}\omega_\mathrm{r}\right)\boldsymbol{\psi}_\mathrm{R} + R_\mathrm{R}\boldsymbol{i}_\mathrm{s}$$

Rotor-Flux Orientation

- d-axis of coordinate system is fixed to the rotor flux

$$\boldsymbol{\psi}_{\mathrm{R}} = \psi_{\mathrm{Rd}} + \mathrm{j}\psi_{\mathrm{Rq}} = \psi_{\mathrm{R}} + \mathrm{j}\cdot 0 \qquad \boldsymbol{i}_{\mathrm{s}} = i_{\mathrm{d}} + \mathrm{j}i_{\mathrm{q}}$$

- Real and imaginary parts of the rotor-flux dynamics

$$\frac{\mathrm{d}\psi_{\mathrm{R}}}{\mathrm{d}t} = -\frac{R_{\mathrm{R}}}{L_{\mathrm{M}}}\psi_{\mathrm{R}} + R_{\mathrm{R}} i_{\mathrm{d}} \qquad \text{(in the steady state } \psi_{\mathrm{R}} = L_{\mathrm{M}} i_{\mathrm{d}}\text{)}$$

$$0 = -\omega_{\mathrm{r}}\psi_{\mathrm{R}} + R_{\mathrm{R}} i_{\mathrm{q}}$$

- Rotor-flux magnitude ψ_{R} follows i_{d} slowly,

$$\psi_{\mathrm{R}}(s) = \frac{L_{\mathrm{M}}}{1 + T_{\mathrm{r}} s} i_{\mathrm{d}}(s) \qquad \text{(in the Laplace domain)}$$

due to the rotor time constant $T_{\mathrm{r}} = L_{\mathrm{M}}/R_{\mathrm{R}}$ (typically 0.1...1.5 s)

Rotor-Flux Orientation

- d-axis of coordinate system is fixed to the rotor flux

$$\boldsymbol{\psi}_\mathrm{R} = \psi_\mathrm{R} + \mathrm{j} \cdot 0 \qquad\qquad \boldsymbol{i}_\mathrm{s} = i_\mathrm{d} + \mathrm{j} i_\mathrm{q}$$

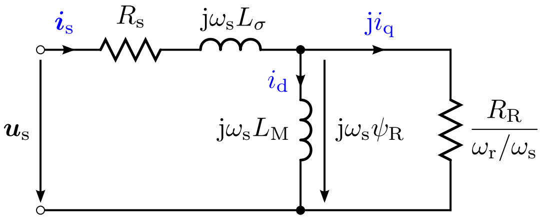
- Electromagnetic torque

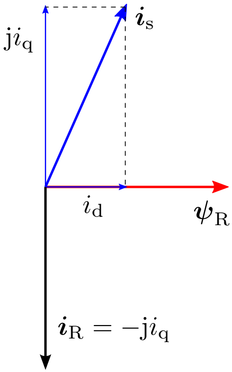
$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2} \,\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{s}\boldsymbol{\psi}_\mathrm{R}^*\right\} = \frac{3n_\mathrm{p}}{2}\psi_\mathrm{R} i_\mathrm{q}$$

- If ψ_R is constant, the torque can be controlled using i_q (without delays)

The coordinate system could be fixed to the stator flux $\boldsymbol{\psi}_\mathrm{s}$ instead of the rotor flux. This stator-flux orientation would simplify the field weakening, but other parts of the control system would become more complicated.

Steady-State Equivalent Circuit in Rotor-Flux Coordinates



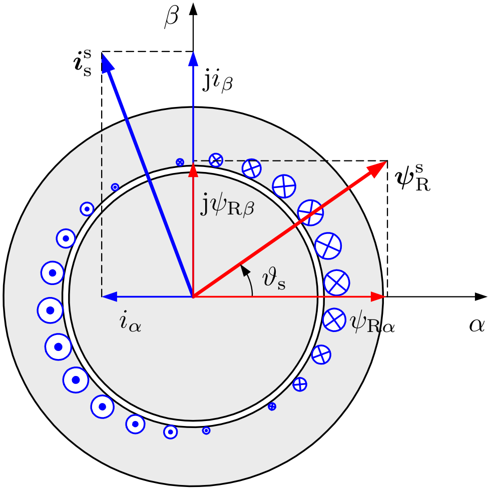


Stator Coordinates ($\alpha\beta$)

- Vectors are rotating (in the steady state $\vartheta_s = \omega_s t$)
- Controlling the torque

$$\tau_M = \frac{3n_p}{2} \operatorname{Im}\{\boldsymbol{i}_s^s (\boldsymbol{\psi}_R^s)^*\} = \frac{3n_p}{2}(i_\beta \psi_{R\alpha} - i_\alpha \psi_{R\beta})$$

would be difficult

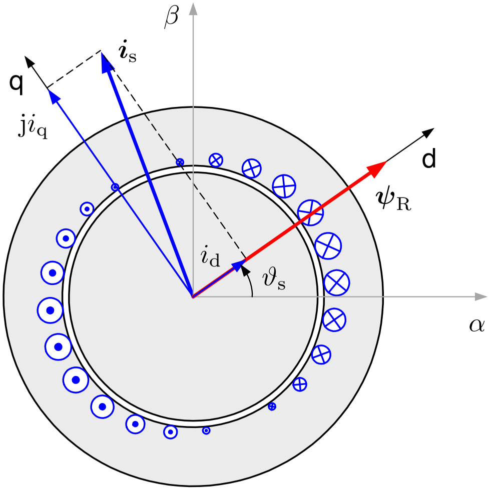


Rotor-Flux Coordinates (dq)

- Variables are constant in the steady state
- Torque

$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{s}\boldsymbol{\psi}_\mathrm{R}^*\right\} = \frac{3n_\mathrm{p}}{2}\psi_\mathrm{R} i_\mathrm{q}$$

easily controllable via i_q

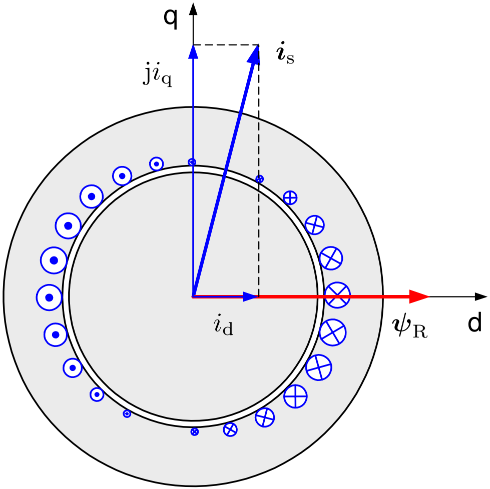


Rotor-Flux Coordinates (dq)

- Variables are constant in the steady state
- Torque

$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{s}\boldsymbol{\psi}_\mathrm{R}^*\right\} = \frac{3n_\mathrm{p}}{2}\psi_\mathrm{R} i_\mathrm{q}$$

easily controllable via i_q

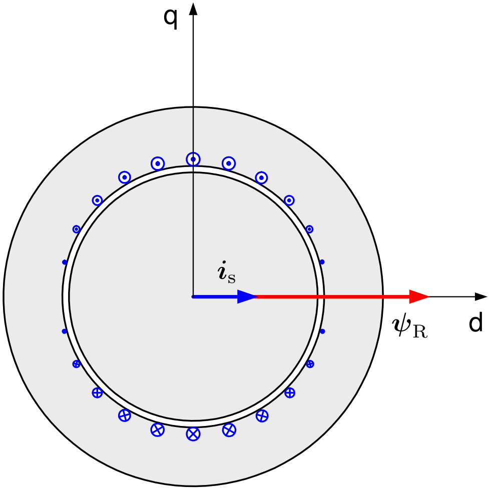


Rotor-Flux Coordinates (dq)

- Variables are constant in the steady state
- Torque

$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{s}\boldsymbol{\psi}_\mathrm{R}^*\right\} = \frac{3n_\mathrm{p}}{2}\psi_\mathrm{R} i_\mathrm{q}$$

easily controllable via i_q

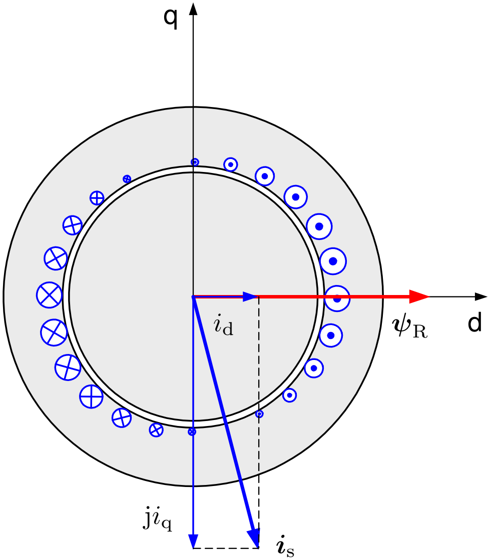


Rotor-Flux Coordinates (dq)

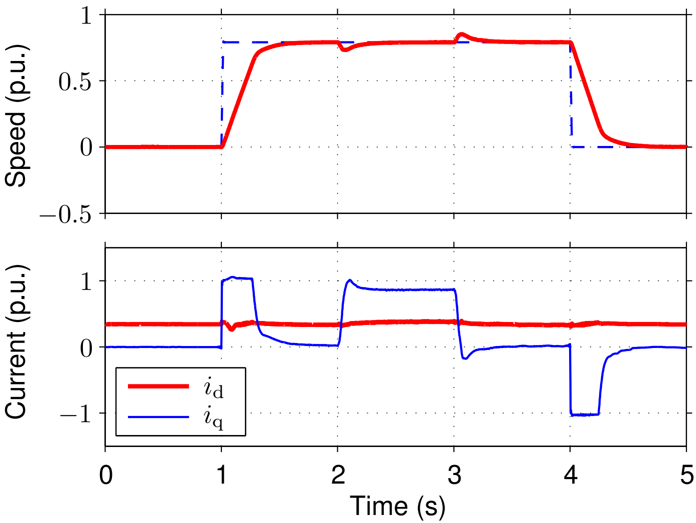
- Variables are constant in the steady state
- Torque

$$\tau_{\mathrm{M}} = \frac{3n_{\mathrm{p}}}{2}\,\mathrm{Im}\left\{\boldsymbol{i}_{\mathrm{s}}\boldsymbol{\psi}_{\mathrm{R}}^{*}\right\} = \frac{3n_{\mathrm{p}}}{2}\psi_{\mathrm{R}} i_{\mathrm{q}}$$

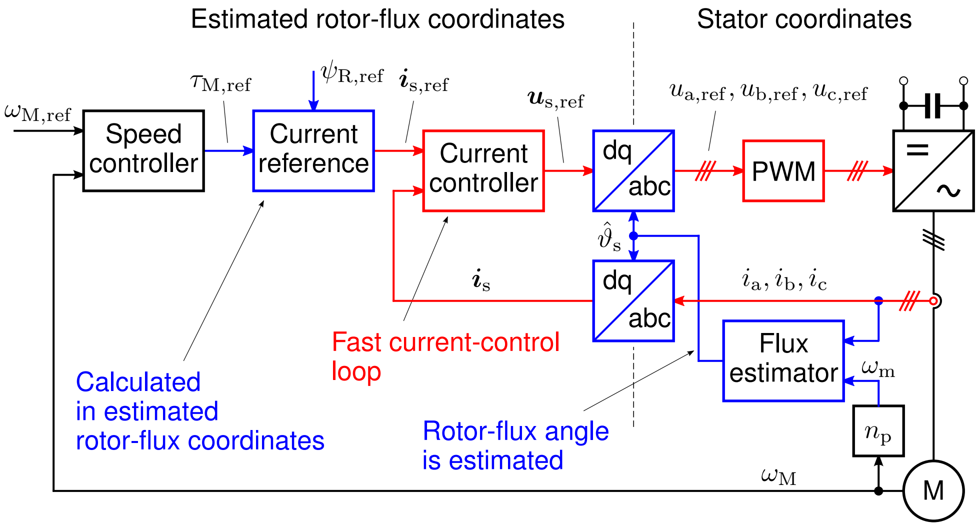
easily controllable via i_{q}



Example Measured Waveforms: 45-kW Induction Motor Drive



Rotor-Flux-Oriented Vector Control



Space-Vector and Coordinate Transformations

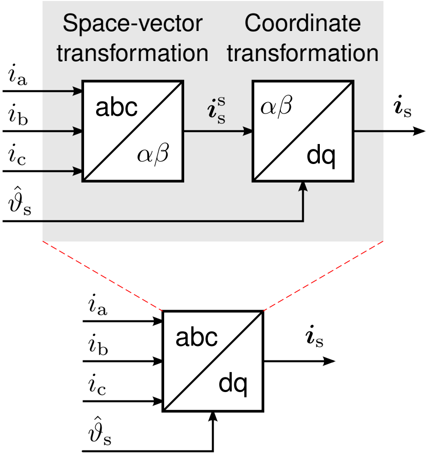
- Space-vector transformation (abc/$\alpha\beta$)

$$\boldsymbol{i}_\mathrm{s}^\mathrm{s} = \frac{2}{3}\left(i_\mathrm{a} + i_\mathrm{b}\mathrm{e}^{\mathrm{j}2\pi/3} + i_\mathrm{c}\mathrm{e}^{\mathrm{j}4\pi/3}\right)$$

- Transformation to rotor flux coordinates ($\alpha\beta$/dq)

$$\boldsymbol{i}_\mathrm{s} = \boldsymbol{i}_\mathrm{s}^\mathrm{s}\mathrm{e}^{-\mathrm{j}\hat{\vartheta}_\mathrm{s}}$$

- Combination of these two transformations is often referred to as an abc/dq transformation
- Similarly, the inverse transformation is referred to as a dq/abc transformation



Current References

1. Flux-producing current reference

$$i_{\mathrm{d,ref}} = \frac{\psi_{\mathrm{R,ref}}}{\hat{L}_{\mathrm{M}}}$$ (where the hat refers to estimates)

- Integral term based on $u_{\max} - |\boldsymbol{u}_{\mathrm{s,ref}}|$ can be used for field weakening
- If fast torque dynamics are not required, the flux level can be optimized according to the load[1]

2. Torque-producing current reference

$$i_{\mathrm{q,ref}} = \frac{2\tau_{\mathrm{M,ref}}}{3n_{\mathrm{p}}\psi_{\mathrm{R,ref}}}$$

- Flux reference $\psi_{\mathrm{R,ref}}$ is often replaced with the estimate $\hat{\psi}_{\mathrm{R}}$

[1] Qu, Ranta, Hinkkanen, *et al.*, "Loss-minimizing flux level control of induction motor drives," *IEEE Trans. Ind. Appl.*, 2012.

State-Space Representation

Principle of Rotor-Flux Orientation

Flux Estimation With the Current Model

Current-Model Flux Estimator in Stator Coordinates

- Current model is based on the rotor voltage equation

$$\frac{\mathrm{d}\hat{\boldsymbol{\psi}}_{\mathrm{R}}^{\mathrm{s}}}{\mathrm{d}t} = -\left(\frac{\hat{R}_{\mathrm{R}}}{\hat{L}_{\mathrm{M}}} - \mathrm{j}\omega_{\mathrm{m}}\right)\hat{\boldsymbol{\psi}}_{\mathrm{R}}^{\mathrm{s}} + \hat{R}_{\mathrm{R}}\boldsymbol{i}_{\mathrm{s}}^{\mathrm{s}}$$

- Corresponding forward Euler approximation

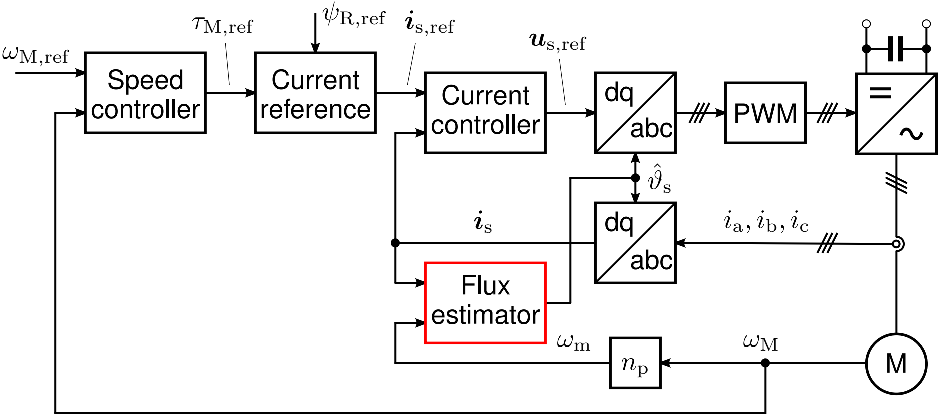
$$\hat{\boldsymbol{\psi}}_{\mathrm{R}}^{\mathrm{s}}(k+1) = \hat{\boldsymbol{\psi}}_{\mathrm{R}}^{\mathrm{s}}(k) + T_{\mathrm{s}}\left\{-\left[\frac{\hat{R}_{\mathrm{R}}}{\hat{L}_{\mathrm{M}}} - \mathrm{j}\omega_{\mathrm{m}}(k)\right]\hat{\boldsymbol{\psi}}_{\mathrm{R}}^{\mathrm{s}}(k) + \hat{R}_{\mathrm{R}}\boldsymbol{i}_{\mathrm{s}}^{\mathrm{s}}(k)\right\}$$

  where T_{s} is the sampling period and k is the discrete-time index
- At each time step, the angle of the flux estimate $\hat{\boldsymbol{\psi}}_{\mathrm{R}}^{\mathrm{s}} = \hat{\psi}_{\mathrm{R}\alpha} + \mathrm{j}\hat{\psi}_{\mathrm{R}\beta}$ is

$$\hat{\vartheta}_{\mathrm{s}} = \mathrm{atan2}\left(\hat{\psi}_{\mathrm{R}\beta}, \hat{\psi}_{\mathrm{R}\alpha}\right)$$

In practice, the forward Euler approximation should not be used in stator coordinates due to its poor accuracy and limited stability

Current Model in Estimated Rotor Flux Coordinates



- Signals fed to the flux estimator are DC in the steady state
- Discrete-time implementation becomes easier

Current-Model Flux Estimator in Estimated Flux Coordinates

$$\frac{\mathrm{d}\hat{\boldsymbol{\psi}}_\mathrm{R}}{\mathrm{d}t} = -\left(\frac{\hat{R}_\mathrm{R}}{\hat{L}_\mathrm{M}} + \mathrm{j}\hat{\omega}_\mathrm{r}\right)\hat{\boldsymbol{\psi}}_\mathrm{R} + \hat{R}_\mathrm{R}\boldsymbol{i}_\mathrm{s} \qquad \hat{\boldsymbol{\psi}}_\mathrm{R} = \hat{\psi}_\mathrm{R} + \mathrm{j}\cdot 0$$

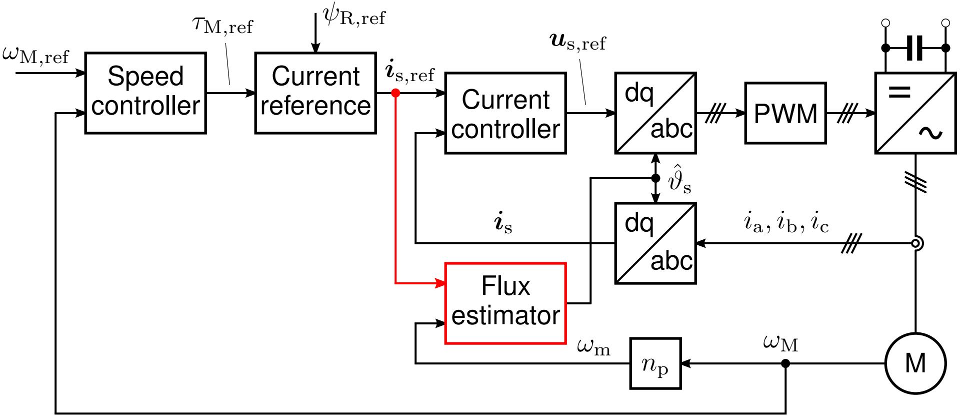
- Real and imaginary parts in estimated flux coordinates

$$\frac{\mathrm{d}\hat{\psi}_\mathrm{R}}{\mathrm{d}t} = -\frac{\hat{R}_\mathrm{R}}{\hat{L}_\mathrm{M}}\hat{\psi}_\mathrm{R} + \hat{R}_\mathrm{R} i_\mathrm{d} \qquad \hat{\omega}_\mathrm{r} = \frac{\hat{R}_\mathrm{R} i_\mathrm{q}}{\hat{\psi}_\mathrm{R}}$$

- Flux-angle estimation

$$\hat{\vartheta}_\mathrm{s} = \int \hat{\omega}_\mathrm{s}\mathrm{d}t = \int (\omega_\mathrm{m} + \hat{\omega}_\mathrm{r})\mathrm{d}t$$

Indirect Field Orientation (IFO)



- Current reference is used as an input of the flux estimator
- Flux estimator is also simplified (see the following slide)

IFO

- Flux-magnitude dynamics are omitted in the slip relation

$$\hat{\omega}_{\mathrm{r}} = \frac{R_{\mathrm{R}} i_{\mathrm{q,ref}}}{\psi_{\mathrm{R,ref}}}$$

- Flux-angle estimation

$$\hat{\vartheta}_{\mathrm{s}} = \int (\omega_{\mathrm{m}} + \hat{\omega}_{\mathrm{r}}) \mathrm{d}t$$

- Poor performance if the flux reference $\psi_{\mathrm{R,ref}}$ is not constant or if the current controller does not work as intended

Properties of the Current Model and IFO

Disadvantages:

- Rotor speed measurement is needed
- Converges slowly (with the rotor time constant), which can be a problem if the flux reference $\psi_{\mathrm{R,ref}}$ is varied
- Inaccurate model parameters $\hat{R}_{\mathrm{R}}$ and $\hat{L}_{\mathrm{M}}$ cause errors in field orientation $\Rightarrow$ degraded control performance

Advantages:

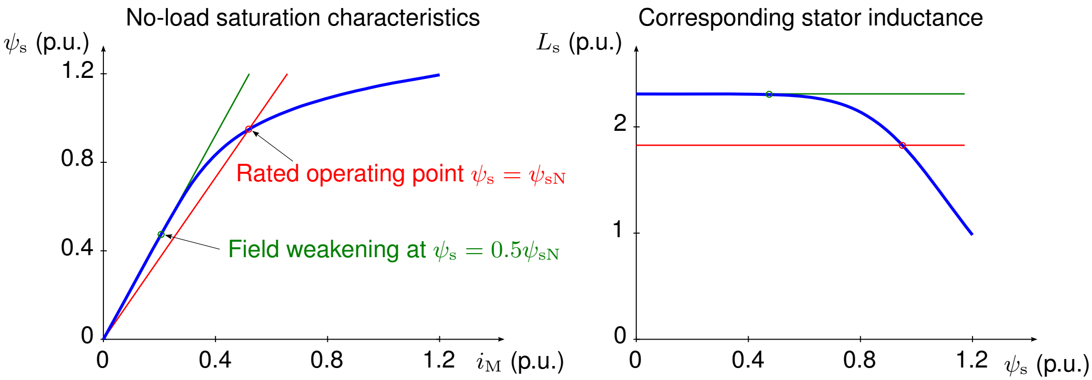
- Simplicity
- Robustness

Reasons for Parameter Detuning: Actual Motor Parameters Vary

- Inductances depend on the magnetic state[2]
  - Stator inductance increases as the flux decreases in the field-weakening region
  - Torque may also affect the inductances
- Resistances depend on
  - Temperature (about 0.4%/K)
  - Frequency due to the skin effect
    (especially the resistances of the rotor bars)
- Some phenomena are omitted in the model but exist in the actual machine (e.g. core losses, deep-bar effect)
- Identification of the motor parameters is never perfect

[2] Mölsä, Saarakkala, Hinkkanen, *et al.*, "A dynamic model for saturated induction machines with closed rotor slots and deep bars," *IEEE Trans. Energy Convers.*, 2020.

Magnetic Saturation: 2.2-kW Motor as an Example



- Stator inductance $L_s = L_\sigma + L_M$ depends on the stator-flux magnitude ψ_s
- Effect should be taken into account in control, if field weakening is used

Summary: Rotor-Flux Orientation

- Decoupled control of the flux and the torque, as in the DC machines
- d-axis of the coordinate system is fixed to the rotor flux vector (or its estimate in practice)
- Rotor-flux magnitude is controlled using the d-component of the stator current
- Torque is controlled using the q-component of the current

- Sensitivity to the parameter errors can be reduced by using more advanced flux observers
- Similar control structure can also be used in sensorless methods (with a suitable flux observer)