Aalto University
School of Electrical
Engineering

Lecture 4: Pulse-Width Modulation and Current Control

ELEC-E8402 Control of Electric Drives and Power Converters

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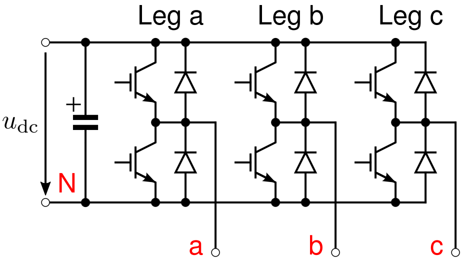
Learning Outcomes

After this lecture and exercises you will be able to:

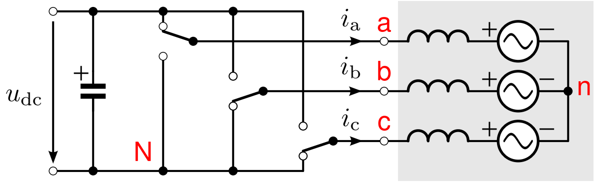
- Explain the difference between the standard suboscillation PWM method and the symmetrical suboscillation PWM method
- Explain the principle of three-phase synchronous-frame current control
- Understand operation of the current controller in Assignment 1

Current control is presented here for induction motors, but it can be almost directly applied to other AC machines and for grid converters (equipped with L filter)

3-Phase Inverter



Space Vector of the Converter Output Voltages



- Zero-sequence voltage does not affect the phase currents
- Reference potential of the phase voltages can be freely chosen

$$\boldsymbol{u}_\mathrm{s}^\mathrm{s} = \frac{2}{3}\left(u_\mathrm{an} + u_\mathrm{bn}\mathrm{e}^{\mathrm{j}2\pi/3} + u_\mathrm{cn}\mathrm{e}^{\mathrm{j}4\pi/3}\right)$$ Neutral n as a reference

$$= \frac{2}{3}\left(u_\mathrm{aN} + u_\mathrm{bN}\mathrm{e}^{\mathrm{j}2\pi/3} + u_\mathrm{cN}\mathrm{e}^{\mathrm{j}4\pi/3}\right)$$ Negative DC bus N as a reference

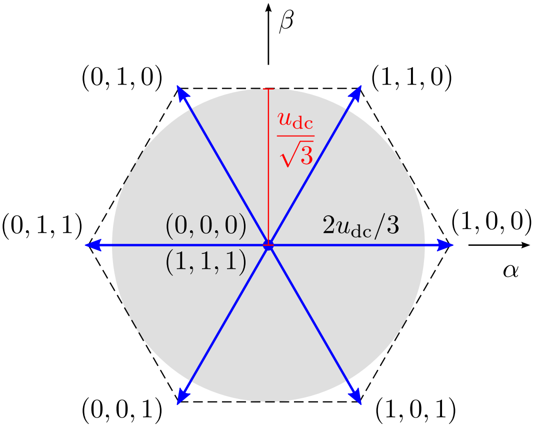
- Converter output voltage vector

$$\boldsymbol{u}_\mathrm{s}^\mathrm{s} = \frac{2}{3}\left(u_\mathrm{aN} + u_\mathrm{bN}\mathrm{e}^{\mathrm{j}2\pi/3} + u_\mathrm{cN}\mathrm{e}^{\mathrm{j}4\pi/3}\right) = \frac{2}{3}\left(q_\mathrm{a} + q_\mathrm{b}\mathrm{e}^{\mathrm{j}2\pi/3} + q_\mathrm{c}\mathrm{e}^{\mathrm{j}4\pi/3}\right) u_\mathrm{dc}$$

  where q_abc are the switching states (either 0 or 1)

- Vector $(1,0,0)$ as an example

$$\boldsymbol{u}_\mathrm{s}^\mathrm{s} = \frac{2u_\mathrm{dc}}{3}$$



Switching-Cycle Averaged Voltage

- Using PWM, any voltage vector inside the voltage hexagon can be produced in average over the switching period

$$\overline{\boldsymbol{u}}_\mathrm{s}^\mathrm{s} = \frac{2}{3}\left(d_\mathrm{a} + d_\mathrm{b}\mathrm{e}^{\mathrm{j}2\pi/3} + d_\mathrm{c}\mathrm{e}^{\mathrm{j}4\pi/3}\right) u_\mathrm{dc}$$

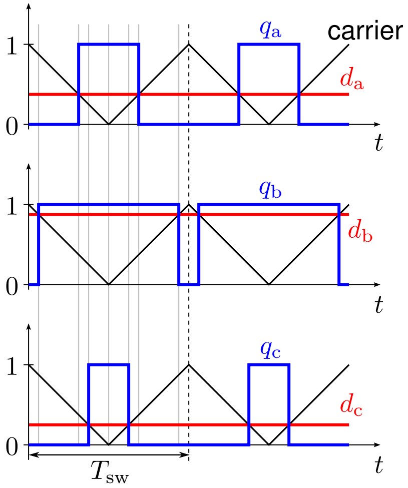
  where d_abc are the duty ratios (between 0. . . 1)
- Maximum magnitude of the voltage vector is $u_\mathrm{max} = u_\mathrm{dc}/\sqrt{3}$ in linear modulation (the circle inside the hexagon)
- PWM can be implemented, e.g., using the carrier comparison
- Mainly switching-cycle averaged quantities will be needed in this course (overlining will be omitted for simplicity)

Suboscillation Method

- Duty ratios for carrier comparison

$$d_\mathrm{a} = \frac{1}{2} + \frac{u_\mathrm{a,ref}}{u_\mathrm{dc}} \quad d_\mathrm{b} = \frac{1}{2} + \frac{u_\mathrm{b,ref}}{u_\mathrm{dc}} \quad d_\mathrm{c} = \frac{1}{2} + \frac{u_\mathrm{c,ref}}{u_\mathrm{dc}}$$

- Voltage vectors during T_sw in the example: $(0,0,0) \rightarrow (0,1,0) \rightarrow (1,1,0) \rightarrow (1,1,1) \rightarrow (1,1,1) \rightarrow (1,1,0) \rightarrow (0,1,0) \rightarrow (0,0,0)$
- Problem: only 87% of the maximum available voltage can be used!
- Proper zero-sequence component should be added to utilize all available voltage



Symmetrical Suboscillation Method

- Zero sequence

$$u_0 = \frac{\min(u_{\mathrm{a,ref}}, u_{\mathrm{b,ref}}, u_{\mathrm{c,ref}}) + \max(u_{\mathrm{a,ref}}, u_{\mathrm{b,ref}}, u_{\mathrm{c,ref}})}{2}$$

- Modified voltage references

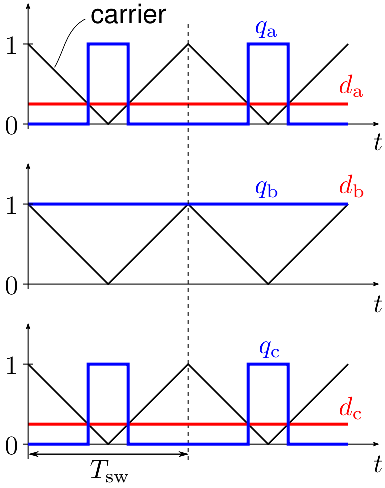
$$u'_{\mathrm{a,ref}} = u_{\mathrm{a,ref}} - u_0 \qquad u'_{\mathrm{b,ref}} = u_{\mathrm{b,ref}} - u_0 \qquad u'_{\mathrm{c,ref}} = u_{\mathrm{c,ref}} - u_0$$

- Duty ratios for carrier comparison

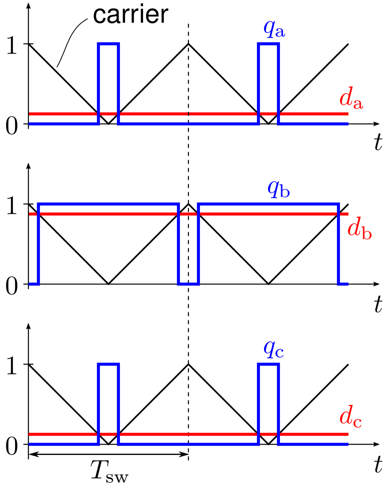
$$d_{\mathrm{a}} = \frac{1}{2} + \frac{u'_{\mathrm{a,ref}}}{u_{\mathrm{dc}}} \qquad d_{\mathrm{b}} = \frac{1}{2} + \frac{u'_{\mathrm{b,ref}}}{u_{\mathrm{dc}}} \qquad d_{\mathrm{c}} = \frac{1}{2} + \frac{u'_{\mathrm{c,ref}}}{u_{\mathrm{dc}}}$$

- Whole voltage hexagon can be now used
- Following example: $u_{\mathrm{a,ref}} = u_{\mathrm{c,ref}} = -u_{\mathrm{dc}}/4$ and $u_{\mathrm{b,ref}} = u_{\mathrm{dc}}/2$

The symmetrical suboscillation method is identical to the continuous space-vector PWM.



Suboscillation method



Symmetrical suboscillation method

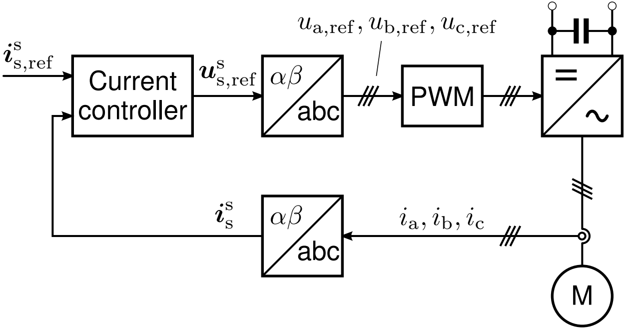
3-Phase Inverter

Pulse-Width Modulation

Current Control

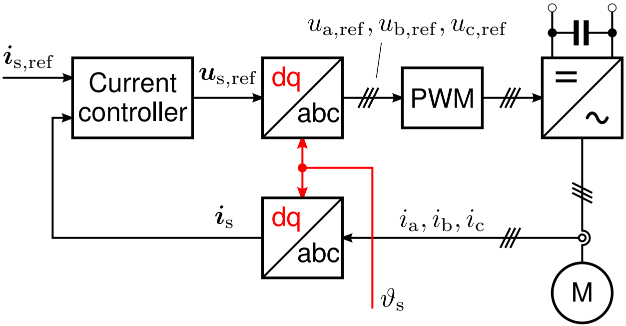
Anti-Windup, Sampling, PWM Update

Current Controller in Stator Coordinates



- PI controller cannot give zero steady-state error for sinusoidal references
- Actual current does not follow its reference in the steady state (phase shift and magnitude error)

Current Controller in Synchronous Coordinates



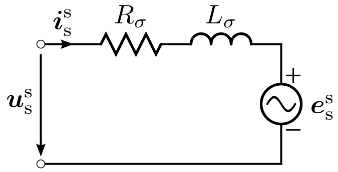
- DC signals in steady state, no steady-state error
- PI controller can be used

State-Space Representation in Synchronous Coordinates

- Stator current and rotor flux as state variables

$$L_\sigma \frac{\mathrm{d}\boldsymbol{i}_\mathrm{s}}{\mathrm{d}t} = \boldsymbol{u}_\mathrm{s} - (R_\sigma + \mathrm{j}\omega_\mathrm{s} L_\sigma)\boldsymbol{i}_\mathrm{s} - \underbrace{\left(\mathrm{j}\omega_\mathrm{m} - \frac{R_\mathrm{R}}{L_\mathrm{M}}\right)\boldsymbol{\psi}_\mathrm{R}}_{\text{back-emf } \boldsymbol{e}_\mathrm{s}}$$

$$\frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{R}}{\mathrm{d}t} = R_\mathrm{R}\boldsymbol{i}_\mathrm{s} - \left(\frac{R_\mathrm{R}}{L_\mathrm{M}} - \mathrm{j}\omega_\mathrm{r}\right)\boldsymbol{\psi}_\mathrm{R}$$



$$R_\sigma = R_\mathrm{s} + R_\mathrm{R}$$

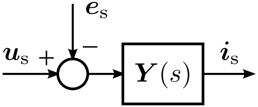
- Rotor-flux magnitude and speed change slowly as compared to the stator current
- Back-emf is a quasi-constant load disturbance for the current controller

Stator Current Dynamics in Open Loop

- System seen by the current control

$$L_\sigma \frac{\mathrm{d}\boldsymbol{i}_\mathrm{s}}{\mathrm{d}t} = \boldsymbol{u}_\mathrm{s} - (R_\sigma + \mathrm{j}\omega_\mathrm{s} L_\sigma)\boldsymbol{i}_\mathrm{s} - \boldsymbol{e}_\mathrm{s}$$

- Term $\mathrm{j}\omega_\mathrm{s} L_\sigma \boldsymbol{i}_\mathrm{s}$ causes cross-coupling between the axes
- Equivalent representation



$$\boldsymbol{i}_\mathrm{s} = \boldsymbol{Y}(s)\,(\boldsymbol{u}_\mathrm{s} - \boldsymbol{e}_\mathrm{s})$$

where

$$\boldsymbol{Y}(s) = \frac{1}{(s + \mathrm{j}\omega_\mathrm{s})L_\sigma + R_\sigma}$$

In the latter equations, the signals and systems can be considered to be in the Laplace domain, but, for simplicity, the argument s for the signals is omitted. Alternatively, they can be considered to be in the time domain, in which case $s = \mathrm{d}/\mathrm{d}t$ is the diffrential operator.

Synchronous-Frame 2DOF PI Controller

- Two-degrees-of-freedom (2DOF) control allows for independent design of disturbance rejection and reference tracking[1]
- State-space form of the synchronous-frame 2DOF PI controller

$$\frac{\mathrm{d}\boldsymbol{u}_\mathrm{i}}{\mathrm{d}t} = \boldsymbol{k}_\mathrm{i}\left(\boldsymbol{i}_\mathrm{s,ref} - \boldsymbol{i}_\mathrm{s}\right)$$

$$\boldsymbol{u}_\mathrm{s,ref} = \boldsymbol{k}_\mathrm{t}\boldsymbol{i}_\mathrm{s,ref} - \boldsymbol{k}_\mathrm{p}\boldsymbol{i}_\mathrm{s} + \boldsymbol{u}_\mathrm{i}$$

  where $\boldsymbol{k}_\mathrm{t}$ is the reference feedforward gain, $\boldsymbol{k}_\mathrm{p}$ is the state-feedback gain, $\boldsymbol{k}_\mathrm{i}$ is the integral gain, and $\boldsymbol{u}_\mathrm{i}$ is the integral state
- Equivalently

$$\boldsymbol{u}_\mathrm{s,ref} = \boldsymbol{k}_\mathrm{t}\boldsymbol{i}_\mathrm{s,ref} - \boldsymbol{k}_\mathrm{p}\boldsymbol{i}_\mathrm{s} + \frac{\boldsymbol{k}_\mathrm{i}}{s}\left(\boldsymbol{i}_\mathrm{s,ref} - \boldsymbol{i}_\mathrm{s}\right)$$

- Notice that selection $\boldsymbol{k}_\mathrm{t} = \boldsymbol{k}_\mathrm{p}$ yields the standard PI controller

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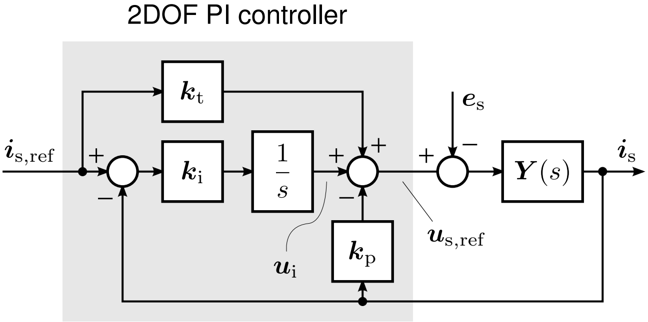
[1] Skogestad and Postlethwaite, *Multivariable Feedback Control: Analysis and Design*. John Wiley and Sons, 1996.

Closed-Loop System



- Ideal voltage production will be first assumed, $\boldsymbol{u}_\mathrm{s} = \boldsymbol{u}_\mathrm{s,ref}$
- Inclusion of voltage saturation and antiwindup will be discussed later

- Resulting closed-loop system

$$\boldsymbol{i}_\mathrm{s} = \boldsymbol{G}_\mathrm{c}(s)\boldsymbol{i}_\mathrm{s,ref} - \boldsymbol{Y}_\mathrm{c}(s)\boldsymbol{e}_\mathrm{s}$$

- Disturbance rejection depends on the closed-loop admittance

$$\boldsymbol{Y}_\mathrm{c}(s) = \frac{s}{L_\sigma s^2 + (R_\sigma + \mathrm{j}\omega_\mathrm{s} L_\sigma + \boldsymbol{k}_\mathrm{p})s + \boldsymbol{k}_\mathrm{i}}$$

- Poles can be placed by means of $\boldsymbol{k}_\mathrm{p}$ and $\boldsymbol{k}_\mathrm{i}$
- Rerefence tracking transfer function

$$\boldsymbol{G}_\mathrm{c}(s) = \frac{s\boldsymbol{k}_\mathrm{t} + \boldsymbol{k}_\mathrm{i}}{L_\sigma s^2 + (R_\sigma + \mathrm{j}\omega_\mathrm{s} L_\sigma + \boldsymbol{k}_\mathrm{p})s + \boldsymbol{k}_\mathrm{i}}$$

- Zero can be placed by means of $\boldsymbol{k}_\mathrm{t}$

Gain Selection

- Assume $\hat{L}_\sigma = L_\sigma$ and $\hat{R}_\sigma = R_\sigma$
- Selecting gains

$$\boldsymbol{k}_\mathrm{i} = \alpha_\mathrm{c}^2 \hat{L}_\sigma \qquad \boldsymbol{k}_\mathrm{t} = \alpha_\mathrm{c} \hat{L}_\sigma \qquad \boldsymbol{k}_\mathrm{p} = (2\alpha_\mathrm{c} - \mathrm{j}\omega_\mathrm{s})\hat{L}_\sigma - \hat{R}_\sigma$$

  results in the closed-loop system

$$\boldsymbol{G}_\mathrm{c}(s) = \frac{\alpha_\mathrm{c}}{s + \alpha_\mathrm{c}} \qquad \boldsymbol{Y}_\mathrm{c}(s) = \frac{s/L_\sigma}{(s + \alpha_\mathrm{c})^2}$$

  where α_c is the closed-loop bandwidth for reference tracking
- Effect of the resistance is negligible, i.e., $\hat{R}_\sigma = 0$ can be chosen
- This is a typical gain selection, but others are also possible[2]

[2] Awan, Saarakkala, and Hinkkanen, “Flux-linkage-based current control of saturated synchronous motors,” *IEEE Trans. Ind. Appl.*, 2019.

3-Phase Inverter

Pulse-Width Modulation

Current Control

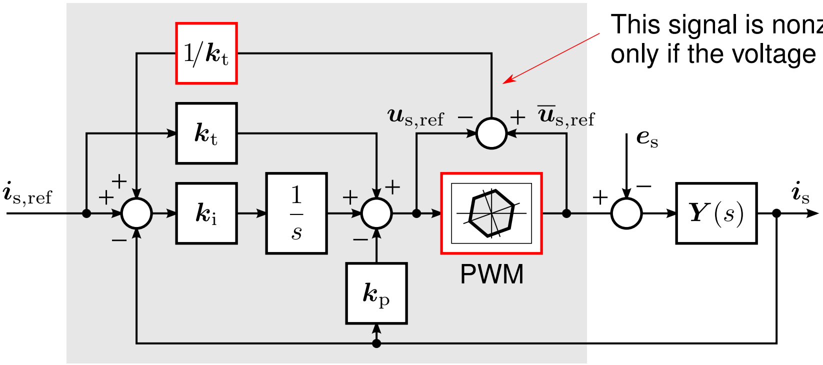
Anti-Windup, Sampling, PWM Update

Inclusion of Anti-Windup

- Maximum converter output voltage is limited: $|\boldsymbol{u}_\mathrm{s}| < u_\mathrm{max}$
- Reference $|\boldsymbol{u}_\mathrm{s,ref}|$ may exceed u_max for large current steps, especially at high rotor speeds due to the large back-emf $|\boldsymbol{e}_\mathrm{s}|$
- Anti-windup is needed

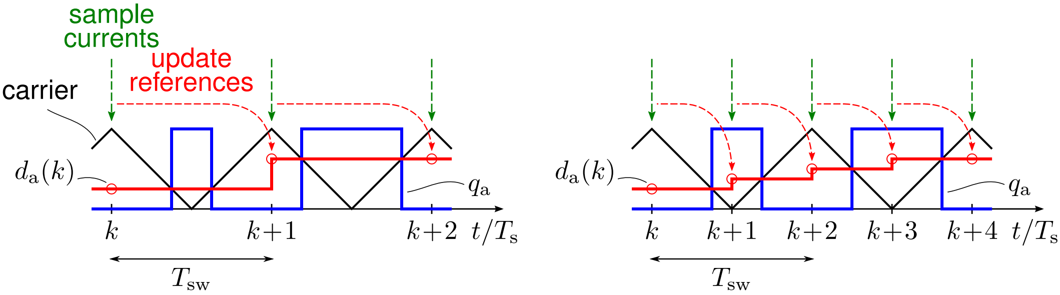
$$
\begin{aligned}
\frac{\mathrm{d}\boldsymbol{u}_\mathrm{i}}{\mathrm{d}t} &= \boldsymbol{k}_\mathrm{i}\left(\boldsymbol{i}_\mathrm{s,ref} - \boldsymbol{i}_\mathrm{s} + \frac{\overline{\boldsymbol{u}}_\mathrm{s,ref} - \boldsymbol{u}_\mathrm{s,ref}}{\boldsymbol{k}_\mathrm{t}}\right) \\
\boldsymbol{u}_\mathrm{s,ref} &= \boldsymbol{k}_\mathrm{t}\boldsymbol{i}_\mathrm{s,ref} - \boldsymbol{k}_\mathrm{p}\boldsymbol{i}_\mathrm{s} + \boldsymbol{u}_\mathrm{i} \\
\overline{\boldsymbol{u}}_\mathrm{s,ref} &= \mathsf{sat}(\boldsymbol{u}_\mathrm{s,ref})
\end{aligned}
$$

where $\overline{\boldsymbol{u}}_\mathrm{s,ref}$ is the realizable voltage vector obtained from the PWM algorithm



This signal is nonzero only if the voltage saturates

Discrete Implementation: Sampling and PWM Update



Single-update PWM

Double-update PWM

- No switching ripple in the current samples due to synchronous sampling
- Duty ratios d_b and d_c are updated simultaneously with d_a

Study the implementation in Assignment 1, where the double-update PWM scheme is used.