Aalto University
School of Electrical
Engineering

Lecture 10: Synchronous Motor Drives

ELEC-E8402 Control of Electric Drives and Power Converters

Marko Hinkkanen

Spring 2023

Learning Outcomes

After this lecture and exercises you will be able to:

- Identify, based on the cross-section of the rotor, if the motor is magnetically anisotropic
- Explain what is the reluctance torque
- Calculate operating points of synchronous motors and draw the corresponding vector diagrams
- Derive and explain the MTPA control principle

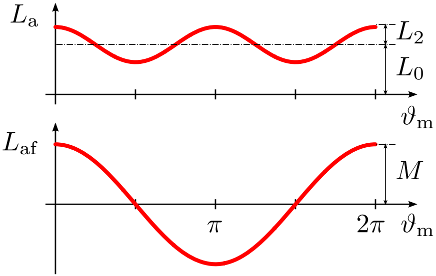
Common 3-Phase AC Motor Types

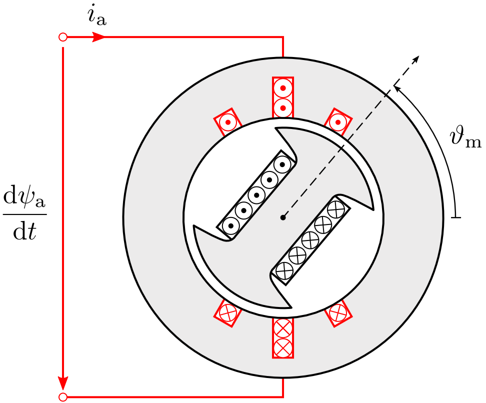
- Asynchronous motors
  - Induction motor with squirrel-cage rotor
  - Wound-rotor induction motor
- Synchronous motors
  - Synchronous motor with a field winding
  - Surface-mounted permanent-magnet synchronous motor (SPMSM)
  - Interior permanent-magnet synchronous motor (IPMSM)
  - Reluctance synchronous motor (SyRM)
  - Permanent-magnet-assisted SyRM (PM-SyRM)
- Same model and similar control can be used for these synchronous motors

Recap: Single-Phase Motor

Assumption: ideal sinusoidal winding distribution

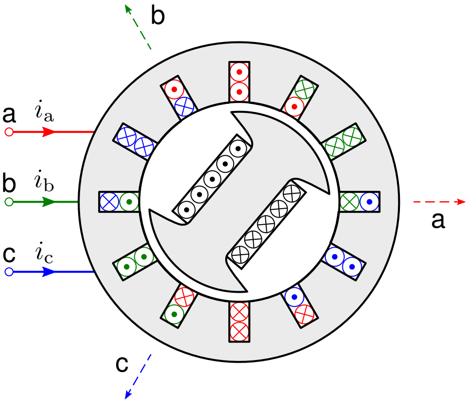
$$\psi_\mathrm{a} = L_\mathrm{a}(\vartheta_\mathrm{m}) i_\mathrm{a} + L_\mathrm{af}(\vartheta_\mathrm{m}) i_\mathrm{f} = [L_0 + L_2 \cos(2\vartheta_\mathrm{m})] i_\mathrm{a} + M \cos(\vartheta_\mathrm{m}) i_\mathrm{f}$$



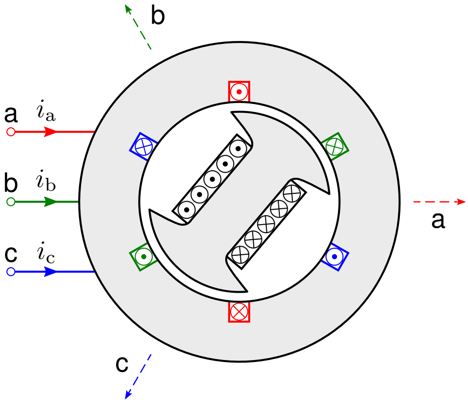


3-Phase Synchronous Motor

Sinusoidal phase windings and constant field-winding current i_f will be assumed

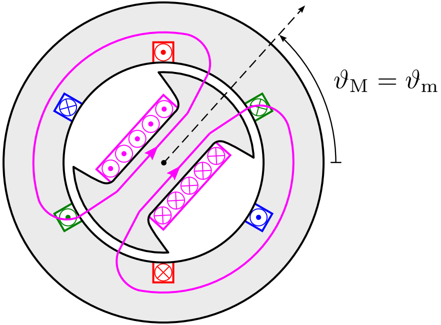


Example of a 3-phase distributed winding
(Y or D connection)

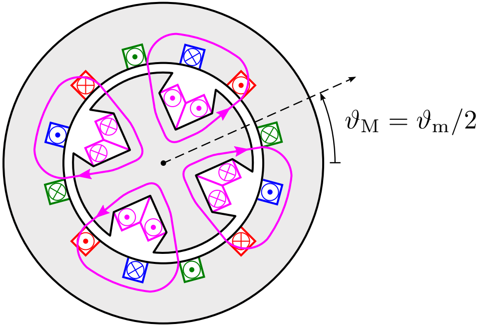


Simplified representation
will be used in the following

Number of Pole Pairs



2 poles ($n_\mathrm{p} = 1$)



4 poles ($n_\mathrm{p} = 2$)

Electrical angular speed $\omega_\mathrm{m} = n_\mathrm{p}\omega_\mathrm{M}$ and electrical angle $\vartheta_\mathrm{m} = n_\mathrm{p}\vartheta_\mathrm{M}$

Note that the stator and the rotor should have the same number of poles. What happens if their pole numbers differ?

Space Vector Transformation

- Instantaneous 3-phase quantities can be transformed to the $\alpha\beta$ components

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} i_\mathrm{a} \\ i_\mathrm{b} \\ i_\mathrm{c} \end{bmatrix}$$

  where currents are used as an example
- Equivalently, the complex space vector transformation can be used

$$\boldsymbol{i}_\mathrm{s} = i_\alpha + \mathrm{j} i_\beta = \frac{2}{3}\left(i_\mathrm{a} + i_\mathrm{b}\mathrm{e}^{\mathrm{j}2\pi/3} + i_\mathrm{c}\mathrm{e}^{\mathrm{j}4\pi/3} \right)$$

  which gives the same components i_α and i_β
- 3-phase motor can be modeled as an equivalent 2-phase motor with no loss of information

Equivalent 2-Phase Motor

- Stator flux linkages

$$\begin{bmatrix} \psi_\alpha \\ \psi_\beta \end{bmatrix} = \begin{bmatrix} L_\alpha & L_{\alpha\beta} \\ L_{\beta\alpha} & L_\beta \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \begin{bmatrix} L_{\alpha\mathrm{f}} \\ L_{\beta\mathrm{f}} \end{bmatrix} i_\mathrm{f}$$

$$L_\alpha = L_0 + L_2\cos(2\vartheta_\mathrm{m})$$

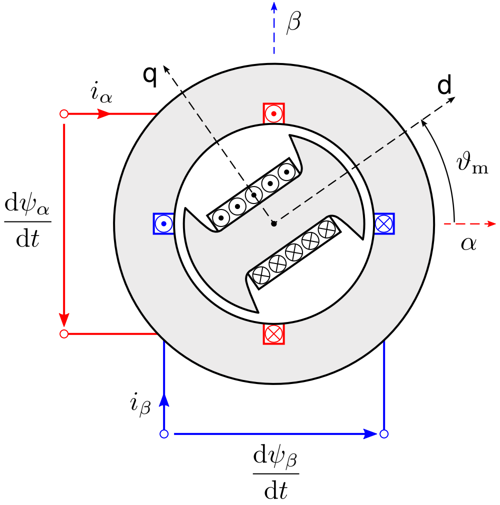
$$L_\beta = L_0 - L_2\cos(2\vartheta_\mathrm{m})$$

$$L_{\alpha\beta} = L_{\beta\alpha} = L_2\sin(2\vartheta_\mathrm{m})$$

$$L_{\alpha\mathrm{f}} = M\cos(\vartheta_\mathrm{m}) \qquad L_{\beta\mathrm{f}} = M\sin(\vartheta_\mathrm{m})$$

- Induced voltages

$$e_\alpha = \frac{\mathrm{d}\psi_\alpha}{\mathrm{d}t} \qquad e_\beta = \frac{\mathrm{d}\psi_\beta}{\mathrm{d}t}$$



Torque could be derived using the approach described in the previous lecture, but transforming the model to rotor coordinates allows us to use a shortcut, as shown in the following slides.

Transformation to Rotor Coordinates

- $\alpha\beta$ components can be transformed to the dq components

$$\begin{bmatrix} i_\mathrm{d} \\ i_\mathrm{q} \end{bmatrix} = \begin{bmatrix} \cos(\vartheta_\mathrm{m}) & \sin(\vartheta_\mathrm{m}) \\ -\sin(\vartheta_\mathrm{m}) & \cos(\vartheta_\mathrm{m}) \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

- Equivalent to the transformation for complex space vectors

$$\begin{aligned} i_\mathrm{d} + \mathrm{j} i_\mathrm{q} &= \boldsymbol{i}_\mathrm{s} = \mathrm{e}^{-\mathrm{j}\vartheta_\mathrm{m}} \boldsymbol{i}_\mathrm{s}^\mathrm{s} \\ &= [\cos(\vartheta_\mathrm{m}) - \mathrm{j}\sin(\vartheta_\mathrm{m})](i_\alpha + \mathrm{j} i_\beta) \\ &= \cos(\vartheta_\mathrm{m}) i_\alpha + \sin(\vartheta_\mathrm{m}) i_\beta + \mathrm{j}[-\sin(\vartheta_\mathrm{m}) i_\alpha + \cos(\vartheta_\mathrm{m}) i_\beta] \end{aligned}$$

- Inverse transformation is obtained similarly

Model in Rotor Coordinates

- Stator flux linkages

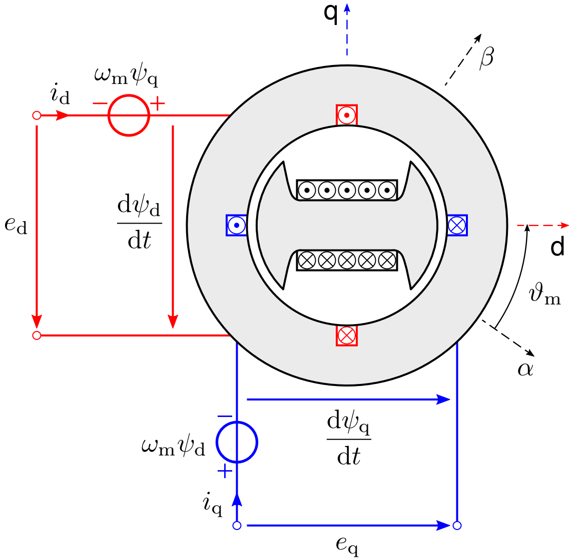
$$\begin{bmatrix} \psi_\mathrm{d} \\ \psi_\mathrm{q} \end{bmatrix} = \begin{bmatrix} L_\mathrm{d} & 0 \\ 0 & L_\mathrm{q} \end{bmatrix} \begin{bmatrix} i_\mathrm{d} \\ i_\mathrm{q} \end{bmatrix} + \begin{bmatrix} L_\mathrm{d} \\ 0 \end{bmatrix} i_\mathrm{F}$$

- Inductances are constant

$$L_\mathrm{d} = L_0 + L_2 \qquad L_\mathrm{q} = L_0 - L_2$$

- Equivalent field-winding current $i_\mathrm{F} = (M/L_\mathrm{d})i_\mathrm{f}$
- Induced voltages

$$e_\mathrm{d} = \frac{\mathrm{d}\psi_\mathrm{d}}{\mathrm{d}t} - \omega_\mathrm{m}\psi_\mathrm{q} \qquad e_\mathrm{q} = \frac{\mathrm{d}\psi_\mathrm{q}}{\mathrm{d}t} + \omega_\mathrm{m}\psi_\mathrm{d}$$



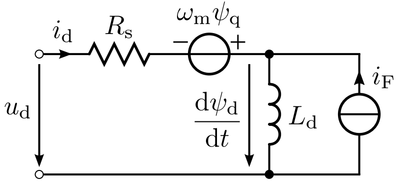
- Model can be expressed using space vectors
- Stator flux linkage

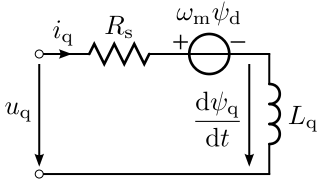
$$\boxed{\boldsymbol{\psi}_\mathrm{s} = L_\mathrm{d} i_\mathrm{d} + \psi_\mathrm{F} + \mathrm{j} L_\mathrm{q} i_\mathrm{q}}$$

  where $\psi_\mathrm{F} = L_\mathrm{d} i_\mathrm{F}$

- Stator voltage

$$\boxed{\boldsymbol{u}_\mathrm{s} = R_\mathrm{s} \boldsymbol{i}_\mathrm{s} + \frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{s}}{\mathrm{d}t} + \mathrm{j}\omega_\mathrm{m} \boldsymbol{\psi}_\mathrm{s}}$$





Alternatively, space vectors could be represented using real-valued column vectors, e.g., $\boldsymbol{i}_\mathrm{s} = [i_\mathrm{d}, i_\mathrm{q}]^\mathrm{T}$ instead of $\boldsymbol{i}_\mathrm{s} = i_\mathrm{d} + \mathrm{j} i_\mathrm{q}$. Real-valued vectors would allow expressing the flux linkage equation in a more convenient form.

Power Balance

$$\frac{3}{2}\,\mathrm{Re}\,\{\boldsymbol{u}_\mathrm{s}\boldsymbol{i}_\mathrm{s}^*\} = \frac{3}{2}R_\mathrm{s}|\boldsymbol{i}_\mathrm{s}|^2 + \frac{3}{2}\,\mathrm{Re}\left\{\frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{s}}{\mathrm{d}t}\boldsymbol{i}_\mathrm{s}^*\right\} + \tau_\mathrm{M}\frac{\omega_\mathrm{m}}{n_\mathrm{p}}$$

- Electromagnetic torque

$$\boxed{\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\,\{\boldsymbol{i}_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}^*\} = \frac{3n_\mathrm{p}}{2}\,[\psi_\mathrm{F} + (L_\mathrm{d} - L_\mathrm{q})i_\mathrm{d}]\, i_\mathrm{q}}$$

- Rate of change of the magnetic field energy

$$\mathrm{Re}\left\{\frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{s}}{\mathrm{d}t}\boldsymbol{i}_\mathrm{s}^*\right\} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}L_\mathrm{d}i_\mathrm{d}^2 + \frac{1}{2}L_\mathrm{q}i_\mathrm{q}^2\right)$$

- This model is valid also for other synchronous motors

Model in a Block Diagram Form

- Magnetic model is obtained from the flux linkage equation

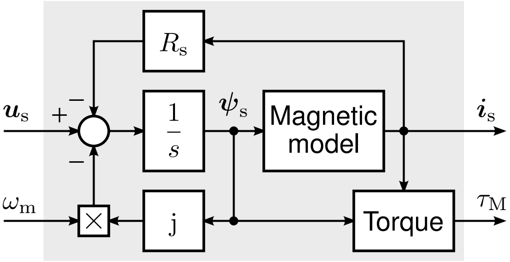
$$\boldsymbol{i}_\mathrm{s} = \frac{\psi_\mathrm{d} - \psi_\mathrm{F}}{L_\mathrm{d}} + \mathrm{j}\frac{\psi_\mathrm{q}}{L_\mathrm{q}}$$

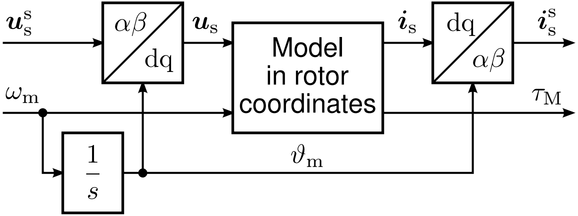
- If needed, magnetic saturation could be modeled in a form

$$\boldsymbol{i}_\mathrm{s} = i_\mathrm{d}(\psi_\mathrm{d}, \psi_\mathrm{q}) + \mathrm{j}i_\mathrm{q}(\psi_\mathrm{d}, \psi_\mathrm{q})$$

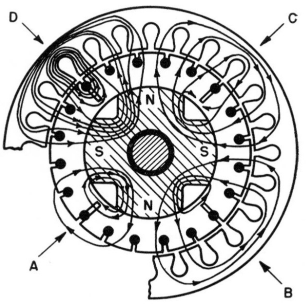
- Mechanical subsystem closes the loop from τ_M to ω_m (not shown)

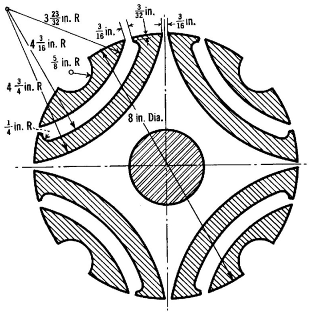
Model in rotor coordinates

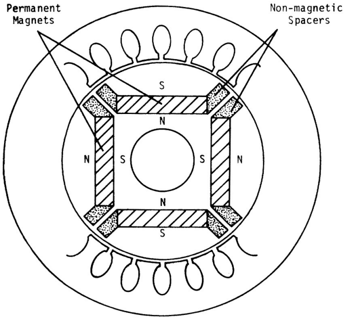




This model could be implemented in Simulink in a similar manner as the induction motor model in Assignment 1.







PM synchronous motor (with a damping cage)[1]

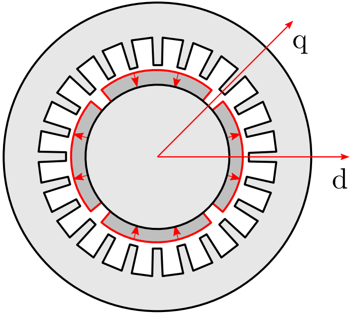
Reluctance synchronous motor[2]

Interior PM synchronous motor[3]

[1] Merrill, "Permanent-magnet excited synchronous motors," *AIEE Trans.*, 1955.

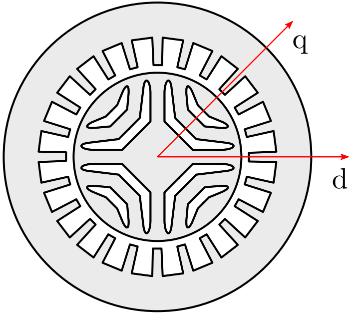
[2] Kostko, "Polyphase reaction synchronous motors," *J. AIEE*, 1923.

[3] Jahns, Kliman, and Neumann, "Interior permanent-magnet synchronous motors for adjustable-speed drives," *IEEE Trans. Ind. Appl.*, 1986.



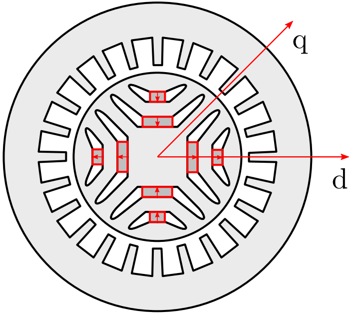
Surface-mounted PM synchronous motor

$L_{\mathrm{d}} = L_{\mathrm{q}}$, $\psi_{\mathrm{F}} =$ const



Reluctance synchronous motor

$L_{\mathrm{d}} > L_{\mathrm{q}}$, $\psi_{\mathrm{F}} = 0$



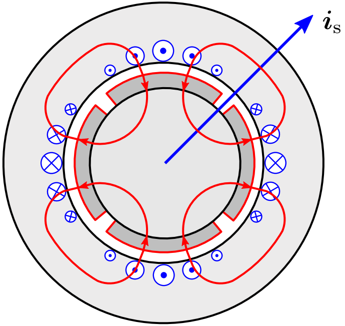
Interior PM synchronous motor

$L_{\mathrm{q}} > L_{\mathrm{d}}$, $\psi_{\mathrm{F}} =$ const

Permeability of PMs ($\mu_{\mathrm{r}} \approx 1.05$) almost equals the permeability of air ($\mu_{\mathrm{r}} \approx 1$)

Surface-Mounted PM Synchronous Motor (SPMSM)

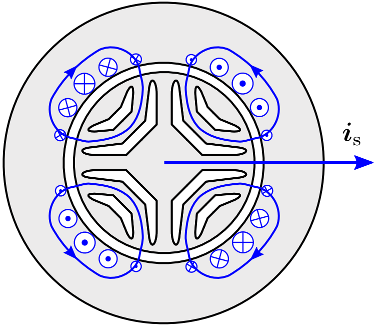
- Either distributed or concentrated 3-phase stator winding
- Rare-earth magnets (NdFeB or SmCo) mounted at the rotor surface
- High efficiency (or power density)
- Limited field-weakening range
- Expensive due to the magnets and manufacturing process
- Typical motor type in servo drives



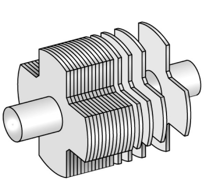
What are the current components i_d and i_q in the figure?

Reluctance Synchronous Motor (SyRM)

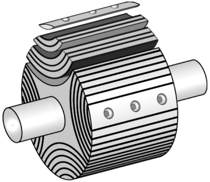
- Distributed 3-phase stator winding
- Transversally laminated rotor
- Flux barriers are shaped to maximize $L_\mathrm{d}/L_\mathrm{q}$
- Rotor tries to find its way to the position that minimizes the magnetic field energy
- Cheaper than PM motors
- More efficient than induction motors
- Poor power factor (means a larger inverter)
- Magnetic saturation has to be taken into account in control
- Competitor to induction motors



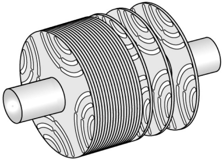
What is the electromagnetic torque in the figure?



Conceptual rotor

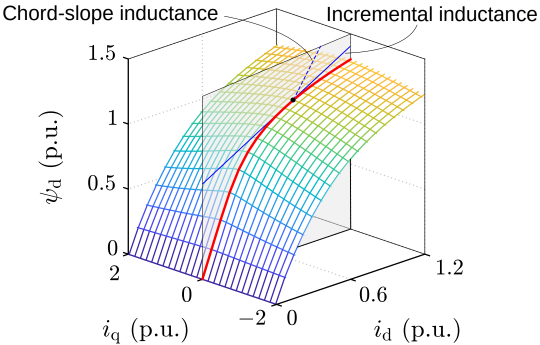


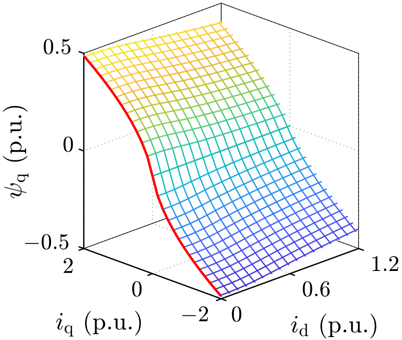
Axially laminated



Transversally laminated

Figures: (left) Fukami, Momiyama, Shima, *et al.*, “Steady-state analysis of a dual-winding reluctance generator with a multiple-barrier rotor,” *IEEE Trans. Energy Conv.*, 2008; (right) ABB.





- Magnetic model of a 6.7-kW motor shown as an example
- Can be measured at constant speed[4] or identified at standstill[5]

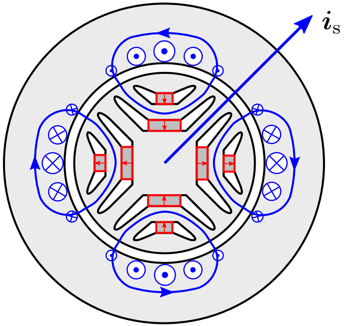
---

[4] Armando, Bojoi, Guglielmi, *et al.*, "Experimental identification of the magnetic model of synchronous machines," *IEEE Trans. Ind. Appl.*, 2013.

[5] Hinkkanen, Pescetto, Mölsä, *et al.*, "Sensorless self-commissioning of synchronous reluctance motors at standstill without rotor locking," *IEEE Trans. Ind. Appl.*, 2017.

Interior PM Synchronous Motors (IPMSM)

- Reluctance synchronous motor can be improved by placing either rare-earth or ferrite magnets inside the flux barriers
- Magnets improve the power factor and contribute to the torque
- Excellent field-weakening performance
- Minor risk of overvoltages due to the low back-emf induced by the magnets
- If the reluctance torque dominates, these motors are called PM-assisted reluctance synchronous motors (PM-SyRM)[6]

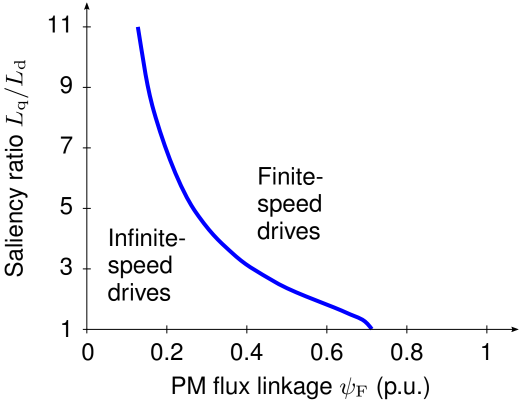


What is the reluctance torque in the figure?

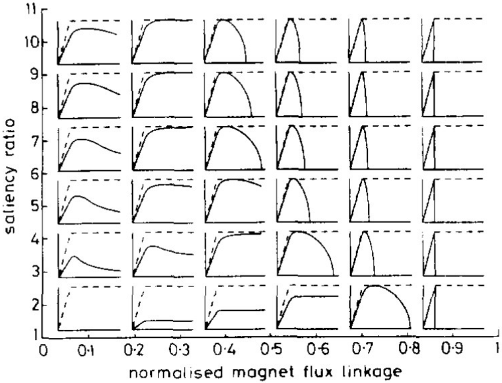
[6] Guglielmi, Pastorelli, Pellegrino, *et al.*, "Position-sensorless control of permanent-magnet-assisted synchronous reluctance motor," *IEEE Trans. Ind. Appl.*, 2004.

Optimal field-weakening design criterion $\psi_{\mathrm{F}} = L_{\mathrm{d}} i_{\mathrm{N}}$, where i_{N} is the rated current

Design parameter plane



Normalized power[7]

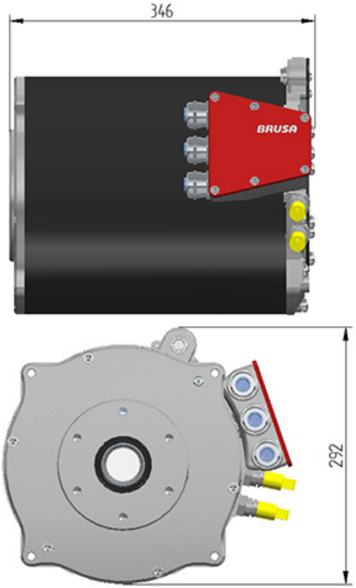


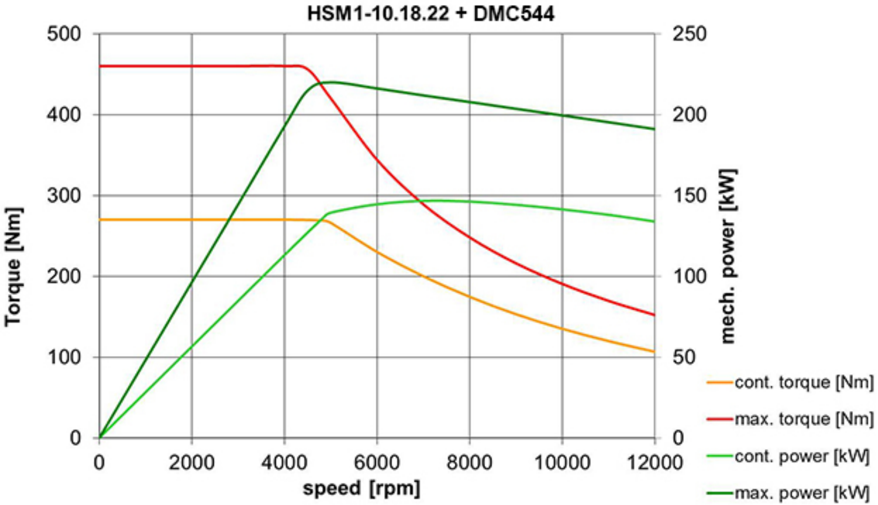
[7] Soong and Miller, "Field-weakening performance of brushless synchronous ac motor drives," *IEE Proc. EPA*, 1994.

Example: Brusa HSM1-10.18.22

- For truck and bus applications
- Low magnetic material
- IPMSM or PM-SyRM?

- Speed: 4 400 r/min (nom), 12 000 r/min (max)
- Torque: 270 Nm (S1), 460 Nm (max)
- Power: 145 kW (S1), 220 kW (max)
- DC-bus voltage: 400 V
- Weight: 76 kg



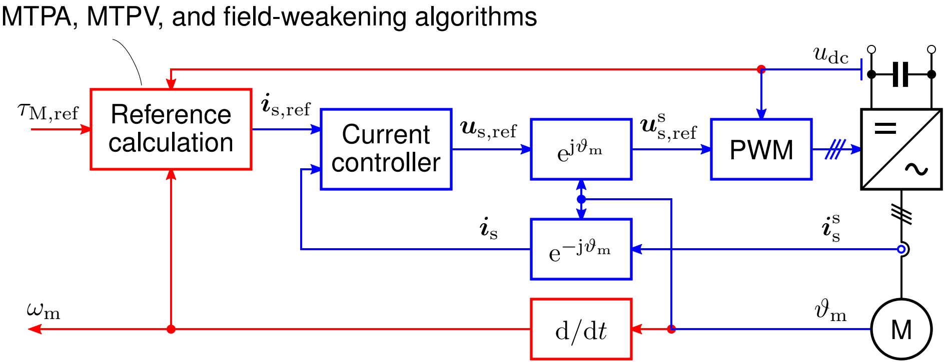

HSM1-10.18.22 + DMC544
500
400
300
200
100
0
250
200
150
100
50
0
0
2000
4000
6000
8000
10000
12000
speed [rpm]
Torque [Nm]
mech. power [kW]
cont. torque [Nm]
max. torque [Nm]
cont. power [kW]
max. power [kW]

3-Phase Synchronous Motor Model

Permanent-Magnet and Reluctance Synchronous Motors

Control of Synchronous Motors

Typical Vector Control System



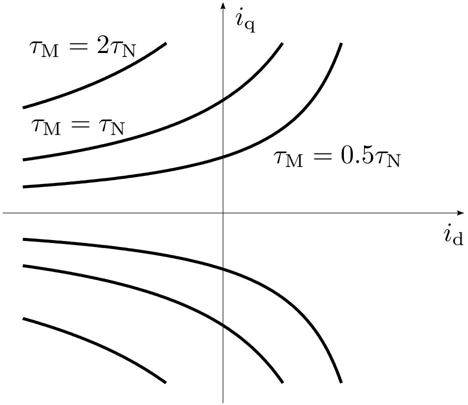
- Fast current-control loop
- Rotor position ϑ_{m} is measured (or estimated)
- Current reference $\boldsymbol{i}_{\mathrm{s,ref}}$ is calculated in rotor coordinates
- Control of PM and reluctance synchronous motors will be considered

Constant Torque Loci in the Current Plane

- Same torque can be produced with different current components

$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\left[\psi_\mathrm{F} + (L_\mathrm{d} - L_\mathrm{q})i_\mathrm{d}\right] i_\mathrm{q}$$

- IPMSM ($L_\mathrm{q}/L_\mathrm{d} = 1.7$ and $\psi_\mathrm{F} = 0.7$ p.u.) is used as an example motor
- How are the loci for $L_\mathrm{d} = L_\mathrm{q}$ (SPMSM) and for $\psi_\mathrm{F} = 0$ (SyRM)?
- How to choose i_d and i_q?



Magnetic saturation will be omitted here, but it should be taken into account at least for SyRMs in practice.

Current and Voltage Limits

- Maximum current

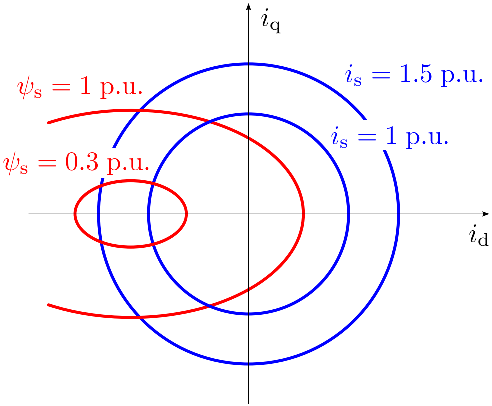
$$i_\mathrm{s} = \sqrt{i_\mathrm{d}^2 + i_\mathrm{q}^2} \leq i_\mathrm{max}$$

- Maximum flux linkage

$$\psi_\mathrm{s} = \sqrt{\psi_\mathrm{d}^2 + \psi_\mathrm{q}^2} \leq \frac{u_\mathrm{max}}{|\omega_\mathrm{m}|}$$

where

$$\psi_\mathrm{d} = L_\mathrm{d} i_\mathrm{d} + \psi_\mathrm{F}$$
$$\psi_\mathrm{q} = L_\mathrm{q} i_\mathrm{q}$$



Control Principle

- Goal is to produce the requested torque at minimum losses and to maximize available torque for the given drive capacity (i_max and u_max)
- Speeds below the base speed
  - Maximum torque per ampere (MTPA) locus minimizes the copper losses
- Higher speeds
  - MTPA locus cannot be used due to the limited voltage
  - To reach higher speeds, the flux linkage ψ_s has to be reduced by negative i_d
  - Maximum torque per volt (MTPV) limit has to be taken into account

Maximum Torque per Ampere (MTPA)

- Current magnitude $i_\mathrm{s} = \sqrt{i_\mathrm{d}^2 + i_\mathrm{q}^2}$
- Torque is expressed as

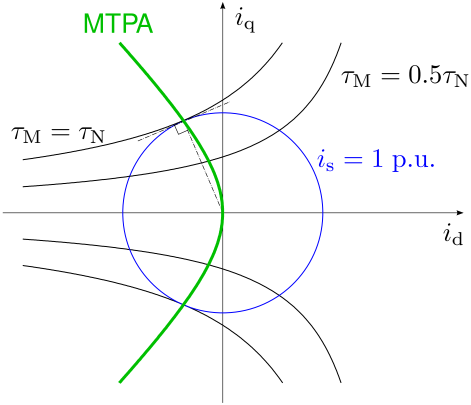
$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\left[\psi_\mathrm{F} + (L_\mathrm{d} - L_\mathrm{q})i_\mathrm{d}\right]\sqrt{i_\mathrm{s}^2 - i_\mathrm{d}^2}$$

- Maximum torque at $\partial\tau_\mathrm{M}/\partial i_\mathrm{d} = 0$

$$i_\mathrm{d}^2 + i_\mathrm{d}\frac{\psi_\mathrm{F}}{L_\mathrm{d} - L_\mathrm{q}} - i_\mathrm{q}^2 = 0$$

- Special cases

$$\begin{aligned} i_\mathrm{d} &= 0 \quad &&\text{for} \quad L_\mathrm{d} = L_\mathrm{q} \text{ (SPMSM)} \\ |i_\mathrm{d}| &= |i_\mathrm{q}| \quad &&\text{for} \quad \psi_\mathrm{F} = 0 \text{ (SyRM)} \end{aligned}$$



Maximum Torque per Volt (MTPV)

- Flux magnitude

$$\psi_\mathrm{s} = \sqrt{(\psi_\mathrm{F} + L_\mathrm{d} i_\mathrm{d})^2 + (L_\mathrm{q} i_\mathrm{q})^2}$$

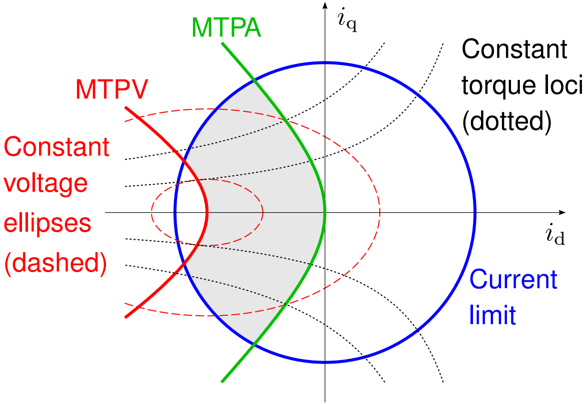
- MTPV condition can be derived similarly as the MTPA condition

$$(\psi_\mathrm{F} + L_\mathrm{d} i_\mathrm{d})^2 + \frac{L_\mathrm{q}}{L_\mathrm{d} - L_\mathrm{q}} \psi_\mathrm{f} (\psi_\mathrm{F} + L_\mathrm{d} i_\mathrm{d}) - (L_\mathrm{q} i_\mathrm{q})^2 = 0$$

- Special cases

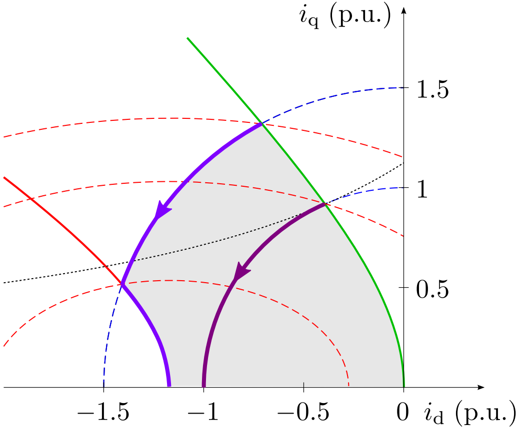
$$\begin{aligned} i_\mathrm{d} &= -\psi_\mathrm{F}/L_\mathrm{d} && \text{for} \quad L_\mathrm{d} = L_\mathrm{q} \text{ (SPMSM)} \\ |\psi_\mathrm{d}| &= |\psi_\mathrm{q}| && \text{for} \quad \psi_\mathrm{F} = 0 \text{ (SyRM)} \end{aligned}$$

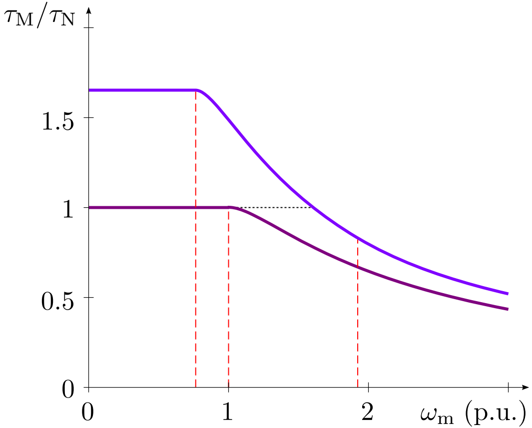
Feasible Operating Area[8]

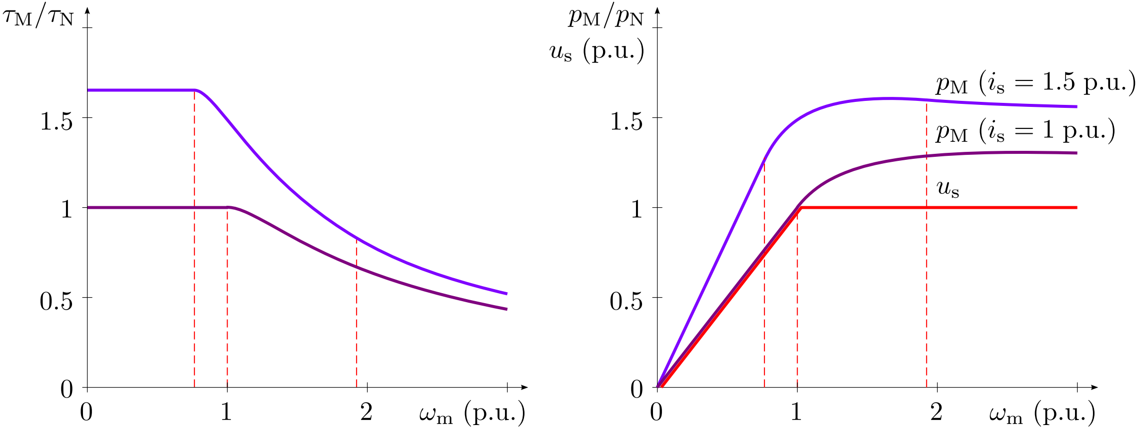


[8] Morimoto, Takeda, Hirasa, *et al.*, "Expansion of operating limits for permanent magnet motor by current vector control considering inverter capacity," *IEEE Trans. Ind. Appl.*, 1990.

Example: Acceleration Loci for i_{max} = 1 p.u. and i_{max} = 1.5 p.u.

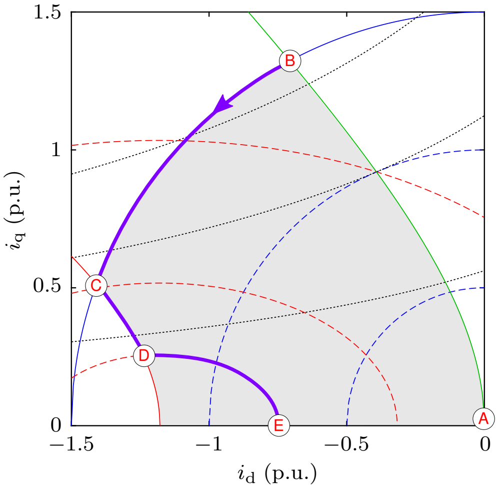


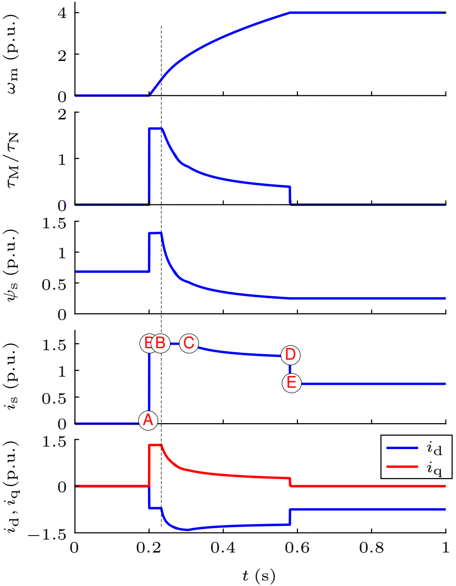




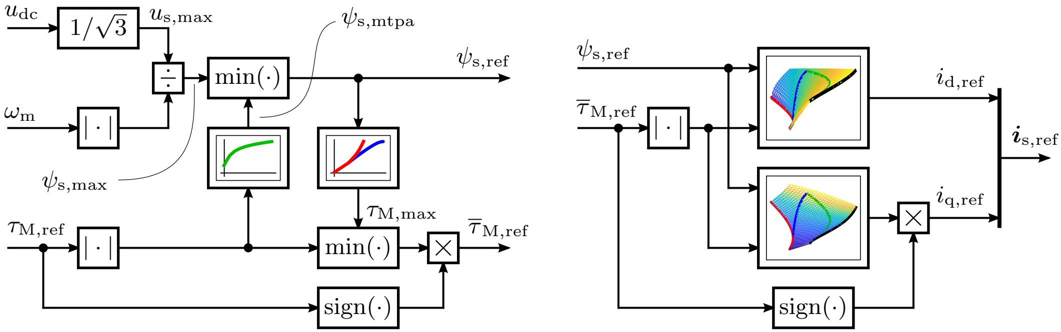
Mechanical power $p_M = \tau_M \omega_m / n_p$

Example: Time-Domain Waveforms





Feedforward Reference Calculation Method[9,10]



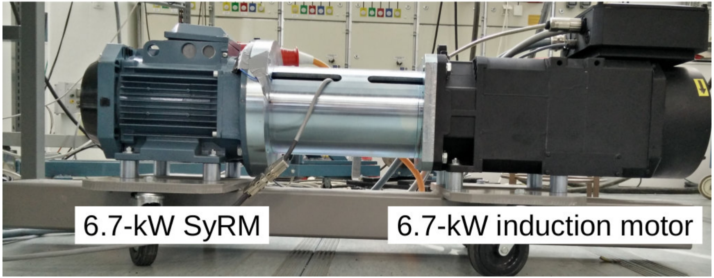
- Control lookup tables numerically solved from the magnetic model
- Other control structures and control variables are possible

[9] Meyer and Böcker, "Optimum control for interior permanent magnet synchronous motors (IPMSM) in constant torque and flux weakening range," in *Proc. EPE-PEMC*, 2006.

[10] Awan, Song, Saarakkala, *et al.*, "Optimal torque control of saturated synchronous motors: Plug-and-play method," *IEEE Trans. Ind. Appl.*, 2018.

Experimental Results: 6.7-kW SyRM

- Rated values: 3175 r/min; 105.8 Hz; 370 V; 15.5 A
- Sampling and switching frequency 5 kHz
- Current-control bandwidth 500 Hz
- Flux linkages used as state variables in the current controller[11]



[11] Awan, Saarakkala, and Hinkkanen, "Flux-linkage-based current control of saturated synchronous motors," *IEEE Trans. Ind. Appl.*, 2019.

- Acceleration to 2 p.u. (212 Hz)
- Control takes the magnetic saturation into account
- Saturation affects significantly the optimal current components