Problem 1: Torque expressions of an induction machine

The electromagnetic torque of an induction machine can be expressed in terms of the stator flux linkage and the stator current as

$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}^*\right\}$$

(a) Formulate the given torque expression in the dq component form.

(b) Using the given torque expression as the starting point, derive the corresponding torque expression in terms of the stator current and the rotor flux linkage.

(c) Derive the torque expression in terms of the stator flux linkage and the rotor flux linkage.

Solution

(a) The notation corresponding to $\boldsymbol{i}_\mathrm{s} = i_\mathrm{d} + \mathrm{j}i_\mathrm{q}$ is used for the space vector components, yielding

$$\begin{aligned}\tau_\mathrm{M} &= \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}^*\right\} = \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{(\psi_\mathrm{d} - \mathrm{j}\psi_\mathrm{q})(i_\mathrm{d} + \mathrm{j}i_\mathrm{q})\right\} \\ &= \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\psi_\mathrm{d}i_\mathrm{d} + \psi_\mathrm{q}i_\mathrm{q} + \mathrm{j}(\psi_\mathrm{d}i_\mathrm{q} - \psi_\mathrm{q}i_\mathrm{d})\right\} \\ &= \frac{3n_\mathrm{p}}{2}(\psi_\mathrm{d}i_\mathrm{q} - \psi_\mathrm{q}i_\mathrm{d})\end{aligned}$$

(b) The inverse-Γ model is used. Inserting the stator flux linkage

$$\boldsymbol{\psi}_\mathrm{s} = L_\sigma\boldsymbol{i}_\mathrm{s} + \boldsymbol{\psi}_\mathrm{R}$$

into the torque expression gives

$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}^*\right\} = \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{s}\left(L_\sigma\boldsymbol{i}_\mathrm{s}^* + \boldsymbol{\psi}_\mathrm{R}^*\right)\right\} = \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{s}\boldsymbol{\psi}_\mathrm{R}^*\right\}$$

since $\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{s}^*\boldsymbol{i}_\mathrm{s}\right\} = 0$.

(c) Based on the previous stator flux equation, the stator current is

$$\boldsymbol{i}_\mathrm{s} = \frac{1}{L_\sigma}\left(\boldsymbol{\psi}_\mathrm{s} - \boldsymbol{\psi}_\mathrm{R}\right)$$

This expression can be inserted into the torque expression,

$$\begin{aligned}\tau_\mathrm{M} &= \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}^*\right\} = \frac{3n_\mathrm{p}}{2}\frac{1}{L_\sigma}\,\mathrm{Im}\left\{\left(\boldsymbol{\psi}_\mathrm{s} - \boldsymbol{\psi}_\mathrm{R}\right)\boldsymbol{\psi}_\mathrm{s}^*\right\} \\ &= -\frac{3n_\mathrm{p}}{2}\frac{1}{L_\sigma}\,\mathrm{Im}\left\{\boldsymbol{\psi}_\mathrm{R}\boldsymbol{\psi}_\mathrm{s}^*\right\} = \frac{3n_\mathrm{p}}{2}\frac{1}{L_\sigma}\,\mathrm{Im}\left\{\boldsymbol{\psi}_\mathrm{s}\boldsymbol{\psi}_\mathrm{R}^*\right\}\end{aligned}$$

Problem 2: Steady-state torque expressions

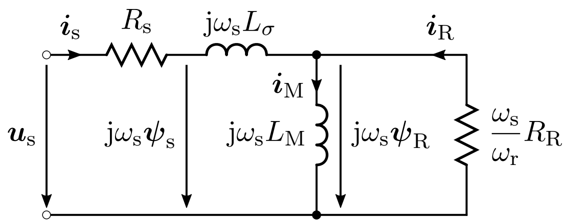
Express the torque τ_M of an induction motor in the steady state as a function of:

(a) the slip angular frequency ω_r and the rotor-flux magnitude ψ_R;

(b) the slip angular frequency ω_r and the stator-flux magnitude ψ_s.

Solution

The steady-state equivalent circuit (valid in any coordinates) is shown in the figure.



(a) Since the rotor flux linkage is $\boldsymbol{\psi}_\mathrm{R} = L_\mathrm{M}(\boldsymbol{i}_\mathrm{s} + \boldsymbol{i}_\mathrm{R})$, the torque can be expressed as

$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{s}\boldsymbol{\psi}_\mathrm{R}^*\right\} = \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\left(\frac{\boldsymbol{\psi}_\mathrm{R}}{L_\mathrm{M}} - \boldsymbol{i}_\mathrm{R}\right)\boldsymbol{\psi}_\mathrm{R}^*\right\} = -\frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{R}\boldsymbol{\psi}_\mathrm{R}^*\right\}$$

The rotor voltage equation in the steady state is

$$0 = R_\mathrm{R}\boldsymbol{i}_\mathrm{R} + \mathrm{j}\omega_\mathrm{r}\boldsymbol{\psi}_\mathrm{R}$$

leading to the rotor current

$$\boldsymbol{i}_\mathrm{R} = -\frac{\mathrm{j}\omega_\mathrm{r}\boldsymbol{\psi}_\mathrm{R}}{R_\mathrm{R}}$$

The desired steady-state torque expression is

$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\frac{\mathrm{j}\omega_\mathrm{r}\boldsymbol{\psi}_\mathrm{R}}{R_\mathrm{R}}\boldsymbol{\psi}_\mathrm{R}^*\right\} = \frac{3n_\mathrm{p}}{2}\frac{\psi_\mathrm{R}^2\omega_\mathrm{r}}{R_\mathrm{R}}$$

where $\psi_\mathrm{R} = |\boldsymbol{\psi}_\mathrm{R}| = \sqrt{\boldsymbol{\psi}_\mathrm{R}\boldsymbol{\psi}_\mathrm{R}^*}$ is the rotor-flux magnitude. It can be seen that the slip is directly proportional to the torque, if the rotor-flux magnitude is constant.

Remark: Peak-valued space vectors are used in this course. This is the most common choice in control applications and in the analysis of transient phenomena. Alternatively, the rms-valued space vectors could be used. For example, the rms-valued rotor current vector is

$$\boldsymbol{I}_\mathrm{R} = -\frac{\mathrm{j}\omega_\mathrm{r}\boldsymbol{\Psi}_\mathrm{R}}{R_\mathrm{R}}$$

where $\boldsymbol{I}_\mathrm{R} = \boldsymbol{i}_\mathrm{R}/\sqrt{2}$ and $\boldsymbol{\Psi}_\mathrm{R} = \boldsymbol{\psi}_\mathrm{R}/\sqrt{2}$. Note that the scaling has an effect on power and torque expressions, e.g.,

$$\tau_\mathrm{M} = 3n_\mathrm{p}\,\mathrm{Im}\left\{\boldsymbol{I}_\mathrm{s}\boldsymbol{\Psi}_\mathrm{s}^*\right\} = -3n_\mathrm{p}\,\mathrm{Im}\left\{\boldsymbol{I}_\mathrm{R}\boldsymbol{\Psi}_\mathrm{R}^*\right\} = 3n_\mathrm{p}\frac{\Psi_\mathrm{R}^2\omega_\mathrm{r}}{R_\mathrm{R}}$$

(b) The stator current in the steady state as a function of the stator flux and the slip angular frequency can be written from the equivalent circuit

$$\boldsymbol{i}_\mathrm{s} = \frac{\mathrm{j}\omega_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}}{\mathrm{j}\omega_\mathrm{s}L_\sigma + \dfrac{\mathrm{j}\omega_\mathrm{s}L_\mathrm{M}(\omega_\mathrm{s}/\omega_\mathrm{r})R_\mathrm{R}}{\mathrm{j}\omega_\mathrm{s}L_\mathrm{M} + (\omega_\mathrm{s}/\omega_\mathrm{r})R_\mathrm{R}}} = \frac{\boldsymbol{\psi}_\mathrm{s}}{L_\sigma + \dfrac{L_\mathrm{M}}{1+\mathrm{j}\omega_\mathrm{r}T_\mathrm{r}}}$$

where $T_\mathrm{r} = L_\mathrm{M}/R_\mathrm{R}$ is the rotor time constant. Note that the denominator in the first expression is simply the impedance seen by the voltage $\mathrm{j}\omega_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}$. The desired steady-state torque expression becomes

$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}^*\right\} = \frac{3n_\mathrm{p}}{2}\psi_\mathrm{s}^2\,\mathrm{Im}\left\{\frac{1}{L_\sigma + \dfrac{L_\mathrm{M}}{1+\mathrm{j}\omega_\mathrm{r}T_\mathrm{r}}}\right\}$$
$$= \frac{3n_\mathrm{p}}{2}\psi_\mathrm{s}^2\frac{L_\mathrm{M}T_\mathrm{r}\omega_\mathrm{r}}{(L_\sigma + L_\mathrm{M})^2 + (L_\sigma T_\mathrm{r}\omega_\mathrm{r})^2}$$

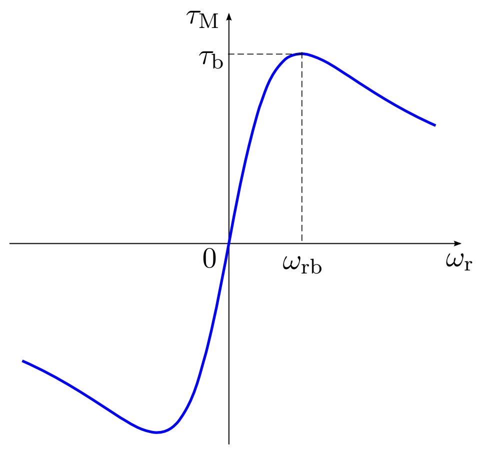
Remark: The above torque expression can be rewritten as

$$\tau_\mathrm{M} = \frac{2\tau_\mathrm{b}}{\omega_\mathrm{r}/\omega_\mathrm{rb} + \omega_\mathrm{rb}/\omega_\mathrm{r}}$$

where the breakdown torque, breakdown slip frequency, and leakage factor are

$$\tau_\mathrm{b} = \frac{3n_\mathrm{p}}{2}\frac{L_\mathrm{M}}{L_\sigma + L_\mathrm{M}}\frac{\psi_\mathrm{s}^2}{2L_\sigma} \qquad \omega_\mathrm{rb} = \frac{1}{\sigma T_\mathrm{r}} \qquad \sigma = \frac{L_\sigma}{L_\sigma + L_\mathrm{M}}$$

respectively. Note that the breakdown torque depends on the square of the stator-flux magnitude and that the breakdown slip is a constant (which depends on the motor parameters).



Problem 3: Determination of induction motor parameters

The parameters of an inverter-fed induction motor are estimated using three tests:

(a) A direct current is commanded into the α-axis at standstill, resulting in the space vectors $\boldsymbol{i}_\mathrm{s}^\mathrm{s} = 4.0$ A and $\boldsymbol{u}_\mathrm{s}^\mathrm{s} = 14.8$ V. Determine the stator resistance. What are the phase currents?

(b) A pulsating sinusoidal current is commanded into the α-axis at standstill, resulting in the space vectors

$$\boldsymbol{i}_\mathrm{s}^\mathrm{s}(t) = i_\mathrm{s}\sin(\omega_\mathrm{s}t + \varphi_\mathrm{i}) \qquad \boldsymbol{u}_\mathrm{s}^\mathrm{s}(t) = u_\mathrm{s}\sin(\omega_\mathrm{s}t + \varphi_\mathrm{u})$$

where $\omega_\mathrm{s} = 2\pi \cdot 80$ rad/s, $i_\mathrm{s} = 4.0$ A, $\varphi_\mathrm{i} = 0$, $u_\mathrm{s} = 48.2$ V, and $\varphi_\mathrm{u} = 61.2°$. Determine the leakage inductance and the rotor resistance. Assume the magnetizing current to be zero, since the excitation frequency ω_s is comparatively high.

(c) The motor is driven at no load in the steady state. The rotating space vectors are

$$\boldsymbol{i}_\mathrm{s}^\mathrm{s}(t) = i_\mathrm{s}\mathrm{e}^{\mathrm{j}(\omega_\mathrm{s}t+\varphi_\mathrm{i})} \qquad \boldsymbol{u}_\mathrm{s}^\mathrm{s}(t) = u_\mathrm{s}\mathrm{e}^{\mathrm{j}(\omega_\mathrm{s}t+\varphi_\mathrm{u})}$$

where $\omega_\mathrm{s} = 2\pi \cdot 30$ rad/s, $i_\mathrm{s} = 4.0$ A, $\varphi_\mathrm{i} = 0$, $u_\mathrm{s} = 185$ V, and $\varphi_\mathrm{u} = 85.5°$. Determine the magnetizing inductance. Assume that the stator resistance is unknown.

Solution

(a) The steady-state stator voltage equation is

$$\boldsymbol{u}_\mathrm{s}^\mathrm{s} = R_\mathrm{s}\boldsymbol{i}_\mathrm{s}^\mathrm{s} + \mathrm{j}\omega_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}^\mathrm{s} \tag{1}$$

Since $\omega_\mathrm{s} = 0$, the stator resistance can be solved as

$$R_\mathrm{s} = \boldsymbol{u}_\mathrm{s}^\mathrm{s}/\boldsymbol{i}_\mathrm{s}^\mathrm{s} = 14.8 \text{ V}/4.0 \text{ A} = 3.7\ \Omega$$

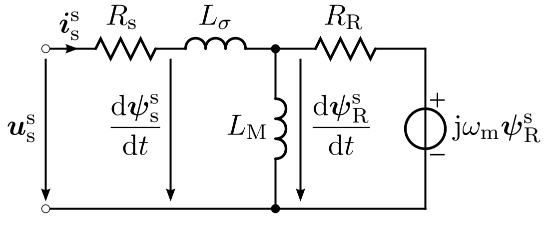
The phase currents are

$$i_\mathrm{a} = \mathrm{Re}\left\{\boldsymbol{i}_\mathrm{s}^\mathrm{s}\right\} = 4.0 \text{ A} \quad i_\mathrm{b} = \mathrm{Re}\left\{\boldsymbol{i}_\mathrm{s}^\mathrm{s}\mathrm{e}^{-2\pi/3}\right\} = -2.0 \text{ A} \quad i_\mathrm{c} = \mathrm{Re}\left\{\boldsymbol{i}_\mathrm{s}^\mathrm{s}\mathrm{e}^{-4\pi/3}\right\} = -2.0 \text{ A}$$

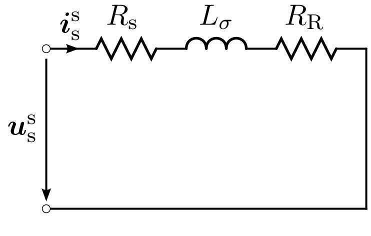
(b) The left figure below shows the dynamic induction motor model. The magnetizing current can be omitted here, since the impedance of the magnetizing branch is much higher than the impedance of the rotor branch at the excitation frequency of 80 Hz. As illustrated in the right figure below, the stator voltage can be approximated as

$$\boldsymbol{u}_\mathrm{s}^\mathrm{s}(t) = R_\sigma\boldsymbol{i}_\mathrm{s}^\mathrm{s}(t) + L_\sigma\frac{\mathrm{d}\boldsymbol{i}_\mathrm{s}^\mathrm{s}(t)}{\mathrm{d}t} \tag{2}$$

where $R_\sigma = R_\mathrm{s} + R_\mathrm{R}$ is the total resistance.



Dynamic model



Approximate dynamic model for $|\omega_\mathrm{s}| \gg |\omega_\mathrm{m}|$

In this problem, only the real part of (2) is needed,

$$u_\alpha(t) = R_\sigma i_\alpha(t) + L_\sigma \frac{\mathrm{d}i_\alpha(t)}{\mathrm{d}t} \tag{3}$$

The current $i_\alpha(t)$ and the voltage $i_\alpha(t)$ vary sinusoidally and they are represented using the peak-valued complex phasors as

$$\boldsymbol{i}_\alpha = i_\mathrm{s}\mathrm{e}^{\mathrm{j}\varphi_\mathrm{i}} \qquad \boldsymbol{u}_\alpha = u_\mathrm{s}\mathrm{e}^{\mathrm{j}\varphi_\mathrm{u}}$$

In the frequency domain, (3) can be expressed as the stator impedance

$$\frac{\boldsymbol{u}_\alpha}{\boldsymbol{i}_\alpha} = R_\sigma + \mathrm{j}\omega_\mathrm{s}L_\sigma$$

Hence, the total resistance is

$$R_\sigma = \mathrm{Re}\left\{\frac{\boldsymbol{u}_\alpha}{\boldsymbol{i}_\alpha}\right\} = \frac{u_\mathrm{s}}{i_\mathrm{s}}\cos(\varphi_\mathrm{u} - \varphi_\mathrm{i}) = \frac{48.2\ \mathrm{V}}{4.0\ \mathrm{A}}\cos(61.2°) = 5.8\ \Omega$$

The rotor resistance is $R_\mathrm{R} = R_\sigma - R_\mathrm{s} = 5.8\ \Omega - 3.7\ \Omega = 2.1\ \Omega$. Similarly, the leakage inductance is

$$L_\sigma = \frac{1}{\omega_\mathrm{s}}\mathrm{Im}\left\{\frac{\boldsymbol{u}_\alpha}{\boldsymbol{i}_\alpha}\right\} = \frac{1}{\omega_\mathrm{s}}\frac{u_\mathrm{s}}{i_\mathrm{s}}\sin(\varphi_\mathrm{u} - \varphi_\mathrm{i}) = \frac{48.2\ \mathrm{V}}{4.0\ \mathrm{A}}\frac{\sin(61.2°)}{2\pi\cdot 80\ \mathrm{rad/s}} = 21\ \mathrm{mH}$$

Remark: The link between the time-domain signal $i_\alpha(t)$ and the complex phasor $\boldsymbol{i}_\alpha$ is

$$i_\alpha(t) = i_\mathrm{s}\cos(\omega_\mathrm{s}t + \varphi_\mathrm{i}) = \mathrm{Re}\left\{\boldsymbol{i}_\alpha\mathrm{e}^{\mathrm{j}\omega_\mathrm{s}t}\right\}$$

and similarly for other signals. In practice, the complex phasors can be easily evaluated from the measured time-domain signals, for example, using the complex discrete Fourier transformation (DFT) at the angular frequency ω_s.

The space vector and the complex phasor are different concepts (although the space-vector models in the steady state typically reduce to the phasor models). The space vector represents a complex-valued signal, which may freely vary in time. On the other hand, the complex phasor represents the magnitude and phase of a sinusoidally varying real-valued signal.

(c) The rotor current is zero at no load. Hence, the stator flux is $\boldsymbol{\psi}_\mathrm{s}^\mathrm{s} = L_\mathrm{s}\boldsymbol{i}_\mathrm{s}^\mathrm{s}$, where $L_\mathrm{s} = L_\sigma + L_\mathrm{M}$ is known as the stator inductance. Based on (1), the stator impedance in the no-load condition is

$$\frac{\boldsymbol{u}_\mathrm{s}^\mathrm{s}}{\boldsymbol{i}_\mathrm{s}^\mathrm{s}} = R_\mathrm{s} + \mathrm{j}\omega_\mathrm{s}L_\mathrm{s}$$

Hence, the stator inductance is

$$L_\mathrm{s} = \frac{1}{\omega_\mathrm{s}}\mathrm{Im}\left\{\frac{\boldsymbol{u}_\mathrm{s}^\mathrm{s}}{\boldsymbol{i}_\mathrm{s}^\mathrm{s}}\right\} = \frac{1}{\omega_\mathrm{s}}\frac{u_\mathrm{s}}{i_\mathrm{s}}\sin(\varphi_\mathrm{u} - \varphi_\mathrm{i}) = \frac{185\ \mathrm{V}}{4.0\ \mathrm{A}}\frac{\sin(85.4°)}{2\pi\cdot 30\ \mathrm{rad/s}} = 245\ \mathrm{mH}$$

The magnetizing inductance is $L_\mathrm{M} = L_\mathrm{s} - L_\sigma = 245\ \mathrm{mH} - 21\ \mathrm{mH} = 224\ \mathrm{mH}$.

Remark: The saturation curve could be determined by repeating the no-load test at different current levels.