## Problem 1: Flux estimation based on the current model

The rotor flux of the induction motor can be estimated in many ways. If the drive is equipped with a speed sensor, a current-model based flux estimator in synchronous coordinates can be used:

$$\frac{\mathrm{d}\hat{\psi}_{\mathrm{R}}}{\mathrm{d}t} = \hat{R}_{\mathrm{R}}\left(i_{\mathrm{d}} - \frac{\hat{\psi}_{\mathrm{R}}}{\hat{L}_{\mathrm{M}}}\right) \quad \text{and} \quad \frac{\mathrm{d}\hat{\vartheta}_{\mathrm{s}}}{\mathrm{d}t} = \omega_{\mathrm{m}} + \frac{\hat{R}_{\mathrm{R}}i_{\mathrm{q}}}{\hat{\psi}_{\mathrm{R}}}$$

where $\hat{\psi}_{\mathrm{R}}$ is the estimate of the flux magnitude and $\hat{\vartheta}_{\mathrm{s}}$ is the estimate of the flux angle. Derive these equations starting from the voltage and flux equations in stator coordinates. Draw also the block diagram of the rotor-flux-oriented control system.

Solution

The dynamic equivalent circuit in stator coordinates is shown in the figure.
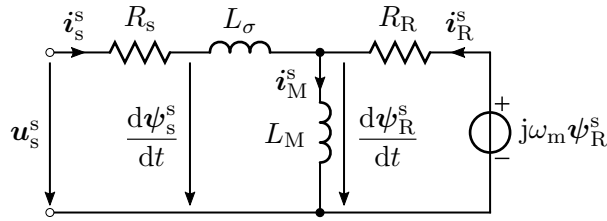
The hats denoting the estimated variables are not used in this solution for simplicity. The rotor voltage equation in stator coordinates is

$$\frac{\mathrm{d}\boldsymbol{\psi}_{\mathrm{R}}^{\mathrm{s}}}{\mathrm{d}t} = -R_{\mathrm{R}}\boldsymbol{i}_{\mathrm{R}}^{\mathrm{s}} + \mathrm{j}\omega_{\mathrm{m}}\boldsymbol{\psi}_{\mathrm{R}}^{\mathrm{s}}$$

which can be expressed in synchronous coordinates by substituting $\mathrm{d}/\mathrm{d}t \to \mathrm{d}/\mathrm{d}t + \mathrm{j}\omega_{\mathrm{s}}$:

$$\frac{\mathrm{d}\boldsymbol{\psi}_{\mathrm{R}}}{\mathrm{d}t} = -R_{\mathrm{R}}\boldsymbol{i}_{\mathrm{R}} - \mathrm{j}(\omega_{\mathrm{s}} - \omega_{\mathrm{m}})\boldsymbol{\psi}_{\mathrm{R}} \tag{1}$$

The rotor flux (in any coordinate system) is

$$\boldsymbol{\psi}_{\mathrm{R}} = L_{\mathrm{M}}(\boldsymbol{i}_{\mathrm{s}} + \boldsymbol{i}_{\mathrm{R}})$$

yielding the rotor current

$$\boldsymbol{i}_{\mathrm{R}} = \frac{\boldsymbol{\psi}_{\mathrm{R}}}{L_{\mathrm{M}}} - \boldsymbol{i}_{\mathrm{s}} \tag{2}$$

Inserting (2) into (1) gives the rotor-flux dynamics as a function of the stator current

$$\frac{\mathrm{d}\boldsymbol{\psi}_{\mathrm{R}}}{\mathrm{d}t} = R_{\mathrm{R}}\boldsymbol{i}_{\mathrm{s}} - \left[\frac{R_{\mathrm{R}}}{L_{\mathrm{M}}} + \mathrm{j}(\omega_{\mathrm{s}} - \omega_{\mathrm{m}})\right]\boldsymbol{\psi}_{\mathrm{R}} \tag{3}$$

In rotor-flux orientation, the d-axis of the coordinate system is fixed to the rotor flux vector, i.e., $\boldsymbol{\psi}_{\mathrm{R}} = \psi_{\mathrm{R}} + \mathrm{j}0$. The real and imaginary parts of (3) are separated

$$\frac{\mathrm{d}\psi_{\mathrm{R}}}{\mathrm{d}t} = R_{\mathrm{R}}(i_{\mathrm{d}} + \mathrm{j}i_{\mathrm{q}}) - \left[\frac{R_{\mathrm{R}}}{L_{\mathrm{M}}} + \mathrm{j}(\omega_{\mathrm{s}} - \omega_{\mathrm{m}})\right]\psi_{\mathrm{R}}$$

$$= R_{\mathrm{R}}i_{\mathrm{d}} - \frac{R_{\mathrm{R}}}{L_{\mathrm{M}}}\psi_{\mathrm{R}} + \mathrm{j}\underbrace{\left[R_{\mathrm{R}}i_{\mathrm{q}} - (\omega_{\mathrm{s}} - \omega_{\mathrm{m}})\psi_{\mathrm{R}}\right]}_{=0}$$

The dynamics of the rotor-flux magnitude are obtained from the real part:

$$\frac{d\psi_R}{dt} = R_R \left(i_d - \frac{\psi_R}{L_M} \right) \tag{4}$$

The angular frequency of the rotor flux is obtained from the imaginary part:

$$\frac{d\vartheta_s}{dt} = \omega_s = \omega_m + \frac{R_R i_q}{\psi_R} \tag{5}$$

The magnitude and angle of the rotor flux can be estimated using (4) and (5). The angle ϑ_s of the rotor flux is obtained by integrating ω_s. The block diagram of the rotor-flux-oriented control system is shown in the figure.
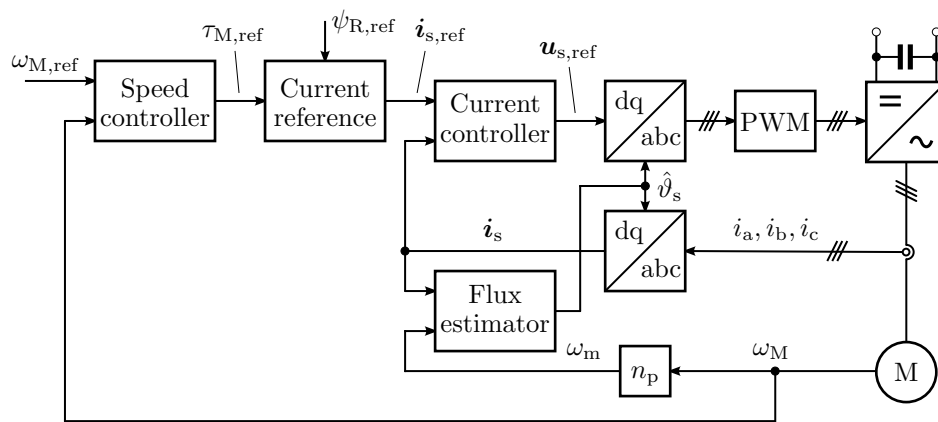
## Problem 2: Calculation of operating points

The rated values of a three-phase induction motor are: $U_N = 400$ V; $f_N = 50$ Hz; $n_N = 1460$ r/min. The motor parameters are:

$$R_s = 0.45 \ \Omega \qquad R_R = 0.28 \ \Omega \qquad L_M = 0.075 \ \text{H} \qquad L_\sigma = 7.1 \ \text{mH}$$

(a) Rotor-flux-oriented vector control is used. Calculate the stator current components i_d and i_q as well as the mechanical power in the rated operating point.

(b) The motor is driven in the field-weakening region. Evaluate the slip angular frequency and the torque in the steady state when i_d is reduced to 50% but i_q is kept constant. Calculate also the stator frequency and the stator voltage assuming that the rotor speed is two times the rated speed.

Solution

(a) The steady-state equivalent circuit is shown in the figure.
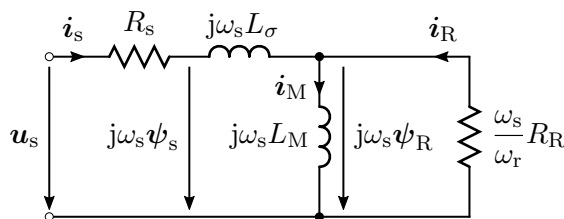
Based on the rated speed and frequency, the number of pole pairs is $n_\mathrm{p} = 2$. In the rated operating point, the angular stator frequency is $\omega_\mathrm{s} = 2\pi \cdot 50$ rad/s. Furthermore, the electrical angular rotor speed is

$$\omega_\mathrm{m} = n_\mathrm{p}\omega_\mathrm{M} = 2 \cdot 2\pi \cdot \frac{1\,460 \text{ r/min}}{60 \text{ s/min}} = 2\pi \cdot 48.67 \text{ rad/s}$$

and the slip angular frequency is $\omega_\mathrm{r} = \omega_\mathrm{s} - \omega_\mathrm{m} = 2\pi \cdot 1.33$ rad/s. The impedance seen from the stator terminals can be expressed as

$$\boldsymbol{Z}_\mathrm{s} = R_\mathrm{s} + \mathrm{j}\omega_\mathrm{s}L_\sigma + \frac{\mathrm{j}\omega_\mathrm{s}L_\mathrm{M}(\omega_\mathrm{s}/\omega_\mathrm{r})R_\mathrm{R}}{\mathrm{j}\omega_\mathrm{s}L_\mathrm{M} + (\omega_\mathrm{s}/\omega_\mathrm{r})R_\mathrm{R}} = R_\mathrm{s} + \mathrm{j}\omega_\mathrm{s}\left(L_\sigma + \frac{L_\mathrm{M}}{1 + \mathrm{j}\omega_\mathrm{r}T_\mathrm{r}}\right)$$

$$= 0.45 \text{ } \Omega + \mathrm{j} \cdot 2\pi \cdot 50 \cdot \left(0.0071 + \frac{0.075}{1 + \mathrm{j} \cdot 2\pi \cdot 1.33 \cdot 0.075/0.28}\right) \text{ } \Omega$$

$$= 11.1\underline{/33.7^\circ} \text{ } \Omega$$

where $T_\mathrm{r} = L_\mathrm{M}/R_\mathrm{R}$ s is the rotor time constant. The magnitude of the rated stator voltage is $u_\mathrm{s} = \sqrt{2/3} \cdot 400$ V $= 326.6$ V. The stator current is

$$\boldsymbol{i}_\mathrm{s} = \frac{\boldsymbol{u}_\mathrm{s}}{\boldsymbol{Z}_\mathrm{s}} = \frac{326.6\underline{/\phi_\mathrm{u}} \text{ V}}{11.1\underline{/33.7^\circ} \text{ } \Omega} = 29.5\underline{/\phi_\mathrm{u} - 33.7^\circ} \text{ A}$$

where ϕ_u is the angle of the stator voltage. Based on the equivalent circuit, the voltage across the parallel connection of the impedances $\mathrm{j}\omega_\mathrm{s}L_\mathrm{M}$ and $(\omega_\mathrm{s}/\omega_\mathrm{r})R_\mathrm{R}$ is

$$\mathrm{j}\omega_\mathrm{s}\boldsymbol{\psi}_\mathrm{R} = \frac{\mathrm{j}\omega_\mathrm{s}L_\mathrm{M}(\omega_\mathrm{s}/\omega_\mathrm{r})R_\mathrm{R}}{\mathrm{j}\omega_\mathrm{s}L_\mathrm{M} + (\omega_\mathrm{s}/\omega_\mathrm{r})R_\mathrm{R}}\boldsymbol{i}_\mathrm{s}$$

Hence, the rotor flux is

$$\boldsymbol{\psi}_\mathrm{R} = \frac{L_\mathrm{M}}{1 + \mathrm{j}\omega_\mathrm{r}T_\mathrm{r}}\boldsymbol{i}_\mathrm{s} = \frac{0.075 \text{ H}}{1 + \mathrm{j} \cdot 2\pi \cdot 1.33 \cdot 0.075/0.28} \cdot 29.5\underline{/\phi_\mathrm{u} - 33.7^\circ} \text{ A}$$

$$= 0.031\underline{/-66.0^\circ} \cdot 29.5\underline{/\phi_\mathrm{u} - 33.7^\circ} \text{ Vs}$$

$$= 0.90\underline{/\phi_\mathrm{u} - 99.6^\circ} \text{ Vs}$$

Let us fix the coordinates to the rotor flux by choosing $\phi_\mathrm{u} = 99.6^\circ$, leading to $\boldsymbol{\psi}_\mathrm{R} = \psi_\mathrm{R} = 0.90$ Vs. In these coordinates, the stator current is

$$\boldsymbol{i}_\mathrm{s} = i_\mathrm{d} + \mathrm{j}i_\mathrm{q} = 29.5\underline{/99.6^\circ - 33.7^\circ} \text{ A} = (12.0 + \mathrm{j}26.9) \text{ A}$$

or the components are $i_\mathrm{d} = 12.0$ A and $i_\mathrm{q} = 26.9$ A. The electromagnetic torque and the mechanical power in the rated operating point are

$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\psi_\mathrm{R}i_\mathrm{q} = \frac{3}{2}\cdot 2 \cdot 0.90 \cdot 26.9 \text{ Nm} = 72.6 \text{ Nm}$$

$$p_\mathrm{M} = \tau_\mathrm{M}\omega_\mathrm{M} = 72.6 \text{ Nm} \cdot \frac{2\pi \cdot 48.67 \text{ rad/s}}{2} = 11.1 \text{ kW}$$

respectively.

(b) When the current component i_d is reduced to half of its rated value, the rotor flux magnitude is half of the rated as well: $\psi_\mathrm{R} = L_\mathrm{M}i_\mathrm{d} = 0.075 \cdot 0.5 \cdot 12.0$ Vs $= 0.45$ Vs. Since the current component i_q is not changed, the electromagnetic torque is half of its rated value

$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\psi_\mathrm{R}i_\mathrm{q} = \frac{3}{2}\cdot 2 \cdot 0.45 \cdot 26.9 \text{ Nm} = 36.3 \text{ Nm}$$

and the slip angular frequency is doubled

$$\omega_\mathrm{r} = \frac{R_\mathrm{R}i_\mathrm{q}}{\psi_\mathrm{R}} = 2 \cdot 2\pi \cdot 1.33 \text{ rad/s} = 2\pi \cdot 2.66 \text{ rad/s}$$

If the angular speed of the rotor is twice the rated value, i.e., $\omega_\mathrm{m} = 2 \cdot 2\pi \cdot 48.67$ rad/s $= 2\pi \cdot 97.33$ rad/s, the corresponding angular stator frequency is doubled:

$$\omega_\mathrm{s} = \omega_\mathrm{m} + \omega_\mathrm{r} = 2 \cdot 2\pi \cdot 50 \text{ rad/s} = 2\pi \cdot 100 \text{ rad/s}$$

According to the equivalent circuit, the stator voltage is

$$\begin{aligned}\boldsymbol{u}_\mathrm{s} &= (R_\mathrm{s} + \mathrm{j}\omega_\mathrm{s}L_\sigma)\boldsymbol{i}_\mathrm{s} + \mathrm{j}\omega_\mathrm{s}\boldsymbol{\psi}_\mathrm{R} \\ &= [(0.45 + \mathrm{j}\cdot 2\pi \cdot 100 \cdot 0.0071)\cdot(6.0 + \mathrm{j}26.9) + \mathrm{j}\cdot 2\pi \cdot 100 \cdot 0.45] \text{ V} \\ &= (-117.3 + \mathrm{j}321.6) \text{ V} = 342.3\underline{/110.0°} \text{ V}\end{aligned}$$

The voltage is slightly larger than in the rated operating point.

Remark: The vector diagrams are shown below. The angle between the stator flux vector and the stator voltage vector is approximately 90° since $\boldsymbol{u}_\mathrm{s} = \mathrm{j}\omega_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}$ if $R_\mathrm{s} = 0$ is assumed. The angle φ corresponding to the displacement power factor $\cos\varphi$ is also drawn on the left figure.


(a)   (b)
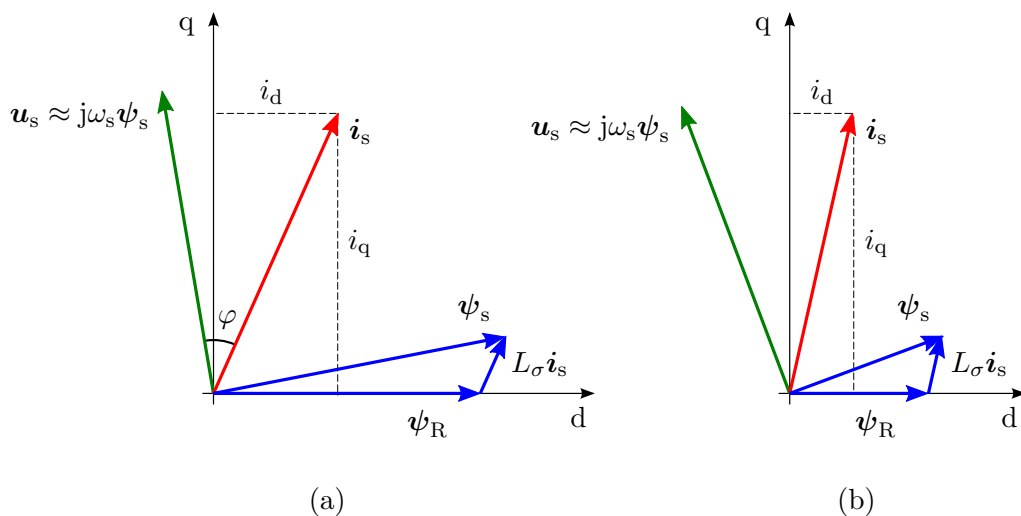
Problem 3: Flux dynamics

The rotor flux of an induction motor is controlled by the stator current component i_d in rotor-flux coordinates. At $t = 0$, a stepwise change Δi_d is assumed in the d-component of the stator current. Derive expressions for the change $\Delta\psi_R(t)$ in the rotor flux and for the change $\Delta i_{Rd}(t)$ in the d-component of the rotor current (that is parallel to the rotor flux).

Solution

The figure shows the original vector diagram and the vector diagram immediately after a stepwise change Δi_d in the d-component of the stator current. The rotor flux of a short-circuited rotor winding cannot change abruptly (whereas the rotor current can, for example, due to abruptly changing flux components imposed from outside the winding). In the beginning of the transient, the d-component of the rotor current changes stepwise, $\Delta i_{Rd} = -\Delta i_d$, and cancels the change in the rotor flux caused by the stator current.
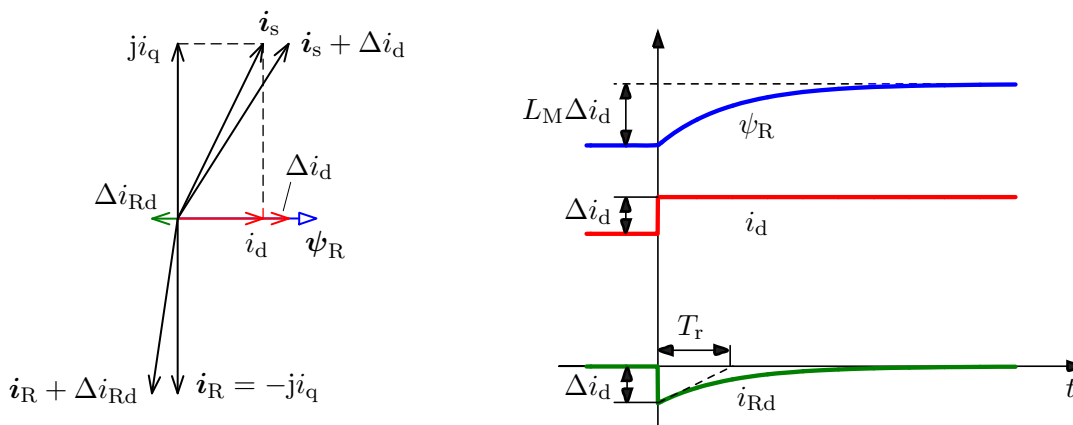
Using the stator current and the rotor flux as variables, the real part of the rotor equation in rotor-flux coordinates is

$$\frac{d\psi_R}{dt} = R_R\left(i_d - \frac{\psi_R}{L_M}\right)$$

The Laplace transform

$$s\Delta\psi_R(s) = R_R[\Delta i_d(s) - \Delta\psi_R(s)/L_M]$$

can be used for deriving the expressions for the changes. Since the changes are considered, the initial values were assumed to be zero. Hence, the change in the rotor flux is

$$\Delta\psi_R(s) = \frac{L_M}{T_r s + 1}\Delta i_d(s) = L_M\frac{1/T_r}{s + 1/T_r}\frac{\Delta i_d}{s}$$

where $\Delta i_d(s) = \Delta i_d/s$ holds for a step change and $T_r = L_M/R_R$ is the rotor time constant. The inverse Laplace transform is

$$\Delta\psi_R(t) = L_M\Delta i_d\left(1 - e^{-t/T_r}\right)$$

The rotor flux approaches its steady-state value at a rate corresponding to the time constant T_r. The change in the rotor current can be calculated using the flux equation $\psi_R = L_M(i_d + i_{Rd})$:

$$\Delta i_{Rd}(t) = \frac{\Delta \psi_R(t)}{L_M} - \Delta i_d = -\Delta i_d e^{-t/T_r}$$

The d-component of the rotor current also approaches its steady-state value (which is zero) at a rate corresponding to the time constant T_r.