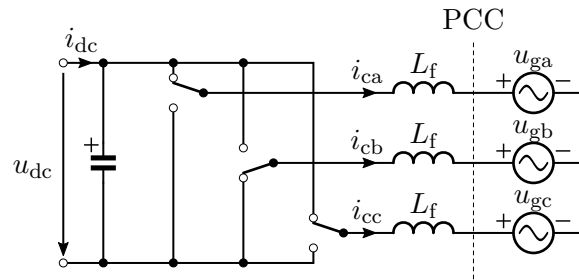


**Problem 1: An operating point of a grid converter**

The figure below shows a grid converter, whose DC voltage is  $u_{dc} = 600$  V, DC current is  $i_{dc} = 10$  A, and filter inductance is  $L_f = 10$  mH. The electric grid is assumed to be a balanced three-phase voltage source with frequency of 50 Hz and phase-to-phase rms voltage of 400 V. The displacement power factor at the PCC is controlled to unity. The converter can be assumed to be lossless and switching-cycle-averaged quantities are considered.

- (a) Calculate the converter current vector in grid-voltage coordinates.
- (b) Calculate the magnitude of the converter output voltage vector.



**Solution**

- (a) In grid-voltage coordinates,

$$\mathbf{u}_g = u_{gd} + j u_{gq} = u_g + j0 = \sqrt{2/3} \cdot 400 \text{ V} = 326.6 \text{ V}$$

holds. The converter output current is  $\mathbf{i}_c = i_{cd} + j i_{cq}$ . Hence, the real power and reactive power at the PCC are

$$p_g = \frac{3}{2} \text{Re} \{ \mathbf{u}_g \mathbf{i}_c^* \} = \frac{3}{2} u_g i_{cd} \quad q_g = \frac{3}{2} \text{Im} \{ \mathbf{u}_g \mathbf{i}_c^* \} = -\frac{3}{2} u_g i_{cq}$$

respectively. The real power in the steady state is

$$p_g = u_{dc} i_{dc} = 600 \text{ V} \cdot 10 \text{ A} = 6 \text{ kW}$$

since the converter is assumed to be lossless. The displacement power factor is unity, corresponding to the reactive power  $q_g = 0$ . The d and q currents are

$$i_{cd} = \frac{2p_g}{3u_g} = \frac{2 \cdot 6 \text{ kW}}{3 \cdot 326.6 \text{ V}} = 12.2 \text{ A} \quad i_{cq} = -\frac{2q_g}{3u_g} = 0$$

**Remark:** The power flows from the DC link to the grid, i.e., the converter operates as an inverter in this operating point. Due to the inductance  $L_f$ , the reactive power at the output of the converter,  $\frac{3}{2} \text{Im} \{ \mathbf{u}_c \mathbf{i}_c^* \}$ , is nonzero, where  $\mathbf{u}_c$  is the converter output voltage, see Part (b).

- (b) The converter output voltage in synchronous coordinates is

$$\mathbf{u}_c = L_f \frac{d\mathbf{i}_c}{dt} + j\omega_g L_f \mathbf{i}_c + \mathbf{u}_g$$

In grid-voltage coordinates, the steady-state voltage is

$$\mathbf{u}_c = j\omega_g L_f \dot{\mathbf{i}}_c + u_g = u_g - \omega_g L_f i_{cq} + j\omega_g L_f i_{cd}$$

Hence, the components are

$$\begin{aligned} u_{cd} &= u_g - \omega_g L_f i_{cq} = 326.6 \text{ V} \\ u_{cq} &= \omega_g L_f i_{cd} = 2\pi \cdot 50 \text{ rad/s} \cdot 0.01 \text{ H} \cdot 12.2 \text{ A} = 38.4 \text{ V} \end{aligned}$$

The magnitude of the converter output voltage is

$$u_c = \sqrt{u_{cd}^2 + u_{cq}^2} = \sqrt{(326.6 \text{ V})^2 + (38.4 \text{ V})^2} = 329 \text{ V}$$

**Remark:** For comparison, the maximum voltage in the linear modulation region is  $u_{dc}/\sqrt{3} = 600 \text{ V}/\sqrt{3} = 346 \text{ V}$ .

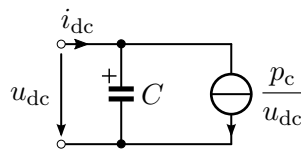
### Problem 2: DC-link voltage controller

A PI controller is used to regulate the DC-link voltage of a power converter,

$$p_{c,\text{ref}} = -k_p(W_{\text{dc},\text{ref}} - W_{\text{dc}}) - k_i \int (W_{\text{dc},\text{ref}} - W_{\text{dc}}) dt$$

where  $p_{c,\text{ref}}$  is the reference of the converter output power,  $W_{\text{dc}} = (C/2)u_{\text{dc}}^2$  is the energy of the DC-link capacitor, and  $W_{\text{dc},\text{ref}} = (C/2)u_{\text{dc},\text{ref}}^2$  is its reference. Power control is assumed to be ideal, i.e.  $p_c = p_{c,\text{ref}}$ . The input power  $p_{\text{dc}} = u_{\text{dc}}i_{\text{dc}}$  is an unknown disturbance.

- Calculate the closed-loop transfer functions  $W_{\text{dc}}(s)/W_{\text{dc},\text{ref}}(s)$  and  $W_{\text{dc}}(s)/p_{\text{dc}}(s)$ .
- Express the controller gains  $k_p$  and  $k_i$  as functions of the damping ratio  $\zeta$  and the undamped natural frequency  $\omega_0$  of the closed-loop characteristic polynomial.



### Solution

- The dynamics of the DC link are governed by

$$\frac{dW_{\text{dc}}}{dt} = p_{\text{dc}} - p_c \quad (1)$$

where  $W_{\text{dc}} = (C/2)u_{\text{dc}}^2$  is the energy of the DC capacitor and the power  $p_{\text{dc}} = u_{\text{dc}}i_{\text{dc}}$  is an unknown load disturbance. The converter output power is

$$p_c = -k_p(W_{\text{dc},\text{ref}} - W_{\text{dc}}) - k_i \int (W_{\text{dc},\text{ref}} - W_{\text{dc}}) dt \quad (2)$$

since ideal power control is assumed. Substituting the control law (2) in (1) and Laplace transforming yields

$$W_{\text{dc}}(s) = \frac{sk_p + k_i}{s^2 + sk_p + k_i} W_{\text{dc,ref}}(s) + \frac{s}{s^2 + sk_p + k_i} p_{\text{dc}}(s)$$

It can be seen that  $W_{\text{dc}} = W_{\text{dc,ref}}$  or  $u_{\text{dc}} = u_{\text{dc,ref}}$  in the steady state ( $s = 0$ ) due to the integral action of the control law. The transient response depends on the controller gains.

- (b) The characteristic polynomial of the second-order system can be expressed as  $s^2 + 2\zeta\omega_0 s + \omega_0^2$ . Hence, the gains are

$$k_p = 2\zeta\omega_0 \quad k_i = \omega_0^2$$

Typically, it is desirable to select the damping ratio in a range  $\zeta = 0.7 \dots 1$ . A critically damped system is obtained if  $\zeta = 1$ , i.e. there is a double pole at  $s = -\omega_0$ . The assumption on the ideal power control holds well if the undamped natural frequency  $\omega_0$  is much lower than the current-control bandwidth.