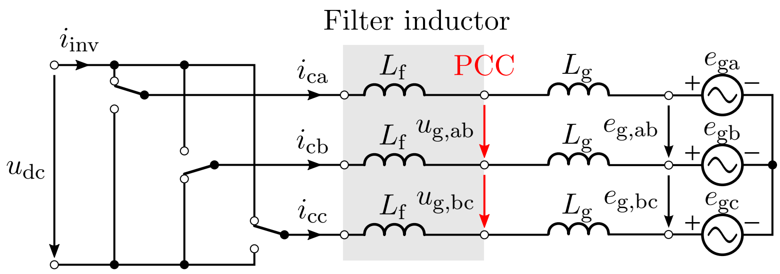
Problem 1: operating point of a grid forming converter

The figure below shows a 10-kVA grid converter, and filter inductance is $L_\mathrm{f} = 5$ mH. The electric grid is assumed to be a balanced three-phase voltage source with frequency of 50 Hz and phase-to-phase rms voltage of 400 V with a grid inductance $L_g = 15$ mH. The converter can be assumed to be lossless and switching-cycle-averaged quantities are considered. For this application, Grid forming (GFM) control method is studied and then compared to grid following (GFL) in terms of operating points. The active power reference is $p_\mathrm{c,ref} = 8$ kW and the voltage magnitude reference u_ref and grid nominal voltage e_g^N are $u_\mathrm{ref} = e_\mathrm{g}^\mathrm{N} = \sqrt{2/3} \cdot 400$ V. The corresponding load angle between the grid voltage $\boldsymbol{e}_\mathrm{g}$ and the converter voltage $\boldsymbol{u}_\mathrm{c}$ is $\delta = \vartheta_\mathrm{c} - \vartheta_\mathrm{g} = 0.32$ rad.

(a) Calculate the converter current vector and the PCC voltage magnitude in steady state when a grid forming (GFM) converter is used.

(b) Calculate the converter current vector and the PCC voltage magnitude in steady state when a grid following (GFL) converter is used with the same active power reference and when the reactive power reference is set to zero.



Solution

(a) In the converter-voltage coordinates with GFM control,

$$\boldsymbol{u}_\mathrm{c} = \boldsymbol{u}_\mathrm{ref} = u_\mathrm{ref} = \sqrt{2/3} \cdot 400 \text{ V} = 326.6 \text{ V}$$

holds. The converter output current is $\boldsymbol{i}_\mathrm{c} = i_\mathrm{cd} + \mathrm{j}i_\mathrm{cq}$, which can be calculated using circuit theory

$$\begin{aligned}
\mathrm{j}\omega_\mathrm{c}(L_\mathrm{f} + L_\mathrm{g})\boldsymbol{i}_\mathrm{c} &= \boldsymbol{u}_\mathrm{c} - \boldsymbol{e}_\mathrm{g} \\
\mathrm{j}\omega_\mathrm{c}(L_\mathrm{f} + L_\mathrm{g})\boldsymbol{i}_\mathrm{c} &= \boldsymbol{u}_\mathrm{ref} - e_\mathrm{g}^\mathrm{N} \cdot e^{-\mathrm{j}\delta} \\
\omega_\mathrm{c}(L_\mathrm{f} + L_\mathrm{g})(-i_\mathrm{cq} + \mathrm{j}i_\mathrm{cd}) &= u_\mathrm{ref} + 0\mathrm{j} - e_\mathrm{g}^\mathrm{N}\cos(\delta) + \mathrm{j}e_\mathrm{g}^\mathrm{N}\sin\delta
\end{aligned}$$

The active and reactive currents can then be calculated as

$$i_\mathrm{cd} = \frac{e_\mathrm{g}^\mathrm{N}\sin\delta}{\omega_\mathrm{c}(L_\mathrm{f} + L_\mathrm{g})} = 16.3 \text{ A} \qquad i_\mathrm{cq} = \frac{e_\mathrm{g}^\mathrm{N}\cos\delta - u_\mathrm{ref}}{\omega_\mathrm{c}(L_\mathrm{f} + L_\mathrm{g})} = -2.6 \text{ A}$$

respectively. The PCC voltage $\boldsymbol{u}_\mathrm{g}$ can then be expressed as

$$\boldsymbol{u}_\mathrm{g} = u_\mathrm{gd} + \mathrm{j}u_\mathrm{gq} = \boldsymbol{u}_\mathrm{c} - \mathrm{j}\omega_\mathrm{c}L_\mathrm{f}\boldsymbol{i}_\mathrm{c} = \boldsymbol{u}_\mathrm{ref} - \mathrm{j}\omega_\mathrm{c}L_\mathrm{f}\boldsymbol{i}_\mathrm{c}$$

The dq components of $\boldsymbol{u}_\mathrm{g}$ are

$$u_\mathrm{gd} = u_\mathrm{ref} + \omega_\mathrm{c} L_\mathrm{f} i_\mathrm{cq} = 322.5 \text{ V} \qquad u_\mathrm{gq} = 0 - \omega_\mathrm{c} L_\mathrm{f} i_\mathrm{cd} = -25.7 \text{ V}$$

The magnitude of the PCC voltage is

$$|\boldsymbol{u}_\mathrm{g}| = \sqrt{u_\mathrm{gd}^2 + u_\mathrm{gq}^2} = \sqrt{(322.5 \text{ V})^2 + (-25.7 \text{ V})^2} = 323.5 \text{ V}$$

Remark: It is interesting to note that in its nominal operating point, the grid forming converter feeds q-axis current even though no reactive power reference was provided. This is caused by the voltage reference used in the scheme ($\boldsymbol{u}_\mathrm{ref}$), which implicitly forces some q-axis current to obtain the desired voltage at the converter output.

(b) This time, the converter is controlled in GFL. The coordinates of this control scheme is synchronized with the PCC voltage through the phase-locked loop (PLL). The current setpoints are

$$i_\mathrm{cd} = \frac{2p_\mathrm{g}}{3u_\mathrm{ref}} = \frac{2 \cdot 8 \text{ kW}}{3 \cdot 326.6 \text{ V}} = 16.3 \text{ A} \qquad i_\mathrm{cq} = -\frac{2q_\mathrm{g}}{3u_\mathrm{ref}} = 0$$

The PCC voltage is

$$\boldsymbol{u}_\mathrm{g} = u_\mathrm{gd} + \mathrm{j}u_\mathrm{gq}$$

Because the converter synchronous frame is aligned with the PCC voltage here

$$u_\mathrm{gd} = |\boldsymbol{u}_\mathrm{g}| \qquad u_\mathrm{gq} = 0$$

And the steady state equations in the PLL frame is

$$\boldsymbol{u}_\mathrm{g} = \mathrm{j}\omega_\mathrm{c} L_\mathrm{g} \boldsymbol{i}_\mathrm{c} + \boldsymbol{e}_\mathrm{g}$$

And the dq components can be expressed as

$$|\boldsymbol{u}_\mathrm{g}| = -\omega_\mathrm{c} L_\mathrm{g} i_\mathrm{cq} + e_\mathrm{gd} \qquad 0 = \omega_\mathrm{c} L_\mathrm{g} i_\mathrm{cd} + e_\mathrm{gq}$$

And e_gq can be calculated

$$e_\mathrm{gq} = -\omega_\mathrm{c} L_\mathrm{g} i_\mathrm{cd} = -77.0 \text{ V}$$

Then, since the grid voltage is constant $|\boldsymbol{e}_\mathrm{g}| = e_\mathrm{g}^\mathrm{N} = \sqrt{2/3} \cdot 400$ V

$$e_\mathrm{gd} = \sqrt{|\boldsymbol{e}_\mathrm{g}|^2 - e_\mathrm{gq}^2} = \sqrt{(326.6 \text{ V})^2 + (-77.0 \text{ V})^2} = 317.4 \text{ V}$$

And finally this gives

$$|\boldsymbol{u}_\mathrm{g}| = -\omega_\mathrm{c} L_\mathrm{g} i_\mathrm{cq} + e_\mathrm{gd} = 317.4 \text{ V}$$

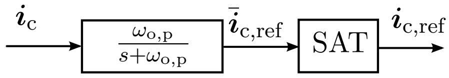
Remark: As it could be expected since the chosen control method affects the q-axis current, the two PCC voltage magnitudes are slightly different.

Problem 2: Current limitation of a grid forming converter

This exercise is about the current limitation action embedded in the control law presented in the lecture. There, an active resistance is used to damp the current oscillations by acting on the converter output voltage

$$\boldsymbol{u}_{\mathrm{c,ref}} = \boldsymbol{u}_{\mathrm{ref}} + R_{\mathrm{a}}(\boldsymbol{i}_{\mathrm{c,ref}} - \boldsymbol{i}_{\mathrm{c}})$$

where R_{a} is the active resistance gain, $\boldsymbol{u}_{\mathrm{ref}} = u_{\mathrm{ref}} + 0\mathrm{j}$ is the voltage reference and $\boldsymbol{i}_{\mathrm{c,ref}}$ is defined in the Figure below, using block diagram.



This current is named the current reference and can be limited to a user-defined value, here denoted by i_{max} in order to mitigate the current transients and to avoid too high overcurrents. In this case q-axis current is prioritized as

$$\boldsymbol{i}_{\mathrm{c,ref}} = \begin{cases} 0 - \mathrm{j}i_{\mathrm{max}} & \text{if} \quad |\bar{\boldsymbol{i}}_{\mathrm{c,ref}}| > i_{\mathrm{max}} \\ \bar{\boldsymbol{i}}_{\mathrm{c,ref}} & \text{otherwise} \end{cases}$$

A three-phase short circuit occurs when the converter is providing zero active power, forcing the grid voltage to zero such that $|\boldsymbol{e}_{\mathrm{g}}| = 0$, thus activating the current limitation action. In this problem, the converter frequency is assumed to be constant in steady state ($\omega_{\mathrm{c}} = \omega_{\mathrm{g,ref}}$). In this problem, $i_{\mathrm{max}} = 30$ A and $R_{\mathrm{a}} = 3.2\ \Omega$.

(a) Calculate the steady-state converter current vector $\boldsymbol{i}_{\mathrm{c}}$ and its magnitude during the fault state ($|\boldsymbol{e}_{\mathrm{g}}| = 0$).

(b) Calculate the corresponding converter output and PCC voltage magnitudes during the fault state.

Solution

(a) When switching-cycle-averaged quantities are considered and delay is omitted

$$\boldsymbol{u}_{\mathrm{c}} = \boldsymbol{u}_{\mathrm{c,ref}}$$

Then, the dynamics of the AC side are given by, in the converter-voltage coordinates

$$\begin{aligned}(L_{\mathrm{f}} + L_{\mathrm{g}})\frac{d\boldsymbol{i}_{\mathrm{c}}}{dt} &= \boldsymbol{u}_{\mathrm{c}} - \boldsymbol{e}_{\mathrm{g}} - \mathrm{j}\omega_{\mathrm{c}}(L_{\mathrm{f}} + L_{\mathrm{g}})\boldsymbol{i}_{\mathrm{c}} \\ &= \boldsymbol{u}_{\mathrm{ref}} + R_{\mathrm{a}}(\boldsymbol{i}_{\mathrm{c,ref}} - \boldsymbol{i}_{\mathrm{c}}) - 0 - \mathrm{j}\omega_{\mathrm{c}}(L_{\mathrm{f}} + L_{\mathrm{g}})\boldsymbol{i}_{\mathrm{c}} \\ &= \boldsymbol{u}_{\mathrm{ref}} + R_{\mathrm{a}}(-\mathrm{j}i_{\mathrm{max}} - \boldsymbol{i}_{\mathrm{c}}) - 0 - \mathrm{j}\omega_{\mathrm{c}}(L_{\mathrm{f}} + L_{\mathrm{g}})\boldsymbol{i}_{\mathrm{c}} = 0\end{aligned}$$

where the notation is similar as in the previous exercise. Converter current is then given as

$$\boldsymbol{i}_{\mathrm{c}} = \frac{\boldsymbol{u}_{\mathrm{ref}} - \mathrm{j}R_{\mathrm{a}}i_{\mathrm{max}}}{R_{\mathrm{a}} + \mathrm{j}\omega_{\mathrm{c}}(L_{\mathrm{f}} + L_{\mathrm{g}})} = 8.9\text{ A} - \mathrm{j}47.5\text{ A}$$

and the corresponding current magnitude is

$$|\boldsymbol{i}_{\mathrm{c}}| = \sqrt{i_{\mathrm{cd}}^2 + i_{\mathrm{cq}}^2} = 48.3\text{ A}$$

(b) This is straightforward from the converter control law

$$\begin{aligned} \boldsymbol{u}_\mathrm{c} &= \boldsymbol{u}_\mathrm{c,ref} = \boldsymbol{u}_\mathrm{ref} + R_\mathrm{a}(\boldsymbol{i}_\mathrm{c,ref} - \boldsymbol{i}_\mathrm{c}) \\ &= 326.6 + 3.2(0 - \mathrm{j}30 - 8.9 + \mathrm{j}47.5) \\ \boldsymbol{u}_\mathrm{c} &= u_\mathrm{cd} + \mathrm{j}u_\mathrm{cq} = 298.2\ \mathrm{V} + \mathrm{j}55.8\ \mathrm{V} \end{aligned}$$

And the converter voltage magnitude is

$$|\boldsymbol{u}_\mathrm{c}| = \sqrt{u_\mathrm{cd}^2 + u_\mathrm{cq}^2} = 303.3\ \mathrm{V}$$

Then the PCC voltage can be calculated accordingly

$$\boldsymbol{u}_\mathrm{g} = \boldsymbol{u}_\mathrm{c} - \mathrm{j}\omega_\mathrm{c} L_\mathrm{f} \boldsymbol{i}_\mathrm{c} = u_\mathrm{gd} + \mathrm{j}u_\mathrm{gq} = 223.6\ \mathrm{V} + \mathrm{j}41.9\ \mathrm{V}$$

Thus the PCC voltage magnitude is

$$|\boldsymbol{u}_\mathrm{g}| = \sqrt{u_\mathrm{cd}^2 + u_\mathrm{cq}^2} = 227.5\ \mathrm{V}$$

Remark: In the fault operating point, it is worth noting that the converter current is not driven to the current reference, which, from a control engineering perspective, means that there is a steady-state error.