Problem 1: Characteristics of an interior-permanent-magnet motor

The data of an interior-permanent-magnet synchronous motor is:

rated voltage	$U_\mathrm{N} = 400$ V	direct-axis inductance	$L_\mathrm{d} = 0.06$ H
rated current	$I_\mathrm{N} = 10$ A	quadrature-axis inductance	$L_\mathrm{q} = 0.10$ H
pole pairs	$n_\mathrm{p} = 2$	permanent-magnet flux	$\psi_\mathrm{F} = 1$ Vs

The stator resistance is omitted. Draw the following characteristics in the i_d–i_q plane:

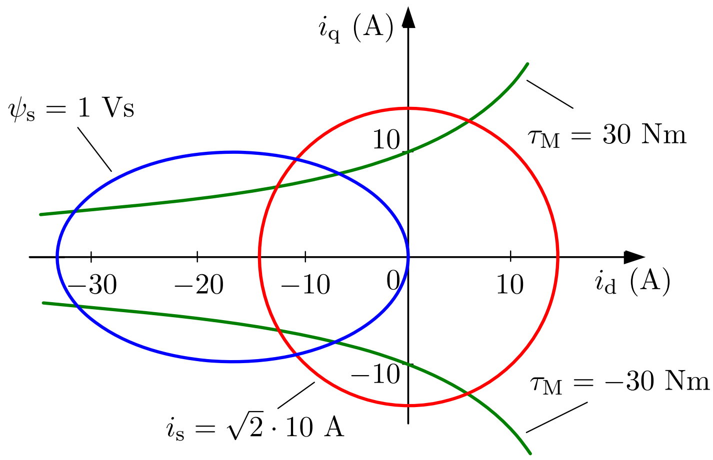
(a) constant current $i_\mathrm{s} = \sqrt{2} I_\mathrm{N}$ (rated value);

(b) constant torque of 30 Nm;

(c) constant stator flux $\psi_\mathrm{s} = 1$ Vs.

Solution

The figure below shows the loci in the i_d–i_q plane corresponding to given constant values.



(a) The square of the stator current can be expressed as

$$i_\mathrm{s}^2 = i_\mathrm{d}^2 + i_\mathrm{q}^2 \quad \text{or} \quad |i_\mathrm{q}| = \sqrt{i_\mathrm{s}^2 - i_\mathrm{d}^2}$$

Hence, the constant stator current in the i_d–i_q plane corresponds to the circle (having the radius of $i_\mathrm{s} = \sqrt{2} \cdot 10$ A in this case). The operating point should be located inside the circle corresponding to the maximum current.

(b) The electromagnetic torque is

$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2} \operatorname{Im}\{\boldsymbol{i}_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}^*\} = \frac{3n_\mathrm{p}}{2} \operatorname{Im}\{(i_\mathrm{d} + \mathrm{j}i_\mathrm{q})(\psi_\mathrm{F} + L_\mathrm{d}i_\mathrm{d} - \mathrm{j}L_\mathrm{q}i_\mathrm{q})\}$$
$$= \frac{3n_\mathrm{p}}{2} [\psi_\mathrm{F}i_\mathrm{q} + (L_\mathrm{d} - L_\mathrm{q})i_\mathrm{d}i_\mathrm{q}]$$

The q-component of the current can be solved as

$$i_\mathrm{q} = \frac{2\tau_\mathrm{M}}{3n_\mathrm{p}[\psi_\mathrm{F} + (L_\mathrm{d} - L_\mathrm{q})i_\mathrm{d}]}$$

The current locus corresponding to the constant torque is a hyperbola. The point of the hyperbola being closest to the origin is an optimal operating point if the stator current is to be minimized.

(c) The square of the stator flux is

$$\psi_\mathrm{s}^2 = \psi_\mathrm{d}^2 + \psi_\mathrm{q}^2 = (\psi_\mathrm{F} + L_\mathrm{d} i_\mathrm{d})^2 + (L_\mathrm{q} i_\mathrm{q})^2$$

from which the q-component of the stator current can be solved as

$$|i_\mathrm{q}| = \frac{\sqrt{\psi_\mathrm{s}^2 - (\psi_\mathrm{F} + L_\mathrm{d} i_\mathrm{d})^2}}{L_\mathrm{q}}$$

The current locus is now an ellipse. The stator-flux magnitude is limited due to the limited voltage ($u_\mathrm{s} \approx \omega_\mathrm{m}\psi_\mathrm{s}$) and the operating point has to be located inside the ellipse. Why does the ellipse touch the origin in this case ($\psi_\mathrm{s} = 1$ Vs)?

Problem 2: Current-minimizing control characteristics

Consider the interior-permanent-magnet motor in the preceding problem.

(a) Derive expressions for the current components i_d and i_q, when the stator current is constant and the torque is maximized.

(b) Calculate the maximum torque at the rated current.

(c) Calculate the rotational base speed corresponding to the rated voltage for the current and torque obtained above.

(d) Calculate the displacement power factor $\cos\varphi$ and draw a vector diagram for the operating point obtained above.

Solution

(a) Maximization of the torque at constant current corresponds to minimization of the current at constant torque. The current component i_q is

$$i_\mathrm{q} = \sqrt{i_\mathrm{s}^2 - i_\mathrm{d}^2} \tag{1}$$

where positive i_q has been assumed. Hence, the torque can be expressed using i_d as

$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\left[\psi_\mathrm{F} + (L_\mathrm{d} - L_\mathrm{q})i_\mathrm{d}\right] i_\mathrm{q} = \frac{3n_\mathrm{p}}{2}\left[\psi_\mathrm{F} + (L_\mathrm{d} - L_\mathrm{q})i_\mathrm{d}\right]\sqrt{i_\mathrm{s}^2 - i_\mathrm{d}^2}$$

where i_s is constant. Differentiation of the torque with respect to i_d leads to

$$\begin{aligned}
\frac{2}{3n_\mathrm{p}}\frac{\mathrm{d}\tau_\mathrm{M}}{\mathrm{d}i_\mathrm{d}} &= \frac{\mathrm{d}}{\mathrm{d}i_\mathrm{d}}\left\{[\psi_\mathrm{F} + (L_\mathrm{d} - L_\mathrm{q})i_\mathrm{d}](i_\mathrm{s}^2 - i_\mathrm{d}^2)^{1/2}\right\} \\
&= (L_\mathrm{d} - L_\mathrm{q})(i_\mathrm{s}^2 - i_\mathrm{d}^2)^{1/2} + [\psi_\mathrm{F} + (L_\mathrm{d} - L_\mathrm{q})i_\mathrm{d}]\cdot\frac{1}{2}\cdot(i_\mathrm{s}^2 - i_\mathrm{d}^2)^{-1/2}\cdot(-2i_\mathrm{d}) \\
&= (L_\mathrm{d} - L_\mathrm{q})\sqrt{i_\mathrm{s}^2 - i_\mathrm{d}^2} - \frac{\psi_\mathrm{F} i_\mathrm{d} + (L_\mathrm{d} - L_\mathrm{q})i_\mathrm{d}^2}{\sqrt{i_\mathrm{s}^2 - i_\mathrm{d}^2}}
\end{aligned}$$

The maximum torque is achieved at $\mathrm{d}\tau_\mathrm{M}/\mathrm{d}i_\mathrm{d} = 0$, leading to

$$i_\mathrm{d}^2 + \frac{\psi_\mathrm{F}}{2(L_\mathrm{d} - L_\mathrm{q})}i_\mathrm{d} - \frac{i_\mathrm{s}^2}{2} = 0 \tag{2}$$

The solutions of this quadratic equation are

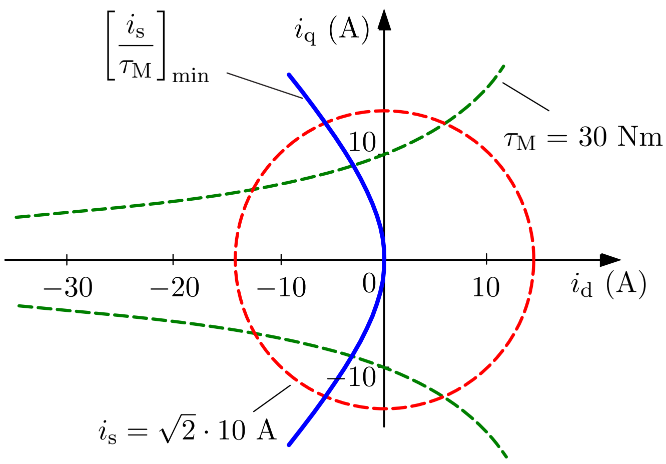
$$i_\mathrm{d} = \frac{\psi_\mathrm{F}}{4(L_\mathrm{q} - L_\mathrm{d})} \pm \sqrt{\frac{\psi_\mathrm{F}^2}{16(L_\mathrm{q} - L_\mathrm{d})^2} + \frac{i_\mathrm{s}^2}{2}} \tag{3}$$

The plus sign in front of the square root can be omitted (at least if $L_\mathrm{q} > L_\mathrm{d}$ as in this problem).

Remark: For plotting the MTPA locus, (2) can be rewritten using (1) as

$$i_\mathrm{q}^2 = i_\mathrm{d}^2 + \frac{\psi_\mathrm{F}}{L_\mathrm{d} - L_\mathrm{q}}i_\mathrm{d} \quad \text{or} \quad i_\mathrm{q} = \pm\sqrt{i_\mathrm{d}^2 + \frac{\psi_\mathrm{F}}{L_\mathrm{d} - L_\mathrm{q}}i_\mathrm{d}}$$

The figure below shows the locus in the i_d–i_q plane.



(b) Based on (3) and (1), the current components are

$$i_d = \frac{\psi_F}{4(L_q - L_d)} - \sqrt{\frac{\psi_F^2}{16(L_q - L_d)^2} + \frac{i_s^2}{2}}$$
$$= \frac{1 \text{ Vs}}{4 \cdot (0.1 \text{ H} - 0.06 \text{ H})} - \sqrt{\frac{(1 \text{ Vs})^2}{16 \cdot (0.1 \text{ H} - 0.06 \text{ H})^2} + \frac{\left(\sqrt{2} \cdot 10 \text{ A}\right)^2}{2}} = -5.54 \text{ A}$$
$$i_q = \sqrt{i_s^2 - i_d^2} = \sqrt{\left(\sqrt{2} \cdot 10\right)^2 - 5.54^2} \text{ A} = 13.0 \text{ A}$$

The maximum torque at the given current is

$$\tau_M = \frac{3n_p}{2}\left[\psi_F + (L_d - L_q)i_d\right] i_q$$
$$= \frac{3}{2} \cdot 2 \cdot [1 \text{ Vs} + (0.06 \text{ H} - 0.1 \text{ H}) \cdot (-5.54 \text{ A})] \cdot 13.0 \text{ A} = 47.7 \text{ Nm}$$

(c) In the steady state, the magnitude of the stator voltage is $u_s = \omega_m \psi_s$, where the stator resistance is omitted. The stator flux components are

$$\psi_d = \psi_F + L_d i_d = 1 \text{ Vs} + 0.06 \text{ H} \cdot (-5.54 \text{ A}) = 0.67 \text{ Vs}$$
$$\psi_q = L_q i_q = 0.1 \text{ H} \cdot 13.0 \text{ A} = 1.30 \text{ Vs}$$

and the flux magnitude is

$$\psi_s = \sqrt{\psi_d^2 + \psi_q^2} = \sqrt{0.67^2 + 1.30^2} \text{ Vs} = 1.46 \text{ Vs}$$

At the base speed, the stator voltage is $u_s = \sqrt{2/3} \cdot 400 \text{ V} = 326.7 \text{ V}$ (peak value of phase-to-neutral voltage). Hence, the base speed as electrical radians is

$$\omega_m = \frac{u_s}{\psi_s} = \frac{326.7 \text{ V}}{1.46 \text{ Vs}} = 223.3 \text{ rad/s}$$

corresponding to the rotational speed

$$n = \frac{\omega_m}{2\pi n_p} = \frac{223.3 \text{ rad/s}}{2\pi \cdot 2} \cdot 60 \text{ s/min} = 1066 \text{ r/min}$$

(d) The output power of the machine is

$$p_M = \tau_M \omega_M = \tau_M \frac{\omega_m}{p}$$

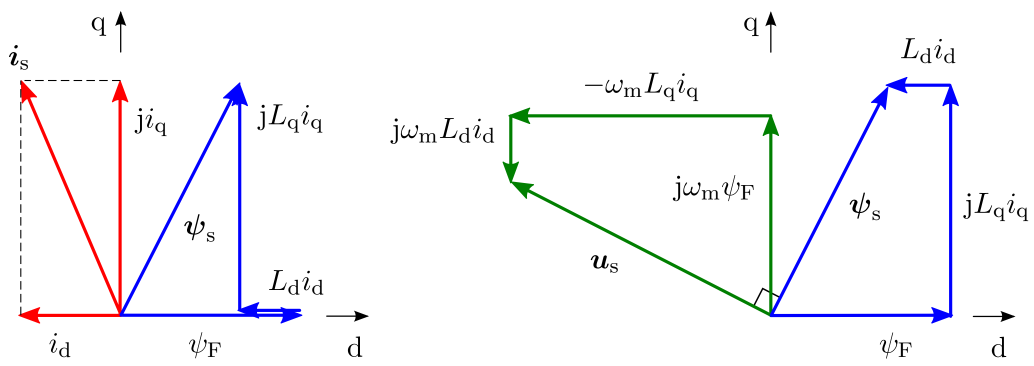
Since the losses are omitted, the output power equals the input power

$$p_s = \frac{3}{2}\mathrm{Re}\{\boldsymbol{u}_s \boldsymbol{i}_s^*\} = 3\frac{U_N}{\sqrt{3}} I_N \cos\varphi$$

The power factor can be solved

$$\cos\varphi = \frac{\tau_M \omega_m}{\sqrt{3} n_p U_N I_N} = \frac{47.7\ \mathrm{Nm} \cdot 223.3\ \mathrm{rad/s}}{\sqrt{3}\cdot 2 \cdot 400\ \mathrm{V}\cdot 10\ \mathrm{A}} = 0.77$$

The vector diagrams are illustrated in the figure below.



Current and flux diagram Flux and voltage diagram

The current, flux, and voltage components used in the diagrams are:

$$\begin{aligned} i_d &= -5.54\ \mathrm{A} & i_q &= 13.0\ \mathrm{A} \\ \psi_d &= 0.67\ \mathrm{Vs} & \psi_q &= 1.30\ \mathrm{Vs} \\ u_d &= -291.5\ \mathrm{V} & u_q &= 149.1\ \mathrm{V} \end{aligned}$$

and the corresponding vectors are

$$\boldsymbol{i}_s = 14.1\underline{/113.1^\circ}\ \mathrm{A} \quad \boldsymbol{\psi}_s = 1.46\underline{/62.8^\circ}\ \mathrm{Vs} \quad \boldsymbol{u}_s = 326.7\underline{/152.8^\circ}\ \mathrm{V}$$

The power factor can also be obtained from the vector diagram:

$$\cos\varphi = \cos(152.8^\circ - 113.1^\circ) = 0.77$$