Aalto University
School of Science
and Technology

PHYS-E0562 Nuclear Engineering, advanced course
Lecture 1 – Introduction to course topics

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Topics of this lecture

Cross sections and reaction types:

- Reaction types: fission, capture, scattering
- Energy-dependence of reaction probabilities

Fission:

- Fission energy, fission neutrons and fission products
- Fissile and fissionable nuclides

Physical operating principles of nuclear reactors:

- Neutron chain reaction with prompt fission chains and delayed neutrons
- Thermal and fast reactors
- Neutron moderation
- Reactivity feedbacks
- Reactivity control
- Fuel depletion

What is expected to be known before the lecture

Nuclear physics:

- $E = mc^2$, binding energy, etc.
- Neutron reactions: scattering, fission, capture
- Cross sections

Reactor physics:

- Basic concepts: multiplication factor, reactivity
- Prompt and delayed neutrons
- Neutron slowing-down, thermal neutrons
- Reactivity feedbacks

Technical:

- Fast and thermal reactors, moderators
- LWR technology: pressurized (PWR) and boiling water reactors (BWR)
- Reactivity control: control rods, boron shim, burnable absorbers

Reaction probabilities: microscopic cross section

Interaction probability between the neutron and a single target nucleus is characterized by the microscopic cross section, σ. The total microscopic cross section can be interpreted as the effective cross-sectional area of the nucleus. The standard unit is barn, 1b = 10^{-24} cm^2.

Total cross section is the sum over partial cross sections:

$$\sigma_{\mathrm{tot}}(E) = \sigma_{\mathrm{capt}}(E) + \sigma_{\mathrm{fiss}}(E) + \sigma_{\mathrm{ela}}(E) + \dots \tag{1}$$

and the conditional probability of reaction mode x is simply:

$$P_x = \frac{\sigma_x(E)}{\sigma_{\mathrm{tot}}(E)} \tag{2}$$

Similar definitions are often used for total absorption, total fission, total scattering, total inelastic scattering, total non-elastic, etc. reaction modes.

Microscopic cross sections are essentially natural constants, which depend on the nuclide, reaction type and neutron energy.[1] Scattering cross sections can also be associated with energy transfer and scattering angle, in which case the parameter is referred to as differential scattering cross section.[2]

[1] To be precise, the relative kinetic energy between neutron and the target.

[2] This topic is revisited in Lecture 2, when formulating the transport equation.

Reaction probabilities: macroscopic cross section

Macroscopic cross section, Σ, describes the neutron interaction probability in a medium, and it is defined using the microscopic cross section σ and the nuclide density N:

$$\Sigma(\mathbf{r}, E) = N(\mathbf{r})\sigma(E) \tag{3}$$

The physical interpretation is that the total macroscopic cross section Σ_{tot} gives the total interaction probability per path length traveled by the neutron. The average neutron mean-free-path (mfp) in a homogeneous medium is given by $1/\Sigma_{\mathrm{tot}}$.

Similar to microscopic cross sections, total macroscopic cross section can be defined as the sum over partial reaction modes:

$$\begin{aligned}\Sigma_{\mathrm{tot}}(\mathbf{r}, E) &= \Sigma_{\mathrm{capt}}(\mathbf{r}, E) + \Sigma_{\mathrm{fiss}}(\mathbf{r}, E) + \Sigma_{\mathrm{ela}}(\mathbf{r}, E) + \dots \\ &= N(\mathbf{r})\left[\sigma_{\mathrm{capt}}(E) + \sigma_{\mathrm{fiss}}(E) + \sigma_{\mathrm{ela}}(E) + \dots\right]\end{aligned} \tag{4}$$

And if the medium consists of multiple nuclides:

$$\Sigma(\mathbf{r}, E) = \sum_i \Sigma_i(\mathbf{r}, E) = \sum_i N_i(\mathbf{r})\sigma_i(E) \tag{5}$$

where index i refers to the nuclide species and cross sections Σ and σ to any partial or total reaction mode.

Reaction probabilities: examples

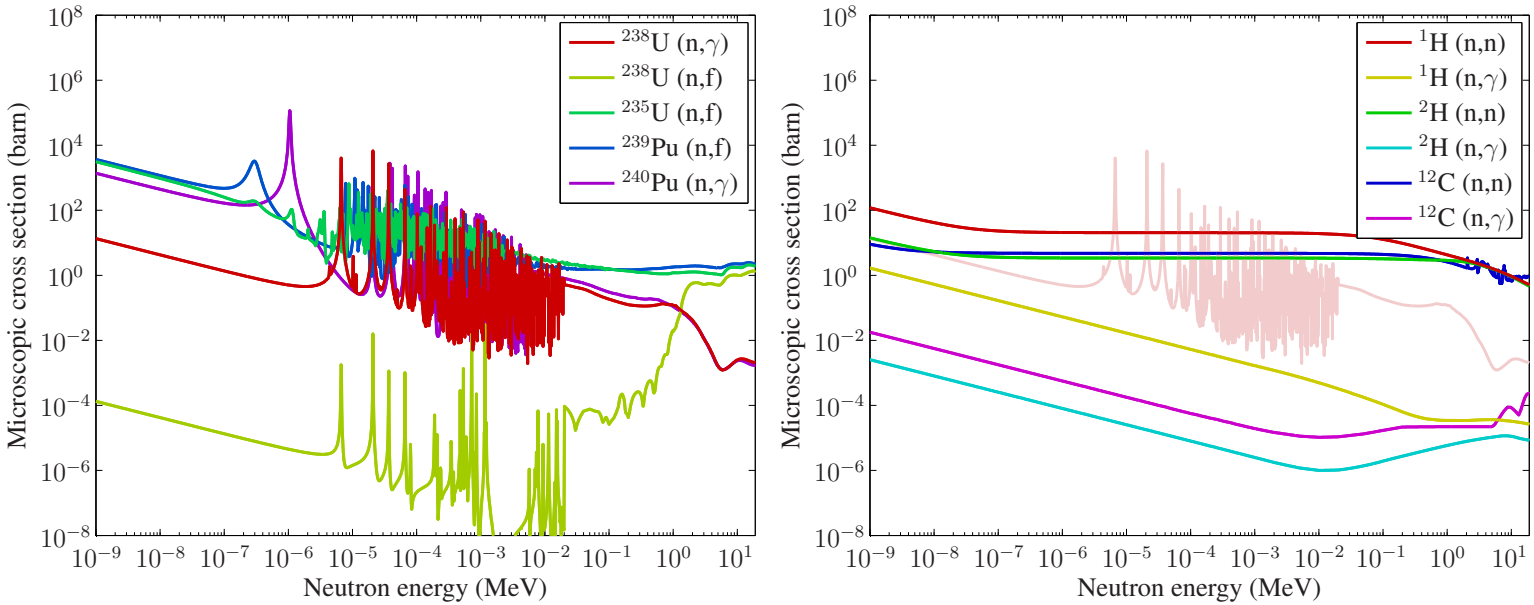

Figure 1: Microscopic cross sections of fuel and moderator isotopes. Left: fission and radiative capture cross sections of actinides. Right: elastic scattering and radiative capture cross sections of hydrogen, deuterium and ^{12}C. Capture cross section of ^{238}U is plotted in the background for comparison. Molecular and lattice bonding has a significant effect for light elements at low energy. This is not reflected in the free-atom scattering cross sections.

Reaction probabilities: examples

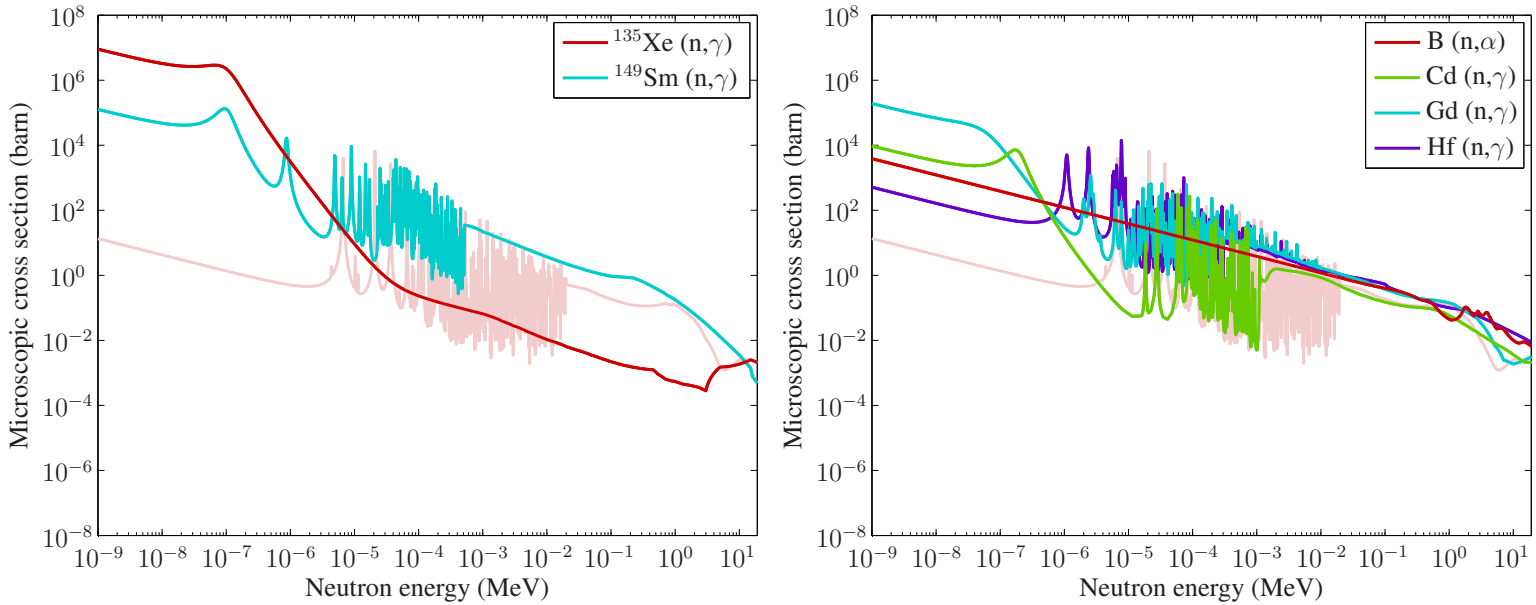

Figure 2: Microscopic cross sections of strong absorbers. Left: Fission product poisons. Right: absorbers used for reactivity control. Capture cross section of ^{238}U is plotted in the background for comparison.

Reaction probabilities: examples




Microscopic cross sections are typically plotted on a logarithmic scale, which does not really illustrate the differences in magnitude. A few examples:

^{2}H $\sigma_{\gamma}(0.1\ \text{eV}) = 2.5{\cdot}10^{-4}$ b

^{1}H $\sigma_{\text{ela}}(0.1\ \text{eV}) = 23.2$ b

^{235}U $\sigma_{\text{fiss}}(0.1\ \text{eV}) = 250$ b

^{135}Xe $\sigma_{\gamma}(0.1\ \text{eV}) = 2.2{\cdot}10^{6}$ b

If the elastic scattering cross section of hydrogen represents the cross-sectional area of a basket ball, the capture cross section of ^{2}H would be about the size of the tip of a ball point pen, ^{235}U fission cross section the size of a designer chair and capture cross section of ^{135}Xe a circle with a diameter of 79 m.

Reaction probabilities: examples

The orders of magnitude can also be visualized by looking at the neutron mean-free-paths, i.e. the average distance the neutron travels before colliding with a nuclide. Since this value depends on the macroscopic cross section $(1/\Sigma)$, it also depends on the density of the medium:

- $\sim$0.3 cm for a 0.05 eV neutron in PWR coolant[3]
- $\sim$2.4 cm for a 0.05 eV neutron in graphite
- $\sim$50 m for a 0.05 eV neutron in air
- $\sim$80 μm for a 6.67 eV neutron in fuel[4]

Mean-free-paths can also be calculated for heterogeneous systems:

- $\sim$0.7 cm for a thermal neutron in PWR core (E < 0.625 eV)
- $\sim$4.3 cm for a high-energy neutron in PWR core (E > 0.821 MeV)
- $\sim$4.5 cm for a thermal neutron in a pebble-bed HTGR core (E < 0.625 eV)
- $\sim$10 cm for a high-energy neutron in a pebble-bed HTGR (E > 0.821 MeV)
- $\sim$3-5 cm for a fast neutron in SFR core

[3] 0.05 eV is the center position of thermal peak in an operating reactor.
[4] 6.67 eV is the position of the lowest ^{238}U resonance.

Reaction types

The energy dependence of reaction cross sections reflects the interaction physics between the neutron and the nucleons of the target nucleus.

In potential scattering, the simplest form of elastic scattering, neutron collides with the target nucleus as if it was a hard sphere (a “billiard ball” type collision). Interaction probability is independent of neutron energy, and potential scattering forms the constant background of elastic scattering cross sections.

Fission and radiative capture are examples of compound nucleus reactions, in which the neutron forms an intermediate bound state with the target:

$$^{238}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \longrightarrow ^{239}_{92}\mathrm{U}^{*} \longrightarrow ^{239}_{92}\mathrm{U} + \gamma \tag{6}$$

Cross sections of compound nucleus reactions have a strong $1/v$-component (reaction probability is inversely proportional to neutron speed), which can be understood by considering the time spent by the neutron inside the nucleus while passing through.[5]

The energy level structure of the compound nucleus forms high resonance peaks in the cross section curves. The resonances of different reaction modes are often located at the same energy points, which reflects the fact that they represent different decay modes of the same compound states.

[5] This $1/v$-component forms a straight line on a log-log scale.

Reaction types

Even though resonances are spread quite uniformly throughout the energy range, their relative width and spacing becomes too small for the resolution of experimental measurements at high energy. Individual peaks cannot be distinguished from each other, and the cross sections are represented either as averaged values or using probability tables. This is known as the unresolved resonance range.

When colliding with a target nucleus, the incident neutron brings both binding and kinetic energy into the interaction. Some reactions can occur only with a sufficient amount of excess kinetic energy. Examples of threshold reactions include:

- Inelastic scattering
- Fission in fissionable actinides
- Absorptions with particle emission: (n,α), (n,p), (n,t), etc.[6]
- Neutron-multiplying scattering reactions: (n,2n), (n,3n), etc.

The only two interaction modes that are possible for all nuclides at all energies are elastic scattering and radiative capture.[7]

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[6] For some light nuclides these reactions may occur without excess energy. The primary absorption reaction in ^{10}B, for example, is (n,α), which is has very high cross section at low energy.

[7] Helium-4 has no measured (n,γ) cross section. The product nuclide ^{5}He decays by neutron emission with a half-life of $\sim 10^{-24}$ s, which makes the capture reaction of ^{4}He indistinguishable from inelastic scattering.

Reaction types

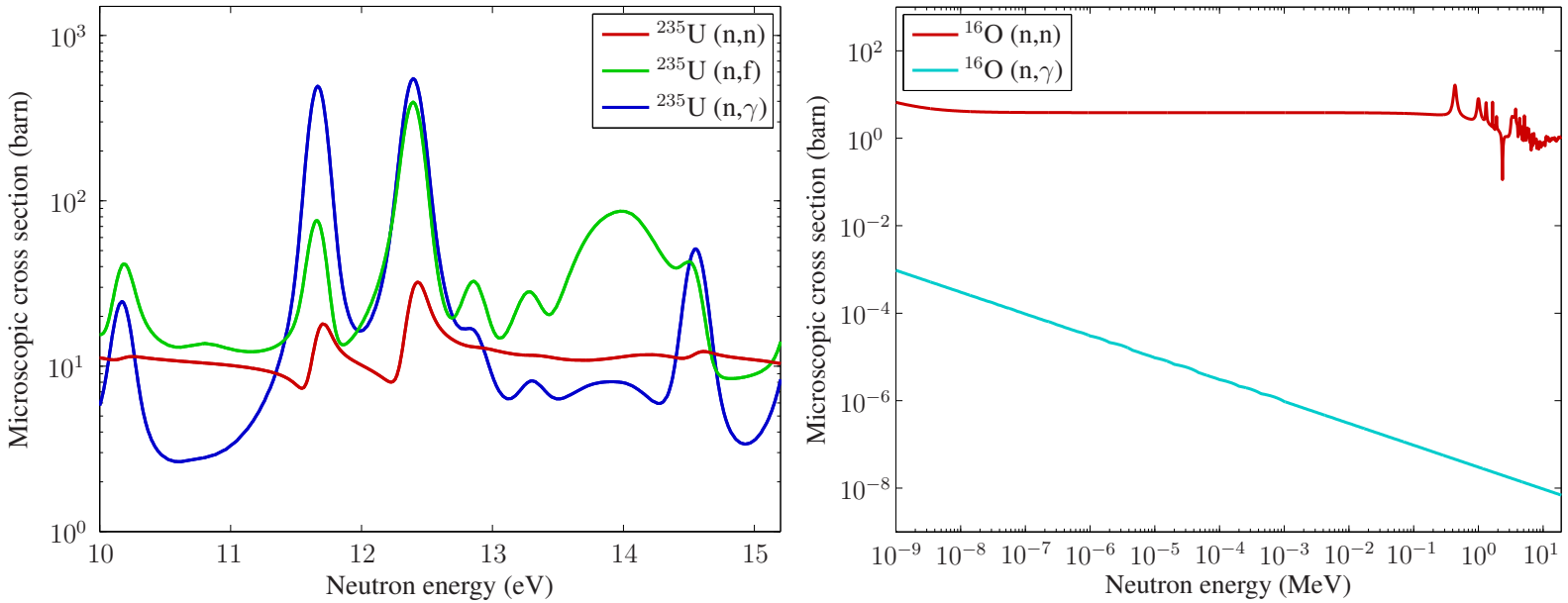

Figure 3: Left: Elastic scattering, fission and radiative capture cross sections of ^{235}U in the low resonance range. The resonance structure depends on the energy levels of the compound nucleus formed during the interaction, which is why the cross sections share similar shape. Right: Elastic scattering and radiative capture cross sections of ^{16}O. The $1/v$ background shape of (n,γ) cross sections is clearly seen in light nuclides without low-energy resonances. Elastic potential scattering cross section is independent of energy, and the increase at very low energy results from thermal motion.

Reaction types: fission

Neutron-induced Fission is typically associated with heavy elements in the actinide series (Z > 90), but the reaction is possible for nuclides as light as lead (Z = 80), provided that the energy of the absorbed neutron is sufficiently high.

Fission is a compound nucleus reaction, in which the absorption of the incident neutron leads to the formation of a short-lived excited state, which decays by splitting the nucleus in two parts. For example:

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \longrightarrow ^{236}_{92}\mathrm{U}^{*} \longrightarrow ^{134}_{52}\mathrm{Te} + ^{99}_{40}\mathrm{Zr} + 3^{1}_{0}\mathrm{n} \tag{7}$$

The split is asymmetric, with two peaks concentrated near mass numbers 90 and 140. In addition to binary fission, the nucleus can also be split in three (ternary fission) or four (quaternary fission) parts, but these reaction types are rare (< 1% of all fissions).

Intermediate-mass fission products are loosely bound because of the excess neutrons left over from the heavy actinide, which makes them susceptible to β^--decay:

$$\begin{gathered} ^{134}_{52}\mathrm{Te} \xrightarrow[41.8\mathrm{m}]{\beta^-} ^{134}_{53}\mathrm{I} \xrightarrow[52.6\mathrm{m}]{\beta^-} ^{134}_{54}\mathrm{Xe} \\ ^{99}_{40}\mathrm{Zr} \xrightarrow[2.4\mathrm{s}]{\beta^-} ^{99}_{41}\mathrm{Nb} \xrightarrow[14.3\mathrm{s}]{\beta^-} ^{99}_{42}\mathrm{Mo} \xrightarrow[2.8\mathrm{d}]{\beta^-} ^{99}_{43}\mathrm{Tc} \xrightarrow[2.1\cdot 10^5\mathrm{y}]{\beta^-} ^{99}_{44}\mathrm{Ru} \end{gathered} \tag{8}$$

Reaction types: fission

Radioactive fission products form decay chains in which the mass number remains unchanged. Some of these chains contain long-lived isotopes.

Fission releases about 200 MeV of energy, the majority of which is received by the two daughter nuclides. About 6% of total energy is released in the decay of radioactive fission products. This fraction forms a major part of the decay heat of irradiated fuel.

Number of emitted fission neutrons is approximately a linear function of neutron energy, and it increases also along with atomic number and nuclide mass. Typical fission releases 2-7 neutrons, which are emitted isotropically. In addition to prompt neutrons, the decay chains of certain fission products release additional neutrons with a considerable delay. The fraction of delayed neutrons ranges from 0.2 to 0.7%, depending on the actinide and neutron energy.

The energy of emitted neutrons resembles a Maxwellian distribution, with an average energy of about 2 MeV. The tails of the distribution extend from below the keV range to about 10 MeV. The energy distribution of fission neutrons forms the high end of reactor flux spectrum.

It should be noted that the reaction usually referred to as fission is actually the sum of multiple reaction channels: first-chance fission, (n,f), second-chance fission, (n,nf), third-chance fission, (n,2nf), etc. The higher fission modes are threshold reactions in which the compound nucleus is fissioned after an inelastic scattering event.

Reaction types: fission

Table 1: Energy release in the neutron-induced fission of ^{235}U. The fraction lost to neutrinos cannot be recovered, which means that the actual deposited energy is slightly below 200 MeV.

	Energy (MeV)	Fraction
Kinetic energy of fission fragments	169.1	83.5%
Kinetic energy of fission neutrons	4.8	2.4 %
Energy of prompt fission gammas	7.0	3.5 %
Decay heat of fission products	12.8	6.3 %
Energy lost to neutrinos	8.8	4.3 %
Total energy release	202.5	100.0 %
Maximum recoverable energy	193.7	95.7%

NOTE: Radiative capture of fission neutrons produces additional 5-10 MeV of energy, which increases the total deposited energy near 200 MeV.

Reaction types: fission

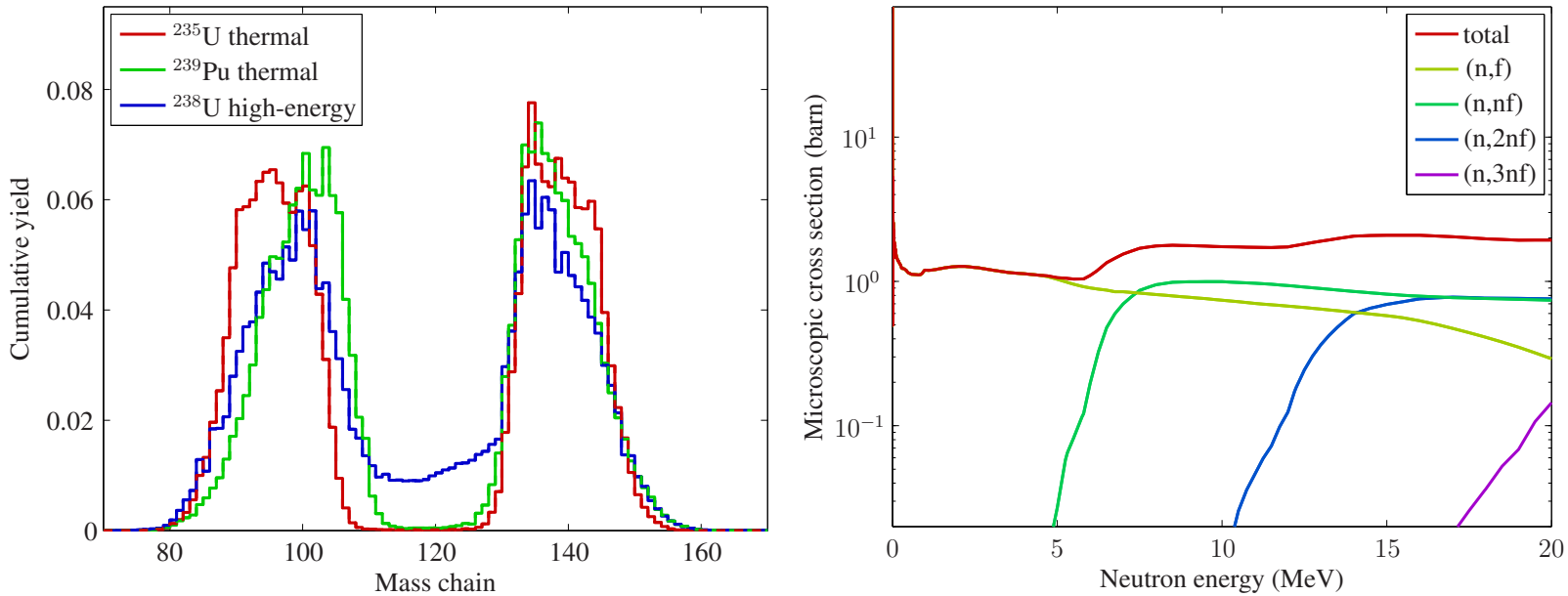

Figure 4: Left: The double-peaked fission product distribution. The yields are calculated as cumulative values over each mass chain. The lower peak shifts to the right as the mass of the fissioned actinide increases. High-energy fission for ^{238}U refers to reaction caused by 14 MeV neutrons. Increasing neutron energy lifts the distribution between the two peaks. Right: Total fission cross section of ^{235}U divided into 1st-, 2nd-, 3rd-, and 4th-chance fission components. The higher modes are threshold reactions that increase the total cross section in the MeV-range.

Reaction types: fission

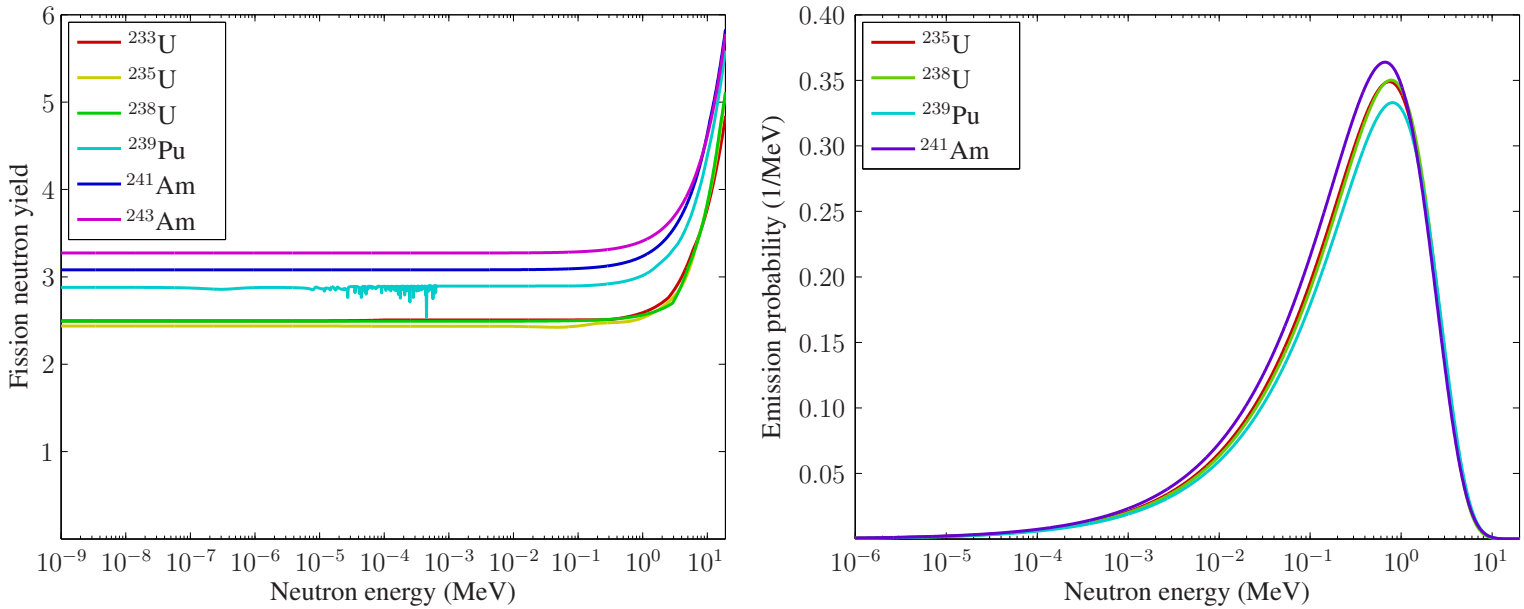

Figure 5: Left: Fission neutron yields of various actinides as function of neutron energy. The yield is actually a linear function of energy, which only appears to increase sharply in the MeV-range because of the logarithmic scale. The number of emitted neutrons increases along with actinide mass.
Right: The Maxwellian energy distributions of ^{235}U, ^{238}U, ^{239}Pu and ^{241}Am fission neutrons. The peak position varies slightly for different actinides, but is practically independent of neutron energy. The average energy for ^{235}U fission neutrons is around 2 MeV.

Reaction types: fission

Neutron-induced fission can occur only if the energy brought into the compound nucleus exceeds the fission barrier. The height of this barrier depends on the actinide, and for ^{235}U and ^{238}U it is around 6 MeV.

The energy brought in by the absorbed neutron consists of:

1) Binding energy component
2) Kinetic energy component

The binding energy of a nuclide depends on its nucleon configuration, and nuclides with even number of protons (even-Z) or neutrons (even-N) tend to be more tightly bound. This is called the the parity effect.

Because of the parity effect, neutron absorption in odd-N nuclides releases more energy, as the resulting even-N isotope is more tightly bound. The compound nucleus is consequently formed at a higher excited state, more likely to exceed the fission barrier.

Actinides are divided into fissile and fissionable nuclides, depending on whether they can undergo fission by low-energy neutron absorption. Most of the fissile isotopes are odd-N nuclides (^{235}U, ^{239}Pu, etc.). Fissionable isotopes (^{238}U, ^{240}Pu, etc.) require $\sim$ 1 MeV of additional kinetic energy for fission to occur.[8]

[8] The fission probability of even-N actinides is non-zero throughout the energy range, but it is practically negligible below the threshold.

Reaction types: fission

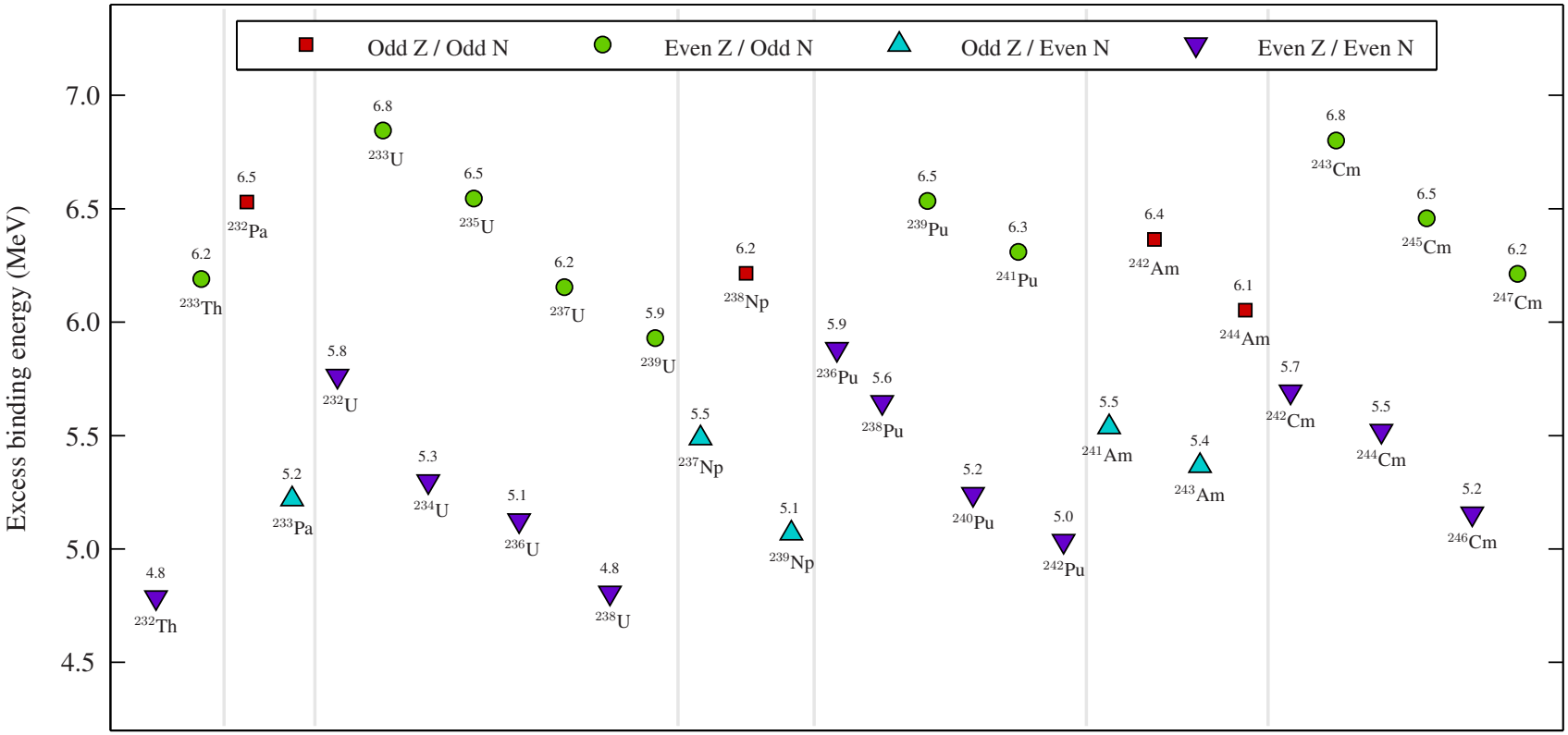

Figure 6: Excess binding energy brought into the compound nucleus by neutron absorption. Calculated as $E = (m_{\mathrm{Z,A}} + m_{\mathrm{n}} - m_{\mathrm{Z,A+1}})c^2$, where $m_{\mathrm{Z,A}}$ is the ground-state rest mass of nuclide ${}^{A}_{Z}\mathrm{X}$ and m_{n} is the rest mass of the neutron. Neutron absorption turns odd-N nuclides into more tightly bound even-N nuclides, releasing more binding energy.

Reaction types: fission

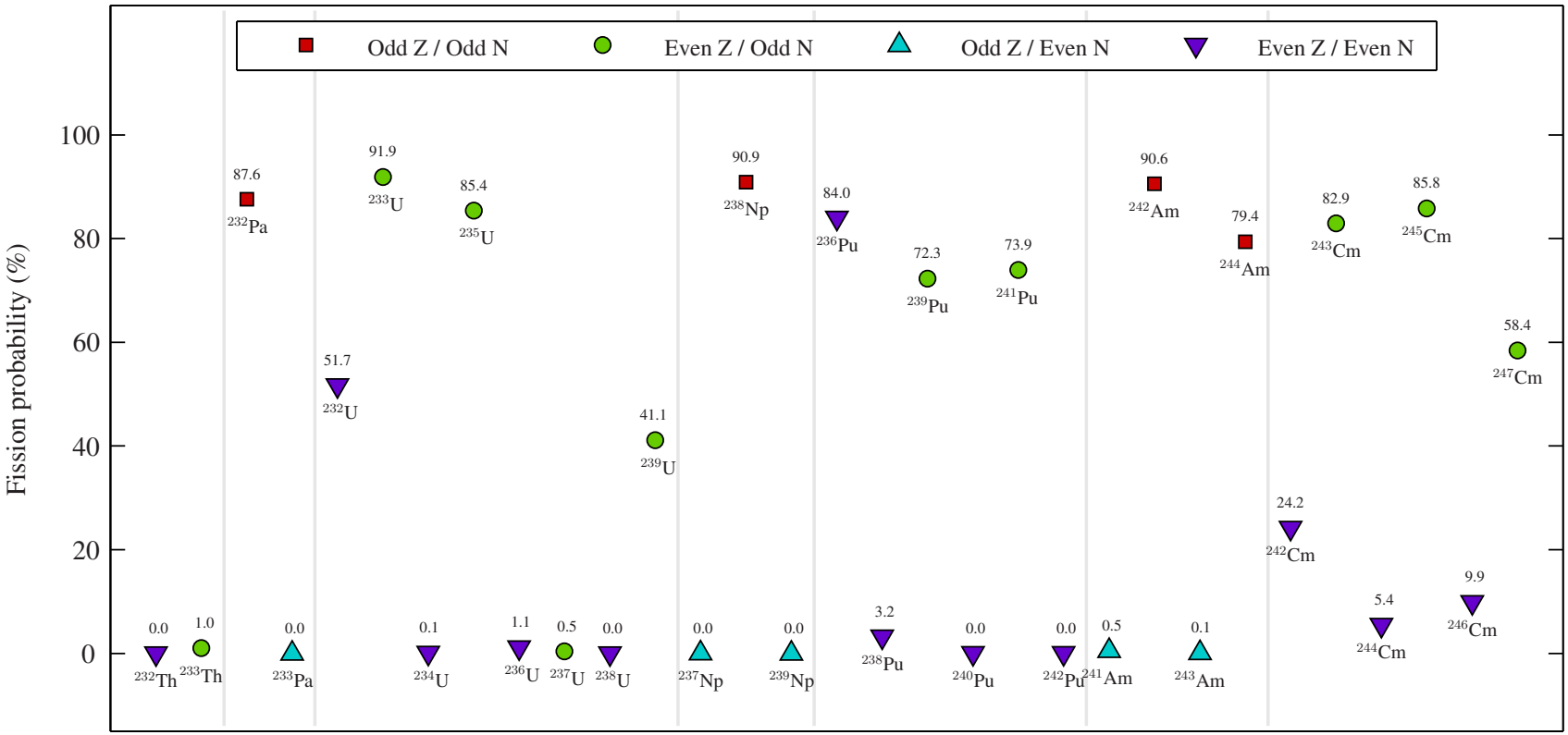

Figure 7: Probability of fission induced by thermal neutrons (Maxwellian energy distribution corresponding to room temperature). If the binding energy released in neutron absorption exceeds the fission barrier, the nuclide can be fissioned by low-energy neutrons. This is more likely to happen with odd-N nuclides, for which the energy release is higher because of the parity effect.

Reaction types: fission

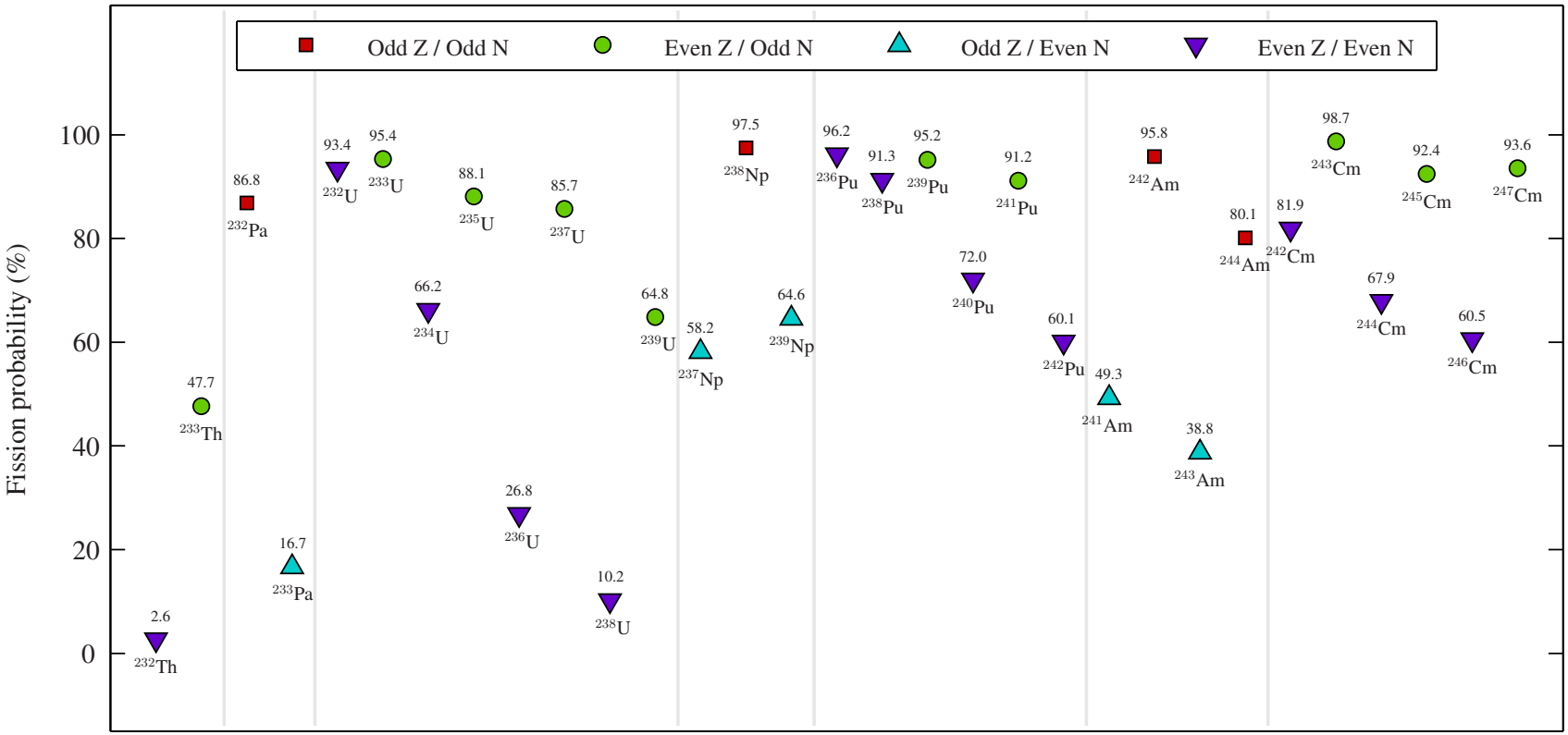

Figure 8: Probability of fission induced by fast neutrons (Maxwellian fission spectrum peaking at 1.3 MeV). The additional kinetic energy of the neutron is sufficient to exceed the fission barrier also for most even-N nuclides.

Reaction types: fission

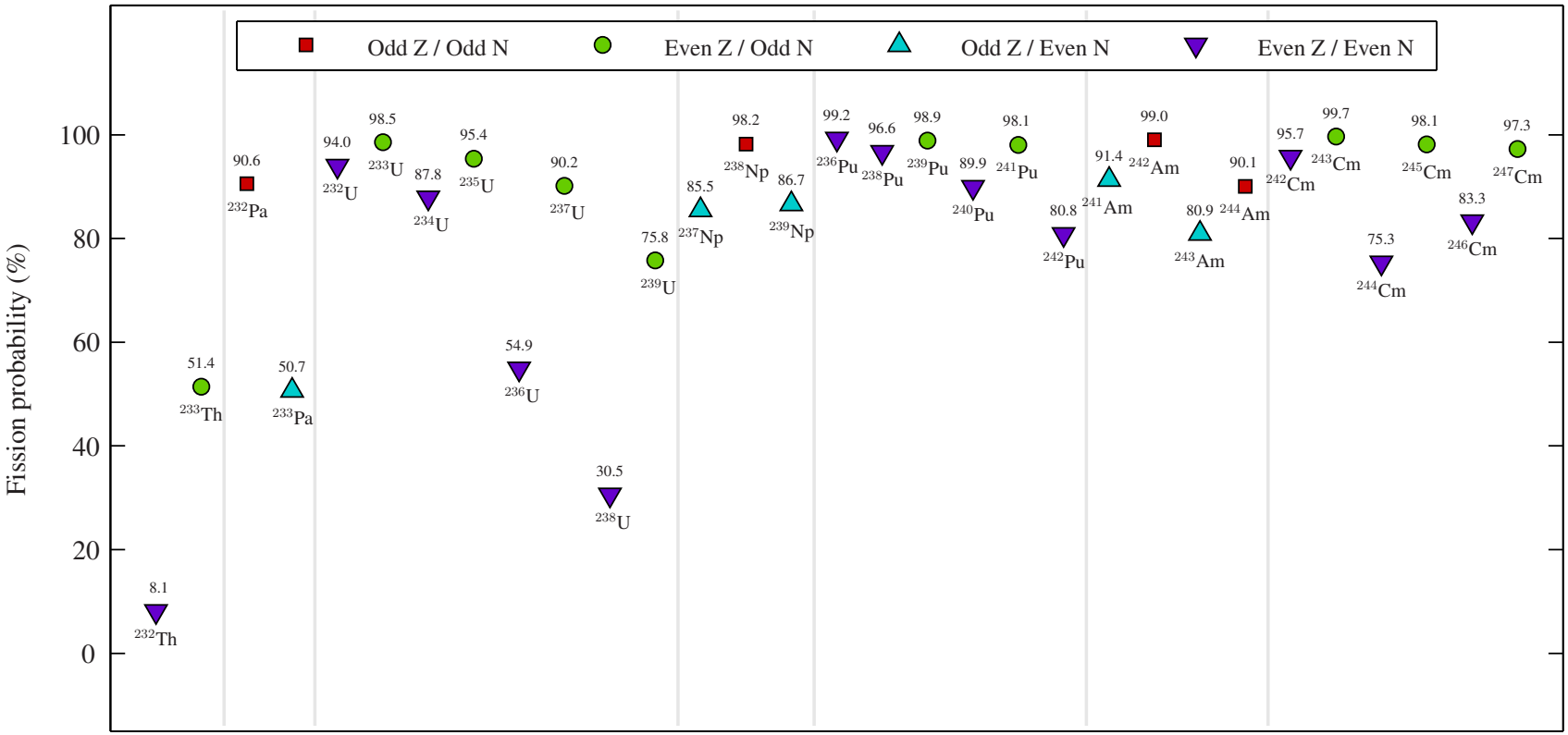

Figure 9: Probability of fission induced by high-energy neutrons (14 MeV). Fission probability becomes even higher when the energy exceeds 10 MeV, but these energies are not very relevant for conventional fission applications.

Reaction types: capture

In neutron transport applications, all reactions in which the incident neutron is lost and no new neutrons are emitted can be categorized as capture,[9] for example:

- Radiative capture: (n,γ)
- Alpha production: (n,α)
- Proton production: (n,p)
- Deuteron production: (n,d)
- Triton production: (n,t)

The only capture mode for actinides with practical significance in reactor applications is (n,γ), which is also the dominant mode for intermediate-weight nuclides. Some light isotopes may have other dominant modes, for example, (n,α) for ^{10}B and (n,t) for ^{6}Li.

Some capture reactions, such as ^{16}O (n,α), are threshold reactions, and some are not, such as ^{10}B (n,α). Radiative capture, together with elastic scattering, are the only reaction modes that are always possible for all energies. Radiative capture releases a few MeV of energy, which is a non-negligible fraction of reactor energy production.

[9] The terminology is not unambiguous. Term “absorption” is often used interchangeably with capture, sometimes including fission in the same category. Capture is also often reserved specifically for radiative capture.

Reaction types: capture

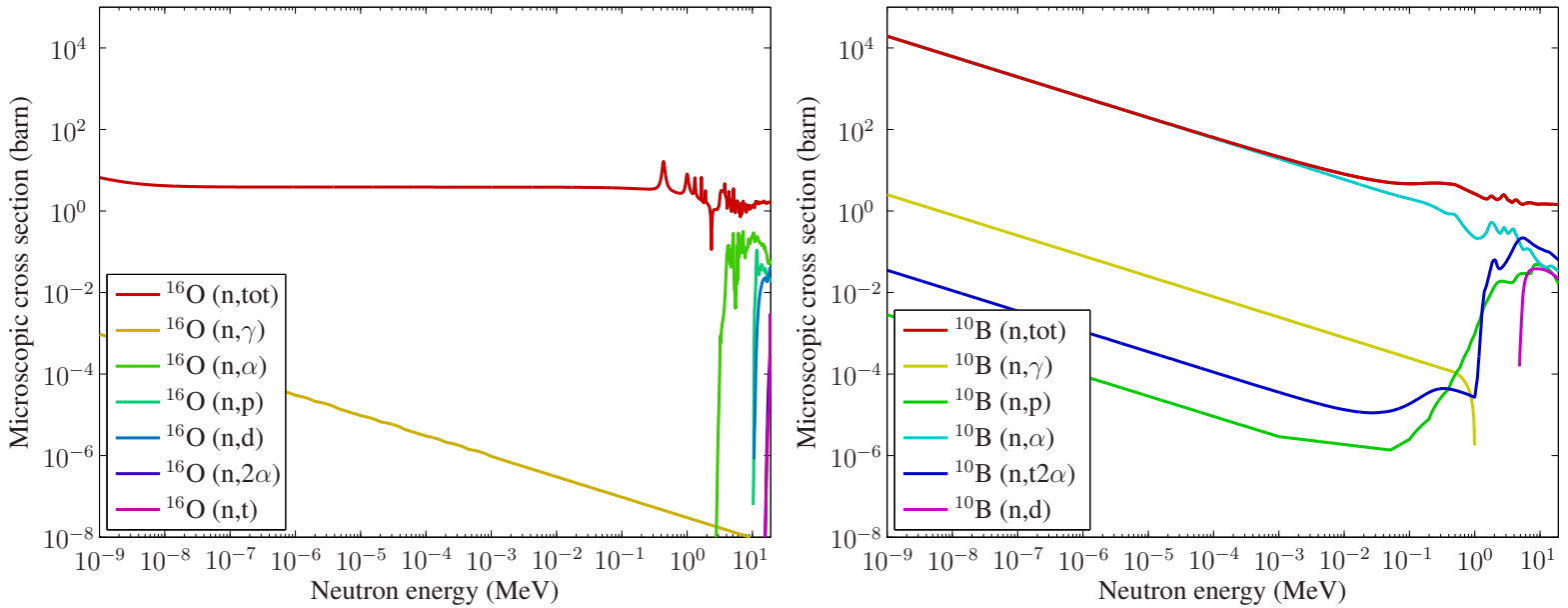

Figure 10: Multitude of capture reactions for light nuclides. Left: The radiative capture cross section of ^{16}O is very low, and the nuclide absorbs mainly fast neutrons by (n,α). Right: ^{10}B has several capture channels available for low-energy neutrons, the most dominant of which is (n,α).

Reaction types: scattering

Scattering includes all reaction modes in which the incident neutron is not lost:

- Elastic scattering
- Inelastic scattering
- Inelastic neutron-multiplying reactions

Elastic scattering preserves the kinetic energy of the system. The most typical elastic scattering mode is potential scattering, which is a “billiard ball” type collision with interaction probability independent of neutron energy.

Compound nucleus formation can also lead to elastic reaction if the excited state decays by neutron emission and the target is left in its ground state. This is a typical reaction near resonances.

Elastic scattering with hydrogen is by far the most common neutron interaction in LWR’s, comprising of more than 95% of all reactions. In heavy water and graphite-moderated reactors this fraction is typically even higher, because of the smaller average energy loss per collision.

Reaction types: scattering

The constant cross section of potential scattering is increased to $1/v$-shape at low energy by the thermal motion of the target nuclides within the medium. This effect is understood by considering the collisions of fast-moving nuclides with a slow-moving neutron.[10]

When the energy of the neutron is comparable to the energy that binds the target in a molecule, the reaction cannot be treated as a collision with a free nucleus. The wavelength of the neutron increases with decreasing energy, and when comparable to the interatomic spacing in a regular crystalline lattice, the neutron interacts with an aggregate of bound nuclei.

These effects become important especially for moderator materials, such as light and heavy water and graphite, and they affect both cross sections and the energy and angular distributions of scattered neutrons.

Potential scattering is typically isotropic in the center-of-mass frame at low energy. Scattering anisotropy increases with neutron energy, especially near resonances.

[10] This topic is revisited in lecture 3, when considering temperature effects on cross sections.

Reaction types: scattering

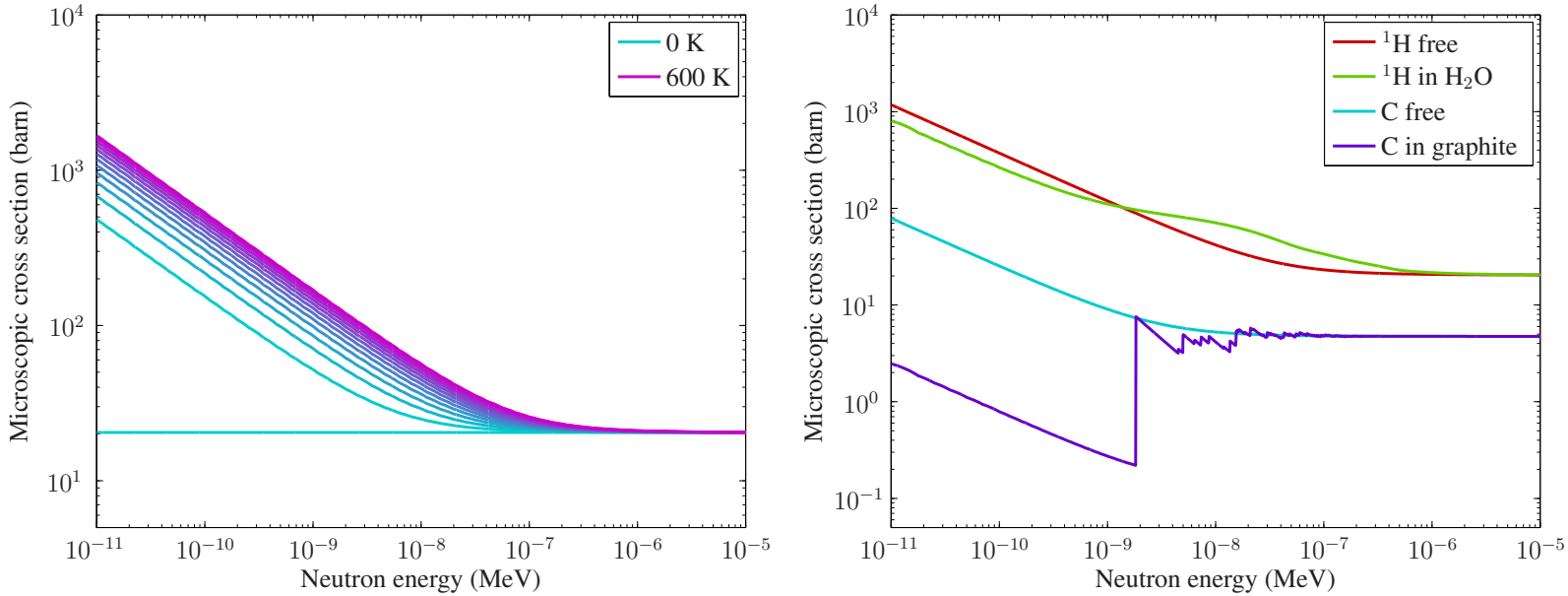

Figure 11: Left: Effect of temperature on low-energy elastic scattering cross section of ^{1}H. The cross sections are plotted for 50 K intervals between 0 and 600 K. Right: Free-atom elastic scattering cross sections of ^{1}H and carbon compared to corresponding bound-atom cross sections in light water and graphite (all cross sections at 300 K). Neutron energies corresponding to wavelenghts equal to multiples of interatomic spacing are reflected in the cross section of graphite.

Reaction types: scattering

In inelastic scattering, part of the incident kinetic energy is transferred to the target nucleus. The excited state decays by gamma or particle emission. Cross sections of inelastic scattering are typically divided into level-scattering to different excited states (1st, 2nd, 3rd, ...), each associated with a discrete Q-value. Unresolved states are combined into a single continuum reaction.

In two-body reactions the energy transfer is coupled to the scattering angle by conservation of energy and momentum. For elastic scattering this coupling can be written as:

$$E'_{\mathrm{L}} = \frac{E_{\mathrm{L}}}{(A+1)^2}\left[\mu_{\mathrm{C}} + \sqrt{A^2 - 1 + \mu_{\mathrm{C}}^2}\right]^2 \tag{9}$$

where E_{L} is the incident neutron energy in the laboratory frame-of-reference (L-frame), μ_{C} is the scattering cosine in the center-of-mass frame-of-reference (C-frame)[11] and A is the atomic weight ratio (ratio of neutron to target mass). Similar coupling can be written for inelastic level scattering.

If the incident energy is sufficiently high ($\gtrsim$ 5 MeV), additional neutrons can be emitted in the reaction. These (n,2n), (n,3n), etc. reactions have noticeable contribution on reactivity.

[11] The reason to use a different frame-of-reference for the scattering angle is that elastic scattering is often isotropic in the C-frame. To be precise, Eq. (9) applies to the target-at-rest frame (T-frame), which is equivalent with L-frame when the neutron scatters from a stationary nuclide. This topic is revisited in Lecture 3.

Reaction types: scattering

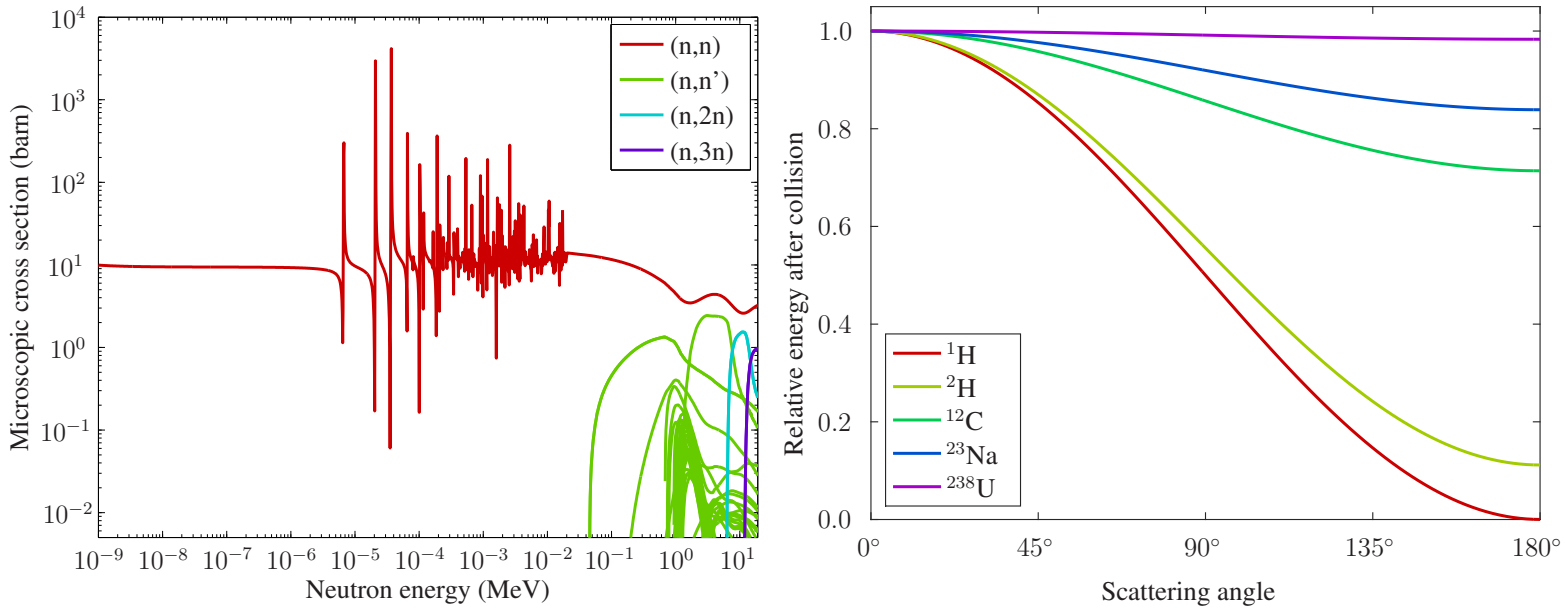

Figure 12: Left: Elastic and inelastic scattering cross sections of ^{238}U. Right: L-frame energy transfer in elastic scattering as function of C-frame scattering angle according to Eq. (9) for target nuclides of different mass.

Evaluated nuclear data

Neutron interaction data is stored and distributed in evaluated nuclear data files, based on the standardized ENDF format. There are currently three major evaluation projects:

- JEFF – Joint Evaluated Fission and Fusion File, coordinated by the OECD/NEA Data Bank, latest release JEFF-3.2 in March 2014.
- ENDF/B – Evaluation of Neutron Data File / Brookhaven, coordinated by the Cross Section Evaluation Working Group (CSEWG) in the US, latest release ENDF/B-VII.1 in December 2011.[12]
- JENDL – Japanese Evaluated Neutron Data Library, coordinated by the Japan Atomic Energy Agency (JAEA), latest release JENDL-4.0 in 2010.

The data files are publicly available and cross sections, angular and energy distributions can be viewed and processed into human-readable format using tools like the Java-based Nuclear Data Information System (Janis): `http://www.oecd-nea.org/janis/`

[12] Because of historical reasons, the ENDF/B file is confusingly named similarly to the standardized ENDF, or more precisely, ENDF-6 file format.

Neutron chain reaction

The operation of all critical reactors is based on a self-sustaining chain reaction, carried on by fission neutrons that form long chains in consecutive generations.

Fission neutrons can be divided into:

1) Prompt neutrons, emitted at the fission event
2) Delayed neutrons, emitted in the decay of certain fission product species, denoted as delayed neutron precursors

It is important to realize that, from the viewpoint of the chain reaction, the roles of prompt and delayed neutrons are fundamentally different, because of their different time-scales.

The operation of a nuclear reactor can be described as follows:[13]

> *Radioactive decay of precursor isotopes forms a source of delayed neutrons, initiating a number of prompt fission chains. When the reactor operates below prompt criticality, these chains are finite in length. In steady state condition, each prompt fission chain produces, on the average, one new delayed neutron precursor.*

[13] The steady-state operating condition is often explained without making difference between prompt and delayed neutrons – the reactor operates at constant power if each fission produces, on the average, one neutron that survives capture and escape to produce a new fission. Separation of delayed neutron emission from prompt fission chains brings some additional insight into reactor operation, especially below prompt criticality.

Neutron chain reaction: prompt fission chains

Concept "prompt fission chain" refers here to the sequence of prompt neutron induced fissions in consecutive generations, started from a single source neutron. Multiplication factor for prompt neutrons can be defined simply as:

$$k_{\mathrm{p}} = \overline{\nu}_{\mathrm{p}} P_{\mathrm{fiss}} \tag{10}$$

where $\overline{\nu}_{\mathrm{p}}$ is the average prompt neutron yield and P_{fiss} is the average probability of an emitted neutron to induce fission.

When a neutron is introduced into a system with prompt multiplication factor k_{p}, the average multiplicity is given by:[14]

$$M = 1 + k_{\mathrm{p}} + k_{\mathrm{p}} k_{\mathrm{p}} + k_{\mathrm{p}} k_{\mathrm{p}} k_{\mathrm{p}} + \cdots = 1 + k_{\mathrm{p}} + k_{\mathrm{p}}^2 + k_{\mathrm{p}}^3 + \cdots = \frac{1}{1 - k_{\mathrm{p}}} \tag{11}$$

provided that $k_{\mathrm{p}} < 1$. All fission chains are finite in length, and die out after a number of generations. This mode of operation corresponds to sub-critical, but also critical and delayed super-critical states.

If $k_{\mathrm{p}} \geq 1$, some chains become infinitely long, multiplying the initiating neutron indefinitely. The system is then in prompt super-critical state.

[14] To be precise, the sum of the series gives the total number of neutrons, including the ones that initiate the chains. It is also assumed that k_{p} is the same for all generations. In reality this may not be the case until the source has converged to its fundamental form. Even so, the sum is finite if the asymptotic value of k_{p} is below unity.

Neutron chain reaction: prompt fission chains

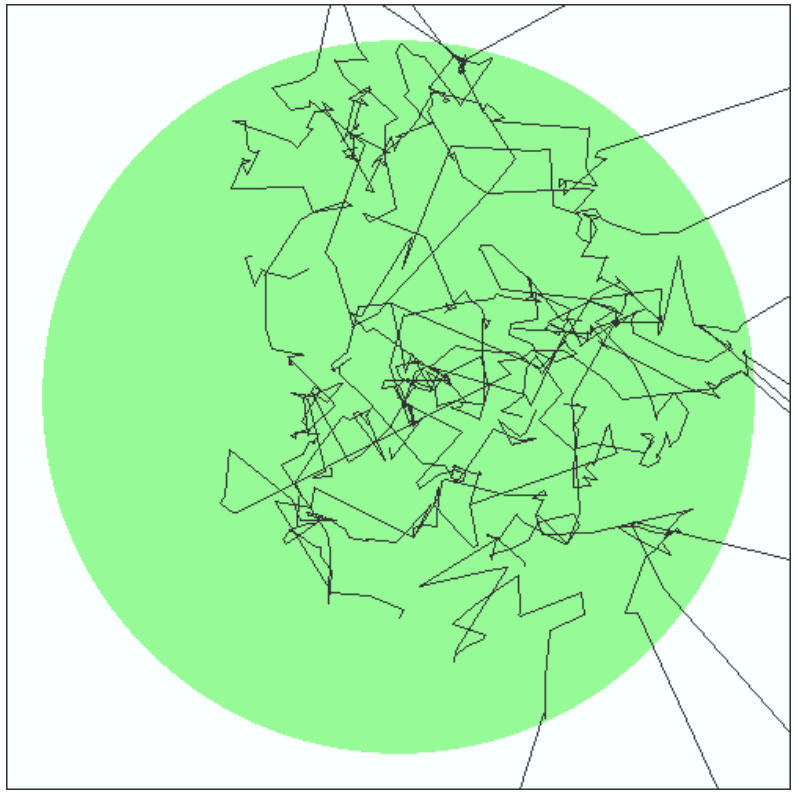

Figure 13: Neutron tracks of a prompt fission chain initiated by a single neutron introduced into a critical system. Infinite uranium cylinder with 6.2 cm diameter and 10% ^{235}U enrichment. The initiating neutron produces 43 descendants and the chain dies out in 0.55 μs. The tracks are animated in `Lecture1_anim1.gif`. For similar animation of a prompt super-critical system, see `Lecture1_anim2.gif`.

Neutron chain reaction: prompt fission chains

The average time between two fission events is characterized by the *prompt neutron lifetime*, l_{p}, which depends on the system:

- $l_{\mathrm{p}} \sim$ 1 - 10 ns for fast criticality experiments (Godiva, Jezebel, etc.)
- $l_{\mathrm{p}} \sim$ 0.5 μs for sodium-cooled fast reactors
- $l_{\mathrm{p}} \sim$ 20 - 50 μs for light water reactors
- $l_{\mathrm{p}} \sim$ 2 ms for high-temperature gas-cooled reactors

Neutron thermalization takes considerable time, which is why the time constants for thermal systems are longer. High-temperature gas-cooled reactors (HTGRs) use graphite moderator, in which the average energy loss per collision is relatively small compared to hydrogen in light water.

The lengths of prompt fission chains depend on multiplication. When close to criticality, the longest branches can extend to hundreds of generations. Consequently, the last neutrons die out within micro- or milliseconds after the initiating event.

Neutron chain reaction: prompt fission chains

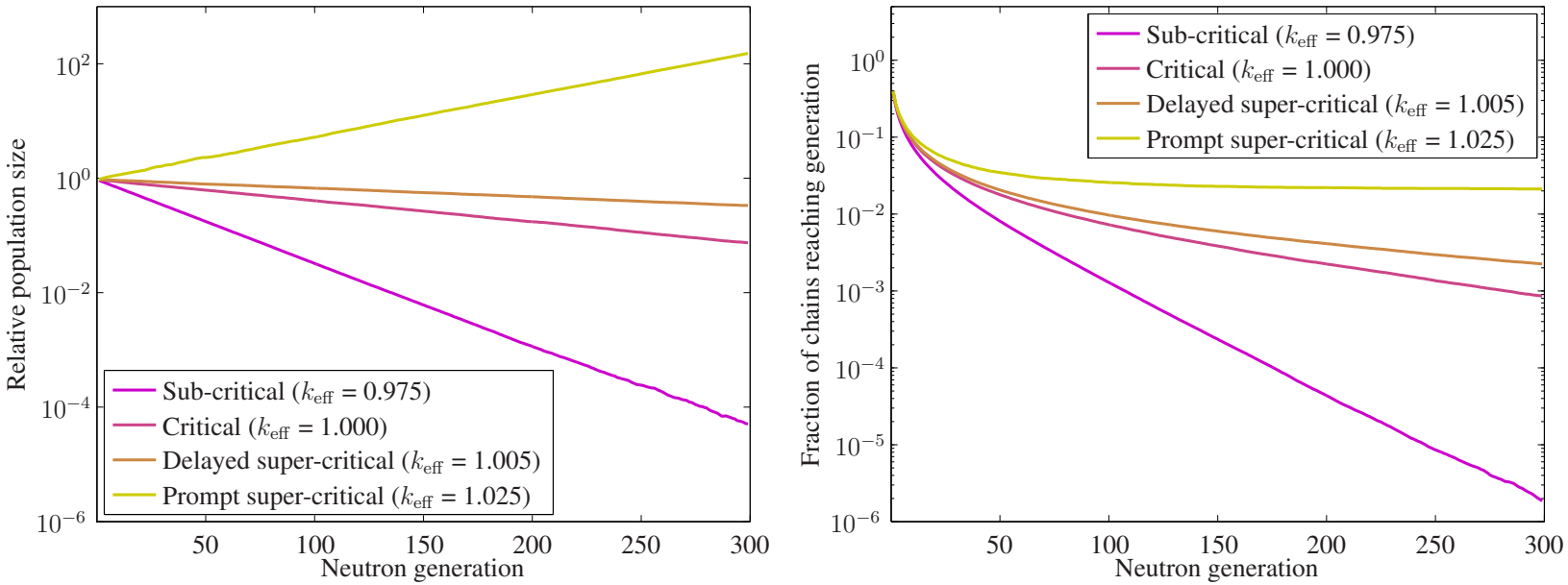

Figure 14: Simulated prompt fission chains in a LWR fuel lattice close to criticality. Left: Relative average population size as function of neutron generation. Right: Fraction of fission chains that reach the given generation. In a critical state, more than 99% of all prompt fission chains die out during the first 100 generations. When the system reaches prompt critical state, about 2% of initiated chains continue to expand indefinitely. Calculated using Monte Carlo simulation.

Neutron chain reaction: delayed neutron emission

Delayed neutrons are produced when a radioactive fission product undergoes beta-decay, and the daughter nucleus is left in a high excited state that releases its energy by neutron emission. For example:

$$^{87}_{35}\mathrm{Br} \xrightarrow[55.7\mathrm{s}]{\beta^-} {}^{87}_{36}\mathrm{Kr}^* \longrightarrow {}^{86}_{36}\mathrm{Kr} + {}^{1}_{0}\mathrm{n} \tag{12}$$

The second reaction is practically instantaneous, and the delay between fission and neutron emission depends on the half-life of the precursor isotope.

There are hundreds of fission products that act as delayed neutron precursors,[15] but most of them are either short lived or have very low fission yields (See Table 2). The half-lives of significant precursor isotopes range from hundreds of milliseconds to almost a minute (for $^{87}\mathrm{Br}$ above).

The fact that delayed neutrons are emitted hundreds of milliseconds, seconds or even minutes after the fission event means that the prompt fission chain that produced them has already died out when the new neutron is introduced into the system. In other words, *delayed neutron emission can be considered the initiating event for a prompt fission chain.*

The system is in a steady-state condition when the delayed neutron source is constant in time, i.e. when each prompt fission chain produces, on the average, one delayed neutron precursor, which after decay initiates a new chain.

[15] Fission yield data typically consists of about 1000 fission product isotopes and isomeric states.

Neutron chain reaction: delayed neutron emission

Table 2: Ten most abundant delayed neutron precursor isotopes in thermal fission of ^{235}U and ^{239}Pu and fast fission of ^{238}U. The yields are cumulative, and they refer to average nuclide production per fission.

Nuclide	Half-life (s)	^{235}U yield	^{239}Pu yield	^{238}U yield
^{87}Br	55.70	0.021	0.007	0.016
^{137}I	24.51	0.036	0.023	0.056
^{88}Br	16.50	0.018	0.005	0.019
^{138}I	6.46	0.015	0.007	0.040
^{93}Rb	5.80	0.035	0.017	0.046
^{89}Br	4.37	0.014	0.003	0.021
^{97}Y	3.75	0.021	0.012	0.032
^{94}Rb	2.70	0.015	0.007	0.034
^{98m}Y	2.00	0.020	0.019	0.026
^{143}Cs	1.79	0.016	0.006	0.035
^{141}Xe	1.73	0.016	0.005	0.034
^{99}Y	1.48	0.019	0.013	0.047
^{97m}Y	1.17	0.028	0.025	0.023
^{97}Sr	0.43	0.017	0.007	0.033
^{95}Rb	0.38	0.007	0.003	0.022

Neutron chain reaction: delayed neutron emission

Criticality condition can be written using the prompt neutron multiplication factor as:

$$k_{\mathrm{p}} = 1 - \beta \tag{13}$$

where β is the delayed neutron fraction, which depends on fuel composition and flux spectrum.

Changes in reactivity are reflected in fission rate in two ways:

1) The change in k_{p} affects the length of prompt fission chains and therefore the number of fissions per chain
2) The build-up or depletion of precursor concentration increases or decreases the rate at which new prompt fission chains are initiated

The coupling between precursor concentration and the production rate of new precursors means that the neutron population increases or decreases at an exponential rate.

As the system approaches prompt criticality, the length of prompt fission chains approaches infinity, and the contribution of initiating delayed neutrons for the growth rate of the population becomes smaller and smaller. Even though delayed and prompt super-critical states are considered two different operating modes, the transition between the two is a continuous process.[16]

[16] This topic is revisited in Lecture 2, with the introduction of the point-kinetics approximation.

Neutron chain reaction: delayed neutron emission

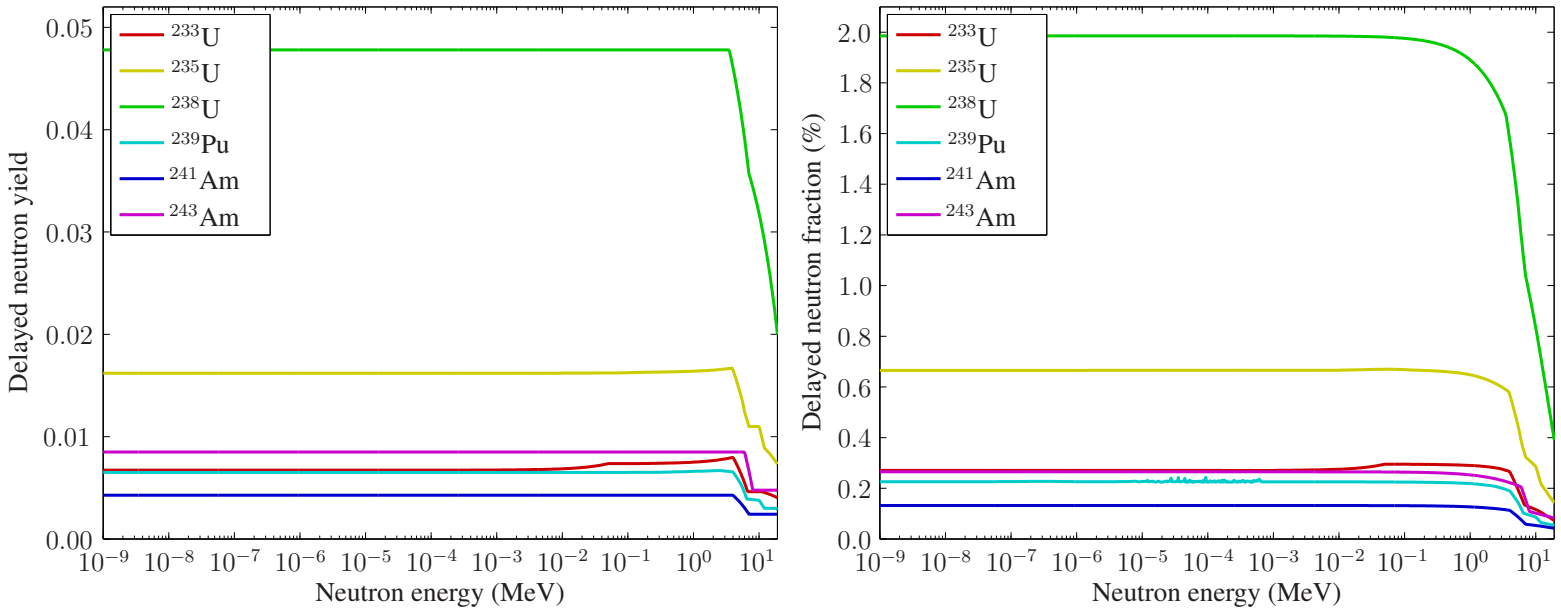

Figure 15: Delayed neutron yield (left) and fraction (right) of selected actinides as function of neutron energy. The yield depends on the probability of producing precursor isotopes, which depends on the fission product distribution of the actinide (see Figure 4) and the neutron energy. The fraction additionally depends on the prompt neutron yield, which varies from nuclide to nuclide and increases practically linearly as function of neutron energy (see Figure 17). Large plutonium and minor actinide content leads to lower delayed neutron fraction compared to fresh uranium fuel. The fraction is reduced even further in fast reactors, operating at the high end of the energy spectrum.

Thermal and fast reactors

Fission neutrons are born at an average energy of about 2 MeV, and they lose energy primarily in elastics scattering collisions. When the neutron energy is high, scattering can be treated as a collision with a free stationary target.

This is not the case when the neutron energy is below about 1 eV. The thermal motion of atoms in the medium follows to a reasonably good approximation the Maxwell-Boltzmann distribution:[17]

$$f(E) = 2\sqrt{\frac{E}{\pi}}\left(\frac{1}{kT}\right)^{3/2}\exp\left(\frac{-E}{kT}\right) \tag{14}$$

where T is the temperature in Kelvin and k = 8.617·10^{-11} MeV/K is the Boltzmann constant. When the kinetic energy of the neutron is comparable to the energy of the target, collisions can lead to up-scattering and increase in neutron energy.

The result is that the slowing-down process is terminated and neutrons collect in the thermal energy range, where they form a distribution resembling the Maxwellian shape.[18]

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[17] This is known as the free-gas approximation, which assumes that elastic scattering can be treated as a collision with an unbound nucleus behaving like an atom or molecule in a gas. The approximations fails for most moderator materials, when the target is bound to a heavier molecule (H_2O, D_2O) or crystalline lattice (graphite). The effect of molecular and lattice binding is also seen in the cross sections in Figure 11.

[18] The energy distribution of thermalized neutrons follows the Maxwell-Boltzmann distribution in the absence of absorption, leakage and source term formed by neutrons scattering down from higher energies. In reality, the shape of the thermal peak is slightly distorted.

Thermal and fast reactors

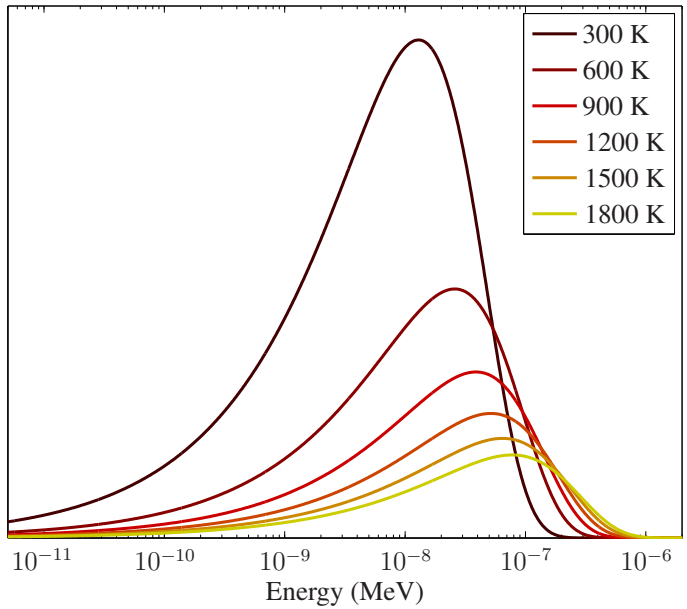

Figure 16: Probability density function of the Maxwell-Boltzmann distribution for energy at different temperatures. The position of the peak shifts towards higher energy as the temperature increases. This distribution forms the basic shape of the energy distribution of thermalized neutrons, although it should be noted that the distribution is not completely Maxwellian, even though it is often referred to as such. The average speed of thermal neutrons at room temperature is about 2200 m/s.

Thermal and fast reactors

Since neutron cross sections depend on energy, so does their capability to maintain the fission chain. Parasitic capture is the dominant reaction mode in the resonance region of actinides, which extends from about 1 eV to high keV range. Fission probability is higher below and above the resonances. Fission neutron yield is practically a linear function of energy, which above 1 MeV compensates for the losses.

There are two practical ways to optimize the neutron economy of the reactor:

1) Keep the average neutron energy high by avoiding scattering with light elements, and increase the content of fissile material to such level that chain reaction can be maintained with fast neutrons.
2) Provide sufficient moderation to slow neutrons past the resonances to sub-eV energies, where they collect and form an equilibrium with the thermal motion of nuclides in the moderator.

These are the operating principles of fast and thermal reactors, respectively.

Thermal and fast reactors

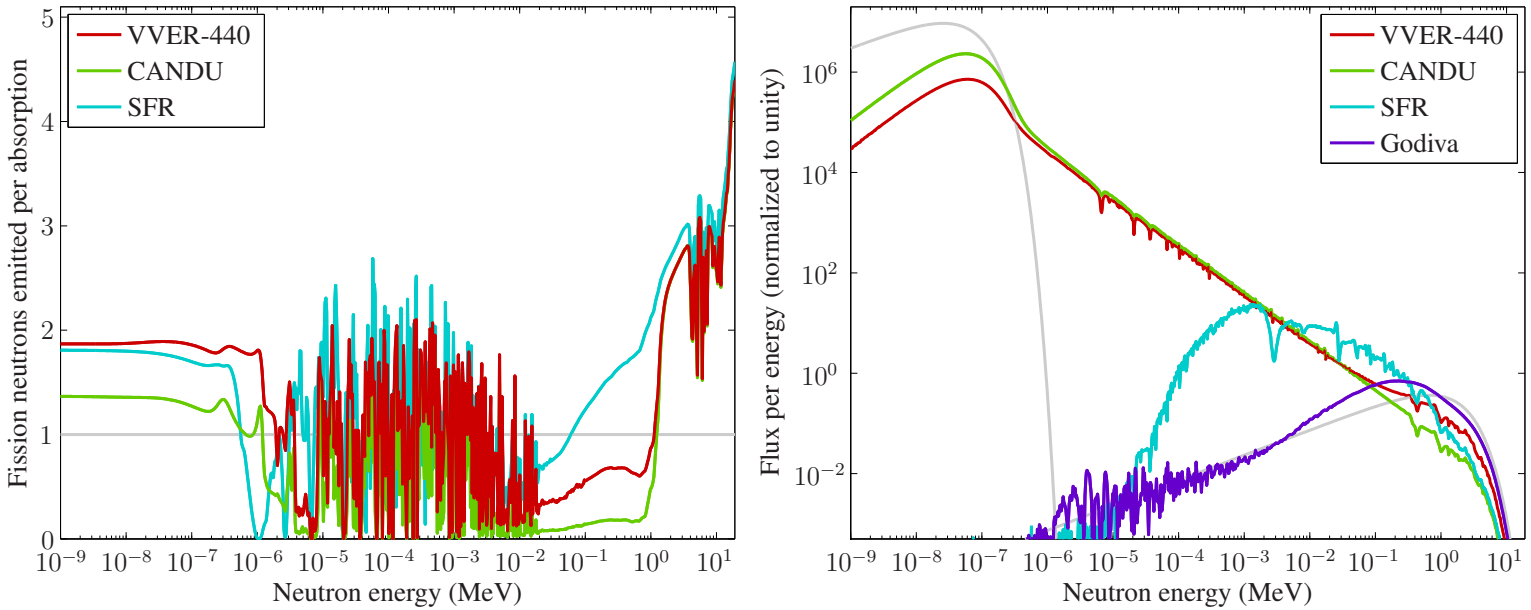

Figure 17: Left: Average number of fission neutrons emitted per absorption to fuel as function of neutron energy in different fuel types. VVER-440 fuel is enriched to 3.6 wt-% ^{235}U and CANDU fuel uses natural uranium (0.72% ^{235}U). The sodium-cooled fast reactor (SFR) fuel is MOX, composed of depleted uranium and recycled plutonium. The fissile plutonium content is 9.6 wt-%. Right: Normalized flux spectra in the three reactor types and a fast criticality experiment (Godiva, bare HEU sphere). Shapes of the Maxwell-Boltzmann distribution at moderator temperature and the energy distribution of fission neutrons are plotted in the background. Light water reactors are typically slightly under-moderated, and the thermal peak is more pronounced in the CANDU fuel. Fast reactors operate on the other end of the flux spectrum, and lower fission cross sections are compensated by higher neutron yield and fissile mass. Even sodium has a noticeable moderating effect on neutrons, and the spectrum is clearly different from the Godiva sphere. Calculated by Monte Carlo simulation.

Thermal reactors: neutron moderation

One of the main advantages of thermal reactors is that they can operate with low-enriched fuel. This results from the high total cross section and fission probability of ^{235}U in the thermal region. In LWR's, more than 80% of fissions occur below 1 eV energy.

Keeping neutrons away from the fuel while slowing past the capture resonances of ^{238}U is an important design goal for thermal reactors. This is accomplished by separating fuel from the moderator. When measured in units of neutron mean-free-path, thermal reactors are clearly more heterogeneous systems compared to fast reactors.

Assuming that scattering is isotropic in the center-of-mass frame, the average fractional energy loss in elastic scattering from a stationary target is constant, and given by:

$$\frac{\Delta E}{E_0} = \frac{1-\alpha}{2} \quad \text{where} \quad \alpha = \left(\frac{1-A}{1+A}\right)^2 \tag{15}$$

and A is the atomic weight ratio of the target nucleus.

This approximation fails when scattering is strongly anisotropic[19] and breaks down completely at thermal energies when up-scattering comes into play.

[19] Elastic scattering of ^{2}H, for example, is slightly backwards-biased between 0.5 MeV and 1.0 MeV, and the energy loss correspondingly larger than what is predicted by Eq. (15).

Thermal reactors: neutron moderation

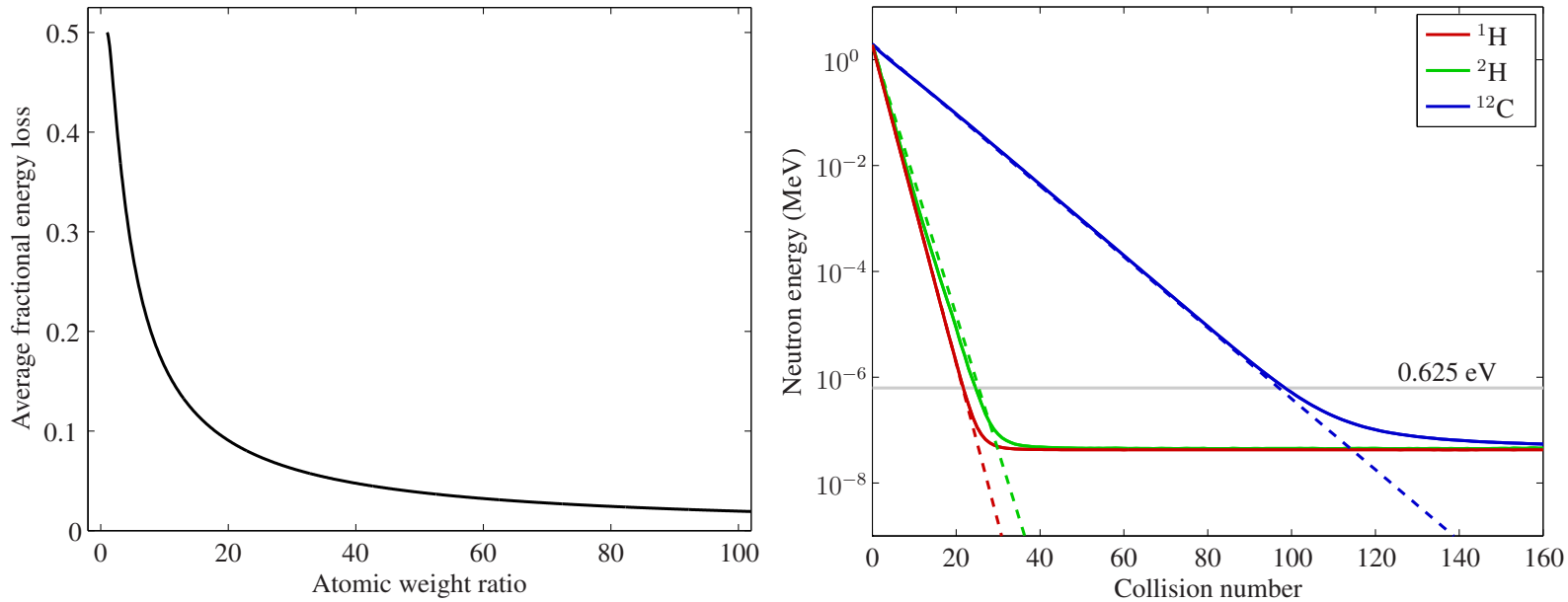

Figure 18: Left: Average fractional energy loss in elastic scattering as function of atomic weight ratio (ratio of target to neutron mass). Calculated using Eq. (15). Right: Average energy of fission neutrons slowed down by collisions from different light nuclides. Slowing-down curves predicted by Eq. (15) are plotted with dashed lines. The moderator is at 300 K temperature, and the slowing-down process is terminated by up-scattering when the neutron energy is comparable to thermal motion of the collision targets. Calculated by Monte Carlo simulation.

Thermal reactors: neutron moderation

Good moderator material is characterized by low absorption rate and high density of light-weight nuclides. When gaseous, chemically unstable and toxic materials are ruled out,[20] there are three practical choices:

- Light water, H_2O
- Heavy water, D_2O
- Graphite

Light water is the most effective of the three in slowing neutrons down with minimum number of collisions, but the (n,γ) cross section of hydrogen is not negligible. The capture probability of a thermal neutron in a collision with ^{1}H is approximately 0.9%.

Capture probability for ^{2}H is much lower, around 0.01% per collision, but because the elastic scattering cross section is also much lower, the required moderator volume is considerably larger.

^{12}C is a massive nucleus compared to hydrogen, and it takes more than four times the number of collisions for the neutron to reach the thermal region. Density of carbon atoms is high in graphite and the cross sections are comparable to ^{2}H.

[20] Liquid hydrogen or helium would make good moderators, but can be ruled out for obvious reasons. Lithium and boron are strong neutron absorbers and beryllium is toxic.

Thermal reactors: neutron moderation

Self-sustained chain reaction cannot be attained with natural uranium (0.72% ^{235}U) and light water moderator, because of neutron absorption in hydrogen. The minimum required enrichment is around 1%.

Heavy water reactors can be run with natural uranium, but the cycle length is somewhat limited. This is most likely one of the original reasons why CANDU reactors were built with pressure tubes, which enables on-line refueling.

The same applies to graphite-moderated reactors, although the required moderator volume is even larger. This makes the moderating properties of graphite sensitive to impurities, in particular boron. Reactor applications require highly purified nuclear-grade graphite (< 5 ppm boron content).

The capability to run with natural uranium was considered a major advantage for the commercialization of nuclear energy at the beginning of the nuclear era, when enrichment technology was expensive and available mainly for military purposes.[21] This is no longer the case and the advantages of LWR technology clearly outweigh the cost of fuel enrichment.

[21] The first LWR's using enriched fuel were naval reactors built to power submarines.

Thermal reactors: neutron moderation

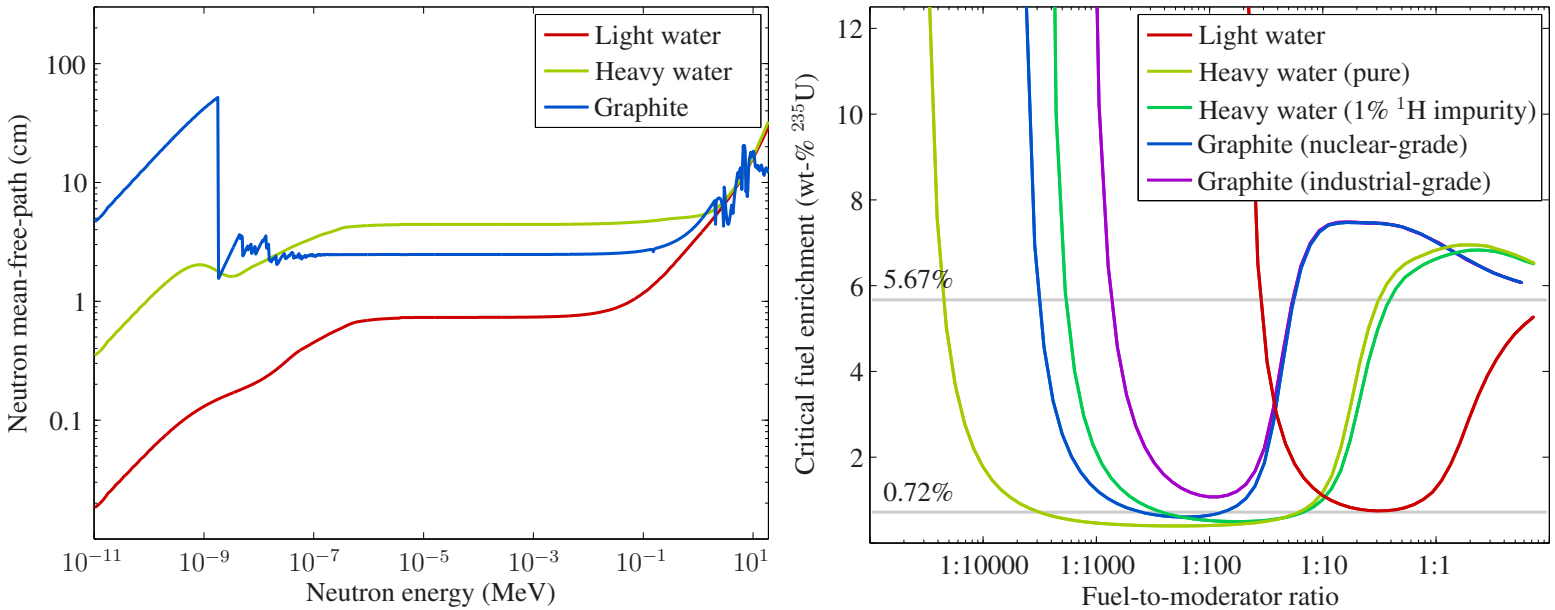

Figure 19: Left: Neutron mean-free-path (mfp) to elastic scattering as function of energy in light and heavy water and graphite. The differences in mfp and the number of collisions required to reach the thermal region explain the differences in moderator volume for different reactor types. Right: Critical enrichments for common moderator materials as function of fuel-to-moderator ratio (FMR). Calculations are performed for a metallic 1 cm diameter uranium pin in an infinite hexagonal lattice surrounded by moderator. FMR is defined as the atomic ratio of uranium to moderator atoms (^{1}H, ^{2}H or ^{12}C). Infinite mass of uranium metal becomes critical at 5.67% enrichment without any moderation. The ^{235}U content in natural uranium is 0.72%. Nuclear- and industrial-grade graphite refer to boron impurities of 1 and 20 ppm, respectively.

Thermal reactors: neutron moderation

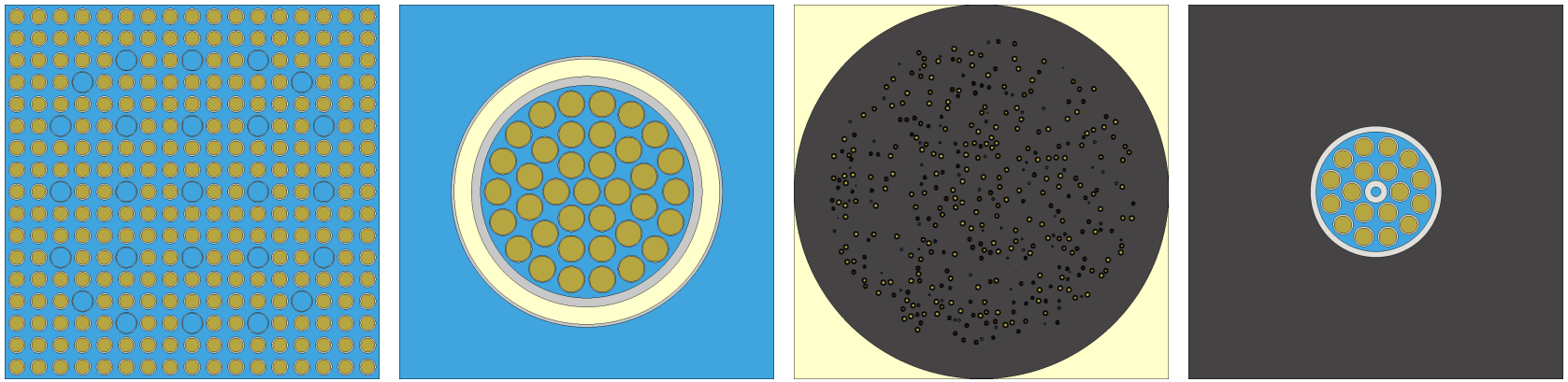

Figure 20: Illustration of typical fuel geometries in light-water, heavy-water and graphite- moderated systems. From left to right: 1) Standard 17$\times$17 PWR fuel assembly, 2) CANDU fuel bundle inside pressure tube and moderator tank, 3) HTGR graphite pebble with 15,000 microscopic fuel particles, 4) RBMK fuel bundle inside pressure tube and graphite block. Moderator volume required to slow neutrons down to thermal region increases with nuclide mass. The fuel pins in figure 1,2 and 4 are 0.8, 1.2 and 1.2 cm in diameter, respectively. The diameter of the fuel pebble is 6.0 cm, and the coated fuel particles are less than 1 mm in size.

Fast reactors

In order to keep neutrons from scattering to the resonance region, fast reactors are cooled by liquid metal or gas. The technology is not in large-scale commercial use, but several research and prototype reactors have been constructed over the years. The development is focused on three concepts:[22]

1) Sodium-cooled fast reactor (SFR)
2) Lead-cooled fast reactor (LFR)[23]
3) Gas-cooled fast reactor (GFR)

Lead-cooled fast reactors were built for submarine propulsion in the USSR in the 1970's (Alfa-class), but no other large-scale prototypes have been constructed.

Several sodium-cooled fast reactor prototypes have been built, for example in France (Phénix and Super-Phénix), Japan (Monju) and Russia and the former Soviet Union (the BN-series). The only operational SFR's are BN-600 and BN-800 in Russia.[24]

[22] These are also the three Gen-IV fast reactor types listed by the Generation IV International Forum (GIF)
[23] In addition to lead, lead-bismuth eutectic is ofte suggested as a coolant.
[24] BN-800 was connected to grid in December 2015.

Fast reactors: breeders

The original incentive for developing fast reactors was the capability to breed fissile material. This was considered essential for the large-scale utilization of nuclear energy in the 1950's, when most of the currently identified uranium resources were yet to be discovered.[25]

The physical basis for breeding is that more than an average of two fission neutrons are emitted per each absorption in fuel – one for maintaining the fission chain and the second for replacing the fissioned nucleus by conversion of a fertile nucleus.

Fast breeder reactors are based on the uranium-plutonium cycle, and the conversion of ^{238}U:

$$^{238}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \longrightarrow ^{239}_{92}\mathrm{U} \longrightarrow ^{239}_{93}\mathrm{Np} \longrightarrow ^{239}_{94}\mathrm{Pu} \tag{16}$$

As seen in Figure 17, achieving sufficient neutron multiplication with ^{239}Pu requires fission with fast neutrons.[26]

To maximize the breeding gain, the reactor core is typically divided into separate seed and blanket regions. After the irradiation cycle is completed, the fuel is reprocessed and fissile material separated from fission products.

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[25] Fast breeder reactor technology has been developed since the beginning of the nuclear era. In fact, EBR-I, the first nuclear reactor to produce electricity in 1951, was a sodium-cooled fast reactor, build for the purpose of demonstrating the basic principles of breeding.

[26] It is also possible attain breeding with thorium-uranium cycle and conversion of fertile ^{232}Th into fissile ^{233}U. Contrary to uranium-plutonium cycle this can be accomplished in thermal spectrum because of the high fission yield of ^{233}U.

Fast reactors: burners

The low thermal fission probabilities of even-N actinides in Figure 7 shows that neutron absorption in LWR flux spectrum, together with beta-decay, leads to accumulation of heavy plutonium, americium and curium isotopes in the fuel, for example:

$$
\begin{aligned}
&{}^{239}_{94}\mathrm{Pu} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{240}_{94}\mathrm{Pu} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{241}_{94}\mathrm{Pu} \xrightarrow{\beta^-} {}^{241}_{95}\mathrm{Am} \\
&{}^{241}_{95}\mathrm{Am} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{242}_{95}\mathrm{Am} \xrightarrow{\beta^-} {}^{242}_{96}\mathrm{Cm} \\
&{}^{242}_{96}\mathrm{Cm} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{243}_{96}\mathrm{Cm} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{244}_{96}\mathrm{Cm}
\end{aligned}
\tag{17}
$$

The build-up of $^{240}\mathrm{Pu}$ and other non-fissile isotopes limits the recyclability of spent fuel. The discharge from LWR fuel cycle inevitably contains long-lived actinides.

This is not necessarily the case for fast reactors. The fission probability of even-N actinides, such as $^{240}\mathrm{Pu}$ and $^{241}\mathrm{Am}$ is relatively high (see Fig 8), which means that the reactor can operate on fuel that has undergone multiple reprocessing cycles. Fast reactors can therefore be used for burning the waste products of LWR's.

Currently the main motivation for developing fast reactor technology is to accomplish closed fuel cycle, in which the utilization of natural resources is maximized and long-lived actinides completely removed from disposed nuclear waste.

Fast reactors: burners

Fast reactor design comes with a few additional technical challenges:

- Material issues: sodium is chemically reactive with air and water and lead is corrosive
- Some of the strong negative reactivity feedbacks that stabilize thermal reactors do not work in fast spectrum
- Possibility of re-criticality cannot be ruled out on physical grounds in severe accidents
- Delayed neutron fractions are low compared to LWR's, because of fast spectrum and plutonium / minor actinide based fuel

The very low delayed neutron fraction (< 2%) of minor actinides, especially ^{241}Am (See Figure 15) reduces the margin to prompt super-criticality, which limits their permissible content in fast reactor fuel. Even though plutonium could be recycled efficiently, it is likely that some long-lived minor actinides would remain in the discharged waste.

Another option for burning plutonium and especially minor actinides (Np, Am, Cm) is to use sub-critical accelerator-driven systems (ADS), in which neutrons from an external source[27] are multiplied in a sub-critical core. Moving from critical to sub-critical systems could remove some of the safety issues related to feedbacks and low delayed neutron fraction, but the technology also introduces a whole new range of problems.

[27] The neutron source in ADS is a particle accelerator, firing protons into a heavy target. The collisions induce spallation reactions, releasing very high-energy neutrons (up to 1 GeV).

Reactivity feedbacks

The chain reaction is stabilized by reactivity feedbacks, which are particularly strong in light water reactors, because the reactor operates on thermal neutrons ($\sim$80% of fissions), and the slowing-down process is sensitive to operating conditions:

- Neutrons are born with a mean energy of 2 MeV, and they require about 20 elastic collisions with hydrogen nuclei to reach the thermal region (< 1 eV)
- If the neutron returns to fuel too early, it is more likely to be absorbed in the capture resonances of ^{238}U than cause fission in ^{235}U
- Since coolant serves the purpose of moderator as well, any change in the reactor operating state is reflected in the slowing-down process

There are two primary mechanisms behind reactivity feedbacks:

(i) The probability of the neutron to reach the thermal region is changed, which affects the number of thermalized neutrons (Doppler- and coolant density feedback)

(ii) The position of the thermal distribution on the energy scale is changed, which affects the fission probability of thermalized neutrons (moderator temperature feedback)

Other effects, less significant for LWR's, include changes in leakage rate and thermal expansion.

Thermal reactors are typically very heterogeneous systems, and keeping neutrons away from the fuel before they are completely thermalized is essential for maintaining the chain reaction.

Reactivity feedbacks: fuel temperature

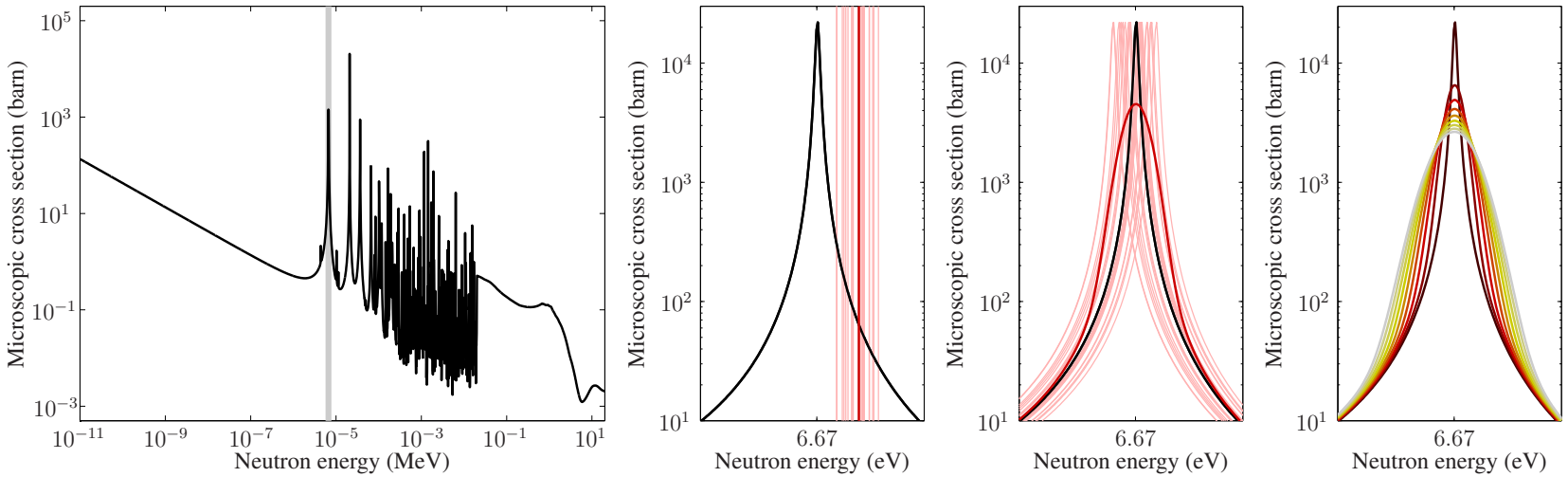

Figure 21: Doppler-broadening of a ^{238}U resonance peak. The thermal motion of atoms causes a random shift in the relative energy between the neutron and the target nuclide at each collision. The average effect is that narrow resonance peaks appear broader, which increases the overall reaction probability. The effect is most pronounced just above the thermal region, where the predominant reaction mode is parasitic capture by ^{238}U. The result is that increasing fuel temperature leads to reduced reactivity and power (negative feedback).

Reactivity feedbacks: moderator density

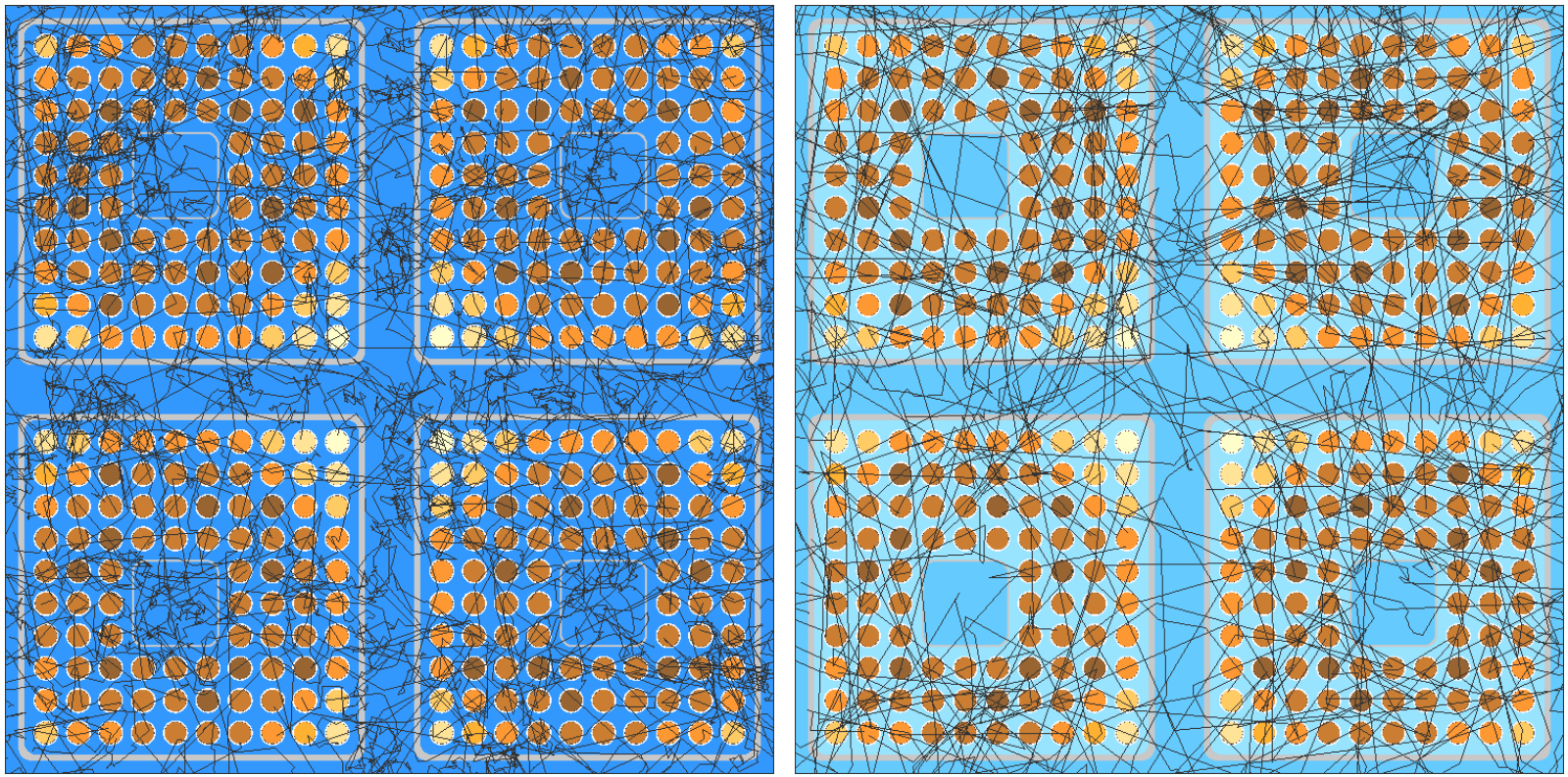

Figure 22: Neutron tracks in BWR fuel. Left: cold core, high coolant density. Right: hot core, low coolant density. Increase in neutron mean-free-path (average distance between two collisions) increases the probability of the neutron returning to the fuel before thermalization is completed. The predominant reaction mode above thermal region is parasitic capture by ^{238}U, so increase in coolant temperature leads to reduced reactivity and power (negative feedback).

Reactivity feedbacks: moderator temperature

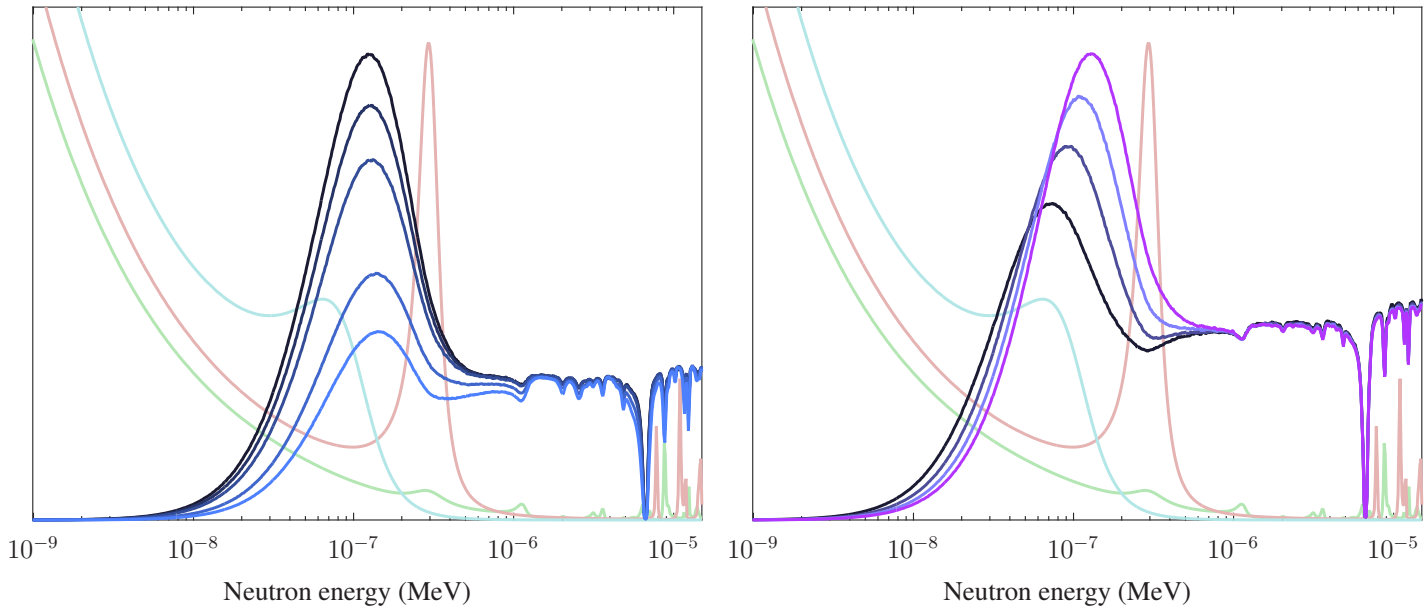

Figure 23: Effect of coolant density and temperature on the distribution of thermalized neutrons. Shapes of ^{235}U, ^{239}Pu fission and ^{135}Xe capture cross sections are plotted on the background in green, red and cyan, respectively (not to scale). Left: decrease in density reduces the height of the thermal peak, leading to reduced fission rate and negative feedback. Right: Effect of coolant temperature on the distribution of thermalized neutrons. Increasing the temperature moves the thermal peak upwards on the energy scale. This reduces the fission rate of ^{235}U. For ^{239}Pu the impact is opposite, because of the peak located right above the distribution. Also the capture rates of high-absorbing fission products are reduced, which results in a positive rectivity effect. The effect of spectral shift is usually not as strong as the moderator density effect, and the net effect in in LWR's is negative feedback.

Reactivity feedbacks

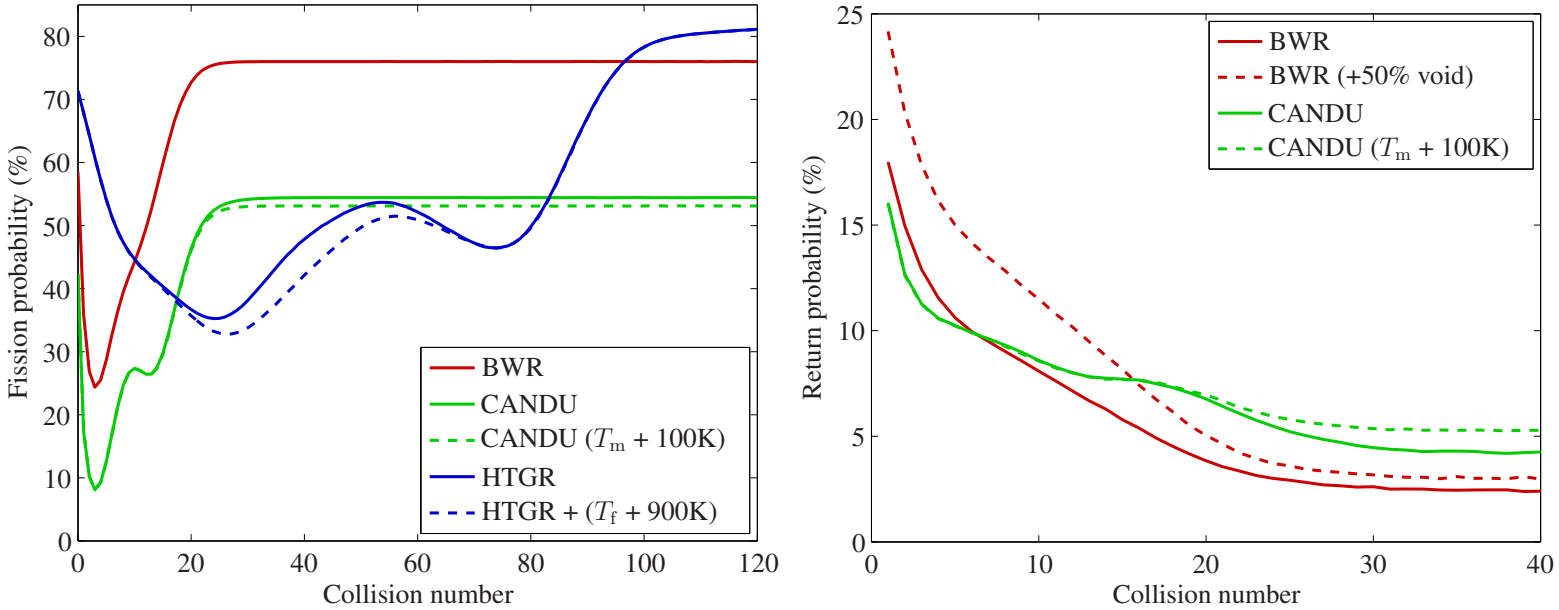

Figure 24: Fission and return probabilities as function of collision number for different reactor types. Left: Probability of a neutron to undergo fission if its next collision occurs in fuel. The probability drops while passing through the resonances of ^{238}U, which requires different number of collisions for different moderators. The dashed blue line shows the mechanism behind the Doppler-feedback, as the fuel temperature in the HTGR case is increased from 300K to 1200K. Moderator temperature feedback is demonstrated in the CANDU case, with a temperature increase from 500K to 600K (see also Fig. 23). Right: Probability of a neutron colliding in the moderator to undergo its next collision in fuel. The dashed red line demonstrates the effect of increasing neutron mean-free-path (mfp) when the void fraction in the BWR case is increased from 25% to 75% (see also Fig. 22). Similar effect is seen for thermalized neutrons in the CANDU case, as the shift in the thermal peak increases the average mfp.

Reactivity feedbacks: positive feedback

Negative feedbacks stabilize reactor operation by resisting deviations in fission power – increase or decrease in heat production is reflected in material temperatures and densities, which cause a counteracting effect in reactivity.

Positive feedbacks have the opposite effect – small deviations in local fission rate begin to grow, and if not terminated by negative feedback or active reactivity control, can lead to run-off chain reaction when reactivity reaches prompt super-critical state.

In thermal reactors, positive feedback can result from:

- Excessive use of soluble absorber, i.e. when the concentration of soluble boron is so high that positive reactivity from reduced absorption exceeds negative reactivity from reduced moderation as the moderator expands
- Over-moderation, i.e. when the moderator volume is so large that the previous effect results from the moderator itself acting as a weak absorber
- Large positive moderator temperature coefficient (see Figure 23)

In fast reactors, coolant expansion and boiling leads to reduced absorption and spectrum hardening, which usually lead to increased reactivity and positive feedback.[28]

[28] Sodium-cooled fast reactors are stabilized by negative feedback resulting from thermal expansion and increased axial leakage following coolant boiling. The effect is maximized if the active part of the core is flat and wide.

Reactivity feedbacks: positive feedback

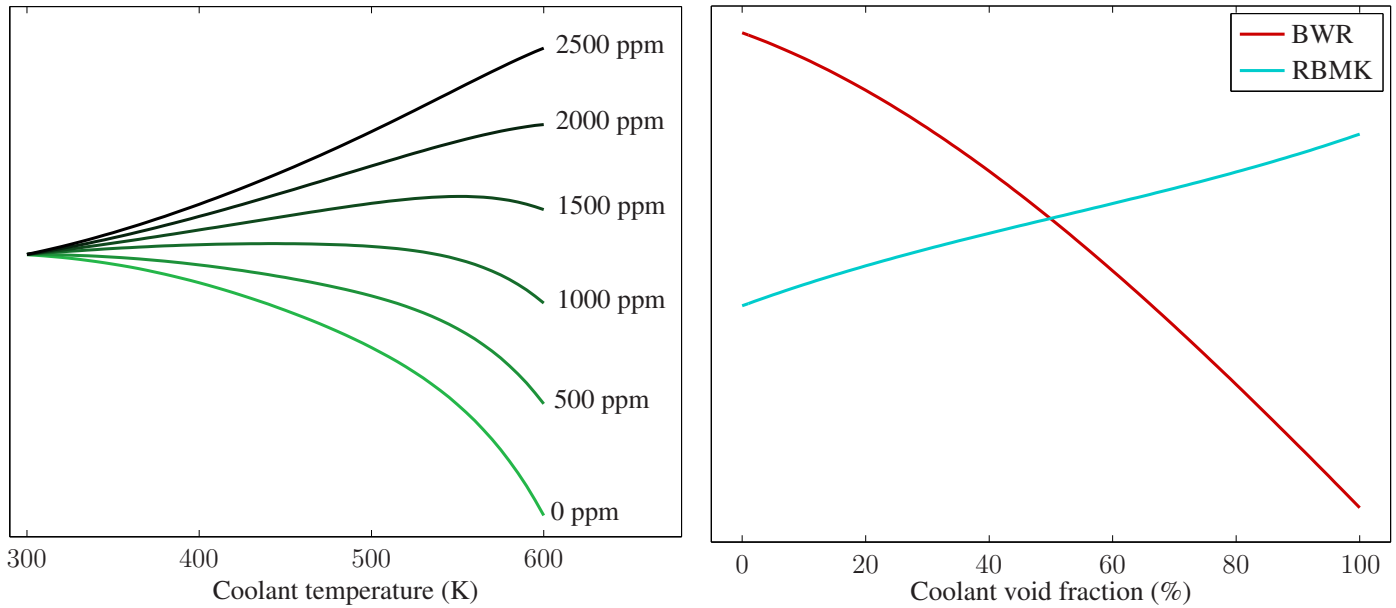

Figure 25: Left: Effect of boron concentration in PWR coolant temperature feedback on reactivity. When the concentration of soluble absorber exceeds a certain limit, the positive reactivity from reduced absorption overwhelms the negative effect from reduced moderation as the coolant expands. Right: Negative (BWR) and positive (RBMK) reactivity feedbacks from coolant boiling. The RBMK reactor is over-moderated by graphite, and water coolant acts as a weak absorber. All calculations were carried out using Monte Carlo simulations in infinite-lattice geometries. Core-level effects and Doppler-feedback from fuel temperature are not taken into account.

Reactivity control

Active reactivity control is accomplished by control rods and soluble absorber, and used for:

- Adjustment of power level, including reactor start-up and shut-down
- Emergency shut-down (scram)
- Compensation of reactivity changes caused by feedbacks and fuel depletion

In BWR's, also the recirculation pumps take active part in reactivity control by adjusting flow rate and void fraction in the reactor core.

Typical control rod absorbers in power reactors include boron (carbide and steel) and AIC (silver-indium-cadmium). Cadmium, hafnium, etc. are used in research and test reactors. Soluble absorber is boric acid, H_3BO_3.

In addition, passive reactivity control in the form of burnable absorbers is used to compensate for the excess reactivity of fresh fuel. Common burnable absorbers include gadolinium, erbium and boron silica glass.

It is important to note that the role of control rods and other active reactivity control is to set and adjust the reactor power level. The stability of the chain reaction is accomplished by negative reactivity feedbacks.

Fuel depletion

Reactivity feedbacks represent fast, almost immediate coupling between neutronics and reactor operating conditions. In the long time-scale, the reactor physical characteristics of nuclear fuel are coupled to changes in its isotopic composition:[29]

- ^{235}U is depleted and replaced by ^{239}Pu as the primary fissile isotope
- Non-fissile plutonium, minor actinides (Np, Am, Cm) and fission products are accumulated in the fuel, increasing absorption
- Burnable absorber used for passive reactivity control is depleted

Nuclear fuel in light water reactors is loaded in the reactor core for the duration of the entire operating cycle, which is typically 12 or 18 months. After the cycle is completed, 1/3 or 1/4 of the fuel assemblies are removed and replaced with fresh fuel, and the core is shuffled to re-optimize the loading pattern.

This means that a single fuel assembly spends 3 to 4 years in the reactor, and each core loading has a history extending several years in the past.

Fuel utilization is measured in units of *burnup*, which refers to the amount of extracted energy per uranium or heavy metal mass (for example, 40 MWd/kgU). Discharge burnup is dictated by the local power level, cycle length and the number of cycles the assembly spends in the reactor core.

[29] Fuel depletion and effects of burnup on reactor operating conditions are covered thoroughly in Lecture 5.

Fuel depletion

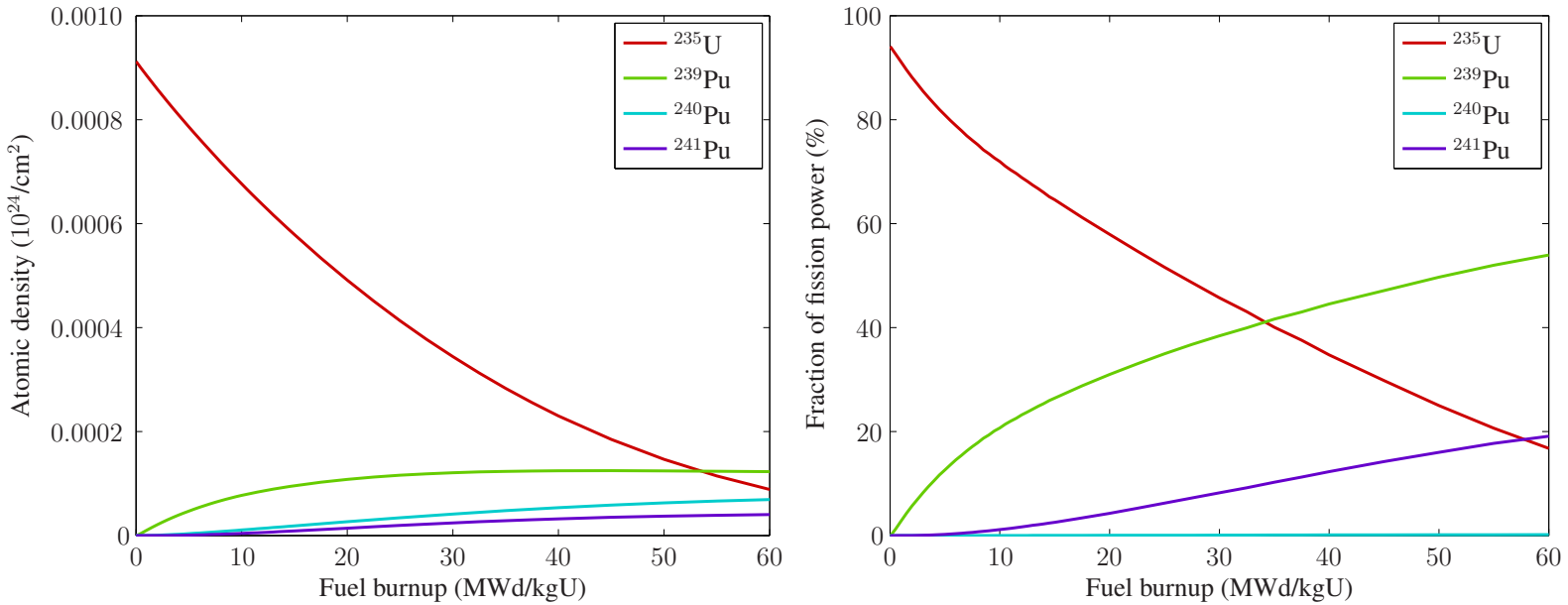

Figure 26: The composition of nuclear fuel changes along with the reactor operating cycle. The depletion of uranium and build-up of plutonium, minor actinides and fission products changes the reactor physical characteristics of the core. Left: depletion of ^{235}U and build-up of plutonium isotopes in PWR fuel as function of burnup. Right: contributions of ^{235}U and plutonium to fission power.

Fuel depletion

Maintaining criticality over the operating cycle requires sufficient amount of excess positive reactivity, and an equal amount of negative reactivity in the capacity of control systems (control rods and boron shim) to compensate for it at beginning of cycle. But there are other limitations:

- Too much boron in the coolant results in positive moderator void coefficient
- Too much absorption in control rods leads to prohibitively high reactivity insertion in rod ejection (PWR) / drop (BWR) transients

The amount of excess reactivity can be compensated for by passive reactivity control using burnable absorbers, i.e. absorbers that are slowly depleted by neutron irradiation. Typical burnable absorbers include:

- Boron silicate glass rods inserted in the control rod guide tubes of selected fuel assemblies (used in PWR's, absorber isotope ^{10}B)
- Gadolinium oxide mixed with uranium oxide in selected fuel pins (used in PWR's and BWR's, absorber isotopes ^{155}Gd and ^{157}Gd)

The absorber is usually depleted during the first quarter of the assembly's life.

Fuel depletion

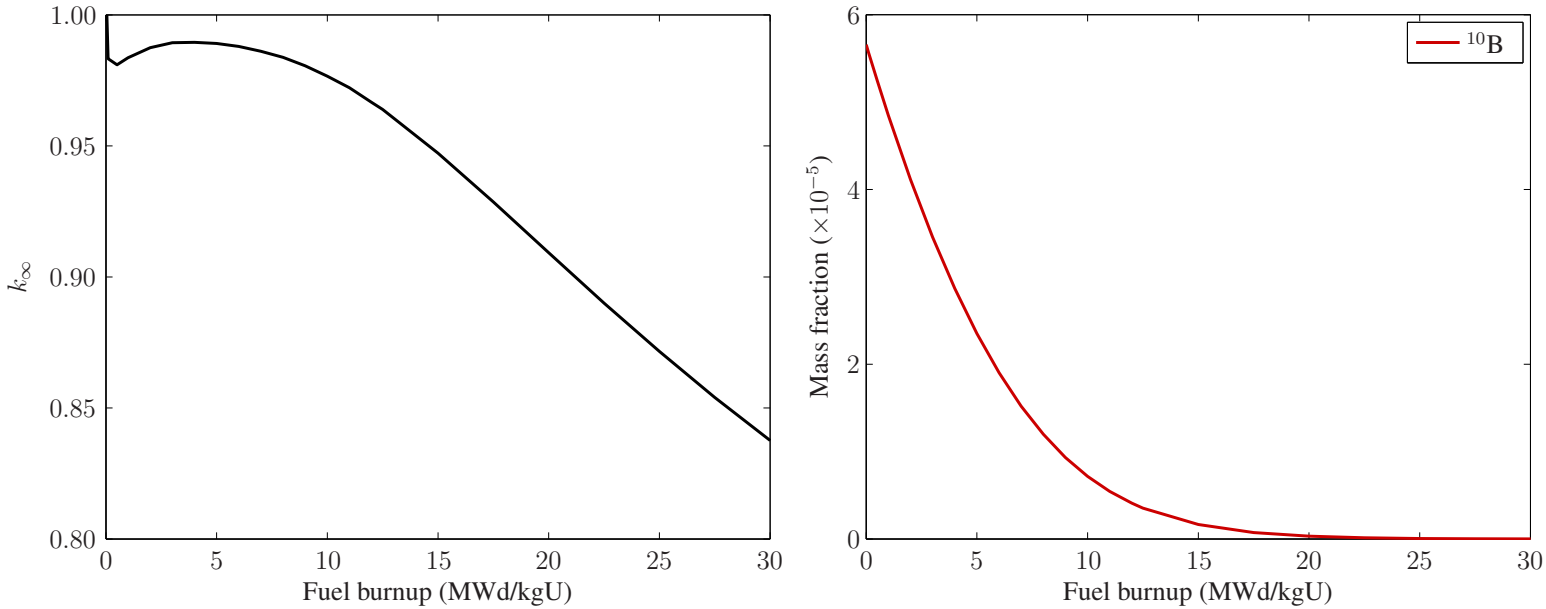

Figure 27: Depletion of a “gray” boron silicate glass burnable absorber in a PWR fuel assembly calculation. Left: Infinite multiplication factor, Right: Absorber concentration. The absorber is natural boron (20% ^{10}B, 80% ^{11}B), but only the high-absorbing isotope is depleted.

Fuel depletion

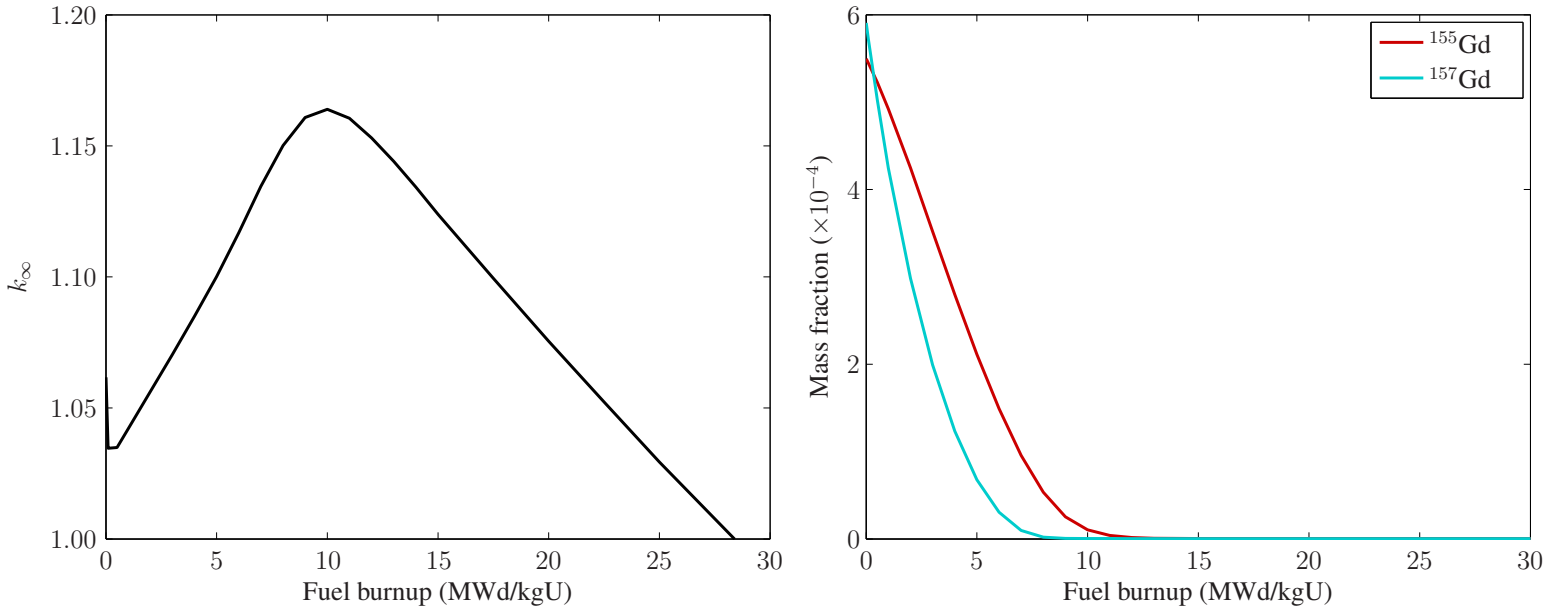

Figure 28: Depletion of a gadolinium oxide burnable absorber in a BWR fuel assembly calculation. Left: Infinite multiplication factor, Right: Absorber concentrations. The absorber is natural gadolinium (0.2% ^{152}Gd, 2.1% ^{154}Gd, 14.8% ^{155}Gd, 20.6% ^{156}Gd, 15.7% ^{157}Gd, 24.8% ^{158}Gd, 21.8% ^{160}Gd), but only the two high-absorbing isotopes are depleted.

Summary of main topics

Neutron interactions can be divided into scattering, capture and fission. The reaction probabilities depend on the target nucleus and vary by several orders of magnitude depending on neutron energy.

Fission produces $\sim$200 MeV of energy, 2-7 new neutrons and two intermediate-mass fission products, that are unstable against beta-decay, because of their high neutron surplus. Fission probability is strongly dependent on the parity-effect, which basically divides actinides into fissile and fissionable isotopes.

Fission reactors operate on a self-sustaining chain reaction, in which the decay of delayed neutron precursors forms a constant source of prompt fission chains. The length of these chains determines the reactor operating condition (sub-critical, critical, delayed super-critical or prompt super-critical).

The operation of thermal reactors relies on the high fission probability of low-energy neutrons. Neutrons are slowed down past the resonance region in a moderator, consisting of light, low-absorbing element. The chain reaction is stabilized by negative reactivity feedbacks, many of which work by disrupting the slowing-down process.

LWR fuel assemblies spend 3-4 years in the reactor core. During this time the reactor physical characteristics of the fuel change as ^{235}U is depleted and plutonium, minor actinides and fission products are build-up. The changes in reactivity are compensated by dilution of soluble absorber (PWR), withdrawal of control rods (BWR) and depletion of burnable absorber.

Topics of next lecture

The next lecture continues with deterministic transport theory.

Specific topics include:

- Basic concepts of deterministic transport theory
- Neutron transport equation in eigenvalue and time-dependent form
- Point-kinetics approximation