General information

The exercise sessions will be held as blackboard sessions, where the participants will present their solutions to the group. As such, the problems should be set up and solved before the session. The focus of the exercises lies on analyzing and discussing the task at hand together with the group: thus, a perfect solution is not required to be awarded points. A point will be awarded for each question, and a person will be chosen to present their solution from the pool.

Exercise 1.

The size of a reactor-scale tokamak

For full derivation of the equations used in this task, please take a look at the Appendix A in reference [1].

Despite the vast complexity of tokamaks, the basic engineering design parameters are determined by relatively simple criteria. A reactor relevant tokamak has to be designed to provide adequate energy confinement, MHD stability, plasma control to avoid frequent disruptions, particle control for fuelling, impurity content, helium exhaust control, power exhaust control to avoid damage and melting of the wall components, adequate shielding of the superconductive coils from nuclear heating and insulator damage, and to support the magnetic stresses imposed on the toroidal field coils.

The energy confinement of the H-mode plasmas has been observed experimentally to follow the ITER, IPB 98 (y,2) scaling [1] (can also found in the "Fusion principles" lecture slides). This scaling indicates that, among some other parameters, the energy confinement increases with the plasma current, I_p, and major radius, R, and is reduced with heating power, P:

$$\tau_E \propto I_p^{0.91} R^{1.5} P^{-0.65} \tag{1}$$

Therefore, in order to maximize the energy confinement, the scaling indicates that as high as possible plasma current and major radius are desired. The capital cost of a tokamak, however, increases strongly as a function of the major radius, and, therefore, from the economical point of view, the reactor should be made as compact as possible. The maximum plasma current, on the other hand, is limited by MHD instabilities. In other words, edge safety factors, q_{95}, of 2.5 or above are required for stable operation. For a robust baseline H-mode scenario, $q_{95} \sim 3$ can be taken as a representative value. The edge safety factor can be approximated by the equation

$$q_{95} = \frac{5a^2 B_T f}{R I_{MA}}, \tag{2}$$

where a is the minor radius of the plasma, B_T, is the strength of the toroidal magnetic field at the magnetic axis, and f is a function of the plasma shape, for which we will use a value of 2.3

here. I_{MA} is the current in MA. The maximum current is, therefore, limited by the machine geometry and the maximum toroidal magnetic field. The maximum toroidal magnetic field is, on the other hand, limited by the requirement to retain superconductivity as well as to resist the magnetic stresses. Therefore, the maximum coil magnetic field achievable with Nb_3Sn coils is about 12 T, which translates to maximum magnetic fields at the magnetic axis about 6 - 7 T.

(a) The ITER final design report (FDR) has developed an ignition criterion for the plasma current and aspect ratio:

$$\frac{I_p R}{a} > 60.$$

This follows from the plasma energy balance and the energy confinement scaling (eq. 1).

By using this ignition criterion, equation (2), and the associated limit $q_{95} \sim 3$, calculate the required minor radius, a, of the tokamak, assuming $B_T \sim 6$ T and $f \sim 2.3$.

The current (reduced cost) ITER reference value is 2.0 meters, while the original design had a minor radius of 2.8 meters. How does the value you calculated compare to these?

(b) The major radius of a tokamak is given by the equation $R_{axis} = R_{coil} + d_{shield} + a$, where R_{coil} is the major radius of the inner edge of the toroidal field coils, d_{shield} is the thickness of the neutron shield required to avoid excessive neutron damage of the toroidal field coils, and a is the minor radius of the tokamak (you can here use $a = 2.6$ meters). The thickness of the required shield is about $d_{shield} \sim 1.3$ m.

Estimate the resulting R_{axis}. First, you need to calculate the ratio of R_{coil}/R_{axis}, which you will get from the assumed ratio of magnetic coil at the axis to the maximum magnetic field at the coil ($B_T/B_{coil} \sim 0.5$) with the relation $B(R) = B_{coil}R_{coil}/R$.

The current (reduced cost) ITER reference value is 6.2 meters, while the original design had a major radius of 8.14 meters. How does the calculated value compare to these reference values?

Solution 1.

(a) Using the ignition criterion:

$$a < \frac{I_P R}{60}$$

The maximum plasma current is limited by the the maximum toroidal field and the required edge safety factor:

$$q_{95} = \frac{5a^2 B_t f}{R I_{MA}} \approx 3 \rightarrow I_{MA} = \frac{5a^2 B_T f}{3R} = \frac{5a^2 \cdot 6 \cdot 2.3}{3R} = \frac{23a^2}{R}$$

Combining the above equations

$$a < \frac{I_P R}{60} = \frac{R}{60}\frac{23a^2}{R} \approx 0.38a^2 \leftrightarrow a > 2.6 \text{ m}$$

The current (reduced cost) ITER reference value is 2.0 m [Progress in the ITER physics basis, Nucl. Fusion, 2007, chapter 1].

The original ITER EDA value was 2.8 m [ITER physics basis, Nucl. Fusion, 1999, chapter 1].

(b) First, we need to find the fraction the total major radius required to the inner edge of the toroidal field coils R_c/R. Using

$$B(R) = B_c \frac{R_{coil}}{R},$$

and the assumed ratio of the magnetic field at the axis to the field at the coil $B_T/B_{coil} = 0.5$

$$B_T(R_{axis}) = 0.5B_{coil} = B_{coil}\frac{R_{coil}}{R_{axis}} \leftrightarrow R_{coil} = \frac{1}{2}R_{axis}$$

The required shield thickness is obtained by numerical simulations calculated the required thickness to attenuate the neutron radiation sufficiently to prevent unacceptable degradation of the superconducting field coils.

The assumed value in this case value is $d_{shield} = 1.3$ m. Using the smallest possible minor radius from the previous sub-task $a \approx 2.6$ m, the major radius becomes:

$$R_{axis} = \frac{1}{2}R_{axis} + 1.3 \text{ m} + 2.6 \text{ m} \leftrightarrow R_{axis} = 2(1.3 \text{ m} + 2.6 \text{ m}) = 7.8 \text{ m}.$$

The current reduced cost ITER value is 6.2 m [Progress in the ITER physics basis, Nucl. Fusion, 2007, chapter 1].

The original EDA ITER value was 8.14 m [ITER physics basis, Nucl. Fusion, 1999, chapter 1].

The original International Thermonuclear Experimental Reactor (ITER) was designed to ignite [ITER physics basis, Nucl. Fusion, 1999, chapter 1, Page 35, Appendix B, section 2]:

"The overall programmatic objective of ITER which shall guide the engineering design activities (EDA), is to demonstrate the scientific and technological feasibility of fusion energy for peaceful purposes. ITER would accomplish this objective by demonstrating controlled ignition and extended burn of deuterium-tritium plasmas, with steady state as an ultimate goal, by demonstrating technologies essential to a reactor in an integrated system, and by performing integrated testing of the high heat flux and nuclear components required to utilize fusion energy for practical purposes."

[Progress in ITER physics basis, Nucl. Fusion, 2007, chapter 1, Page 4]: "However, for financial reasons, the ITER parties recognized the need of a new design to meet revised technical objectives and a **cost reduction target of about 50%** of the previously accepted cost estimate"

The revised goals of ITER are, instead of controlled ignition and extended burn, to

(a) achieve extended burn in inductively driven D-T plasma operation with Q $>$ 10, not precluding ignition, with a burn duration of between 300 and 500 s.

(b) aim at demonstrating steady-state operation using non-inductive current drive with Q $>$ 5.

Non-inductive current drive is actually not absolutely necessary for reactors [European Fusion Roadmap, EFDA, 2012, ISBN 978-3-00-040720-8]. However, pulse type reactors would not be economically as attractive as steady-state ones.

Furthermore, in terms of engineering performance and testing, the design should:

(a) demonstrate availability and integration of essential fusion technologies

(b) test components for a future reactor and

(c) test tritium breeding module concepts; with a 17 MeV neutron power load on the first wall $>$ 0.5 MW m^{-2} and fluence $>$ 0.3 MW a m^{-2}.

In addition, the device should:

(a) use as fas as possible technical solutions and concepts developed and qualified during the previous period of the EDA

(b) cost about 50% of the direct capital cost of the 1998 ITER design.

The present cost estimate of the project is 25.6 billion euros, shared by the 7 ITER members and including a 10% contingency to account for overruns.
`https://physicstoday.scitation.org/do/10.1063/PT.6.2.20180416a/full/`

For comparison, the estimated cost of the Iraq, Afghanistan, and Pakistan wars for US is about a few trillion dollars [N. Crawford and C. Lutz, "Economic and Budgetary Costs of the Wars in Afghanistan, Iraq, and Pakistan to the United States: A summary", Brown University, 20 July 2011]. **This equals about 100 ITERs.**

The annual global drug trade is about 300 – 400 billion dollars [United Nations Office on Drug and Crime]. **This equals about 15 ITERs.**

Table 1. ITER design features and parameters for reference ignited ELMy H-mode operation

Parameter	Value
Major/minor radius	8.14 m/2.80 m
Plasma configuration	Single null divertor
Plasma vertical elongation/triangularity (at 95% poloidal flux)	1.6/0.24
Plasma volume	$\sim$2000 m^3
Plasma surface area	$\sim$1200 m^2
Nominal plasma current	21 MA
Electron density	0.98×10^{20} m^{-3}
Volume average temperature	12.9 keV
Toroidal field	5.68 T (at $R = 8.14$ m)
MHD safety factor (q_{95})	$\sim$3.0 (at 21 MA)
Volume average β/β_N	0.030/2.29
Fusion power (ignited, nominal)	1.5 GW
Plasma thermal energy content	1.07 GJ
Plasma magnetic energy content	1.1 GJ
Confinement mode	ELMy H-mode
Radiation from plasma core	118 MW
Transport power loss	182 MW
Transport energy confinement time τ_E	5.9 s
$P_{transport}/P_{L \to H}$	1.4
Species concentrations % He/Be/Ar	10/2/0.16
Z_{eff} (effective ion charge)	1.9
Average neutron wall loading	$\sim$1 MW$\cdot$m^{-2} (at 1.5 GW)
Lifetime neutron fluence	$\geq$1 MW$\cdot$a$\cdot$m^{-2}
Burn duration (ignited, inductive current drive)	$\geq$1000 s
Available auxiliary heating power	100–150 MW
In vessel tritium inventory safety limit	1 kg

Figure 1: ITER 1998 Design [ITER physics basis, Nucl. Fusion, 1999]

Table 2. ITER parameters and operational capabilities.

Parameter	Attributes
Fusion power	500 MW (700 MW)[a]
Fusion power gain (Q)	$\geqslant$10 (for 400 s inductively driven burn); $\geqslant$5 (steady-state objective)
Plasma major radius (R)	6.2 m
Plasma minor radius (a)	2.0 m
Plasma vertical elongation (95% flux surface/separatrix)	1.70/1.85
Plasma triangularity (95% flux surface/separatrix)	0.33/0.48
Plasma current (I_p)	15 MA (17 MA)[a]
Safety factor at 95% flux surface	3 (at I_p of 15 MA)
Toroidal field at 6.2 m radius	5.3 T
Installed auxiliary heating/current-drive power	73 MW (110 MW)[b]
Plasma volume	830 m^3
Plasma surface area	680 m^2
Plasma cross section area	22 m^2

[a] Increase possible with limitation on burn duration.
[b] A total plasma heating power of 110 MW may be installed in subsequent operation phases.

Figure 2: The reduced cost, current ITER design [Progress in the ITER physics basis, Nucl. Fusion, 2007]

Exercise 2.
Tritium retention

Accumulation of radioactive material in the reactor structures could be a real issue when using D-T fusion. Due to nuclear safety reasons, the maximum amount of tritium retained inside ITER is limited to about 700 g. If this ceiling is reached, operations will be stopped until sufficient tritium cleaning actions have been conducted. Figure 1 shows the fuel retention rate (number of particles deposited in the wall per second) in the JET tokamak for various operation modes. The graph includes values for the previous full carbon plasma-facing components (PFCs) and for the present ITER-like PFC (tungsten divertor and mostly beryllium main chamber wall). The values are for total number of particles, and have to be halved to account for the tritium in D-T plasma.

Assuming that the absolute retention rate in ITER is a factor of 4 higher than in JET, due to its larger size, estimate the number of type-I H-mode pulses (duration 400 seconds) in ITER required to reach the tritium ceiling. Consider both full carbon and tungsten/beryllium PFC materials.

Compare the estimations to the values given in the figure 2. If the device is shut down once a year and the cleaning can be conducted then, is carbon a viable wall material for ITER from the tritium retention point of view? How about the combination of tungsten and beryllium?

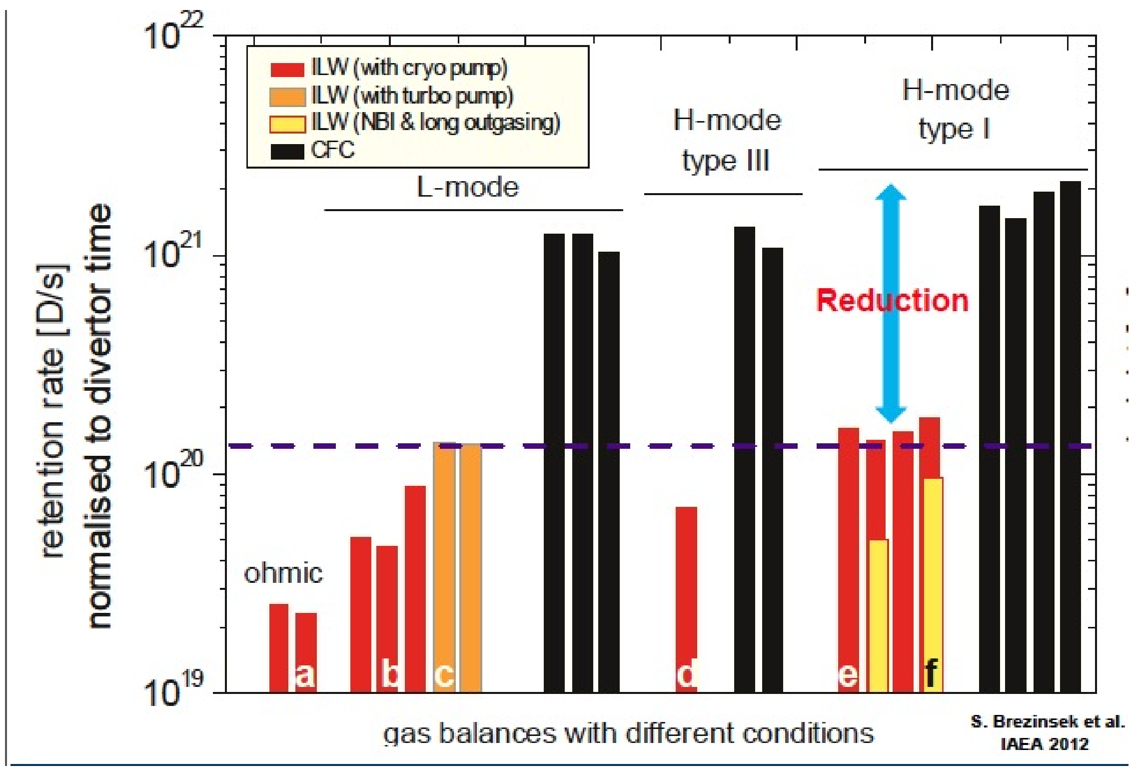

Figure 3: Fuel retention rate (particles per second) in JET tokamak in various operation modes in completely carbon (CFC) wall and Iter-like wall (ILW). [2]

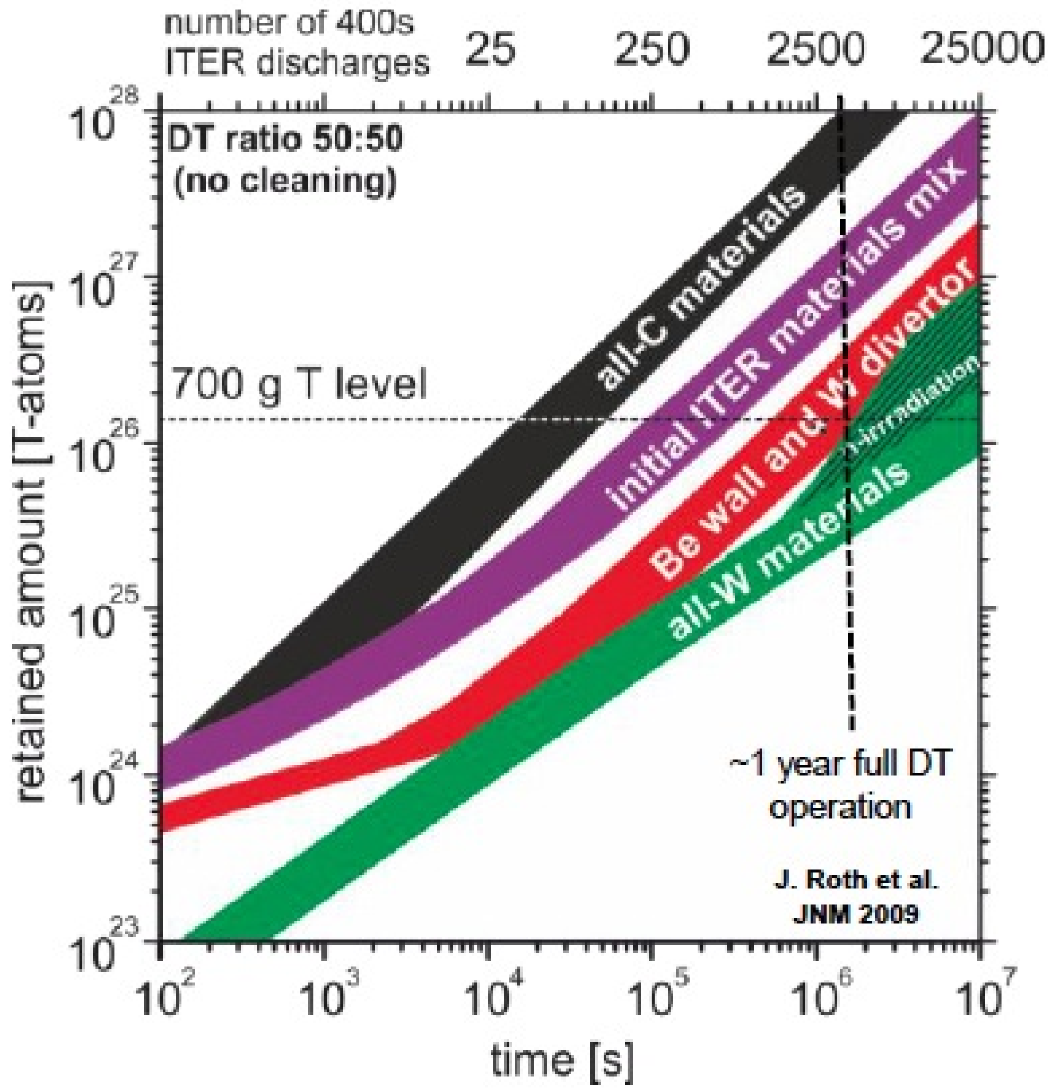

Figure 4: Estimated time to reach the 700 g tritium ceiling in ITER for the different PFC material configurations. [3]

Solution 2.

The measured fuel retention rate in JET type-I ELMy H-mode pulses is about 2×10^{21} particles per second with the full CFC covering, and 2×10^{20} particles per second with the tungsten/beryllium wall materials. Therefore, the estimated fuel retention rate in a full CFC ITER size device would be 8×10^{21} particles per second and in the ITER-like wall material configuration of about 8×10^{20} particles per second.

Since only half to the fuel particles are tritium, the final tritium retention values are 4×10^{21} for CFC and 4×10^{20} for ILW components. The mass of a tritium nucleus is about 3 times the mass of proton: $m_T \approx 3m_p$. Therefore, 700 grams of tritium contains approximately $N_T = 700\ \mathrm{g}/3m_p \approx 1.4 \times 10^{26}$.

Accordingly, a full-CFC ITER would reach this ceiling in $t_{\mathrm{CFC}} \approx 1.46 \times 10^{26}/4 \times 10^{21}$ s $\approx$ 35000 s, which equals to of about 88 experimental pulses of 400 seconds. The corresponding time for the ITER-like wall covering is $t_{\mathrm{ILW}} \approx 350000s$, which equals to of about 880 experimental pulses of 400 seconds.

The orders of magnitude are very close to the ones presented in the figure 4. Based on these calculations, the full CFC covering for ITER would be unacceptable if sophisticated cleaning methods are not developed. This is because 88 ITER pulses would be a too low number for a pragmatic experimental campaign in ITER. The ITER-like value is acceptable for an experimental reactor, where the cleaning can be conducted during an **annual shut down**.

Additional material

The tritium is retained in the device in the form of **implantation** and **co-deposition**. In the former, the fuel particles are implanted into the surface layer of the PFC components due to plasma impact. In the latter, the tritium is co-deposited with eroded material into remote areas, which are not susceptible to plasma impact.

The co-deposition is very strong for carbon based wall materials due to formation of carbohydrates. Therefore, tritium retention does not saturate with carbon based wall materials, since nothing limits the growth of the co-deposition surfaces in the remote areas. The implantation based retention, as is presumably the case with tungsten surfaces, is likely to saturate, since once the surface layer is saturated with tritium, any further impact into the surface is likely to release as much tritium particles as is impacting the surface. This is the **main reason why carbon based wall materials are not favoured in experimental fusion devices operating with tritium**.

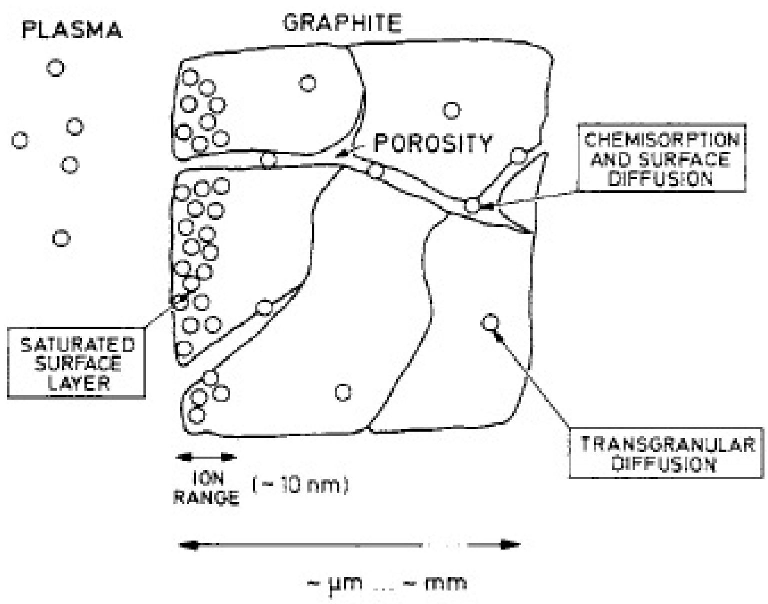

Figure 5: **Implantation physics in graphite**. Formation of hydrogen inventory inside a graphite bulk.

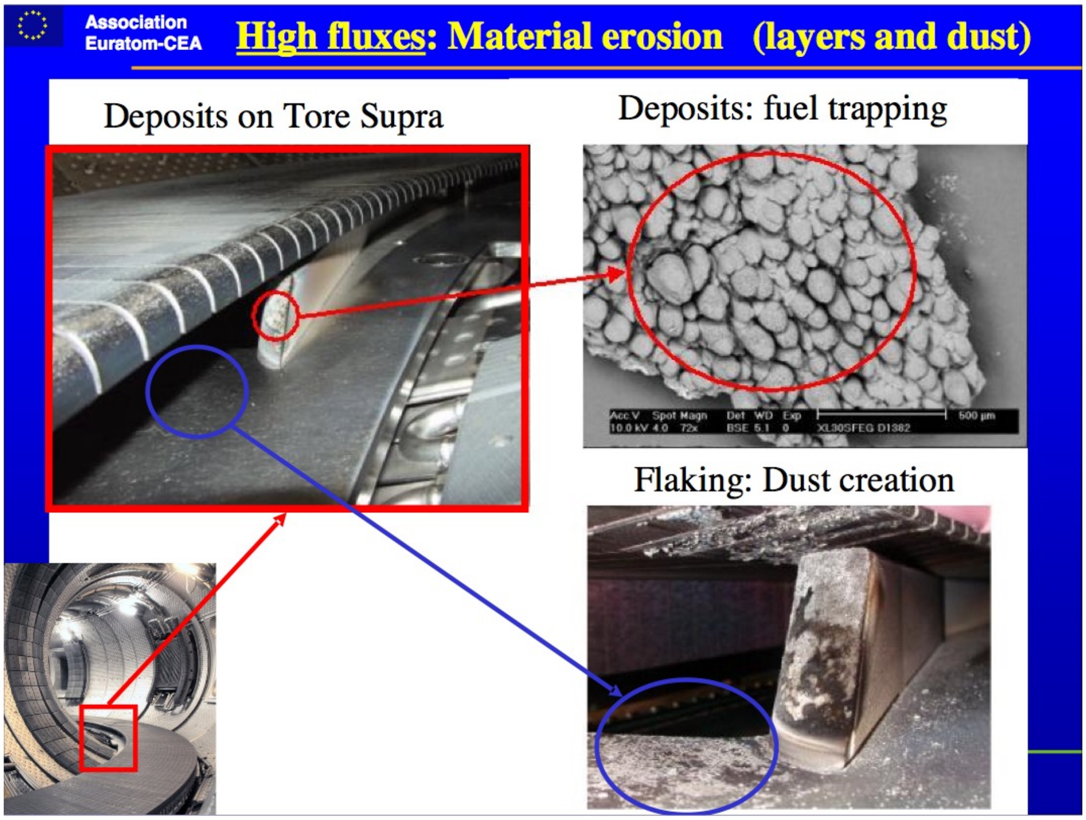

Figure 6: Examples of dust deposits on Tore Supra tokamak [C Grisolia, PWI in Tokamaks (talk), Luxembourg 13.11.2007]

Exercise 3.

Divertor component lifetime How long does it takes to erode 10 mm of the divertor plasma facing components assuming carbon and tungsten divertor target plates?

Assume a peak divertor surface particle flux about 10^{23} ions m^{-2} s^{-1}, effective carbon erosion yield about 1 %, and effective tungsten erosion yield about 10^{-4}. The effective erosion yield represents the number of surface particles eroded per an incident fuel particle. Density is about 1.1×10^{29} particles m^{-3} for the carbon divertor and 6.3×10^{28} particles m^{-3} for the tungsten divertor.

What is the erosion rate for the materials? Is carbon a viable reactor PFC material from the point of view of surface erosion? How about tungsten? How many 400s ITER pulses can be conducted before replacing the PFCs?

Solution 3.

For CFC PFCs:

- We assume a peak divertor particle flux about $\times 10^{23}$ ions m^{-2} s^{-1}.
- The carbon erosion yields are typically of the order of a few %.
- Therefore, the peak gross erosion of the CFC PFCs is estimated to be about $\times 10^{21}$ ions m^{-2} s^{-1}.

- To calculate the erosion rate, we use the density of graphite $\rho_{graphite} \approx 2100$ kg m^{-3}.
- Therefore, the particle density of graphite (assuming ^{12}C), is about 1.1×10^{29} particles m^{-3}.
- Accordingly, the erosion rate of the graphite strike-points is of about 9.1 nm s^{-1}.
- Therefore, 10 mm of the strike-points will be eroded in 1.1×10^6 s, which equal of about 3000 experimental ITER pulses of 400 seconds.
- Therefore, based on these calculations, carbon is a viable test reactor material, but for a power plant, the erosion rate would be unacceptably high.

Let us repeat the calculations for tungsten PFCs:

- The tungsten sputtering yield is much lower than the carbon value due to high surface binding energy of tungsten and lack of chemical reactivity with hydrogenic species.
- The resulting tungsten erosion yield is about $\times 10^{19}$ ions m^{-2} s^{-1}.
- The tungsten density is about $\rho_{graphite} \approx 19250$ kg m^{-3}.
- Therefore, the particle density of tungsten is about 6.3×10^{28} particles m^{-3}.
- The erosion rate of tungsten is, accordingly, 0.16 nm s^{-1}.
- Therefore, 10 mm of the strike-point will be eroded in 62.5×10^6 s, which equals of about 2 years of continuous operation or 156000 full pulses.
- Accordingly, the erosion performance of tungsten PFCs seems to be of the reactor relevant order.

Additional material
Surface erosion is driven by:

(a) **physical sputtering processes**

(b) **chemical sputtering processes**

(c) melting processes

(d) evaporation processes

(e) arcing (formation of a local discharge between the plasma and the wall material)

In a steady-state reactor relevant plasma, the sputtering processes drive the wall erosion. Melting, evaporation, and arcing might happen during transients, such as disruptions or uncontrolled ELMs. Especially melting should be strictly avoided, since the resulting deformation of the PFC components may strongly impact the subsequent plasma performance and operation.

Note the significantly lower tungsten sputtering compared to the carbon and beryllium values! As a result, tungsten is one of the leading candidate (but not the only one) as the fusion reactor wall materials to be used in the entire first wall.

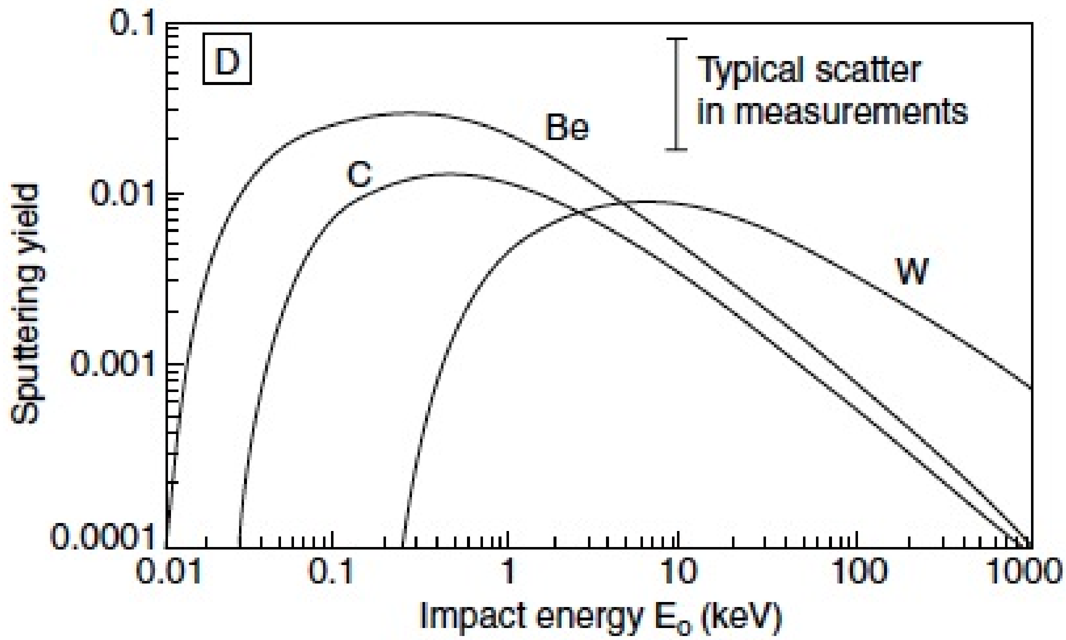

Figure 7: Simulated physical sputtering yields for Beryllium, Carbon, and Tungsten surfaces. [P. C. Stangeby, The Plasma Boundary of Magnetic Fusion Devices]

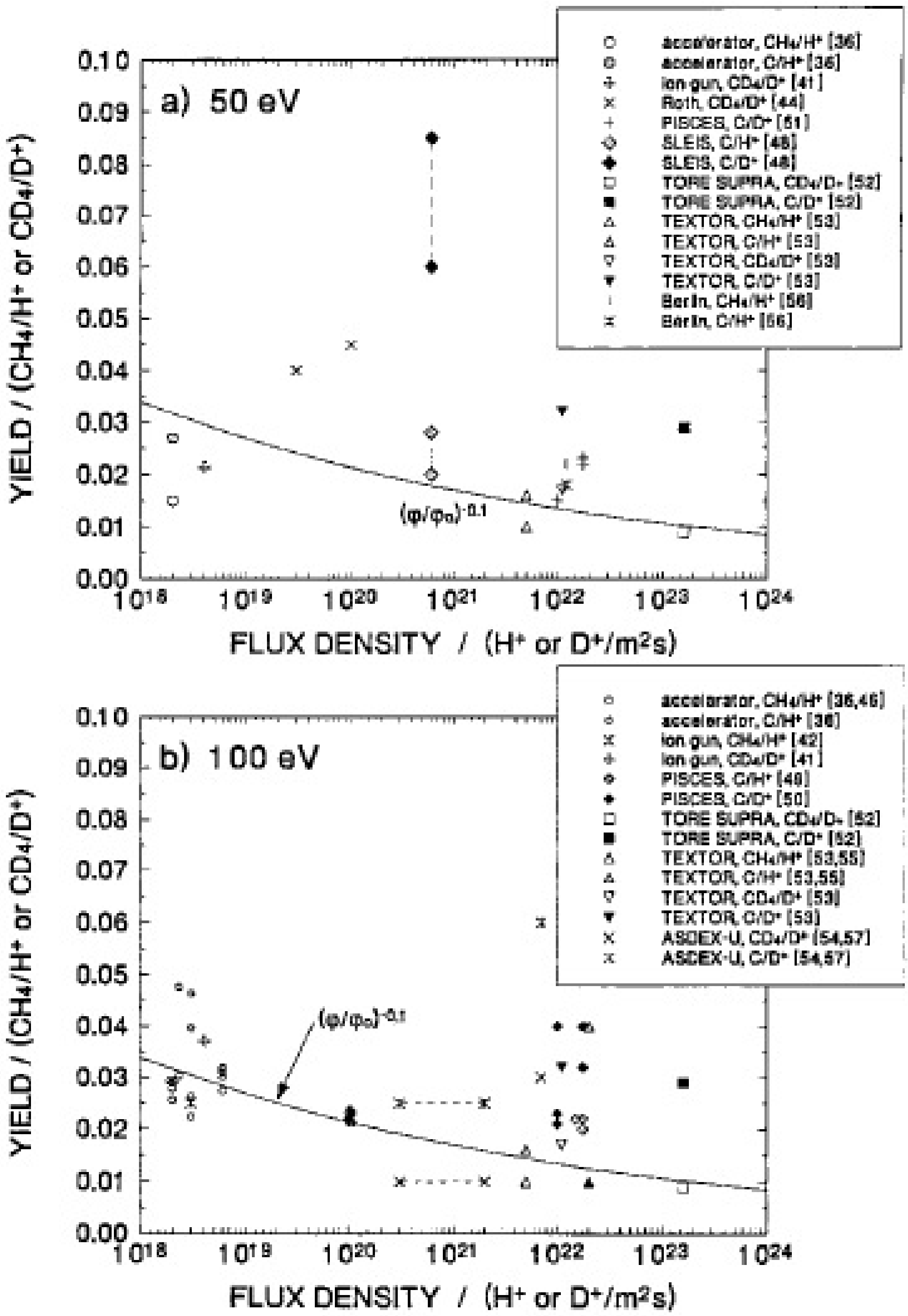

Figure 8: Chemical sputtering yields for carbon surfaces. **Of the order of a few percent!**

Exercise 4.
Tungsten contamination

Tungsten contamination of the main plasma can be a showstopper for reactor relevant operation. Assume a total divertor ion current of 10^{23} ions/s and an effective tungsten erosion yield of 10^{-5}. Calculate the total gross erosion of the tungsten divertor PFCs. Assume that 1 – 10 % of this eroded tungsten is eventually transported into the confined plasmas.

(a) If the tungsten confinement time of the confined plasma is about 1 – 100 ms, how high is the steady-state tungsten content in the confined plasma?

(b) If the nominal fuel density in the confined plasma is 1.5×10^{20} m^{-3}, and the volume of the confined plasma is about 800 m^3 (ITER reference), how high is the resulting tungsten concentration (n_W/n_e)? Based on the earlier calculations in this course, is this concentration acceptable in high performance operation?

Solution 4.

- Tungsten contamination of the main plasma can be a showstopper for reactor relevant operation.
- Estimating a total divertor particle current of about 1×10^{23} ions s^{-1} and effective tungsten yield about 10^{-5}, the primary tungsten source magnitude becomes of about 1×10^{18} particles s^{-1}.
- If 1 – 10% of this tungsten ends up into the central plasma, the contamination rate becomes $1 \times 10^{16} - 1 \times 10^{17}$ particles s^{-1}.
- Assuming tungsten confinement times about 1 – 100 ms, the steady-state tungsten content in the confined region becomes about $1 \times 10^{13} - 1 \times 10^{16}$ tungsten particles.
- The ITER plasma volume is of about 800 m^3.
- Therefore, the total fuel particle content in the confined region is of about 1.2×10^{23}.
- Therefore, the resulting tungsten concentrations, in these calculations, are well below the 10^{-5} threshold and acceptable for reactors. The assumptions made in this exercise represent an ideal scenario with a low divertor ion current and erosion yield. In reality tungsten erosion could be orders of magnitude higher if the plasma is not detached from the divertor or if ELMs are not properly mitigated.

Additional material Although the erosion of tungsten is anticipated to be 2 – 3 orders of magnitude lower than the erosion of carbon, tungsten is 3 – 4 orders of magnitude more efficient radiator than carbon at fusion temperatures. Therefore, while the thermal balance of plasma can withstand very high carbon fractions about a few %, tungsten can lead to a radiative instability with relatively modest concentrations of the order of 0.01 %.

In reactor scale plasmas, the overall tungsten content in the plasma is determined by the dynamics of the **sputtering, transport, and confinement**. In engineering terms, these translate to **source (Φ_W), contamination efficiency (f), and confinement efficiency** (τ_W^{conf}). The overall tungsten content in the device is then calculated by $N_W = \tau_W^{conf} f \Phi_W$.

Therefore, the tungsten control can be impacted by controlling the confinement, i.e., transport and exhaust of tungsten, by controlling the central plasma density and temperature profiles (transport), as well as, the ELM frequency and magnitude (exhaust).

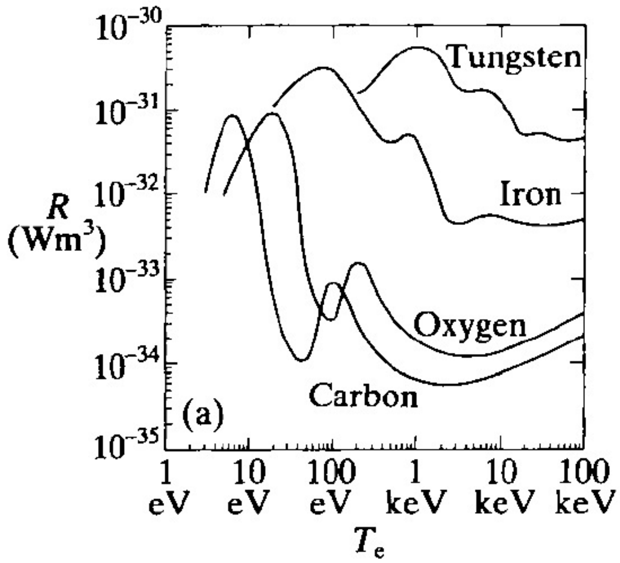

Figure 9: Radiative power function (in coronal equilibrium) for carbon, oxygen, iron, and tungsten particles. [J. Wesson, Tokamaks]

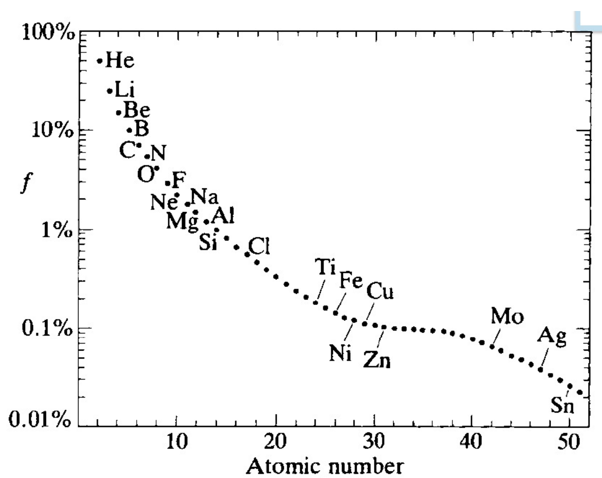

Figure 10: Fractional impurity level, which produces radiation losses equal to half of the alpha-heating power. [J. Wesson, Tokamaks]

References:

[1] Iter physics basis, Nucl. Fusion, (1999), 39, 2137
[2] S. Brezinsek et al., Nucl. Fusion, (2013), 53, 083023
[3] J. Roth et al., Journal of Nuclear Materials, (2009), 390 – 391, 1-9