Design of State Variable Feedback Systems

In the previous lecture...

You:

- Learned how to plot the *open-loop* frequency response of the return ratio $L(s)$, with the imaginary part $\mathrm{Im}\left(L(j\omega)\right)$ plotted against the real part $\mathrm{Re}\left(L(j\omega)\right)$ on an Argand diagram.
- Used the Nyquist diagram to ascertain stability of the *closed-loop* system.
- Inferred more detailed information about the behavior of the closed-loop system:
  (a) No need to explicitly compute the poles of the system, so it can be applied to systems defined by non-rational functions.
  (b) Relatively easy to see how changing $C(s)$ affects $L(s)$, but difficult to see how $C(s)$ affects $L(s)/(1 + L(s))$ directly.
  (c) *Gain* and *phase* margins measure how close the closed-loop system is to instability.

Learning outcomes...

...the student will:

- Become familiar with the concepts of controllability and observability.
- Be able to design full-state feedback controllers and observers.
- Appreciate pole-placement methods and the application of Ackermann's formula.
- Understand the separation principle and how to construct state variable controllers.

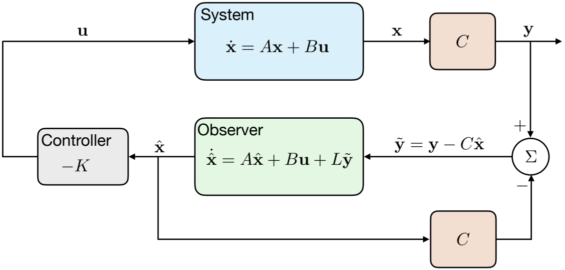
Introduction

State variable design typically consists of *three* steps.

1. We assume that all the state variables are measurable and utilize them in a full-state feedback control law
   - Not practical because it is not possible (in general) to measure all the states - in practice, only certain states (or linear combinations of them) are measured.

2. Construct an observer to estimate the states that are not directly sensed and available as outputs.

3. **Design process:** appropriately connect the observer to the full-state feedback control law.



Fundamental questions:

1. Is it possible to steer a system from a given initial state to any other state?

2. Is it possible to determine a state from observations?

state   given initial state

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t), \quad \mathbf{x}(0) = \mathbf{x}_0$$
$$\mathbf{y}(t) = C\mathbf{x}(t)$$

observation

These questions were posed and answered by Kalman (1930–2016), who also introduced the concepts of **controllability** and **observability**

Rudolph Kalman was a central figure in the development of mathematical systems theory upon which much of the subject of state variable methods rests. Kalman was well known for his role in the development of the so-called Kalman filter, which was instrumental in the successful Apollo moon landings.

Controllability and Observability

They can be roughly defined as follows:

> **Controllability:** In order to be able to take the current state to whichever state we want with the given dynamic system under control input, the system must be controllable
>
> **Observability:** In order to see what is going on inside the system under observation, the system must be observable

In this lecture we:

- show that the concepts of controllability and observability are related to linear systems of algebraic equations
  - a solvable system of linear algebraic equations has a solution *if and only if* the rank of the system matrix is full
- Observability and controllability tests will be connected to the rank tests of certain matrices: the controllability and observability matrices

Controllability

Definition (Controllability)

A linear time-invariant (LTI) system is controllable, if for every $\mathbf{x}^*(t)$ and every finite time $T > 0$, there exists an input function $\mathbf{u}(t)$, $0 < t < T$, such that the system state goes from $\mathbf{x}(0) = \mathbf{x}_0$ to $\mathbf{x}(T) = \mathbf{x}^*$.

- The state of the system at any time t is given by

$$\mathbf{x}(t) = e^{At}\mathbf{x}(0) + \int_0^t e^{A(t-\tau)} B u(\tau) d\tau$$

- Without loss of generality, assume that $\mathbf{x}(0) = \mathbf{0}$[1]. Need only to consider the forced solution to study controllability

$$\mathbf{x}_f(t) = \int_0^t e^{A(t-\tau)} B u(\tau) d\tau$$

[1] Starting at $\mathbf{0}$ is not a special case – if we can get to any state in finite time from the origin, then we can get from any initial condition to that state in finite time as well.

- Change of variables $\tau_2 = t - \tau$, $d\tau = -d\tau_2$ gives a form that is a little easier to work with:

$$\mathbf{x}_f(t) = \int_0^t e^{A\tau_2} B u(t - \tau_2) d\tau_2$$

- Assume system has n states (i.e., $\mathbf{x}(t) \in \mathbb{R}^n$) and m inputs (i.e., $\mathbf{u}(t) \in \mathbb{R}^m$).

- Note that, regardless of the eigenstructure of A, the Cayley-Hamilton theorem gives

$$e^{At} = \sum_{i=0}^{n-1} A^i \alpha_i(t)$$

for some computable scalars $\alpha_i(t)$, so that

$$\mathbf{x}_f(t) = \sum_{i=0}^{n-1} A^i B \int_0^t \alpha_i(\tau_2) u(t - \tau_2) d\tau_2 = \sum_{i=0}^{n-1} A^i B \boldsymbol{\beta}_i(t)$$

for coefficients $\boldsymbol{\beta}_i(t)$ that depend on the input $u(\tau)$, $0 < \tau < t$.

- Result can be interpreted as meaning that the state $\mathbf{x}_f(t)$ is a linear combination of the nm vectors $A^i B$ (with m inputs)
  - All linear combinations of these nm vectors is the range space of the matrix formed from the $A^i B$ column vectors, called the controllability matrix:

$$P_c = \begin{bmatrix} B & AB & A^2B & \cdots A^{n-1}B \end{bmatrix}$$

- **Definition:** Range space of P_c is the controllable subspace of the system
  - If a state $\mathbf{x}_c(t)$ is not in the range space of P_c (i.e., it is not a linear combination of these columns) $\Rightarrow$ it is impossible for $\mathbf{x}_f(t)$ to ever become equal to $\mathbf{x}_c(t)$ – called uncontrollable state

Theorem

A linear time-invariant (LTI) system is controllable, if and only if (iff) it has no uncontrollable states.

- Necessary and sufficient condition for controllability is that

$$\text{rank}(P_c) \triangleq \text{rank} \begin{bmatrix} B & AB & A^2B & \cdots A^{n-1}B \end{bmatrix} = n$$

Remark: *For a single-input, single-output system, the controllability matrix P_c is an $n \times n$ matrix. Hence, if the determinant of P_c is nonzero, the system is controllable.*

- When the system is not completely controllable, but where the states (or linear combinations of them) that cannot be controlled are inherently stable, then these systems are classified as stabilizable
  - If a system is completely controllable, it is also stabilizable (but not vice versa!)

Example: Let us consider the system

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}$$

Is the system controllable?

Answer: First, we find the controllability matrix:

$$P_c = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -a_2 \\ 1 & -a_2 & a_2^2 - a_1 \end{bmatrix}$$

Then, we compute the determinant:

$$\det(P_c) = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & -a_2 \\ 1 & -a_2 & a_2^2 - a_1 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 1 & -a_2 \end{vmatrix} = -1 \neq 0 \quad \text{(system is controllable)}$$

Observability

Definition (Observability)

A linear time-invariant (LTI) system is observable, if the initial state $\mathbf{x}(0)$ can be uniquely deduced from the knowledge of the input $\mathbf{u}(t)$ and output $\mathbf{y}(t)$ for all t between 0 and any finite $T > 0$.

- If $\mathbf{x}(0)$ can be deduced, then we can reconstruct $\mathbf{x}(t)$ exactly because we know $\mathbf{u}(t)$ $\Rightarrow$ we can find $\mathbf{x}(t)$ $\forall t$
- Thus, it is sufficient to only consider the zero-input (homogeneous) solution to study observability

$$\mathbf{y}(t) = Ce^{At}\mathbf{x}(0)$$

- **Definition:** A state $\mathbf{x}^* \neq \mathbf{0}$ is said to be unobservable if the zero-input solution $\mathbf{y}(t)$, with $\mathbf{x}(0) = \mathbf{x}^*$, is zero for all $t \geq 0$
  - Equivalent to saying that $\mathbf{x}^*$ is an unobservable state if

$$Ce^{At}\mathbf{x}^* = 0 \;\forall\; t \geq 0$$

Example: Let us consider the system

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 3 & 0 \end{bmatrix} \mathbf{x}$$

Is the state $\mathbf{x}^* = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$ observable?

Answer:

$$Ce^{At}\mathbf{x}^* = \begin{bmatrix} 3 & 0 \end{bmatrix} \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3e^{-2t} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 \ \forall \ t$$

So, $\mathbf{x}^* = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$ is an unobservable state of this system.

Theorem

A linear time-invariant (LTI) system is observable iff it has no unobservable states.

Proof (by a simple example): Let $\mathbf{x}^* \neq \mathbf{0}$ be an unobservable state and compute the output of the initial conditions $\mathbf{x}_1(0)$ and $\mathbf{x}_2(0) = \mathbf{x}_1(0) + \mathbf{x}^*$. Then,

$$\mathbf{y}_2(t) = Ce^{At}\mathbf{x}_2(0) = Ce^{At}(\mathbf{x}_1(0) + \mathbf{x}^*)$$

$$\underbrace{Ce^{At}\mathbf{x}_1(0)}_{=\mathbf{y}_1(t)} + \underbrace{Ce^{At}\mathbf{x}^*}_{=0} = \mathbf{y}_1(t)$$

Thus, two different initial conditions give the same output $\mathbf{y}(t)$, so it would be impossible for us to deduce the actual initial condition of the system ($\mathbf{x}_1(0)$ or $\mathbf{x}_2(0)$) given $\mathbf{y}_1(t)$

- Testing system observability by searching for a vector $\mathbf{x}(0)$ such that $Ce^{At}\mathbf{x}(0)\ \forall t$ is feasible, but very hard in general – better tests are available.

Theorem

A LTI system is observable, iff it has no unobservable states.

- Necessary and sufficient condition for observability is that

$$\operatorname{rank}(P_o) \triangleq \operatorname{rank} \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} = n$$

Sketch of the proof: If a vector $\mathbf{x}^*$ is an unobservable state, then

$$Ce^{At}\mathbf{x}^* = 0 \ \forall t \geq 0$$

But all the derivatives of Ce^{At} exist and for this condition to hold, all derivatives must be zero at $t = 0$. Hence,

$$Ce^{At}\mathbf{x}^*\Big|_{t=0} = 0 \Rightarrow C\mathbf{x}^* = 0$$

$$\frac{d}{dt}Ce^{At}\mathbf{x}^*\Big|_{t=0} = 0 \Rightarrow C\frac{d}{dt}\left(e^{At}\right)\mathbf{x}^*\Big|_{t=0} = 0$$

$$\Rightarrow CAe^{At}\mathbf{x}^*\Big|_{t=0} = 0 \Rightarrow CA\mathbf{x}^* = 0$$

$$\frac{d^2}{dt^2} Ce^{At}\mathbf{x}^*\bigg|_{t=0} = 0 \Rightarrow CA^2 e^{At}\mathbf{x}^*\Big|_{t=0} = 0 \Rightarrow CA^2\mathbf{x}^* = 0$$

$$\vdots$$

$$\frac{d^k}{dt^k} Ce^{At}\mathbf{x}^*\bigg|_{t=0} = 0 \Rightarrow CA^k e^{At}\mathbf{x}^*\Big|_{t=0} = 0 \Rightarrow CA^k\mathbf{x}^* = 0$$

We only need retain up to the $(n-1)^{\text{th}}$ derivative because of the Cayley-Hamilton theorem.

- **Why does this make sense?**
  - The requirement for an unobservable state is that for $\mathbf{x}^* \neq 0$, $P_o\mathbf{x}^* = \mathbf{0}$, which is equivalent to saying that $\mathbf{x}^*$ is orthogonal to each row of P_o
  - But if the rows of P_o are considered to be vectors and these span the full n-dimensional space, then it is not possible to find an n-dimensional vector $\mathbf{x}^*$ that is orthogonal to each of these
  - To determine if the n rows of P_o span the full n-dimensional space, we need to test their linear independence, which is equivalent to the rank test
- When the system is not completely observable, but where the states (or linear combinations of them) that cannot be observed are inherently stable, are classified as detectable. If a system is completely observable, it is also detectable.

Example: Let us consider the system

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}$$

Is the system observable?

Answer: First, we find the observability matrix:

$$P_o = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then, we compute the determinant:

$$\det(P_c) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \neq 0 \quad \text{(system is observable)}$$

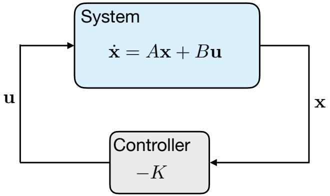
Full state feedback control design

- We consider full-state variable feedback to achieve the desired pole locations of the closed-loop system
- 1st step: we assume that all the states are available for feedback, i.e., we have access to the complete state $\mathbf{x}(t)$ for all t
- The system input $\mathbf{u}(t)$ is given by

$$\mathbf{u}(t) = -K\mathbf{x}(t)$$

Objective: Determine the gain matrix K.

- The full-state feedback block diagram is illustrated in the figure below



- We find the closed-loop system to be

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} = A\mathbf{x} - BK\mathbf{x} = (A - BK)\mathbf{x} \quad (1)$$

- The characteristic equation associated with equation (1) is

$$\det\Big(\lambda I - (A - BK)\Big) = 0$$

  If all the roots lie in the left half-plane, then the closed-loop system is stable. Hence, for any initial condition $\mathbf{x}(0)$, it follows that

$$\mathbf{x}(t) = e^{(A-BK)t}\mathbf{x}(0) \to 0 \quad \text{as } t \to \infty$$

- Given the pair (A, B), we can always determine K to place all the system closed-loop poles in the left half-plane *if and only if* the system is controllable

- The addition of a reference input can be written as

$$\mathbf{u}(t) = -K\mathbf{x}(t) + N\mathbf{r}(t)$$

  where $\mathbf{r}(t)$ is the reference input.

  - When $\mathbf{r}(t) = \mathbf{0}$ for all $t > 0$, the control design problem is known as the regulator problem, i.e., we want to compute K so that all initial conditions are driven to 0 in a specified fashion (as determined by the design specifications)

- When using this state variable feedback, the roots of the characteristic equation are placed where the transient performance meets the desired response

Ackermann's formula for the state variable feedback matrix

- For a single-input, single-output system, Ackermann's formula is useful for determining the state variable feedback matrix

$$K = \begin{bmatrix} k_1 & k_2 & \dots & k_n \end{bmatrix}$$

where $\mathbf{u}(t) = -K\mathbf{x}(t)$.

- Given the *desired* characteristic equation

$$q(\lambda) = \lambda^n + a_{n-1}\lambda^{n-1} + \ldots + a_1\lambda + a_0$$

the state feedback gain matrix is given by

$$\boxed{K = \begin{bmatrix} 0 & 0 & \dots & 1 \end{bmatrix} P_c^{-1} q(A)}$$

where

$$q(A) = A^n + a_{n-1}A^{n-1} + \ldots + a_1 A + a_0 I$$

and P_c is the controllability matrix.

Example: Consider the second-order system

$$\frac{Y(s)}{U(s)} = G(s) = \frac{1}{s^2}$$

and determine the feedback gain to place the closed-loop poles at $s = -1 \pm j$.

Answer: We require that

$$q(\lambda) = [\lambda - (-1 + j)][\lambda - (-1 - j)] = \lambda^2 - (-1 + j - 1 - j)\lambda + 2 = \lambda^2 + 2\lambda + 2$$

Hence,

$$q(A) = A^2 + 2A + 2I = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}^2 + 2\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$$

With $x_1 = y$ and $x_2 = \dot{x}_1$, the matrix equation of the system $G(s)$ becomes

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

The controllability matrix P_c is

$$P_c = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow P_c^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Therefore,

$$K = \begin{bmatrix} 0 & 1 \end{bmatrix} P_c^{-1} q(A) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Observer design

- Generally speaking, only a subset of the states are readily measurable and available for feedback.
- Even if all the states were available for feedback, these states should be measured with a sensor or sensor combinations.
  - The cost and complexity of the control system increase as the number of required sensors increases $\Rightarrow$ even in situations where extra sensors are available, it may not be cost effective to employ them
- According to Luenberger, the full-state observer for the system

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu$$
$$y = C\mathbf{x}$$

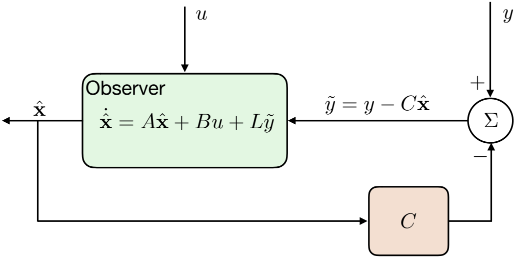
is given by

$$\dot{\hat{\mathbf{x}}} = A\hat{\mathbf{x}} + Bu + L\Big(y - C\hat{\mathbf{x}}\Big)$$

where $\hat{\mathbf{x}}$ denotes the estimate of the state $\mathbf{x}$

- The matrix L is the observer gain matrix and is to be determined as part of the observer design procedure

- The observer has two inputs, u and y, and one output, $\hat{\mathbf{x}}$, as depicted in the figure below:



- *Objective:* provide an estimate $\hat{\mathbf{x}}$ so that $\hat{\mathbf{x}} \to \mathbf{x}$ as $t \to \infty$

- We do not know $\mathbf{x}(0)$ precisely; therefore we must provide an initial estimate $\hat{\mathbf{x}}(0)$ to the observer

- Define the observer estimation error as

$$\mathbf{e}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t)$$

  The observer design should produce an observer with the property that $\mathbf{e}(t) \to 0$ as $t \to \infty$

- One of the main results of systems theory is that if the system is observable, we can always find L so that the tracking error is asymptotically stable, as desired.

- Taking the time-derivative of the estimation error

$$\dot{\mathbf{e}}(t) = \dot{\mathbf{x}}(t) - \dot{\hat{\mathbf{x}}}(t)$$

- Using the system model and the observer

$$\begin{aligned}\dot{\mathbf{e}}(t) &= \Big(A\mathbf{x} + Bu\Big) - \Big(A\hat{\mathbf{x}} + Bu + L(y - C\hat{\mathbf{x}})\Big) \\ &= (A - LC)(\mathbf{x}(t) - \hat{\mathbf{x}}(t)) = (A - LC)\mathbf{e}(t)\end{aligned}$$

  We can guarantee that $\mathbf{e}(t) \to 0$ as $t \to \infty$ for any initial tracking error $\mathbf{e}(0)$ if the characteristic equation

$$\det\Big(\lambda I - (A - LC)\Big) = 0$$

  has all its roots in the left half-plane.

- This can always be accomplished if the system is observable; i.e, if the observability matrix P_o has full rank

Ackermann's formula for the observer gain matrix

- For a single-input, single-output system, Ackermann's formula is useful for determining the observer gain matrix

$$L = \begin{bmatrix} l_1 & l_2 & \dots & l_n \end{bmatrix}^T$$

- Given the *desired* characteristic equation

$$p(\lambda) = \lambda^n + b_{n-1}\lambda^{n-1} + \dots + b_1\lambda + b_0$$

the observer gain matrix is given by

$$\boxed{L = p(A)P_o^{-1} \begin{bmatrix} 0 & 0 & \dots & 1 \end{bmatrix}^T}$$

where

$$p(A) = A^n + b_{n-1}A^{n-1} + \dots + b_1 A + b_0 I$$

and P_o is the observability matrix.

Integrated full-state feedback and observer

- We now consider the feedback law
$$\mathbf{u}(t) = -K\hat{\mathbf{x}}(t) \tag{2}$$
We need to verify that, when using the feedback control law in equation (2), we retain the stability of the closed-loop system.

- Consider the observer
$$\dot{\hat{\mathbf{x}}} = A\hat{\mathbf{x}} + Bu + L\Big(y - C\hat{\mathbf{x}}\Big)$$

- Substituting the feedback law in equation (2) and rearranging terms in the observer yields
$$\dot{\hat{\mathbf{x}}} = (A - BK - LC)\hat{\mathbf{x}} + Ly$$

- Computing the estimation error
$$\begin{aligned}\dot{\mathbf{e}}(t) &= \Big(A\mathbf{x} - BK\hat{\mathbf{x}}\Big) - \Big((A - BK - LC)\hat{\mathbf{x}} + Ly\Big) \\ &= (A - LC)(\mathbf{x}(t) - \hat{\mathbf{x}}(t)) = (A - LC)\mathbf{e}(t) \end{aligned} \tag{3}$$

- This is the same result as we obtained for the estimation error before, because **the estimation error does not depend on the input**

- Recall that the underlying system model is given by

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu$$
$$y = C\mathbf{x}$$

Substituting the feedback law $\mathbf{u}(t) = -K\hat{\mathbf{x}}(t)$ into the system model yield

$$\begin{aligned}\dot{\mathbf{x}} &= A\mathbf{x} - BK\hat{\mathbf{x}}(t) = A\mathbf{x} - BK(\mathbf{x}(t) - \mathbf{e}) \\ &= (A - BK)\mathbf{x} + BK\mathbf{e} \qquad (4)\end{aligned}$$

- Combining equations (3) and (4) in matrix form, we have

$$\begin{bmatrix}\dot{\mathbf{x}} \\ \dot{\mathbf{e}}\end{bmatrix} = \begin{bmatrix}A - BK & BK \\ 0 & A - LC\end{bmatrix}\begin{bmatrix}\mathbf{x} \\ \mathbf{e}\end{bmatrix} \qquad (5)$$

- The characteristic equation associated with equation (5) is

$$\Delta(\lambda) = \det\big(\lambda I - (A - BK)\big)\det\big(\lambda I - (A - LC)\big)$$

  - If the roots of $\det\big(\lambda I - (A - BK)\big) = 0$ lie in the left half-plane (which they do by design of the full-state feedback law), and
  - if the roots of $\det\big(\lambda I - (A - LC)\big) = 0$ lie in the left half-plane (which they do by design of the observer),

then the overall system is stable! The fact that the full-state feedback law and the observer can be designed independently is an illustration of the **separation principle**

Learning outcomes...

...the student will:

- Become familiar with the concepts of controllability and observability.
- Be able to design full-state feedback controllers and observers.
- Appreciate pole-placement methods and the application of Ackermann's formula.
- Understand the separation principle and how to construct state variable controllers.