Introduction to simulation with COMSOL®

Lecture 4: Quantum circuits
08.03.2023

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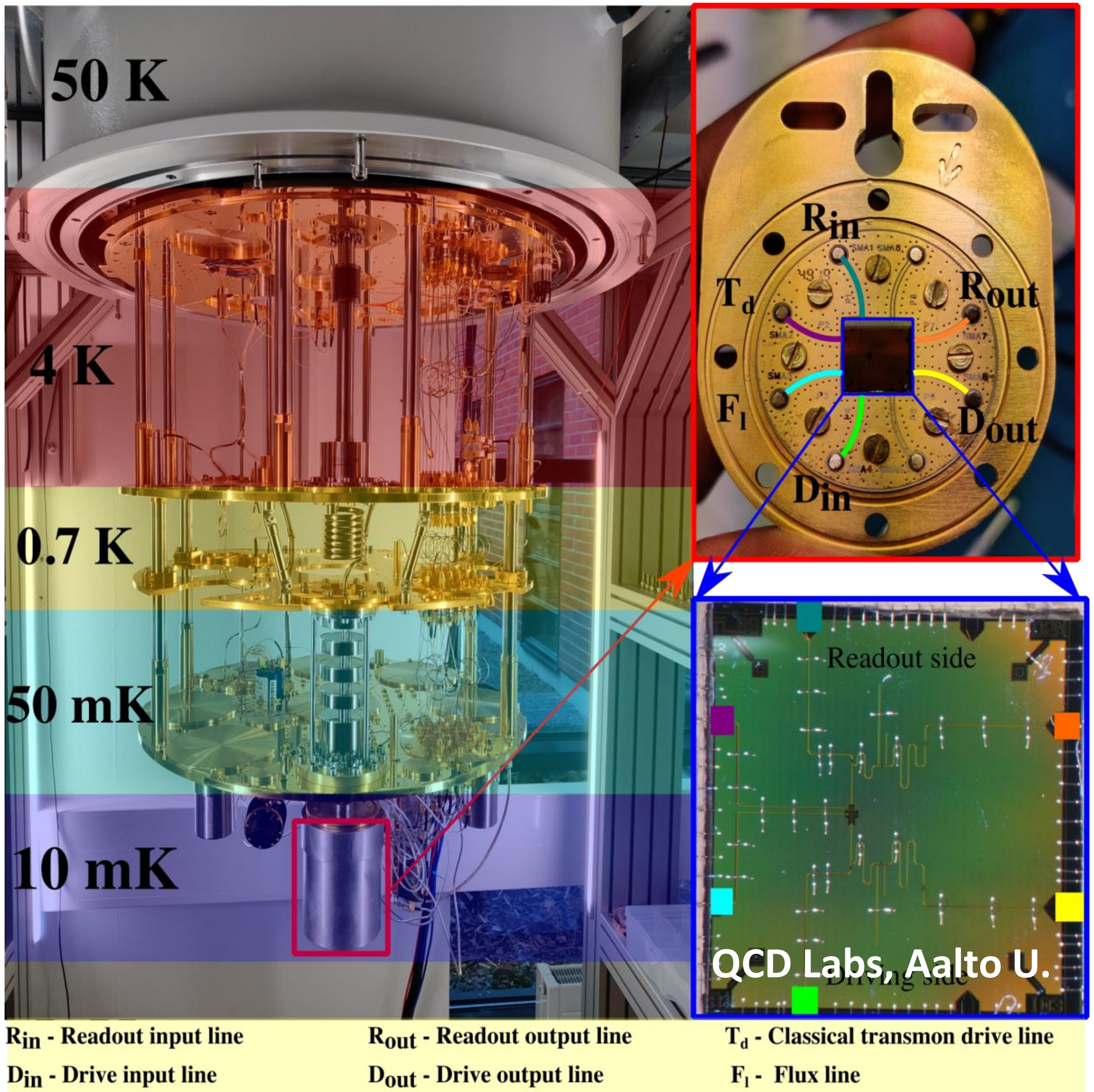
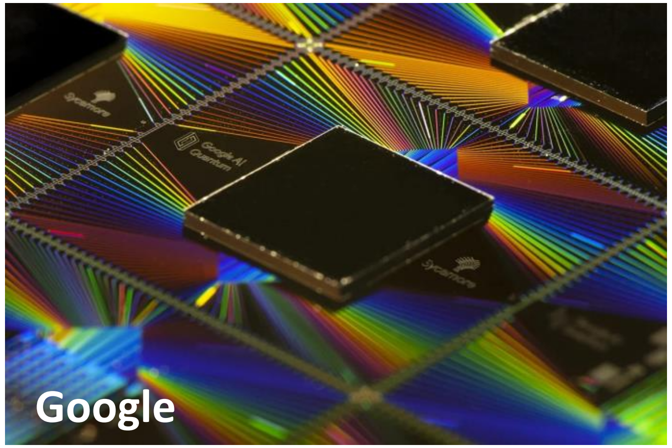
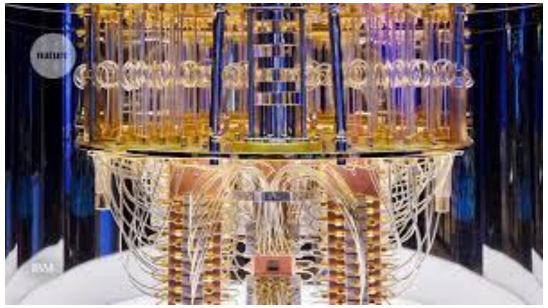
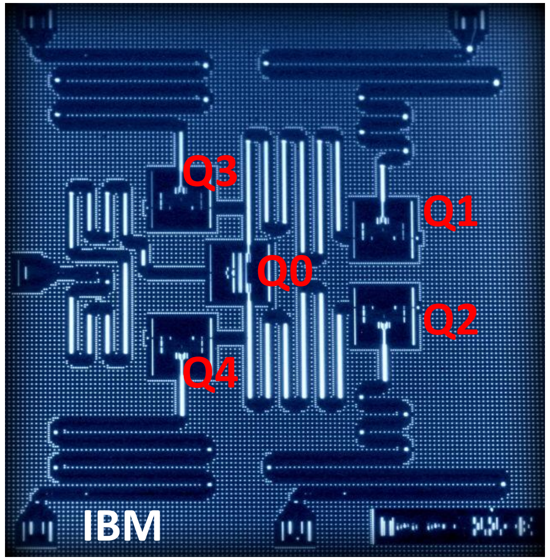
School of Science

Simulation….Why?



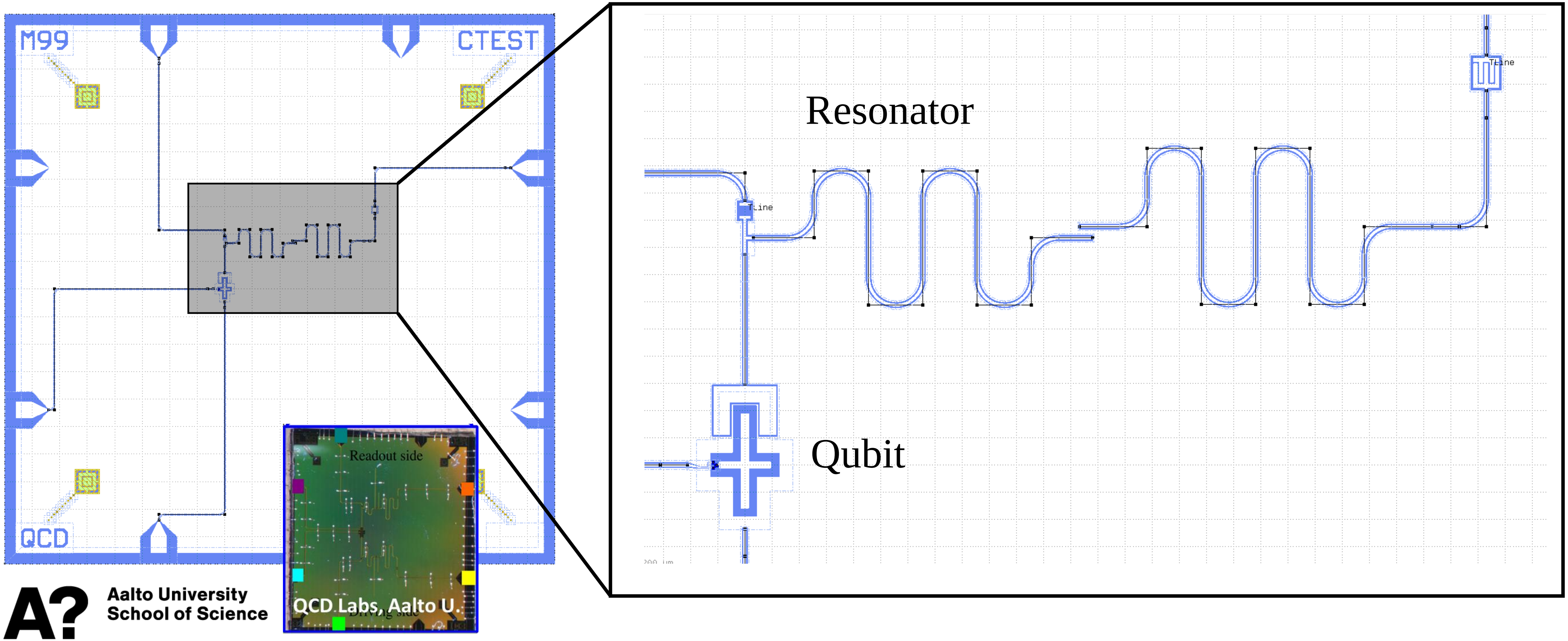






Single qubit chip design

Single-qubit quantum processor



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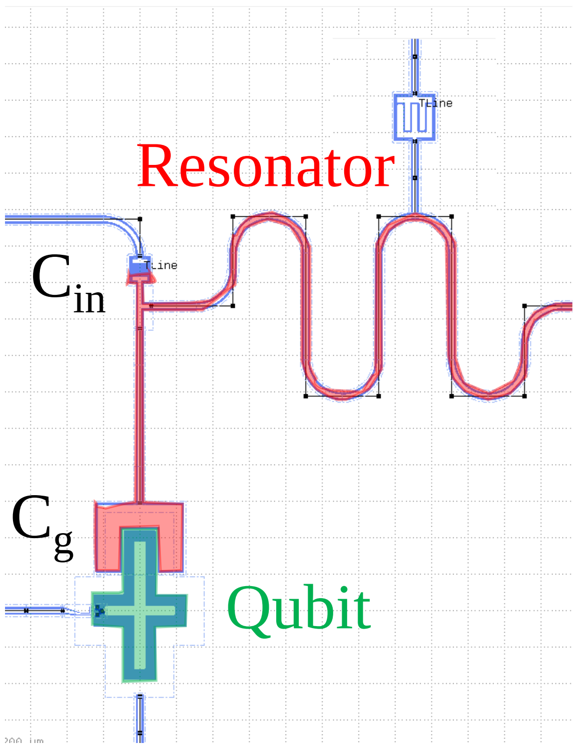
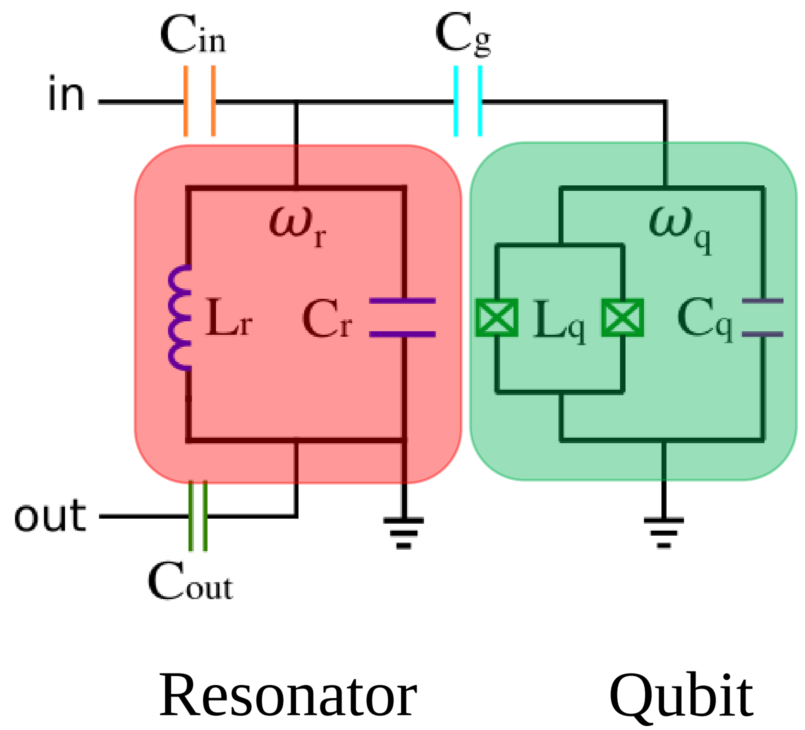
Circuit model for single qubit chip

Single-qubit quantum processor



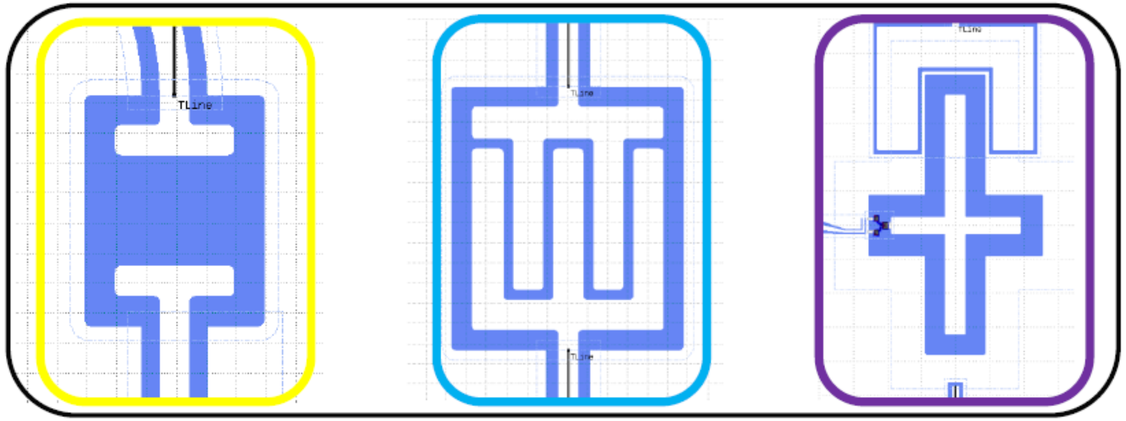
Lumped model



Bare minimum for a single-qubit quantum processor!

Simulations using Comsol

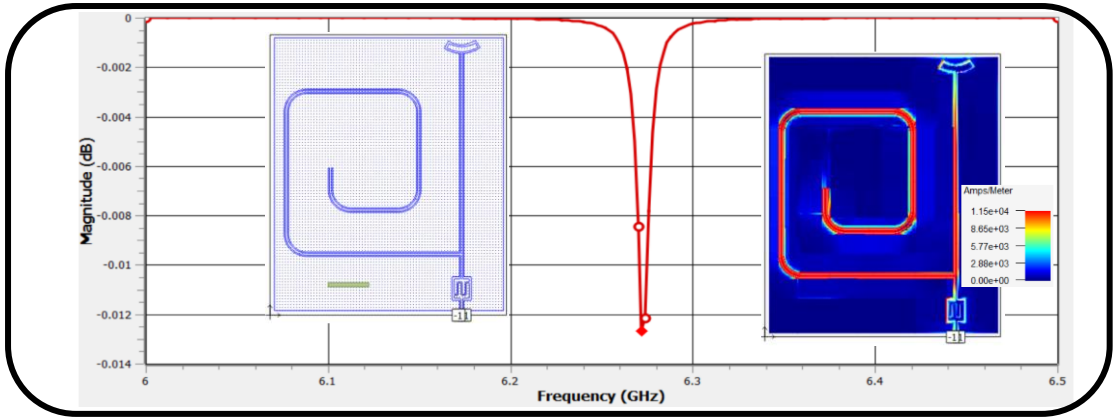
Capacitances



COMSOL Electrostatic simulation:

1. Design
2. Physics: Electrostatic
3. Study: Stationary
4. Define materials and terminals
5. Mesh the structure
6. Derived results(capacitance)

EM frequency domain



COMSOL EM simulation:

1. Design
2. Physics: Electromagnetic waves, frequency domain (emw)
3. Study: Frequency domain
4. Define materials and ports
5. Mesh the structure
6. Derived results
   - S parameter, EM fields etc.

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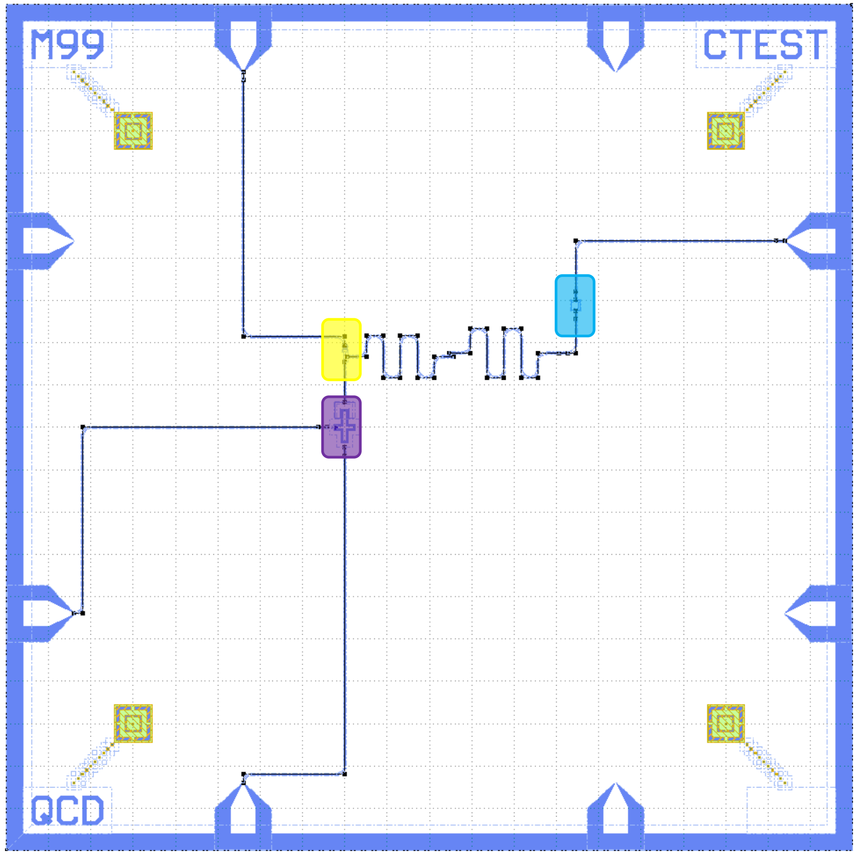
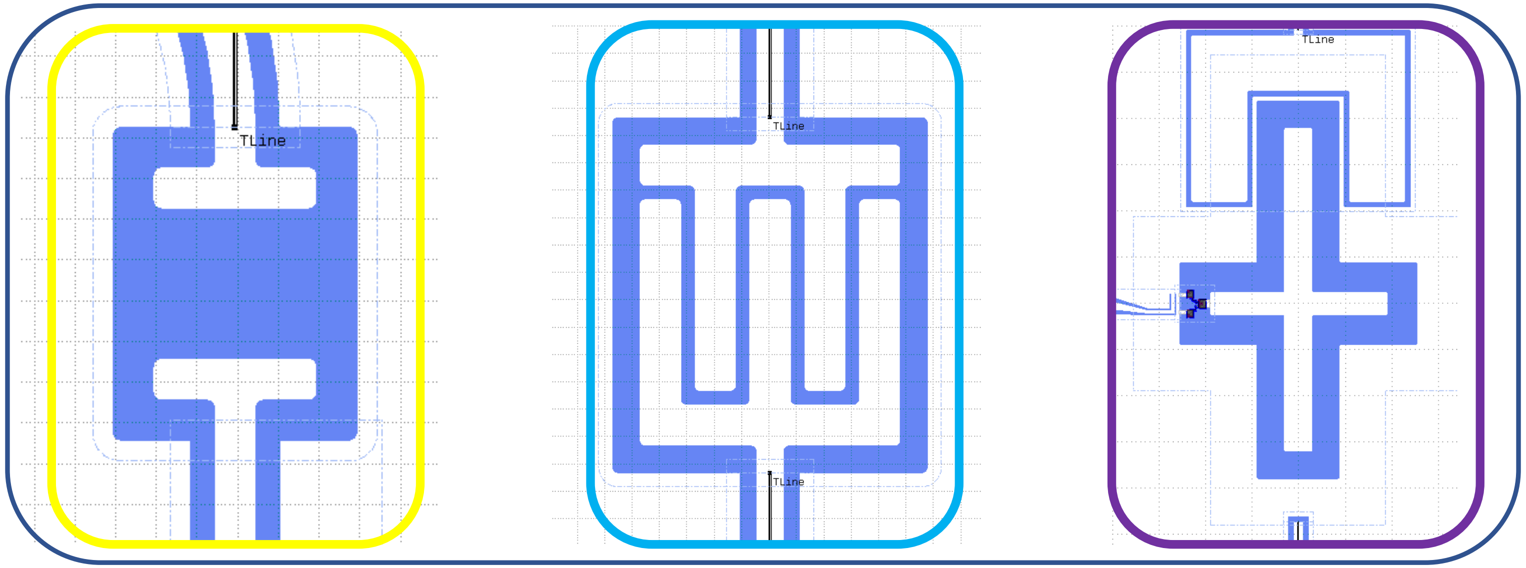
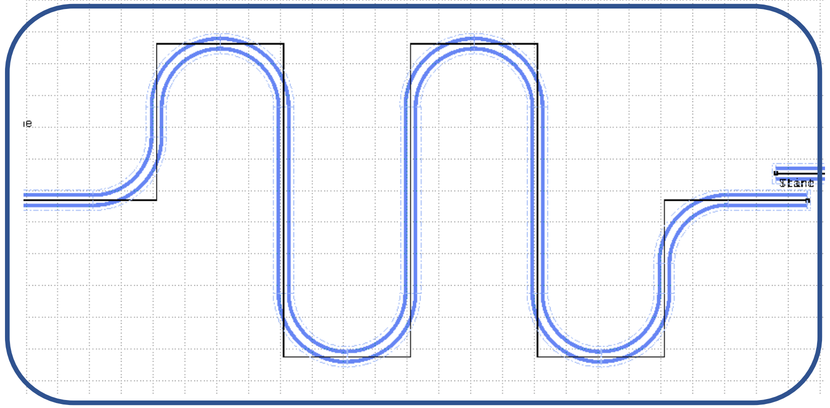
Capacitor simulation

Single-qubit quantum processor



Capacitors





Coplanar waveguide (CPW) resonator

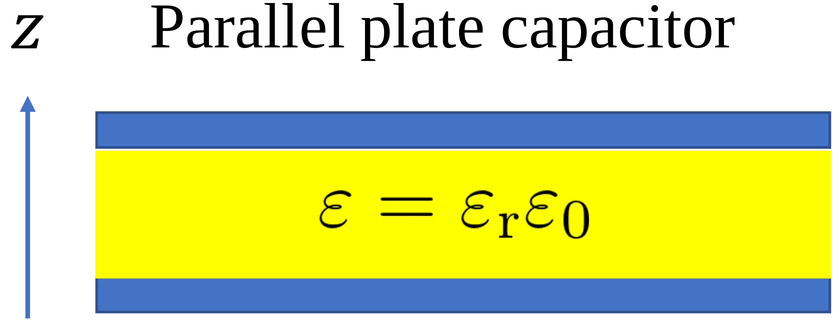
Capacitors

Partial differential equation (PDE)

- Laplace equation $$\nabla^2 V = \frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{d^2V}{dz^2} = 0$$

- Boundary value problem $$\begin{cases} V(z=d) = V_0 \\ V(z=0) = 0 \end{cases}$$

Parallel plate capacitor



$$\frac{d^2V}{dz^2} = 0 \quad \Rightarrow \quad V(z) = \frac{V_0}{d} z$$

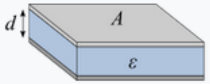
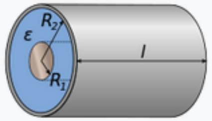
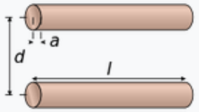
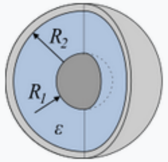
$$\vec{E} = -\nabla V = -\frac{V_0}{d}\hat{z}$$

$$\oiint \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon} \iint \rho \, dA$$

$$\Rightarrow \quad C = \varepsilon \frac{A}{d}$$

Nice closed form solution!

Capacitors

Type	Capacitance	Comment
Parallel-plate capacitor	$\varepsilon A/d$	 ε: Permittivity
Concentric cylinders	$\frac{2\pi\varepsilon\ell}{\ln(R_2/R_1)}$	 ε: Permittivity
Pair of parallel wires[13]	$\frac{\pi\varepsilon\ell}{\operatorname{arcosh}\left(\frac{d}{2a}\right)} = \frac{\pi\varepsilon\ell}{\ln\left(\frac{d}{2a}+\sqrt{\frac{d^2}{4a^2}-1}\right)}$	
Wire parallel to wall[13]	$\frac{2\pi\varepsilon\ell}{\operatorname{arcosh}\left(\frac{d}{a}\right)} = \frac{2\pi\varepsilon\ell}{\ln\left(\frac{d}{a}+\sqrt{\frac{d^2}{a^2}-1}\right)}$	a: Wire radius d: Distance, $d > a$ ℓ: Wire length
Two parallel coplanar strips[14]	$\varepsilon\ell\frac{K\left(\sqrt{1-k^2}\right)}{K(k)}$	d: Distance w_1, w_2: Strip width k_m: $d/(2w_m+d)$ k^2: k_1k_2 K: Complete elliptic integral of the first kind ℓ: Length
Concentric spheres	$\frac{4\pi\varepsilon}{\frac{1}{R_1}-\frac{1}{R_2}}$	 ε: Permittivity

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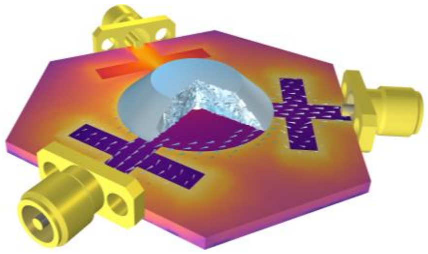
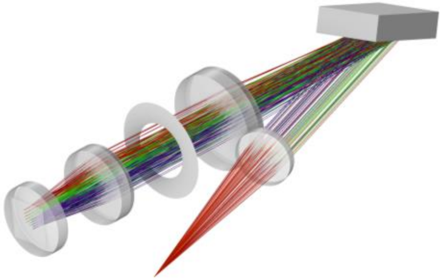
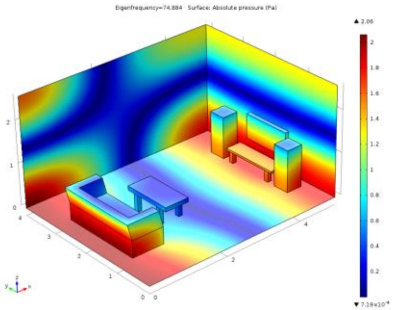
COMSOL Multiphysics



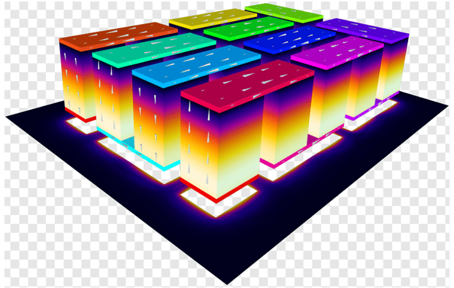
Electromagnetism



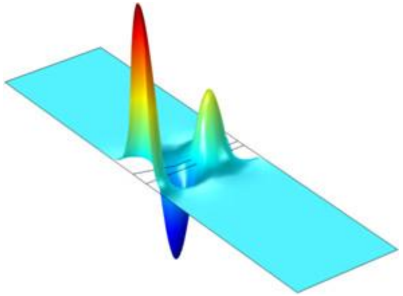
Optics



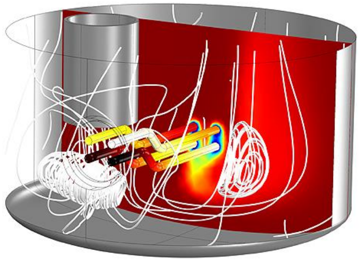
Acoustics



Heat transfer



Schrödinger equation



Chemical reactions

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COMSOL Multiphysics

Physics modules

Electromagnetics

Fluid Dynamics

Solid Mechanics

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Mathematical models

Partial Differential Equations (PDEs)

Ordinary Differential Equations (ODEs)

Differential-Algebraic Equations (DAEs)

⋮

Numerical models

Finite Element Method (FEM)

Boundary Element Method (BEM)

Finite Difference Method (FDM)

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COMSOL Workshop

You can download the software from https://download.aalto.fi

Check out the installation instruction from mycourses webpage *Announcement* forum

Let's move to COMSOL demo…