Lectures 4: Labor Demand

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Lecture Slides

Labor market

- Labor supply: how many workers choose to enter labor market (and how many hours they choose to work)
- Labor demand: How willing firms are to hire these workers...
- Labor market outcomes depends on both (labor market is in equilibrium when supply equals demand)

Today:

- Labor Demand (Application: Minimum wages)
- Equilibrium, Minimum Wages, Payroll taxes

Labor demand

- Labor demand is "derived demand"
  - Firms hire workers and buy capital to produce goods and services that consumers want
- Labor demand is important
  - strong demand leads to high wages and low unemployment
- ... and subject to a lot of regulation
  - minimum wages, maximum hours, safety regulation, anti-discrimination laws etc.

Road map:

1. Introduction
2. Production function
3. Employment decision in the short-run (firms can adjust only labor inputs)
4. Employment decision in the long-run (firms can adjust all inputs)
5. Elasticity of substitution
6. Marshall's rules of derived demand

Note: Notation from Hamermesh 1996 and Borjas 2006.

Production function

Production function

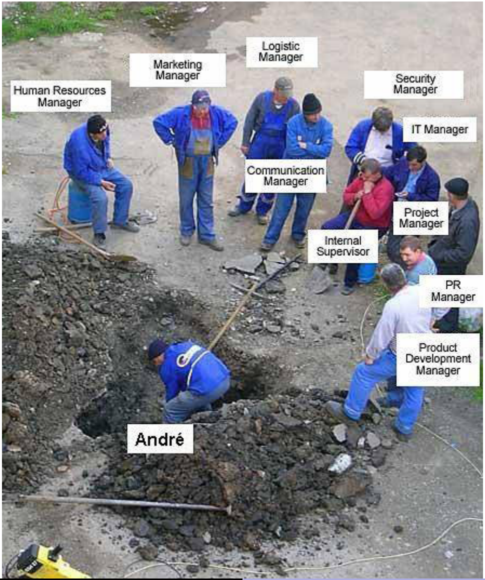
$$q = f(E, K)$$

where E is employee-hours and K is the capital stock

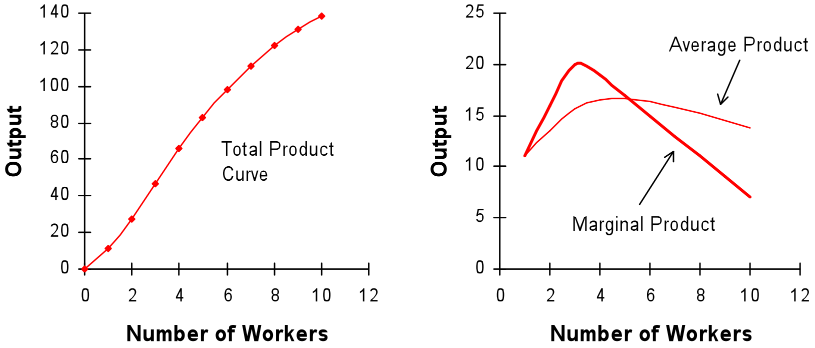
Several simplifying assumptions:

1. Workers and capital are homogenous
2. E = # workers * # hours
3. Decreasing returns to labor and to capital
    - First few workers may increase output substantially. As more and more workers are added to a fixed capital stock, the gains from specializarion declines...

Decreasing Marginal Product of Labor



Marginal and average product



The total product curve gives the relationship between output and the number of workers hired by the firm (holding capital fixed). The marginal product curve shows the output produced by each additional worker, and the average product curve shows output per worker.

Labor Demand in the Short-Run

Firm chooses the level of employment to maximize its profits.

Firm's problem

$$\max_{\{E\}} pf\left(E, \overline{K}\right) - wE - r\overline{K}$$

where p the price of the output, $f\left(E, \overline{K}\right)$ is the production function, E is employee-hours, $\overline{K}$ the (fixed) capital stock, w wage rate and r rental rate of capital

- Short-term: $\overline{K}$ is given, only E is a choice
- The (perfectly competitive) firm is a small player in the industry. It does not affect:
  - price in the output market (p)
  - the price of labor (w)
  - the price of capital (r)

Labor Demand in the Short-Run

First order condition is obtained by setting the derivative of the profit to zero (with respect to E).

First-Order Condition

$$pf'\left(E^{*},\overline{K}\right)=w$$

i.e. value of the marginal product of labor (the dollar increase in revenue generated by additional worker) equals the price of the unit of labor .

At the profit maximising solution, the dollar gain from hiring an additional worker equals the costs of that hire.

Labor Demand in the Short-Run

Second-Order Condition

$$pf''\left(E^*, \overline{K}\right) < 0$$

i.e. the marginal product of labor is decreasing.

Non-negative profits

$$pf\left(E, \overline{K}\right) \geq wE \Rightarrow \frac{pf\left(E, \overline{K}\right)}{E} \geq w$$

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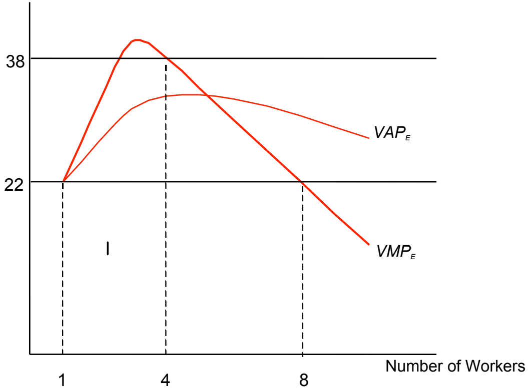


where $VMP_E = pf'(.)$ and $VAP_E = \frac{pf(E,\overline{K})}{E}$

A profit-maximizing firm hires workers up to the point where the wage rate equals the value of marginal product of labor (left panel). For example, if the wage is \$22, the firm hires eight workers. However, if the wage is \$38, the value of average product of labor would be less than the wage → the firm does not hire anyone

Firm-level short-run labor demand curve

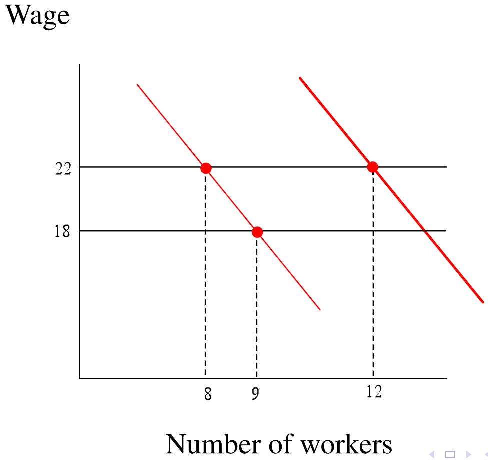
This yields the short-run labor demand curve for the firm ($E_{sr,f}$):

$$E^d_{sr,f} = E\left(w, p, \overline{K}\right)$$

1. $\frac{\delta E^d_{sr,f}}{\delta w} < 0$ (downward sloping due to decreasing marginal product of labor)
2. $\frac{\delta E^d_{sr,f}}{\delta p} > 0$ (increase in output price $\rightarrow$ higher value of marginal product)

Firm-level short-run labor demand curve

Drop in wage from 22 to 18 increases employment (from 8 to 9)
An increase in price of output shifts the marginal product demand curve upward and increases employment



Short-run elasticity of labor demand

Percentage change in (short-run) employment (E^d_{sr}) resulting from one percent change in wage

Short-run elasticity of labor demand

$$\delta_w^{E^d_{sr}} = \left[\frac{\Delta(E^d_{sr})}{E^d_{sr}}\right] / \left[\frac{\Delta(w)}{w}\right] = \frac{dln(E^d_{sr})}{dln(w)} < 0$$

$|\delta_w^{E^d_{sr}}| > 1 \rightarrow$ elastic demand
$|\delta_w^{E^d_{sr}}| < 1 \rightarrow$ inelastic demand.

Product market is not perfectly competitive: Short-Run

Firm's problem

$$\max_{\{E\}} p\left[f\left(E,\overline{K}\right)\right]\cdot f\left(E,\overline{K}\right) - wE - r\overline{K}$$

First-Order Condition

$$p'(E^*)\cdot f'\left(E^*,\overline{K}\right)\cdot f\left(E^*,\overline{K}\right) + p(E^*)\cdot f'\left(E^*,\overline{K}\right) = w$$

which, by multiplying the first term by P/P and remembering the definition of an elasticity, is

$$f'\left(E^*,\overline{K}\right)\cdot(1-\frac{1}{\eta}) = \frac{w}{p}$$

where η is the absolute value of the elasticity of product demand
$\eta \to \infty$ for a perfectly competitive firm

Product market is not perfectly competitive: Short-Run

Labor demand curve for the firm with market power

Firms with market power in the product market have a more inelastic labor demand than firms that operate in a competitive product market:

$$\left.\frac{dE}{dw}\right|_{\text{p. c.}} = \frac{1}{p \cdot f''\left(E^{*}, \overline{K}\right)} < \frac{1}{p \cdot f''\left(E^{*}, \overline{K}\right) \cdot \left(1 - \frac{1}{\eta}\right)} = \left.\frac{dE}{dw}\right|_{\text{m.p.}} < 0$$

where *p.c.* stands for ‘perfect competition’ and *m.p.* stands for ‘market power’

Labor Demand in the Long-Run

Firm's capital stock is not fixed

Firm's problem

$$\max_{\{E,K\}} pf(E,K) - wE - rK$$

[as before, but now the firm chooses the level of labor *and* capital]

Equilibrium

The FOCs are $pf_E(E^*,K^*) = w$ and $pf_K(E^*,K^*) = r$ Firm sets vaue of marginal product of each factor equal its price, hence

$$\frac{f_E(E^*,K^*)}{f_K(E^*,K^*)} = \frac{w}{r}$$

The **marginal rate of technical substitution** equals **the factor-price ratio**
i.e. the slope of the **isoquant** curve equals the slope of the **isocost** line

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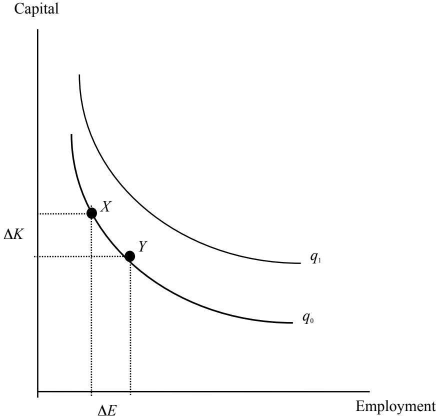
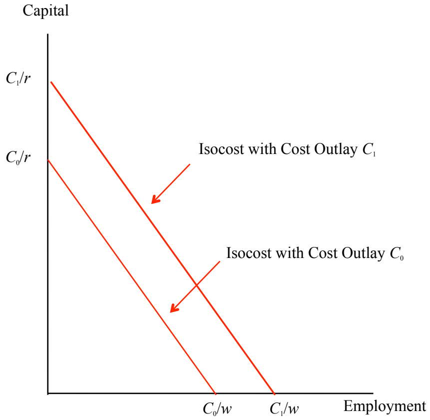
Isoquant Curves



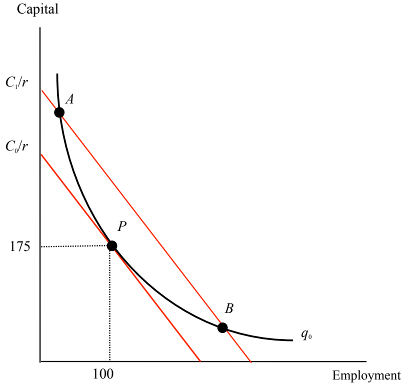
All capital-labor combinations that lie on a single isoquant produce the same level of output. The input combinations at points X and Y produce q_0 units of output. Combinations of input bundles that lie on higher isoquants must produce more output. The slope of the isoquant curve corresponds to the **marginal rate of technical substitution**

Isocost Lines



All capital-labor combinations that lie on a single isocost curve are equally costly. Capital-labor combinations that lie on a higher isocost curve are more costly. The slope of an isocost equals the ratio of input prices $-\frac{w}{r}$. (Derive)

The Firm's Optimal Combination of Inputs



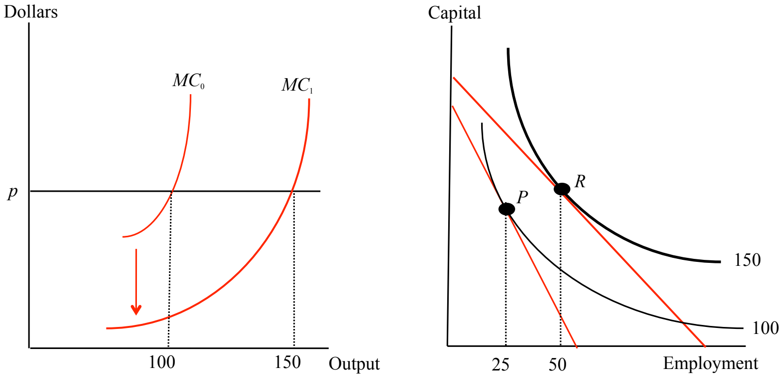
A firm minimizes the cost of producing q_0 units of output by using the capital-labor combination at point P, where the isoquant is tangent to the isocost. All other capital-labor combinations (such as those given by points A and B) lie on a higher isocost curve.

Impact of Wage Decrease on the Long Run Demand for Labor

1. **Substitution effect**: The firm takes advantage of the wage change by rearranging its mix of inputs
2. **Scale effect**: The firm takes advantage of the lower price of labor by expanding production

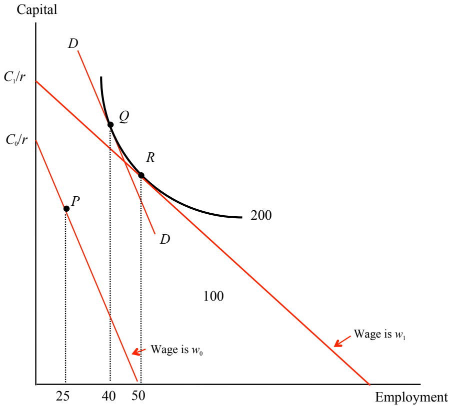
In addition to expandings its scale, the wage change encourages the firm to adapt different method of production

The Impact of a Wage Reduction on the Output and Employment of a Profit-Maximizing Firm



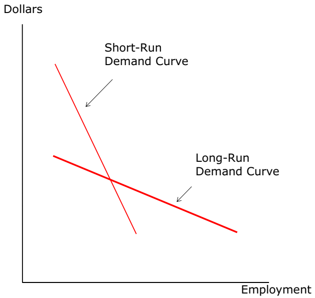
A wage cut reduces the marginal cost of production and encourages the firm to expand (from producing 100 to 150 units). The firm moves from point P to point R, increasing the number of workers hired from 25 to 50.

Substitution and Scale Effects



A wage cut generates substitution and scale effects. The scale effect (from P to Q) encourages the firm to expand, increasing the firm's employment. The substitution effect (from Q to R) encourages the firm to use a more labor-intensive method of production, further increasing employment.

Short- and Long-Run Demand Curves



In the long run, the firm can take full advantage of the economic opportunities introduced by a change in the wage as it can adjust both capital and labor.
As a result, the long-run demand curve is more elastic than the short-run demand curve.

Elasticity of Substitution

The elasticity of substitution

$$\sigma = \left[\frac{\Delta\left(K/E\right)}{K/E}\right] / \left[\frac{\Delta\left(w/r\right)}{w/r}\right] = \frac{dln(K/E)}{dln(w/r)} > 0$$

- The percentage change in the capital to labor ratio given a percentage change in the price ratio (wages to real interest)
- Note that, given the FOC, the elasticity of substitution reflects the production function and it is equal to the percentage change in the capital to labor ratio given a percentage change in the marginal rate of technical substitution:

$$\sigma = \frac{dln(K/E)}{dln(f_E/f_K)}$$

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Conditional factor demand elasticities

Contant output labor demand elasticity

$$\delta_w^E = -(1-s)\sigma < 0$$

where σ is the elasticity of subsitution between production factors. and $s = wE/Y$, the share of labor in total revenue.)

(Constant output) cross-elasticity of demand for labor

$$\delta_r^E = (1-s)\sigma > 0$$

In response to change in the price of capital.

Total (long-run) labor demand elasticities

Changes in factor prices affect costs, and eventually product price.
Total factor demand elasticities include scale and subsitution effects.

Long-run elasticity of labor demand

$$\delta_w^{E_{lr}} = \left[\frac{\Delta(E_{lr}^d)}{E_{lr}^d}\right] / \left[\frac{\Delta(w)}{w}\right] = \frac{dln(E_{lr})}{dln(w)} = -(1-s)\sigma - s\eta_p^Y$$

where

- $s = wE/Y$ is the share of labor cost in total revenue
- σ : elasticity of substitution between production factors
- η_p^Y : absolute value of the elasticity of product demand.

(Exact equality holds for firms in a competitive market with constant returns to scale production function.)

Marshall's Rules of Derived Demand

Long-run elasticity of labor demand :

$$\delta_w^{E_{lr}} = -(1-s)\sigma - s\eta_p^Y$$

Marshall's Rules: Labor Demand is more elastic ($|\delta_w^{E_{lr}}| \uparrow$) when:

1. elasticity of substitution is greater ($\sigma \uparrow$)
2. elasticity of product demand is greater ($\eta_p^Y \uparrow$)
   Higher wages lead to higher prices $\rightarrow$ elastic demand means large cut in output $\rightarrow$ firms cut employment heavily.
3. the labor's share in total costs of production is greater ($s\uparrow$)
   Even a small increase in wages substantially increases marginal cost of production $\rightarrow$ prices go up.
4. the supply of other inputs (substitutes) is more elastic
   The demadn curve for labor is more elastic the easier it is to increase capital stock.

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Factor Demand with Many Inputs

- So far we have assumed, that labor is homogenous.
- Firms may use different labor inputs in production: Skilled and unskilled labor, old and young workers, old and new machines...

$$q = f\left(x_1, x_2, x_3, ..., x_n\right)$$

Factor Demand with Many Inputs

Many different inputs

- What happens to demand of other input iif price of one input changes?

Cross-elasticity of factor demand

$$\delta_{w_j}^{x_i} = \frac{dln(x_i)}{dln(w_j)}$$

i.e. the percent change in the demand for input i given a percentage change in the price of input j

- Substitutes: cross-elasticity positive
- Complements: cross-elasticity negative

Empirical examples

- Estimates of labor demand elasticity
  - Requires exogenous variation in labor costs
  - Policies changes on payrol taxes/minimum wages
- Effect of shocks (trade and technology) on labor demand
  - What happens if price of machines (technological change) falls?
  - Does this benefit some skills groups particularly? Skill biased technological change
- In practise difficult to isolate the effects on labor demand from the effects on labor supply...

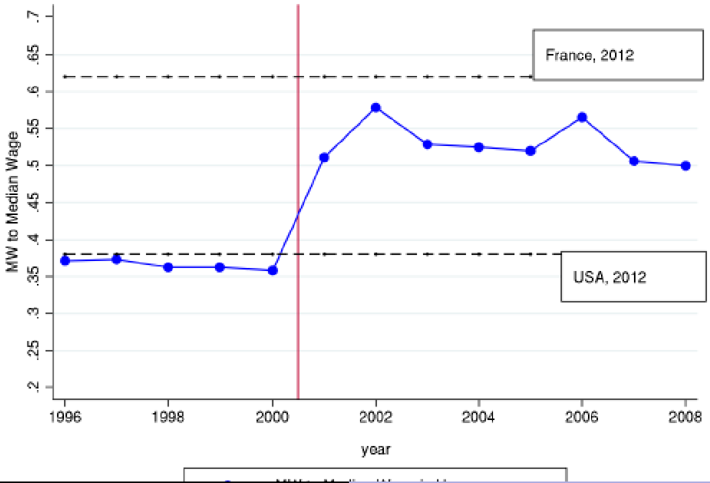
Application 2: Minimum Wages: Minimum Wage Increase in Hungary

Harasztosi and Lindner (2019 AER) “Who Pays for the Minimum Wage?"

- Minimum wages were raised from 25,500 HUF to 40,000 HUF in January 2001
- Estimate the employment (and profit) effects of minimum wage by comparing evolution of firms with many workers affected by the minimum wage increase to those firms with few affected workers
- We will return to minimum wages in next lecture!

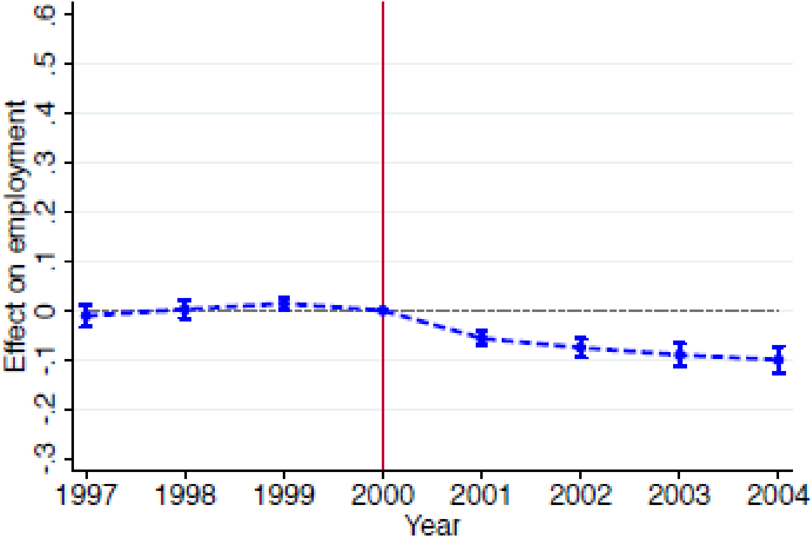
Minimum wage increase in Hungary

Figure 1: Minimum Wage in Hungary



Minimum wage increase in Hungary: Wages

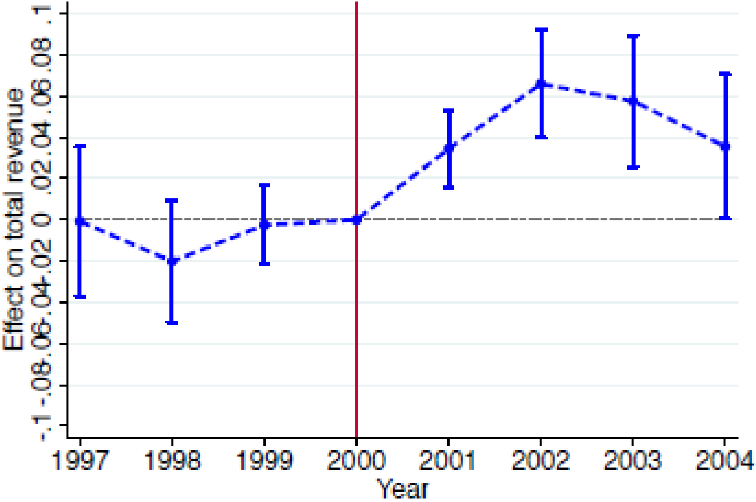
Minimum wage increase in Hungary: Employment



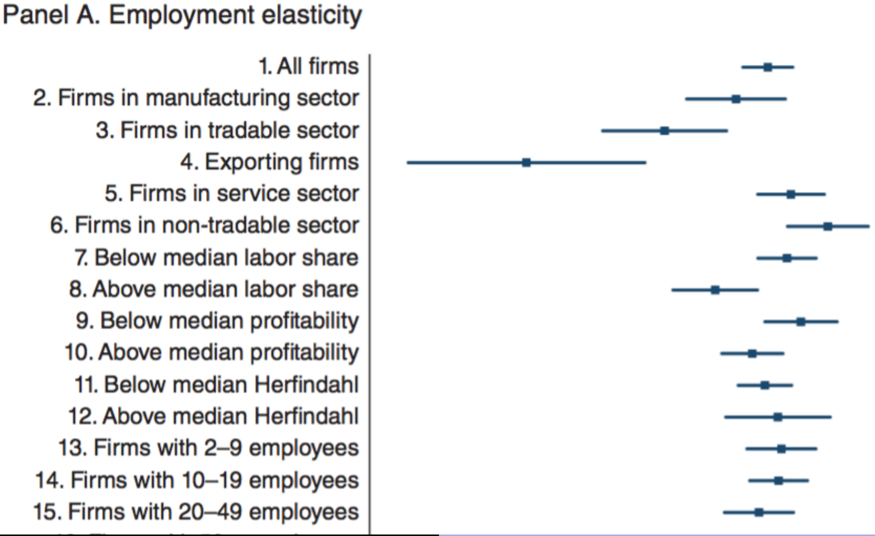
Minimum wage increase in Hungary: Elasticity

	(1)	(2)	(3)	(4)	(5)	(6)
	Main Changes between 2000 and 2002		Main Changes between 2000 and 2004		Placebo Changes between 2000 and 1998	
Panel A: Change in Firm-Level Employment						
Fraction Affected	-0.078 (0.008)	-0.076 (0.010)	-0.093 (0.012)	-0.100 (0.012)	-0.003 (0.008)	0.002 (0.009)
Constant	-0.050 (0.005)		-0.105 (0.007)		0.046 (0.005)	
Observations	19,485	19,485	19,485	19,485	19,485	19,485
Employment elasticity wrt. *MW* (directly affected)	-0.11 (0.01)	-0.10 (0.01)	-0.15 (0.02)	-0.15 (0.02)		
Panel B: Change in Firm-Level Average Wage						
Fraction Affected	0.53 (0.01)	0.58 (0.01)	0.48 (0.01)	0.54 (0.01)	-0.02 (0.003)	-0.03 (0.01)
Constant	0.08 (0.002)		0.16 (0.01)		-0.08 (0.001)	
Observations	18,415	18,415	16,980	16,980	19,485	19,485
Employment elasticity wrt. wage	-0.15 (0.02)	-0.13 (0.02)	-0.20 (0.03)	-0.18 (0.03)		

Minimum wage increase in Hungary: Revenue



Minimum wage increase in Hungary: Elasticity



Minimum wage increase in Hungary: Main findings

Harasztosi and Lindner (forthcoming AER) “Who Pays for the Minimum Wage?"

- Questions in Assigment 2
  - Identification strategy
  - Elasticity
  - What determines labor demand elasticity (Marshall’s rule of derived demand)