General information

The exercise sessions will be held as blackboard sessions, where the participants will present their solutions to the group. As such, the problems should be set up and solved before the session. The focus of the exercises lies on analyzing and discussing the task at hand together with the group: thus, a perfect solution is not required to be awarded points. A point will be awarded for each question, and a person will be chosen to present their solution from the pool.

Exercise 1.

Fusion plasma heating – Pathway to 20 keV

Significant heating is crucial in order to achieve the high temperatures required for magnetic confinement fusion.

(a) Explain the physics principles used in the conventional plasma heating methods, including both intrinsic and extrinsic methods.

(b) What are their advantages and limitations?

(c) How significant are the methods in the existing machines (e.g. JET and ASDEX Upgrade) and in future devices (ITER and DEMO)?

Solution 1.

(a) The conventional plasma heating methods are neutral beam injection, radio-frequency heating, and ohmic heating. One could also talk about plasma self-heating, when the fusion reaction rate becomes significant.

- **Ohmic heating:** Current is driven through the plasma. This leads to resistive heating of the plasma. Heat is deposited mainly on electrons due to the significant mass difference between ions and electrons (about a factor of 2000).
- **Radio frequency heating:** Electromagnetic waves of resonant frequencies (Landau cyclotron resonance, ion cyclotron, electron cyclotron, lower hybrid) are emitted into the plasma. Resonant particles extract energy from the emitted wave. Background plasma is then heated as the accelerated particles thermalize.
- **Neutral beam injection:** Fast hydrogenic neutrals are injected into the plasma. The hydrogen neutrals ionize once entering the plasma and deposit their energy to the background plasma as they ionize. The collisions heat both electrons and ions. The balance between the electron and ion heating depends on the beam energy. At low energies (< 90 keV), ion heating dominates. When the energy increases (> 90 keV) electron heating begins to dominate.

- **Alpha self-heating:** The D-T fusion alphas carry an initial energy of about 3.5 MeV. Thermalization of these with the background plasma heats the background plasma. Therefore, one would need to confine these particle well enough, such that they have enough time to thermalize. However, one should also pump them out fast enough, such that the fusion fuel does not get diluted.

(b) The pros and cons of different heating methods are

- **Ohmic heating:** Intrinsic in tokamaks (always present). Insufficient for reaching ignition temperatures, because the resistivity of the plasma is reduced as $T_e^{-3/2}$ with increasing temperature. Therefore, the temperatures with ohmic heating saturate (exercise 2).
- **Ion cyclotron resonance heating (ICRH):** Central heating (fast wave). Wave needs to tunnel through the edge plasma, which limits coupling efficiency. Antenna must be placed close to the plasma, and they are therefore exposed to the edge plasma interaction. Since the cyclotron frequency depends on the magnetic field strength, the heating system can be focused to a specific region of the plasma (central heating to avoid impurity accumulation in JET for example!). Requires high density plasmas ($n_e > 2 \times 10^{19}$ m^{-3}).
- **Electron cyclotron resonance heating (ECRH):** Reliable, flexible, very localized heating + current drive due to small wavelength, high density cutoffs, electron heating (requires collisional coupling to ions). Due to the high localization of the system, ECE can be used for active control of neo-classical tearing modes (NTMs). By imposing localized heating into the magnetic island, the growth of the island can be quenched and the NTM eliminated. Availability of high-power gyrotron sources may be an issue.
- **Lower hybrid:** Efficient (off-axis) heating and current drive, antenna must be close to plasma. Klystron wave sources are readily available.
- **NBI** Reliable (the base heating scheme in most of the existing standard tokamaks, such as ASDEX and JET). The NBI-injectors need to be close to torus and have large ports, which may be an issue in high neutron yield plasmas. Furthermore, negative ion NBIs are necessary for reactors to reach sufficient neutralization efficiency at the energy levels needed in reactors.
- **Alpha self-heating** The exploration of these plasmas is the fundamental target of ITER. The self-heating provides access to self-sustained fusion burn. However, there is little direct control over the alpha heating source, and, therefore, a self-heated plasma will largely determine its own profiles and characteristics.

(c) Significance relative to each other in the existing machines and in the next step devices:

- In existing baseline tokamaks, the main "engine" is typically NBI. For example at JET the maximum available power levels are $P_{\mathrm{NBI}} \approx 32$ MW, $P_{\mathrm{ICRH}} \approx 6-10$ MW and $P_{\mathrm{Ohmic}} < 1$ MW.
- In ITER, the mixture will be a bit more balanced: $P_{\mathrm{NBI}} \approx 33$ MW, $P_{\mathrm{RF}} \approx 40$ MW.

Exercise 2.

Ohmic heating

Ohmic heating is the most basic heating method for tokamaks. It utilizes the resistivity of the plasma by running a current through the plasma. The resistivity is given approximately by the Spitzer resistivity

$$\eta_s = \frac{\pi Z e^2 m_e^{1/2} \ln\Lambda}{(4\pi\epsilon_0)^2 (k_B T)^{3/2}},$$

where $\ln\Lambda$ is the Coulomb logarithm. In this exercise, assume that $\ln\Lambda \approx 15$ (a reasonable approximation for a fusion-relevant plasma).

(a) Explain qualitatively why the plasma resistivity drops (and conductivity improves) strongly with increasing plasma temperature ($\eta_s \sim T^{-3/2}$)?

(b) Derive the power balance including only transport losses $P_T = 3nk_BT/\tau_E$ and ohmic heating $P_O = \eta j^2$, assuming some current density j, and solve the equilibrium temperature.

(c) Assume ITER-like values $a = 2$ m, $I_p = 15$ MA, $n_e = 10^{20}$ m^{-3}, and $\tau = 8$ s and calculate the equilibrium temperature. How does this compare to fusion relevant temperatures? For the purposes of this exercise, assume that the current density is constant across the plasma cross section, and that the plasma cross section is approximately circular.

Solution 2.

(a) The collisionality of the plasma scales proportionally to nT^{-2}. This follows from the fact that the Coulomb collision cross-section is inversely proportional to the energy of the colliding particles. Due to the same physics reasons, the resistivity of the plasma, which is determined by the level at which the electrons are collisionally coupled to the background ions, scales proportionally to $T^{-3/2}$.

(b) Assuming only Ohmic heating and transport losses, the power balance reads

$$P_{\text{trans}} = P_O$$
$$\frac{3nk_BT}{\tau_E} = \eta j^2$$

Solving for k_BT

$$\frac{3nk_BT}{\tau_E} = \frac{\pi Z e^2 m_e^{1/2} \ln\Lambda}{(4\pi\epsilon_0)^2 (k_B T)^{3/2}} j^2$$
$$(k_BT)^{5/2} = \frac{\tau_E}{3n} \frac{\pi Z e^2 m_e^{1/2} \ln\Lambda}{(4\pi\epsilon_0)^2} j^2$$

Raising both sides to the power of $2/5$ yields

$$k_BT = \left(\frac{\tau_E}{3n} \frac{\pi Z e^2 m_e^{1/2} \ln\Lambda}{(4\pi\epsilon_0)^2} j^2 \right)^{2/5} \tag{1}$$

(c) Assuming an even distribution of current over a circular plasma cross-section gives

$$k_B T = \left(\frac{\tau_E}{3n} \frac{\pi Z e^2 m_e^{1/2} \ln \Lambda}{(4\pi\epsilon_0)^2} \left(\frac{I_p}{\pi a^2} \right)^2 \right)^{2/5}$$

Plugging in the values yields $k_B T \approx 2.6\,\mathrm{keV}$, which is short of the required temperatures of $10\,\mathrm{keV}$ to $15\,\mathrm{keV}$.

Exercise 3.

RF heating

As seen from above, Ohmic heating is insufficient to sustain fusion reactions and other means of heating, such as RF heating, are required.

(a) How is heat transferred from the injected RF wave to the plasma particles?

(b) List the various RF-heating methods, and their characteristic frequencies.

(c) What are their main limitations and advantages?

(d) What is the main application of lower hybrid heating systems?

(e) Why does ion cyclotron resonance heating antenna have to be located close to the plasma?

Solution 3.

(a) The resonant particles absorb the energy of the wave. They are accelerated to suprathermal velocities and they transfer their energy to the plasma with collisional coupling to the background

(b) The primary classes are electron cyclotron, ion cyclotron and lower-hybrid heating/current drive. EC operates at hundreds of GHz, IC at tens of MHz, and LH at a few GHz.

(c) Consult the model solution to question 1 (b) for details.

(d) Lower-hybrid waves can be used to drive a non-inductive off-axis current. This is useful for extending the length of a tokamak discharge and for tailoring the current profile for the best possible confinement. However, the current drive efficiency (in terms of Amperes of current per watts of power consumed) is likely not high enough for commercial applications, even though it is higher for LHCD than other current drive mechanisms.

(e) The fast IC wave, which does not have an upper density limit and can therefore be used for plasma heating, has a lower density limit and must tunnel through the edge plasma. The bigger the gap the larger the loss in the tunnelling process.

Exercise 4.
Neutral beam heating

(a) Why are neutral particles needed for the beam heating of a tokamak plasma?

(b) How is the heat transferred from the NBI-particles to the plasma particles?

(c) Why are the neutral beams sometimes arranged tangentially to the plasma? What can you say about the possibility to drive plasma current with NBIs?

(d) Describe the main components and operating principle of an NBI-injector.

(e) What limits the performance of NBI-injectors?

(f) What are the main differences between positive and negative NBI-systems?

Solution 4.

(a) Charged particles are deflected by magnetic fields and cannot penetrate the plasma.

(b) The NBI particles collide with the plasma electrons and ions and release their energy. The collisions heat both electrons and ions. The balance between the electron and ion heating depends on the beam energy. At low energies (< 90 keV), ion heating dominates. When the energy increases (> 90 keV) electron heating begins to dominate.

(c) Tangential NBI exerts a torque on the plasma, making it rotate. This can help in avoiding certain instabilities (so-called locked modes). Tangential NBI can also be used to drive non-inductive plasma current, up to 2 MA in ITER. The NBI current drive efficiency saturates at high beam energy, but increases with temperature.

(d)
  (1) In the first subsystems, neutral gas is ionized.
  (2) Then the ionized gas is accelerated in an electric field to high energies.
  (3) The accelerated ion beam is directed into a neutralization chamber, where CX-reactions with neutral deuterium (tritium) atoms are used to neutralize the fast ions.
  (4) After this section deflection magnetics are used to collect the un-neutralized ions
  (5) The un-neutralized ions are collected onto a dump plate, whereas the neutral particles are directed into the plasma.
  (6) Vacuum pump is used to minimize the ambient neutral density in front of the path of the high energy neutrals.

(e) The overarching challenge of NBIs is to reach a sufficient neutralization efficiency. Especially as the beam energy increases, as is needed for bigger machines, such as ITER and DEMO. Therefore, one needs to develop negative NBIs, which can reach higher neutralization efficiency at high beam energies.

(f) Negative ions have a higher neutralization efficiency at high energies. Negative-ion driven systems have typically lower current densities.