General information

The exercise sessions will be held as blackboard sessions, where the participants will present their solutions to the group. As such, the problems should be set up and solved before the session. The focus of the exercises lies on analyzing and discussing the task at hand together with the group: thus, a perfect solution is not required to be awarded points. A point will be awarded for each question, and a person will be chosen to present their solution from the pool.

Exercise 1.

Power exhaust in tokamaks Consider an ITER-sized tokamak of $R = 6$ m and $r = 2$ m with 50 MW of external heating power supplied, operating at steady-state with $Q \approx 10$ using D-T fusion. According to conservation of energy all power must me exhausted from the device while remaining within the material limitations of the plasma facing components, taken to be ~ 10 MW m^{-2}.

a) What are the heat loads on the vessel walls, assuming the power to be exhausted isotropically. How does this value compare to the material limitations.

b) Next, consider the realistic situation in a diverted device where only the fusion power carried by the neutrons is exhausted isotropically. Approximate the heat loads on the divertor target as the sum of the α-power and the auxiliary heating power. Assume $v_\perp \approx 10$ ms^{-1}, $v_\parallel \approx c_s \approx 10^5$ ms^{-1} and $L \approx 100$m, where $v_\perp$ is the effective cross-field velocity, $v_\parallel$ is the velocity along the field lines - assumed to be sonic speed c_s, and L is the distance from the plasma source to the divertor targets along the field lines (Fig. 1). How does this value compare to the material limitations?

Solution 1.

a)

$$A_{torus} = 2\pi R \times 2\pi r = 2\pi 6 \text{ m}^2 \times 2\pi 2 \text{ m}^2 \approx 473 \text{ m}^2 \tag{1}$$

$$P_{device} = (1 + Q)P_{heating} \approx 550 \text{ MW} \tag{2}$$

$$q_{isotropic} = \frac{550 \text{ MW}}{473 \text{ m}^2} \approx 1.2 \text{ MW m}^2 \tag{3}$$

This value is well within the material limitations.

b)

$$L_\perp = v_\perp \frac{L}{\mathrm{v}_\parallel} \approx 10\ \mathrm{ms}^{-1} \frac{100\ \mathrm{ms}}{10^5}\ \mathrm{ms}^{-1} \approx 0.01\ \mathrm{m} \tag{4}$$

$$A_{div} \approx 2 L_\perp L_{tor} = 2 \times 0.01\ \mathrm{m} \times 2\pi 6 \approx 0.75\ \mathrm{m}^2 \tag{5}$$

where the factor of 2 accounts for the inner and outer targets.

$$P_\alpha \approx \frac{1}{5} P_f \approx 100\ \mathrm{MW} \tag{6}$$

$$q_{div} = \frac{P_\alpha + P_{aux}}{A_{div}} = \frac{150\ \mathrm{MW}}{0.75\ \mathrm{m}^2} \approx 199\ \mathrm{MW\ m}^2 \tag{7}$$

Well above material limitations!

Exercise 2.
Challenges in power exhaust

a) What additional caveats regarding power exhaust and the material limitations can you think of?

b) Based on these caveats and exercise 1, what can you say about the operation and PWI of reactor-size tokamaks?

c) How can the divertor heat loads be decreased?

Solution 2.

a) This calculation assumes steady-state operation: ITER will operate in H-mode, where transient events, ELMs, will occur, exhausting a significant amount of the core energy within a short time span

b) ELMs need to be controlled in order to assure that the heat loads are manageable and controlled. A large portion of the power exhausted from the core needs to be lost isotropically. The main process for isotropic power losses is radiation.

c) There are two alternatives: increase the surface area or decrease power. The surface area can be increased by altering the magnetic topology, i.e. expanding the magnetic fields in the divertor, spreading out the power. Additionally the targets can be tilted, allowing the projection of incident power to be spread over larger areas. However, this might lead to difficulties with leading edges. The other path is to decrease the power incident on the targets. The process to achieve this is by radiation. One method is to inject seeding impurities into the divertor volume, which will radiate away power strongly. However, these seeding impurities must be confined in the divertor, because if they made it into the core they would radiate away the fusion power and quench the core. Another principle is by volumetric radiative processes in the divertor such as recombination, ionization and dissociation of molecules.

Exercise 3.

The two-point model Here we derive a simple analytic expression relating the divertor target conditions to those at the upstream location. On lecture slide 44 the basic equations are presented, assuming conductive transport, particle, pressure, and momentum conservation along the parallel distance. Using this slide, derive the following as functions of upstream density (n_u) and heat flux density ($q_\parallel$), assuming significant temperature gradients along the parallel distance L ($T_t \ll T_u$). You will need to use $c_{st} = (2kT_t/m_i)^{1/2}$.

a) An expression for upstream temperature (T_u).

b) An expression for target temperature (T_t).

c) An expression for target density (n_t).

d) An expression for target plasma particle flux density ($\Gamma_t = q_\parallel/(\gamma e T_t)$).

Solution 3.

a)

$$T_u^{7/2} = T_t^{7/2} + \frac{7}{2}\frac{q_\parallel L}{\kappa_{0,e}} \rightarrow T_u = \left(\frac{7}{2}\frac{q_\parallel L}{\kappa_{0,e}}\right)^{2/7}, \tag{8}$$

since $T_u \gg T_t$

b) Combining

$$2n_tT_t = n_uT_u \rightarrow T_t = \frac{n_uT_u}{2n_t} \tag{9}$$

$$q_\parallel = \gamma n_t k T_t c_{st} \rightarrow n_t = \frac{q_\parallel}{\gamma k T_t \sqrt{2kT_t/m_i}} \tag{10}$$

where $k = e$, and solving for T_t:

$$T_t = \frac{n_uT_u}{2}\frac{T_t\gamma k\sqrt{2kT_t/m_i}}{q_\parallel} \rightarrow \frac{2q_\parallel}{n_uT_u\gamma e} = \sqrt{\frac{2kT_t}{m_i}} \leftrightarrow T_t = \frac{2q_\parallel^2 m_i}{n_u^2T_u^2\gamma^2e^3}. \tag{11}$$

However, equation 11 is still dependent on T_u, which is defined in equation 8. Combining these, the expression is only dependent on n_u and $q_\parallel$:

$$T_t = \frac{2q_\parallel^2 m_i}{n_u^2\gamma^2e^3}\left(\frac{7}{2}\frac{q_\parallel L}{\kappa_{0,e}}\right)^{-4/7} = \frac{2m_i}{\gamma^2e^3}\left(\frac{7}{2}\frac{L}{\kappa_{0,e}}\right)^{-4/7}\frac{q_\parallel^{10/7}}{n_u^2}. \tag{12}$$

c) Combining

$$2n_tT_t = n_uT_u \to n_t = \frac{n_uT_u}{2T_t} \tag{13}$$

and equations 12 and 8 and solving for n_t:

$$n_t = \frac{n_u}{2m_i}\left(\frac{7}{2}\frac{q_\parallel L}{\kappa_{0,e}}\right)^{2/7}\frac{\gamma^2e^3}{2}\left(\frac{7}{2}\frac{L}{\kappa_{0,e}}\right)^{4/7}\frac{n_u^2}{q_\parallel^{10/7}} = \frac{n_u^3}{q_\parallel^2}\left(\frac{7}{2}\frac{q_\parallel L}{\kappa_{0,e}}\right)^{6/7}\frac{\gamma^2e^3}{4m_i}. \tag{14}$$

d) Using $\Gamma_t = q_\parallel/\gamma eT_t$ and equation 12:

$$\Gamma_t = \frac{n_u^2}{q_\parallel}\left(\frac{7}{2}\frac{q_\parallel L}{\kappa_{0,e}}\right)^{4/7}\frac{\gamma e^2}{2m_i} \tag{15}$$

Exercise 4.

The extended two-point model and detachment Now, we no longer assume the quantities to be conserved and introduce *ad hoc* conservation factors (lecture slide 45). Now f_{mom} describes the fraction of momentum conserved, f_{cond} the fraction of the heat flux which is conducted, and f_{power} the power lost in the flux tube.

a) Now, repeat task 3 in order to find the scaling factors of the target parameters. Note that the formula for the target particle flux is now modified: $\Gamma_t = (1 - f_{pow})q_\parallel/(\gamma eT_t))$

b) Detachment is crucial in alleviating the heat loads incident on the targets. Detachment occurs when the target temperatures are sufficiently low ($T_t \approx 1$ eV) for volumetric recombination to occur. This reduces the plasma flux incident on the wall. Using the extended two-point model, describe which process drive detachment.

Solution 4.

a) Following the same derivation as above, now including the loss factors, the following dependencies are derived:

$$T_u \propto f_{cond}^{2/7} \tag{16}$$

$$T_t \propto \frac{(1-f_{pow})^2}{f_{mom}^2 f_{cond}^{4/7}} \tag{17}$$

$$n_t \propto \frac{f_{mom}^3 f_{cond}^{6/7}}{(1-f_{pow})^2} \tag{18}$$

$$\Gamma_t \propto \frac{f_{mom}^2 f_{cond}^{4/7}}{1-f_{pow}} \tag{19}$$

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b) From equations 16–19 it is seen that f_{cond} has a small impact on the target conditions, compared to f_{mom} and f_{pow}. The same equations suggest that power and momentum losses are competing: power losses decrease the target temperature, but increase the ion flux incident on the target. On the other hand, momentum losses increase temperature but decrease ion flux.