$$G_e(j\omega) = K \frac{j\frac{\omega}{\omega_0}+1}{j\frac{\omega}{K\omega_0}+1}$$

$$\measuredangle G_e = \arctan\left(\frac{\omega}{\omega_0}\right) - \arctan\left(\frac{\omega}{K\omega_0}\right)$$

$$\frac{d}{d\omega}(\measuredangle G_e) = \frac{1}{\omega_0}\frac{1}{1+\left(\frac{\omega}{\omega_0}\right)^2} - \frac{1}{K\omega_0}\frac{1}{1+\left(\frac{\omega}{K\omega_0}\right)^2} = 0$$

kun $\omega = \omega_0\sqrt{K} = \omega_m$

Tarkennus: $\frac{1}{\omega_0}\frac{1}{1+K} - \frac{1}{K\omega_0}\frac{1}{1+\left(\frac{\sqrt{K}}{K}\right)^2}$

$$= \frac{1}{\omega_0}\frac{1}{1+K} - \frac{1}{\omega_0}\frac{1}{K+1} = 0$$

Olkoon $\alpha = \arctan(u)$, $\beta = \arctan(v)$

Tunnetusti $\tan(\alpha+\beta) = \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\cdot\tan(\beta)}$

$$\Rightarrow \tan\left[\arctan(u)+\arctan(v)\right] = \frac{u+v}{1-uv}$$

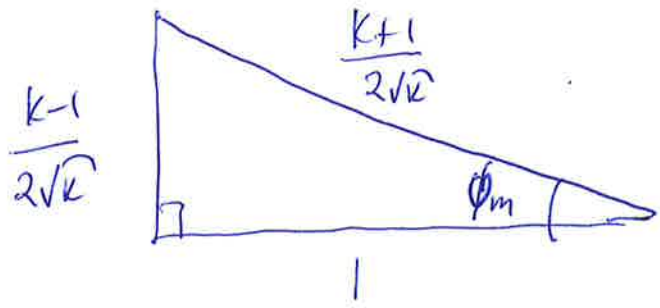
$$\Rightarrow \arctan(u)+\arctan(v) = \arctan\left(\frac{u+v}{1-uv}\right)$$

$$\angle G_{0l} = \arctan\left(\frac{w}{w_0}\right) - \arctan\left(\frac{w}{Kw_0}\right)$$

$$= \arctan \frac{\frac{w}{w_0}\left(1 - \frac{1}{K}\right)}{1 + \left(\frac{w}{w_0}\right)^2 \frac{1}{K}}$$

$$w = w_m = \sqrt{K}\, w_0$$

$$\Rightarrow \quad \phi_m = \arctan \frac{\sqrt{K}\left(1 - \frac{1}{K}\right)}{2} = \arctan \frac{K-1}{2\sqrt{K}}$$



$$\Rightarrow \sin \phi_m = \frac{K-1}{K+1}$$