ECON-C4200 - Econometrics II: Capstone

Lecture 8: Regression Discontinuity

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Learning outcomes

- At the end of lecture 8, you understand

1. what **Regression Discontinuity (RD)** is
2. what the difference between **sharp** and **fuzzy** RD is
3. the important role graphs play in RD
4. how to implement a simple RD in a regression framework

The idea behind Regression Discontinuity

- To identify the causal effect of D on Y, we look for "identifying variation" in observational data.
- We have seen that an instrumental variable or a DID can provide such variation.
- Neither is however always easily aligned with an experiment.
- RD builds on variation that is "close to random" by utilizing man-made variation in the assignment of treatment status.

Basic building blocks

RD builds on there being

1. a variable **score** or **running variable** which ranks the units;
2. a **cutoff** above which a unit receives (with a higher probability); and
3. a **treatment** which some units get and other units don't.

Some material

- Sources:
  - Cattaneo, M., Idrobo, N. & Titiunik, R. (2020). A practical introduction to regression discontinuity designs: Foundations. *Cambridge elements: Quantitative and computational methods for social sciences*. CUP
  - Santoleri, P., Mina, A., Di Minin, A. & Martelli, I. (2023). The causal effects of r&d grants: Evidence from a regression discontinuity. *Review of Economics and Statistics*, forthcoming
  - Kerr, W. R., Lerner, J. & Schoar, A. (2014). The consequences of entrepreneurial finance: Evidence from angel financings. *The Review of Financial Studies*, *27*(1), 20–55 (not really RD, but close)

The role of the score

- Example: your comprehensive school GPA defined which high school(s) you were admitted to.
- Silliman, M. & Virtanen, H. (2022). Labor market returns to vocational secondary education. *American Economic Journal: Applied Economics*, *14*(1), 197–224
- Think of the pupils at the **cut-off**: one has a marginally higher GPA and gets in, the other a marginally lower GPA and does not.
- They are for all practical purposes **equally good** $\rightarrow$ the cut-off acts as a randomization device.
- Note: it is OK if the score affects the outcome.

The score, the cut-off and receiving the treatment

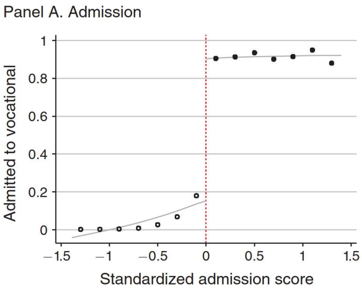


Figure 4. Cutoffs

Source: Silliman and Virtanen, 2022

Score, cut-off and treatment assignment more formally

- Let's denote the score with X_i.
- Let's denote the cut-off c.
- **Important**: always normalize the cut-off to zero.
- Unit i gets the treatment if and only if $X_i \geq c$.
- Let's denote the treatment status with D_i.
- D_i is defined as

$$D_i = \mathbb{1}[X_i \geq c]$$

- A unit for which $D_i = \mathbb{1}[X_i \geq c]$ is **assigned** to the treatment group.

Treatment assignment vs. receiving the treatment

- It is one thing to be assigned to the treatment group.
- One may thereafter either receive or not receive the treatment.
- **Sharp RD**: assignment = received treatment.
- **Fuzzy RD**: assignment $\neq$ received treatment.

The counterfactual: how to get the causal impact of the treatment

The fundamental problem of causal inference:

- For each observation unit, we only ever observe the outcome when the unit (does not) receive the treatment.
- What would have been the outcome of the unit selected into the treatment, had it not received the treatment?
- What would have been the outcome of the unit that was selected out of treatment, had it received the treatment?

The Potential Outcomes Framework

- It is one thing to be assigned to the treatment group.
- One may thereafter either receive or not receive the treatment.
- **Sharp RD**: assignment = received treatment.
- **Fuzzy RD**: assignment $\neq$ received treatment.

$$Y_i = (1 - D_i) \times Y_i(0) + D_i(1) \tag{1}$$

$$Y_i = Y_i(1) \textit{ iff } X_i \geq c$$

$$Y_i = Y_i(0) \textit{ iff } X_i < c$$

- The 1 or 0 in $Y(.)$ is the indicator for receiving (not receiving) the treatment.

Defining the counterfactual in Sharp RD

- Think of units with scores just above and just below the cut-off c.
- For all practical purposes they are the same regarding their score.
- If the minuscule difference in their score is not informative, they should identical in all possible respects but one: Their **treatment assignment**.
- Therefore one can assume that those individuals at the cut-off but just below provide the right counterfactual for the outcome of interest.
- The following then holds (under given assumptions):

$$\tau_{SRD} = \mathbb{E}[Y_i(1) - Y_i(0)|X_i = c] \tag{2}$$

$$\tau_{SRD} = lim_{\epsilon \to 0}\mathbb{E}[Y_i(1)|X_i = c + \epsilon] - lim_{\epsilon \to 0}\mathbb{E}[Y_i(0)|X_i = c - \epsilon]$$

Why use regression?

- Trade-off: units close to the cut-off very similar, but there are few of them.
- $\rightarrow$ variance (excellent comparison, small N) -bias (less excellent comparison, (much) larger N) - trade-off.
- If you include in your sample units further away from the cut-off c, you want to / need to control for their differences $\rightarrow$ regression.
- The simplest example with linear control of the score:

$$Y_i = \beta_0 + \tau D_i + \beta_1 D_i (X_i - c) + \beta_2 (1 - D_i)(X_i - c) + \epsilon_i \quad (3)$$

- Notice from equation ( ) why you need to normalize the score to be zero at the cutoff.

Why use regression? / Interpretation of the estimate

- Important decisions:
  1. What functional form to use for the "control function"?
  2. What data points to include, i.e., how far to go from the cut-off?
- With sufficient data, one should use **non-parametric** methods such as **local polynomial regression**.
- Nature of the estimate: it is a **local** of the average treatment effect, i.e., one cannot (easily) extrapolate it to observation away from the cut-off.

Santoleri, P., Mina, A., Di Minin, A. & Martelli, I. (2023). The causal effects of r&d grants: Evidence from a regression discontinuity. *Review of Economics and Statistics*, forthcoming

- What is the effect of R&D grants (subsidies) in inventive outcomes?
- Why would you want to subsidies private R&D? Externalities...
- Even if there is an effect, what is the mechanism? Funding problems, lower mc, certification, ...?

Setting

- EU Small and Medium Enterprise Instrument.
- 0.5 - 2.5M euros in funding (note: far too little information on the distribution of granted funding).
- External experts rank the applications, winners selected on budget availability.
- Competitions 2014 - 2017; outcomes measured in 2019.
- Outcomes:
  1. Investment
  2. Investment in intangibles
  3. Patents (quality weights)
  4. Follow-on equity investment
  5. Firm growth
  6. Firm failure

Descriptive statistics

Source: Santoleri et al., 2023

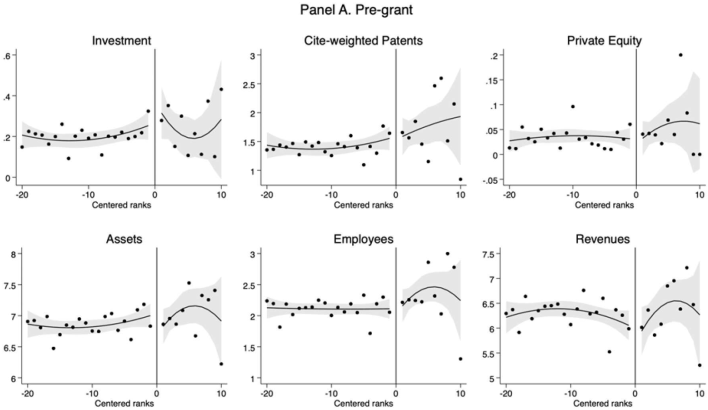
Table 1: Descriptive statistics on SME Instrument competitions and applicants

	Mean	SD	p50	N
Panel A: competitions (raw data)				
	Mean	SD	p50	N
# competitions				176
# applicants per competition	84.68	74.35	68	14904
# winning applicants per competition	4.09	3.08	3	719
Panel B: competitions (cleaned data)				
	Mean	SD	p50	N
# competitions				176
# applicants per competition	63.04	56.97	50	11095
# winning applicants per competition	2.66	2.17	2	468
Panel C: applicants characteristics				
	Mean	SD	p50	N
Patents^{Pre}	4.03	8.13	0	11095
Citw patents^{Pre}	30.84	84.70	0	11095
Private Equity^{Pre} (d)	0.04	0.18	0	8352
Private Equity^{Pre} (1,000 €)	170	1940	0	8352
Revenues^{Pre} (1,000 €)	2944	7832	554	6238
Employees^{Pre}	19.40	29.96	8	6700
Assets^{Pre} (1,000 €)	2932	5337	994	8411
Age^{Pre}	8.83	11.62	5	11313
High-Tech (d)	0.57	0.50	1	11024
Failure (d)	0.06	0.24	0	11402
IPO (d)	0.00	0.05	0	8432

Comparing treated and control firms

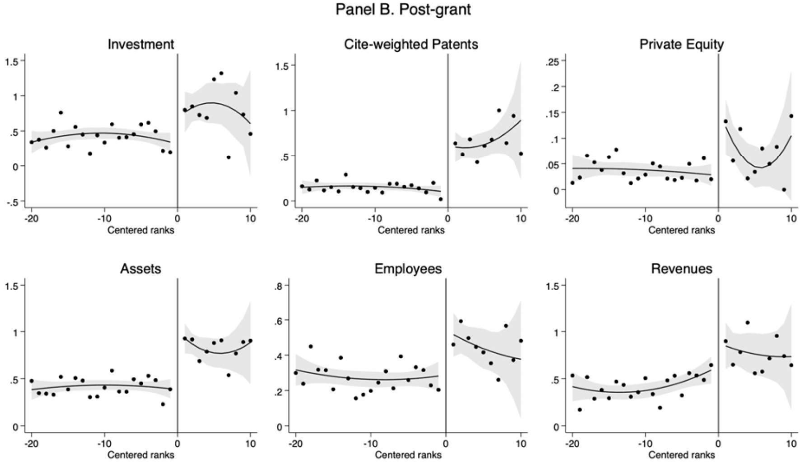
Source: Santoleri et al., 2023

Fig. 1: RDD plots before and after the grant



Treatment plots

Source: Santoleri et al., 2023



RD estimation equation

- The authors estimate the following model:

$$Y_{ic}^{Post} = \alpha + \beta Grant_{ic} + f(Rank_{ic}) + \gamma Y_{ic}^{pre} + \delta_c epsilon_{ic} \quad (4)$$

  where $-r \leq Rank_{ic} \leq r$.

- $Grant_{ic}$ = treatment status of firm i in competition c.
- Y_{ic}^{Post} = post-treatment outcome of firm i in competition c.
- Y_{ic}^{Pre} = pre-treatment outcome of firm i (to reduce variance).
- $Rank_{ic}$ = centered rank of firm i in rank c.
- $f(Rank_{ic})$ = polynomial control for centered ranks, allowed to differ on either side of cut-off. Either linear of quadratic.
- r = **bandwidth**, i.e., how far from the cut-off lies the furthest observation included in the estimation sample.

RD estimates

Source: Santoleri et al., 2023

Table 3: The effects on investment

	(1) All	(2) All	(3) All	(4) ± 10	(5) ± 10	(6) ± 5	(7) ± 5
Grant	0.437***	0.369*	0.388***	0.481***	0.481*	0.677***	1.595***
	(0.129)	(0.211)	(0.090)	(0.169)	(0.274)	(0.224)	(0.524)
Rank $\times$ Grant	Yes	Yes	No	Yes	Yes	Yes	Yes
$\text{Rank}^2 \times$ Grant	No	Yes	No	No	Yes	No	Yes
Rank quintiles	No	No	Yes	No	No	No	No
N	6873	6873	6873	1241	1241	698	698
R-squared	0.05	0.05	0.05	0.20	0.20	0.26	0.27
BIC	20231.97	20241.51	20242.34	3760.74	3770.39	2116.04	2122.26

RD estimates

Source: Santoleri et al., 2023

Table 4: The effects on cite-weighted patents and external equity

Panel A: Citw-patents	(1) All	(2) All	(3) All	(4) ± 10	(5) ± 10	(6) ± 5	(7) ± 5
Grant	0.203^{***}	0.282^{**}	0.148^{***}	0.147^{*}	0.236^{*}	0.314^{***}	0.390^{*}
	(0.068)	(0.117)	(0.051)	(0.085)	(0.138)	(0.113)	(0.230)
Rank $\times$ Grant	Yes	Yes	No	Yes	Yes	Yes	Yes
$\text{Rank}^2 \times$ Grant	No	Yes	No	No	Yes	No	Yes
Rank quintiles	No	No	Yes	No	No	No	No
N	11095	11095	11095	1822	1822	1050	1050
R-squared	0.36	0.36	0.36	0.45	0.45	0.51	0.51
BIC	23502.73	23516.83	23509.32	4221.02	4234.39	2318.97	2332.66
Panel B: Private Equity	(1) All	(2) All	(3) All	(4) ± 10	(5) ± 10	(6) ± 5	(7) ± 5
Grant	0.070^{**}	0.126^{***}	0.036^{**}	0.080^{***}	0.123^{***}	0.117^{***}	0.157^{*}
	(0.028)	(0.045)	(0.015)	(0.027)	(0.047)	(0.039)	(0.085)
Rank $\times$ Grant	Yes	Yes	No	Yes	Yes	Yes	Yes
$\text{Rank}^2 \times$ Grant	No	Yes	No	No	Yes	No	Yes
Rank quintiles	No	No	Yes	No	No	No	No
N	8352	8352	8352	1358	1358	784	784
R-squared	0.07	0.07	0.07	0.17	0.17	0.27	0.27
BIC	-5077.46	-5071.33	-5058.36	-600.21	-588.55	-337.13	-324.29

RD estimates

Source: Santoleri et al., 2023

Table 5: The effects on firm growth

Panel A: Assets	(1) All	(2) All	(3) All	(4) ±10	(5) ±10	(6) ±5	(7) ±5
Grant	0.561***	0.578***	0.437***	0.477***	0.570***	0.545***	1.037***
	(0.065)	(0.099)	(0.050)	(0.095)	(0.150)	(0.138)	(0.321)
Rank × Grant	Yes	Yes	No	Yes	Yes	Yes	Yes
Rank^2 × Grant	No	Yes	No	No	Yes	No	Yes
Rank quintiles	No	No	Yes	No	No	No	No
N	7306	7306	7306	1311	1311	743	743
R-squared	0.77	0.77	0.77	0.74	0.74	0.74	0.74
BIC	17860.70	17875.35	17862.13	2990.32	3002.53	1682.91	1691.63
Panel B: Employees	(1) All	(2) All	(3) All	(4) ±10	(5) ±10	(6) ±5	(7) ±5
Grant	0.330***	0.256***	0.219***	0.283***	0.318**	0.242**	0.234
	(0.062)	(0.092)	(0.038)	(0.081)	(0.132)	(0.120)	(0.222)
Rank × Grant	Yes	Yes	No	Yes	Yes	Yes	Yes
Rank^2 × Grant	No	Yes	No	No	Yes	No	Yes
Rank quintiles	No	No	Yes	No	No	No	No
N	5493	5493	5493	962	962	548	548
R-squared	0.79	0.79	0.79	0.80	0.80	0.83	0.83
BIC	9093.99	9109.37	9108.84	1472.89	1485.45	730.64	743.24

Generalizations / robustness tests

- (Above median) firm age, firm size (proxies for "financial vulnerability").
- Country of origin / NUTS2 - above or below median GDP/capita.
- Grant size: effects increasing in grant size.
- Note: is it OK to model the treatment as 0/1?