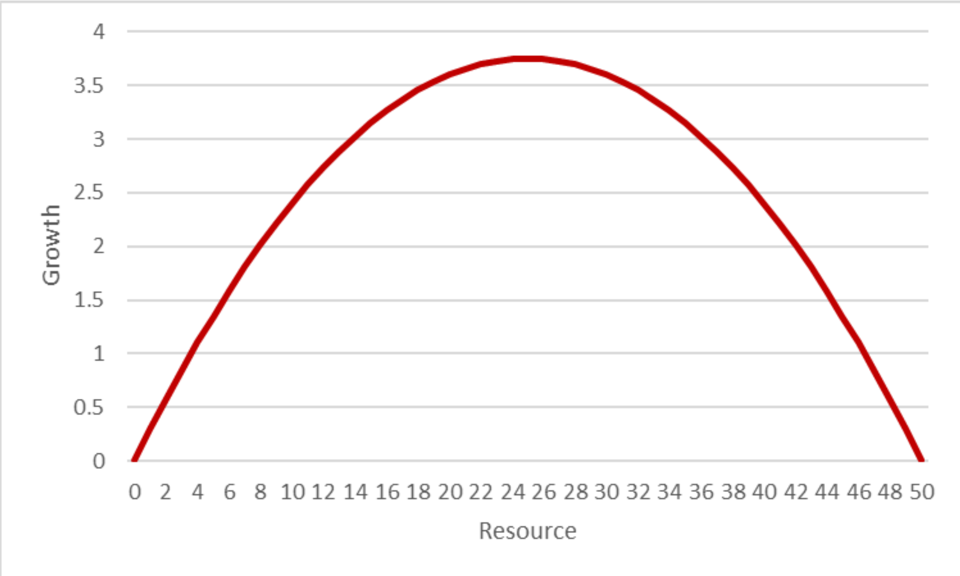
# Problem set 1 – model solutions

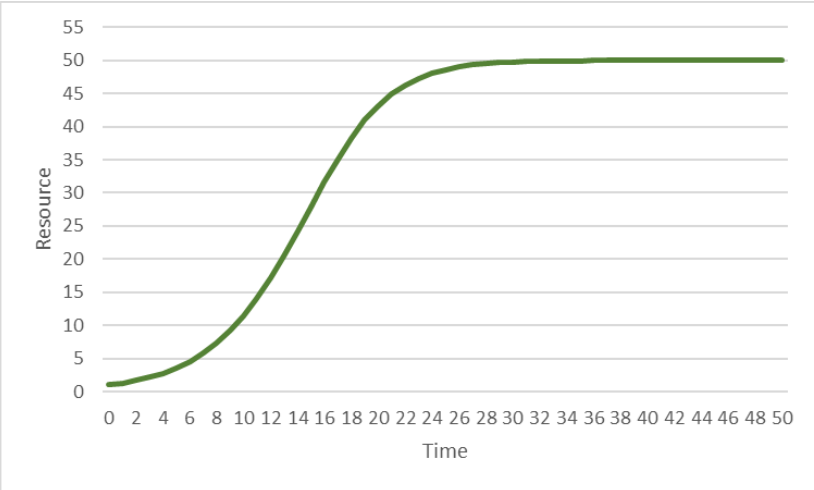
## Question 1

- Resource (x) is on the vertical axis and growth, a function $F(x) = rx(1 - \frac{x}{K})$ of the resource x, is on the horizontal axis. Intrinsic growth rate r = 0.3 and carrying capacity K = 50.

  The biological equilibrium is the point in which the growth is zero. This point is either x = 0 or x = 50, which is the carrying capacity. (If we supposed that there initially was a positive amount of the resource, the equilibrium would be 50.)

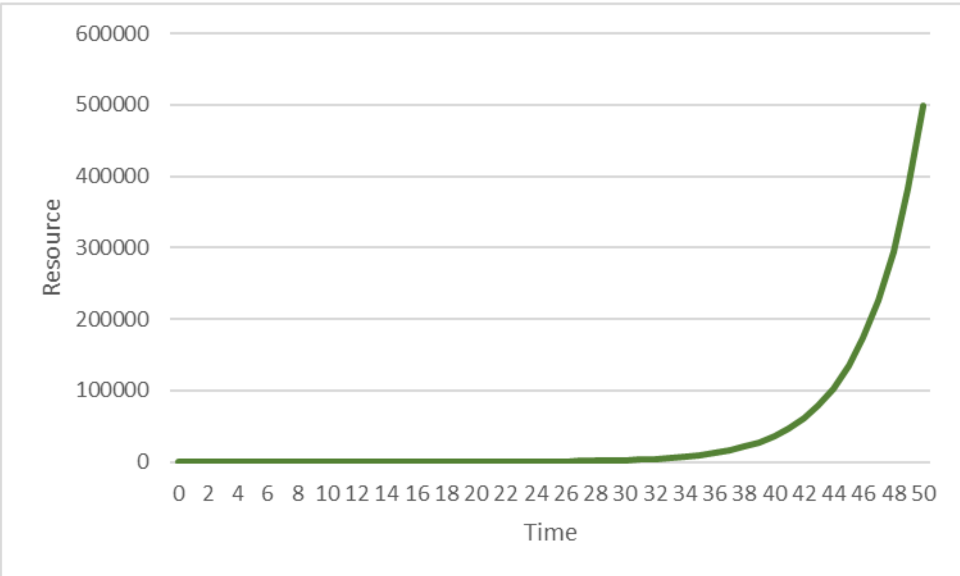



- We set the resource to be x = 1 at time point 0, and depict resource as a function of time. This is done so that we calculate the growth when x = 1. This is $F(x) = 0.3 * 1 * \left(1 - \frac{1}{50}\right) =$ $0.294$. Then we add this growth to the initial value 1 to get the value, 1.294, of the resource at time point 1. Then we again calculate the growth: $F(x) = 0.3 * 1.294 * \left(1.294 - \frac{1.294}{50}\right) \approx$ $0.378$ and add it to 1.294 to get the resource at time point 2. This is continued; you can do it easily with Excel, for instance. In this graph, the value of the resource is drawn up to time point 50. We can see that the resource as a function of time resembles an S-curve: growing fast and with accelerating pace at the beginning but later on the growth slows down and almost disappears. The growth continues forever, but the value of the resource has a limit 50, i.e., the value approaches 50 when time -> ∞, but never approaches or exceeds that value.




The instructions also suggested you try value K = ∞ for the carrying capacity. In this case, the growth function reduces to $F(x) = rx$ since $\frac{x}{K} = 0$ when K = ∞. In this case, growth becomes exponential, and the value of the resource has no limit, which can be seen from the graph below. The situation escalates quite quickly; at time point 10 the resource has about value 13, but at time point 20 the value is already 190. At time point 50, the value is about 498,000!

Think about it: would this kind of growth be realistic, or sustainable? Well, it depends. Quite surely no and certainly no if the resource is of physical character.




- The highest growth is at the maximum of the growth curve. This point is x = 25. You can also find the maximum by derivating the growth function, setting the derivative to zero and solving x (and knowing that this is the global maximum since the growth function is concave and has only one extreme point):

$$D\{F(x)\}dx = D\left[rx\left(1 - \frac{x}{K}\right)\right]dx = r - \frac{2rx}{K}$$

$$r - \frac{2rx}{K} = 0 <=> x = \frac{K}{2} = \frac{50}{2} = 25$$

- The equilibrium resource and harvest levels are found in the point in which harvest equals growth, i.e., the net growth is zero. This point can be found by solving:

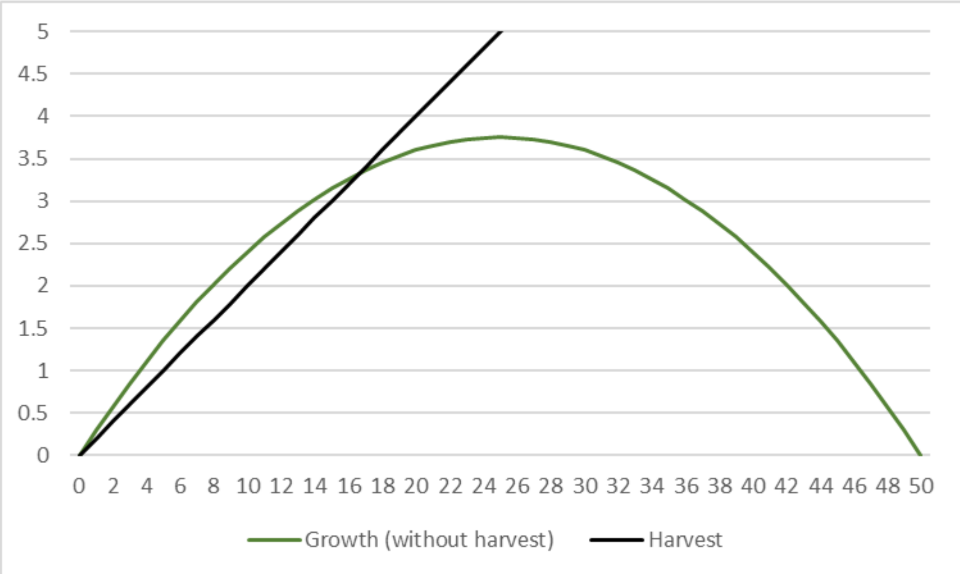
$$F(x) = 0.3x\left(1 - \frac{x}{50}\right) = 0.2x = Ex$$

$$1 - \frac{x}{50} = \frac{2}{3}$$

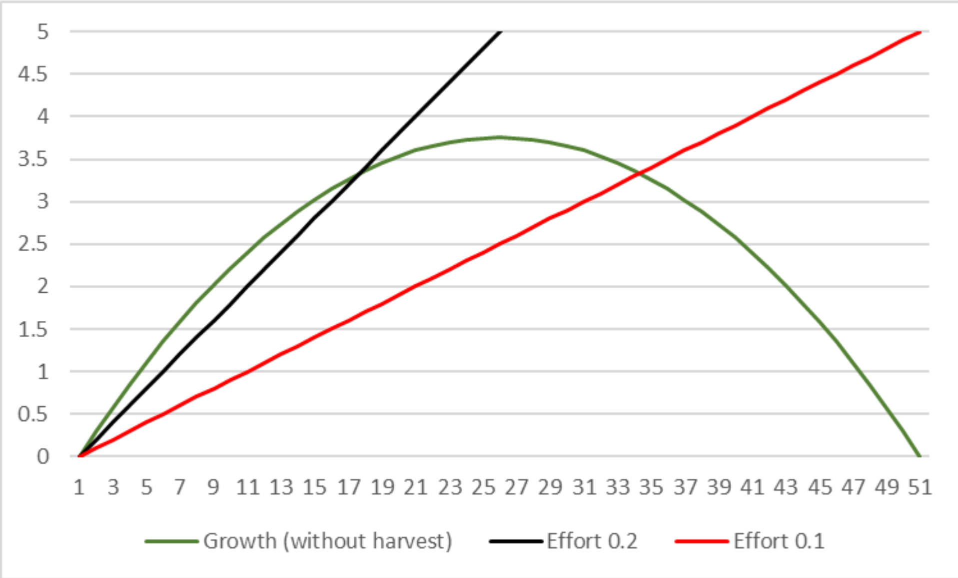
$$x = \frac{50}{3} \approx 16.67$$

$$Ex = 0.2 * \frac{50}{3} = \frac{10}{3} \approx 3.33$$

So the equilibrium resource level is $\frac{50}{3}$ and the equilibrium harvest is $\frac{10}{3}$. It is enough if you based your answer on graphical approximation only.



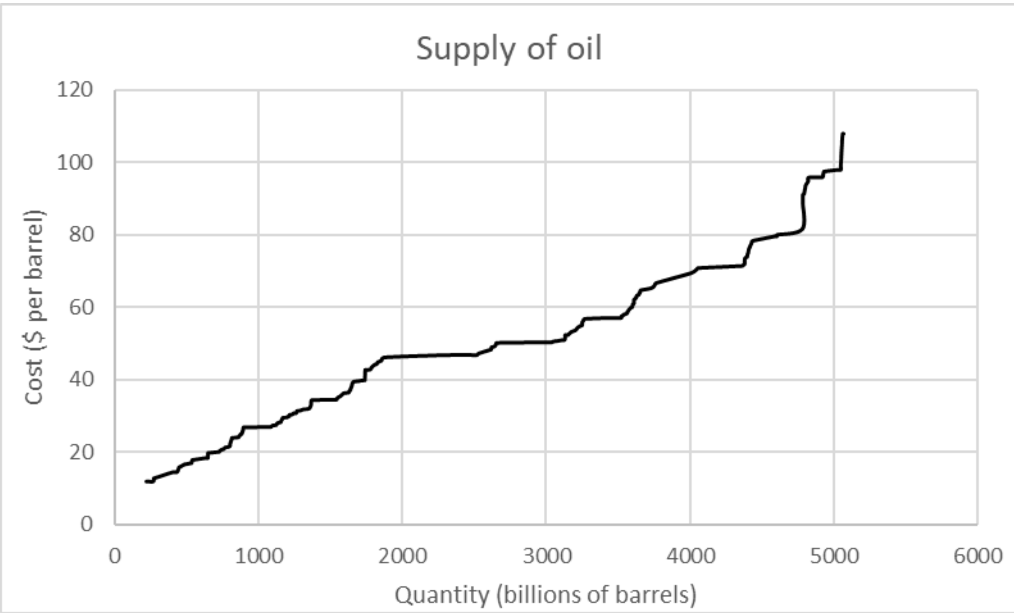

- The growth and harvest at resource value $\frac{50}{3}$ is $\frac{10}{3}$, the equilibrium defined at the previous point. The growth function takes this value $\frac{10}{3}$ also at point x = $\frac{100}{3}$. In the equilibrium, the harvest can then be $\frac{10}{3}$, which can be obtained with effort level: $E * \frac{100}{3} = \frac{10}{3} <=> E = 0.1$ So effort level 0.1 would be enough to get the same harvest if the resource was allowed to recover to level $\frac{100}{3}$. So in this new equilibrium the resource level $\frac{100}{3}$ is larger and the harvest is the same $\frac{10}{3}$, but the harvest is obtained with an effort level 0.1, half of the previous one. So investing in the form of allowing the resource the recover pays back! The graph is shown below:

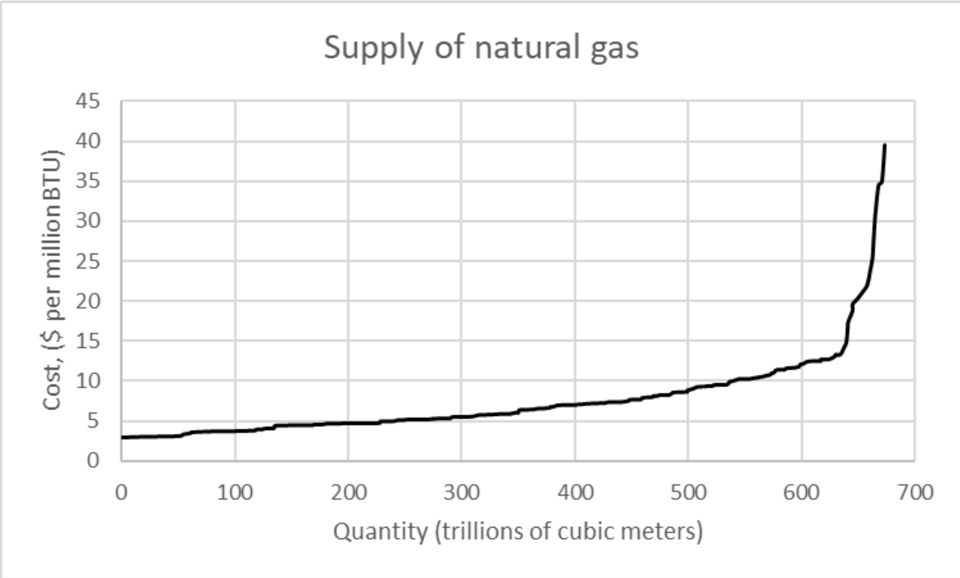


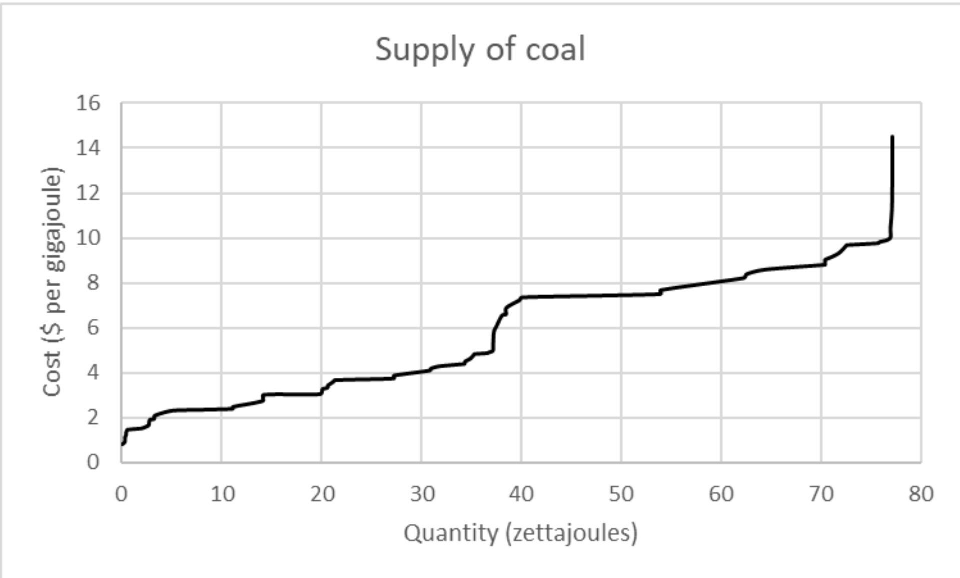

- Based on the earlier calculations, the growth of the resource stock from $\frac{50}{3}$ to $\frac{100}{3}$ would take approximately five time periods. You can see this from the spreadsheet used in the second point. Check the time period in which the resource stock is $\frac{50}{3}$ (somewhere between periods 11 and 12) and compare to the time period in which the resource stock has increased to $\frac{100}{3}$ (somewhere between periods 16 and 17).

## Question 2

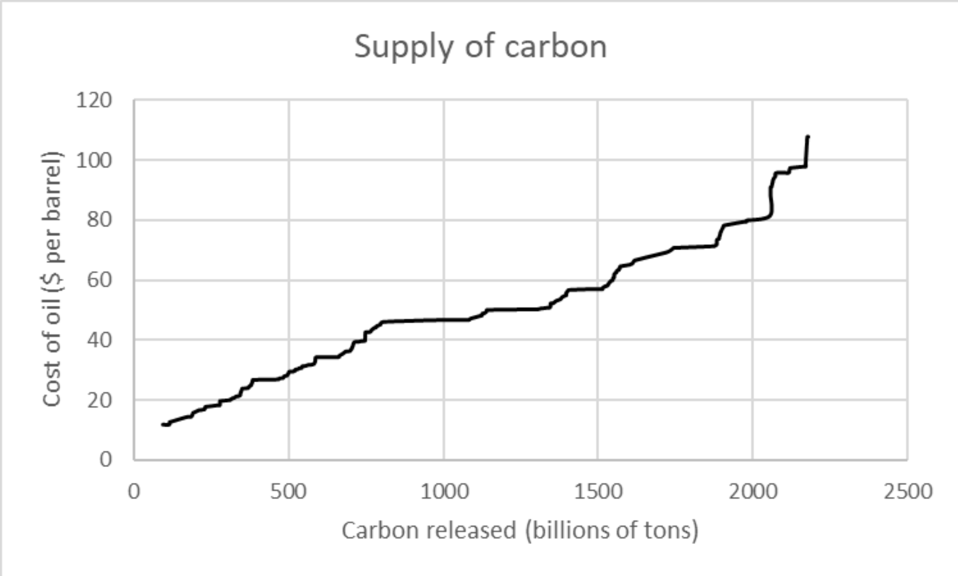
1. Supply curves are drawn below:






2. Supply of carbon (in metric tons) is calculated by multiplying the cumulative quantity of oil barrels by 0.43.




- The total possible supply of oil is 5065.743 billion barrels, which releases 2178.269 billion tons of carbon. With a 50 % carbon budget, about 1089.135 tons of carbon can be released. This means that at maximum 2532.872 billion barrels of oil can be produced. Based on the supply and cost data, the price corresponding this amount is $47.184 (then the supply of oil would be 2528.141 billion barrels and supply of carbon 1087.101 billion tons, a bit under the target).

  There are two basic solutions for achieving the carbon target: quantity and price-based ones. The quantity-based solution is to set a production quota corresponding the maximum amount of oil that can be produced. For this oil to be used efficiently, the quota should be implemented with emission trading. In emission trading, the rights to release carbon are traded on the market. There is a fixed amount of emission rights on the market (amount corresponding the carbon target), and agents are free to buy and sell the rights. By this way, the emission rights end up to agents that value the right to emit carbon the most, that is, for whom it is the most costly to cut carbon emissions. Those agents that can inexpensively cut their emissions, end up doing so instead of buying emissions rights.

  The price-based means to achieve the carbon budget would be to set a carbon tax that the emitters of carbon must pay. The tax should be so high that the marginal cost of oil production at the desired production level would equal the maximum willingness to pay for oil. By this way, there would not be oil supply in excess of the desired amount, since producing the marginal unit of oil would cost more than anyone would be willing to pay for it. So, the tax would have to satisfy: $Tax = WTP - market\ price$. By this way, the price paid for oil, $tax + market\ price$, would correspond the maximum willingness to pay.

**Question 3**

1. Total emissions are capped by the 50 % reduction target. That is, emissions are allowed to total 100 tons at most, and firms must abate by 100 tons. How do the firms divide the abatement? The division is solved by setting firms' marginal costs of abatement equal subject to the required total abatement. We have thew following system of equations:

$$\begin{cases} MPCA_1 = MPCA_2 \\ A_1 + A_2 = 100 \end{cases} <=> \begin{cases} \frac{2}{3}A_1 = \frac{3}{4}A_2 \\ A_1 + A_2 = 100 \end{cases}$$

We get $A_2 = \frac{8}{9}A_1$ so $A_1 + \frac{8}{9}A_1 = 100 <=> A_1 = \frac{900}{17} \approx 52.9$
and $A_2 = \frac{8}{9}\frac{900}{17} = \frac{800}{17} \approx 47.1$

The tax must equal the marginal cost of abatement:

$$Tax = \frac{2}{3}A_1 = \frac{3}{4}A_2 = \frac{2}{3}\frac{900}{17} = \frac{600}{17} \approx 35.3$$

So, the tax level is $Tax = \frac{600}{17} \approx 35.3$

2. Let's round the abatement values to integers, so firm 1 abates by 53 units and firm 2 by 47 units.
   Abatement costs of firm 1: $\sum_{A=0}^{53} \frac{2}{3}A = \frac{2}{3} * 1431 = 954$
   Abatement costs of firm 2: $\sum_{A=0}^{47} \frac{3}{4}A = \frac{3}{4} * 1128 = 846$
   Total abatement cost: 954+846 = 1800

   Then, suppose the tax is the same for all units. Firm 1 emits $100 - \frac{900}{17} = \frac{800}{17}$ units and pays taxes $\frac{800}{17} * \frac{600}{17} = \frac{480000}{289} \approx 1661$. Firm 2 emits $\frac{900}{17}$ units and pays taxes $\frac{900}{17} * \frac{600}{17} = \frac{540000}{289} \approx 1869$. Then total tax receipts are $\frac{480000+540000}{289} = \frac{1020000}{289} \approx 3529$

   Abatement costs of firm 1 are 954 and it pays taxes 1661. Abatement costs of firm 2 are 846 and it pays taxes 1869. So, abatement costs are 1800 and taxes are 3529.
   Then, the total costs for the firms are 5329. The government receives 3529 as tax revenue.

   Note that firms pay the same total costs (not exactly due to rounding), but the costs are divided to abatement costs and taxes differently. Firm 1 has larger abatement costs but firm 2 pays more taxes.

3. Both firms have initially a right to emit 50 units, so they should both abate 50 units. This cannot be the equilibrium, since the marginal costs of abatement is different for the firms ate this value. Trade occurs up to the point in which the marginal costs of abatement, given the required total abatement, are the same. At this point, the price of the emission right equals the marginal cost of abatement. We already solved those in point 1. So, the price of emissions is $\frac{600}{17} \approx 35.3$. Firm 1 abates $\frac{900}{17}$ units and firm 2 abates $\frac{800}{17}$ units. This means that firm 2 buys $50 - \frac{800}{17} = \frac{50}{17} \approx 2.94$ units from firm 1.

   Abatement costs of firm 1 are $\sum_{A=0}^{53} \frac{2}{3}A = \frac{2}{3} * 1431 = 954$ and it receives revenue from selling the emission rights by amount $\frac{600}{17} * \frac{50}{17} \approx 104$. Total costs of firm 1 are 850
   Abatements costs of firm 2 are $\sum_{A=0}^{47} \frac{3}{4}A = \frac{3}{4} * 1128 = 846$ and it pays $\frac{600}{17} * \frac{50}{17} \approx 104$ for the additional emission rights. Total costs of firm 2 are 950.

   The aggregate emission payments are 0 since firm 2 pays for firm 1. The total abatement costs are 946+846 = 1800.

   We see that cap-and-trade costs the firms much less than the price-based solution since they need not pay taxes on top of the abatement costs.

## Question 4

- The formula for the present value sum is: $PV = \sum_{t=0}^{T} \frac{y_t}{(1+r)^t} = \delta^t y_t$
- Let us assume that the value to which damages occur is 1000 as in the video. The discount rate is r = 0.01, so the discount factor for period t is $\frac{1}{1.01^t}$. The present value of damages is: $PV = \sum_{t=0}^{50} \frac{1000*p_t}{1.01^t}$ in which we denote with $p_t$ the damage percent in period t. Using the data in Excel sheet we get $PV \approx 134.84$
- Consider now the damages for 100 periods. Using the data we get:
$$PV = \sum_{t=0}^{100} \frac{1000 * p_t}{1.01^t} \approx 196.16$$
So, the 50 additional periods bring an additional damage of 196.16-134.84 = 57.32 units. The damages 50 additional periods are considerably "cheaper" in present value terms, but the total damages increase by about 43%. The longer time window is used for evaluating the effects of the climate change, the more severe the effects are considered.
- Using the same formulas and data as above, the 50-period present value of damages would be about 67.60 and the present value of 100-period damages would be about 71.71. So, the present values of 100-period damages are only about 4.11 units, or 6%, larger than the 50-period damages. Using a larger discount rate affect the results drastically; now the damages occurring in the future have considerably smaller present value. The present values for damages occurring after 50 years are almost zero. This shows that if you do not value the future much (high discount rate), you pay only little attention to what happens in the future. We can conclude that if we want people to care about climate change, we get them appreciate the future more (think about the children!)

## Question 5

1. The hold-up problem means that one bargaining party can exert large bargaining power over the other. In this case, the last individual trading with the firm can ask require the firm to pay an amount corresponding the difference between firm's willingness to pay (100,000) and the total price of the previous pollution right (60,000), so the last individual can ask 40,000. Since the last individual has such a large bargaining power, everyone would obviously like to be the last seller. It can happen, that there is no trade: each individual holds up and does not sell her right, since she wishes to be able to get a larger sum by waiting. It would be efficient that the firm bought the pollution rights and the individuals moved away, since firm's abatement costs are larger than individuals' moving costs.
   One solution for the problem could be that there is no distinct bargaining between individuals and the firm, but the individuals pool the pollution rights into one piece that is sold for the firm. This could be done by forming a representative for the individuals and this representative deals with the firm over the transaction.

2. The free-rider problem means that a party can benefit from what others do without contributing herself. Therefore, every party has an incentive not to contribute and an action that would benefit everyone is not taken. In the case of the lecture slides, the polluter would stop if individuals paid at least 20,000. Individuals together are ready to

pay 50,000 at most, 500 per head. If everyone contributes, the individuals will pay 200 each. The free-rider problem arises since if one individual does not pay, the other would be still ready to pay 20,000/99 ≈ 202 each. As long a the per-head cost is at most 500, those paying are ready to pay, so 20,000/500 = 40 contributing individuals is enough. However, since everyone has an incentive not to contribute, no one contributes and the firm continues polluting. This is an inefficient outcome, since firm's abatement costs are smaller than individuals' moving costs.
One solution for the problem could be that individuals have no option not to contribute. The individuals could set up an authority that collect 200 from each village member and the sum is paid for the firm. This is how taxes work.