MAXSURF CONNECT Edition V22
MOSES CONNECT Edition V11
SACS CONNECT Edition V14

User Manual

MAXSURF Motions
MOSES Motions
SACS Motions

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Motions Program & User Manual
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Contents

Differences between versions

There are several different versions of Motions; here the differences between them are explained.

Feature Comparison

The following table lists the differences between the versions of Motions:

Feature	MAXSURF Motions		MOSES Motions	SACS Motions
	Basic	**Advanced**		
Solver				
Strip Theory	yes	yes	no	yes
Panel method	no	yes	no	yes
MOSES strip	no	no	yes	no
MOSES panel	no	no	yes	no
Martec panel	no	no	no	yes
User defined RAOs	yes	yes	no	yes
Other Features				
Reporting	yes	yes	yes	yes
SACS Cargo	no	no	yes	yes
COM Automation	no	yes	yes	yes

[**] second-order low-frequency Newman approximation drift forces

The results produced by the different solvers are listed below:

	Strip theory	**Panel method**	**MOSES strip**	**MOSES radiation-diffraction**	**Martec**	**User defined RAOs**
Available in	MAXSURF, SACS	MAXSURF, SACS	MOSES only	MOSES only	SACS only	MAXSURF, SACS
Result						
RAOs	3 dof	yes	yes	yes	yes	n/a
Velocities	3 dof	yes	yes	yes	yes	yes
Accelerations	3 dof	yes	yes	yes	yes	yes
MSI	yes	yes	yes	yes	yes	yes
Panel pressures	no	yes	yes	yes	yes	no
Remote location	yes	yes	yes	yes	yes	yes
Drift forces[**]	no	yes	yes	yes	yes	no
Wave forces	yes	yes	yes	yes	yes	no
Added mass, damping	yes	yes	yes	yes	yes	no

Collectively the "Panel Method", "MOSES radiation diffraction" and "Martec" solvers are zero-speed 3D radiation diffraction methods and are referred to as "Panel Methods" in the rest of this manual.

Coordinate System

All Modules: Body plan, view from behind (stbd side of vessel on right-hand side of screen)
All Modules with the exception of Stability:Plan view from bottom (stbd side of vessel on upper half of screen); Stability has the Plan view from top (stbd side of vessel on lower half of screen)
All Modules:Profile view from stbd (bow on right-hand side of screen)

Motions
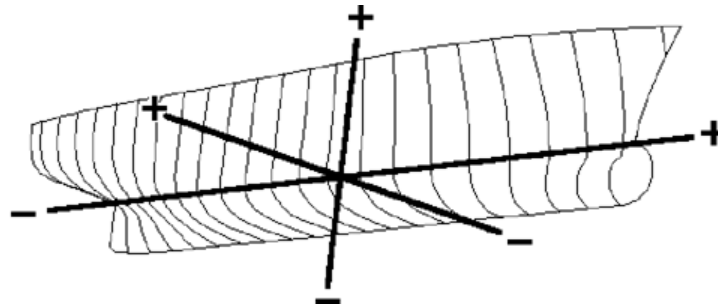
Longitudinal:	+ve forward	-ve aft
Transverse:	+ve stbd	-ve port
Vertical:	+ve up	-ve down

The zero-point can be defined at any location in the model

MOSES Motions
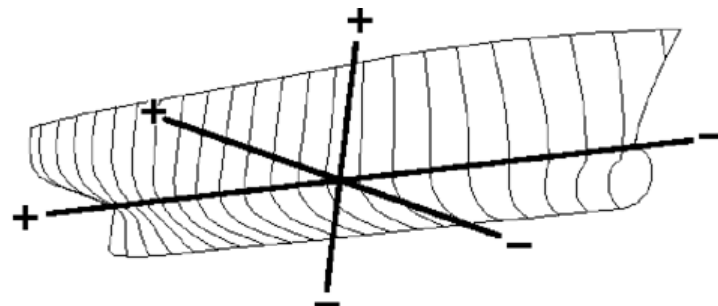
x-Axis (longitudinal):	+ve aft	-ve forward
y-Axis (horizontal):	+ve stbd	-ve port
z-Axis (vertical):	+ve up	-ve down

The zero-point should be defined to coincide with the furthest forward and the lowest part of the model.

SACS Motions

The coordinate system used by SACS Motions is the same as for Hull Modeler. The coordinate system has the origin at the user defined zero point.
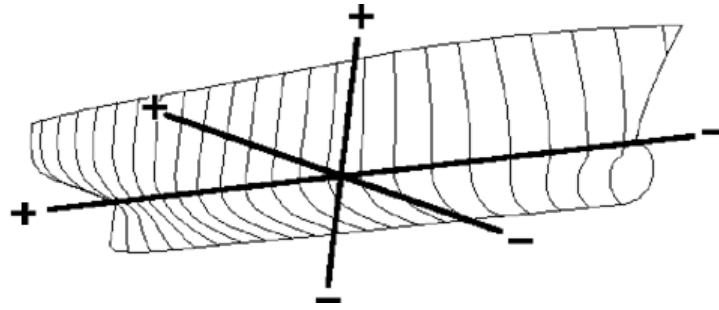
SACS Motions User Coordinate System and view Windows are the same as for Hull Modeler

X-axis	+ve aft	-ve forward
Y-axis	+ve starboard	-ve port
Z-axis	+ve up	-ve down

About this Manual

This manual describes Motions, an application which may be used to predict the motion and seakeeping performance of vessels designed using Modeler (or Hull Modeler). This manual is organised into the following chapters:

Chapter 1 Introduction
Contains a description of Motions, its principal features and mode of operation.

Chapter 2 Using Motions
Explains how to use Motions and provides step-by-step examples to guide the user through the operation of the program.

Chapter 3 Motions Reference
Gives details of each of Motions's menu commands and provides a more detailed guide to the calculation methods and strip theory algorithm used in Motions.

Chapter 4 Theoretical Reference
Gives a summary of the theoretical background to Motions.

Appendix A – Strip Theory Formulation
Presents more in depth explanation of the theoretical model used in Motions.

Appendix B – Strip Theory Validation
Gives details of some of the validation work carried out with Motions.

Appendix C Panel Method Theory
Presents more in depth explanation of the theoretical model used in SACS Motions.

Appendix D Panel Method Validation
Gives details of some of the validation work carried out with SACS Motions.

Appendix E RAO text file format
Gives details of format required for user-defined RAO data.

Chapter 1 Introduction

Motions uses the .msd geometry file to calculate the response of the vessel to user-defined sea conditions. Multiple methods are available to calculate the vessels response: a linear strip theory method and a series of radiation diffraction solvers collectively termed panel methods. The panel methods include a 6 DOF Panel Method, a Martec Solver and a MOSES solver. Which solvers are available depend on the version of Motions being used.

The linear strip theory method is based on the work of Salvesen et al, it is used to calculate the coupled heave and pitch response of the vessel. The roll response is calculated using linear roll damping theory. In addition to graphical and tabular output of numerical results data, Motions is also able to provide an animation of the vessel's response to specified sea conditions.

The panel methods are first-order diffraction/radiation hydrodynamic analysis in which a constant panel based boundary element method (BEM) is used. The panel methods generate their analysis elements based on the geometry from the NURBS surfaces in the .msd design file. The panel methods generates Response Amplitude Operators (RAOs) for all six degrees of freedom: surge, sway, heave, roll, pitch and yaw. The panel methods are valid for a very large range of geometries. The Panel Method and Martec solvers are restricted to zero fwd speed. In addition to outputting RAOs, the panel method outputs also includes hydrodynamic added mass and damping, first order wave exciting forces and moments, mean drift forces and moments and pressure on the vessel wetted surface.

Strip theory along with the Panel methods, and hence Motions, is able to provide reasonably accurate seakeeping predictions for a wide range of vessel types. The speed of the analysis and its integration into the rest of the suite, make Motions particularly useful in initial design stage. For a greater insight into the accuracy that can be achieved by Motions, please refer to the validation appendices at the end of this manual.
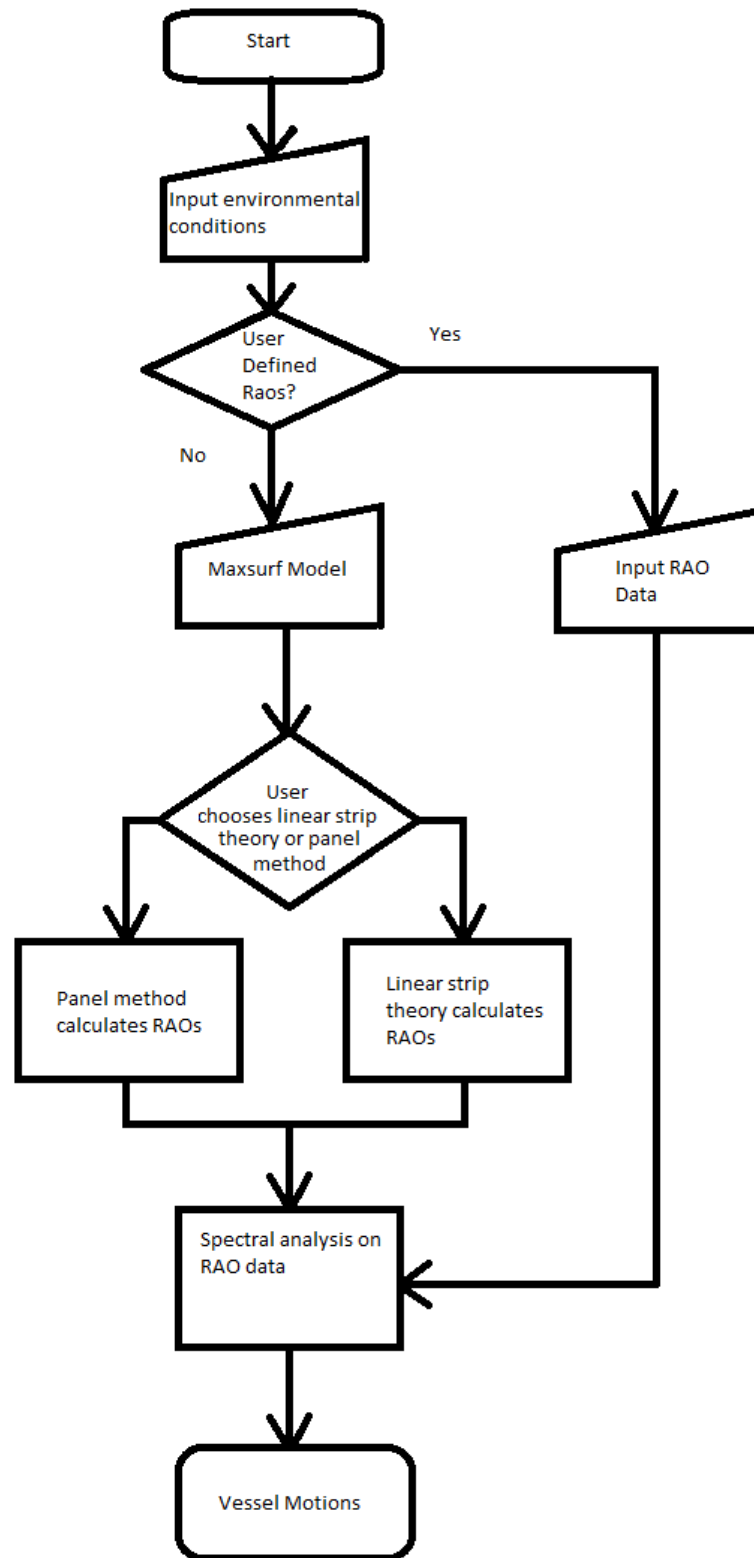
Motions workflow

Seakeeping Fundamentals

In this section we will outline some of the fundamental principles used in seakeeping analysis.

- **Assumptions**
- **Coordinate System**
- **Wave Spectra**
- **Characterising Vessel Response**
- **Calculating Vessel Motions**
- **Statistical Measures**
- **Computational Methods**
- **Visualisation**

Users familiar with these concepts may wish to go straight to Chapter 2 Using Motions.

Assumptions (Linear Strip Theory)

When linear strip theory is used to compute the coupled heave and pitch motions of the vessel, the following underlying assumptions are implied:

- **Slender ship: Length is much greater than beam or draft and beam is much less than the wavelength).**
- **Hull is rigid.**
- **Speed is moderate with no lift from forward speed.**
- **Motions are small and linear with respect to wave amplitude.**
- **Hull sections are wall-sided.**
- **Water depth is much greater than wavelength so that deep-water wave approximations may be applied.**
- **The hull has no effect on the incident waves (so called Froude-Kriloff hypothesis).**

A simplified forced, damped mass-spring system is assumed for the uncoupled roll motions. This assumes the following

- **An added inertia in roll is used which is assumed to be a constant proportion of the roll inertia.**
- **A constant user-specified linear damping is used.**

Please see Appendix A for further details on the strip theory formulation for coupled heave and pitch and the simplified method used for roll.

Assumptions (Panel methods)

The usual linear potential theory assumptions apply:

- **Wave height and steepness are assumed to be small so that linear wave theory may be used.**
- **Fluid is assumed to be inviscid and incompressible.**
- **Fluid flow is assumed to be irrotational.**
- **Water depth is much greater than wavelength so that deep-water wave approximations may be applied**

Coordinate System

The coordinate system has the origin at the user defined zero point. Please refer to the section at the start of this manual explaining the differences between the different versions of Motions for information relating to the coordinate system used in Motions.

When calculating motions at remote locations, the vessel is assumed to rotate about the centre of gravity. Hence the distance of the remote location from the centre of gravity is of interest. Motions calculates this distance internally and all positions are measured in the coordinate system described above.

A vessel has six degrees of freedom, three linear and three angular. These are: surge, sway, heave (linear motions in x, y, z axes respectively) and roll, pitch, yaw (angular motions about the x, y, z axes respectively). For convenience, the degrees of freedom are often given the subscripts 1 to 6; thus heave motion would have a subscript 3 and pitch 5.
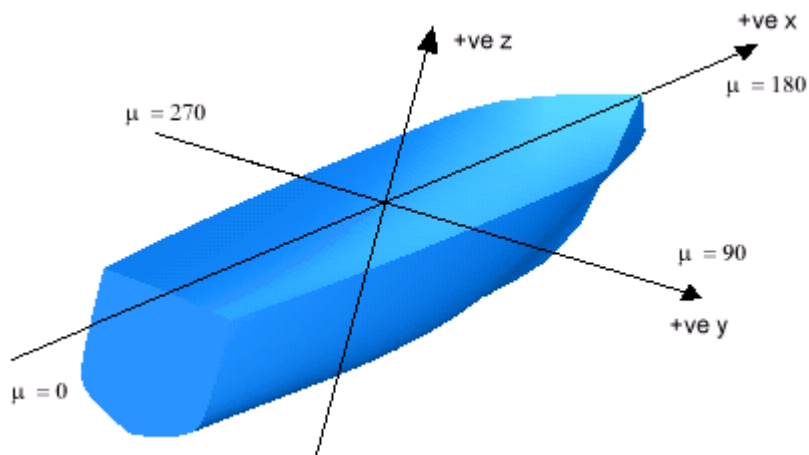
Wave direction is measured relative to the vessel track and is given the symbol μ. Thus following waves are at μ = 0°; starboard beam seas are 90°; head seas 180° and port beam seas 270°.

Wave Spectra

Irregular ocean waves are often characterised by a "wave spectrum", this describes the distribution of wave energy (height) with frequency.

Characterisation

Ocean waves are often characterised by statistical analysis of the time history of the irregular waves. Typical parameters used to classify irregular wave spectra are listed below:

Characterisation of irregular wave time history

$\bar{\zeta}$ mean of many wave amplitude measurements

$\bar{H}$ mean of many wave height measurements

$\overline{T}_P$	mean of many wave period measurements between successive peaks
$\overline{T}_T$	mean of many wave period measurements between successive troughs
$\overline{T}_{Z+}$	mean of many wave period measurements between successive zero up-crossings
$\overline{T}_{Z-}$	mean of many wave period measurements between successive zero down-crossings
$\overline{T}$	mean of many wave period
$\overline{T}_0$	modal wave period
$\overline{\zeta}_{1/3}$	mean of highest third amplitudes or significant amplitude
$\overline{H}_{1/3}$	mean of highest third wave heights or significant wave height
m_0	variance of the surface elevation relative to the mean (mean square)
σ_0	standard deviation of surface elevation to the mean (root mean square)

Sea State Codes

In 1970 the World Meteorological Organisation agreed the standard sea state code – see below. Each code represents a range of wave heights but there is no indication of the corresponding wave periods.

World Meteorological Organisation sea state code

Sea State Code	Significant wave height [m]		Description
	Range	**Mean**	
0	0	0	Calm (glassy)
1	0.0 - 0.1	0.05	Calm (rippled)
2	0.1 - 0.5	0.3	Smooth (wavelets)
3	0.5 - 1.25	0.875	Slight
4	1.25 - 2.5	1.875	Moderate
5	2.5 - 4.0	3.25	Rough
6	4.0 - 6.0	5.0	Very rough
7	6.0 - 9.0	7.5	High
8	9.0 - 14.0	11.5	Very high
9	over 14.0	over 14.0	Phenomenal

Sea state data may also be obtained for specific sea areas and times of year. This can be useful routing information. One of the best sources of this information is Hogben and Lumb (1967).

Wave Spectra Representation

Irregular ocean waves are typically described in terms of a wave spectrum. This describes a wave energy distribution as a function of wave frequency. The continuous frequency domain representation shows the power density variation of the waves with frequency and is known as the wave amplitude energy density spectrum, or more commonly referred to as the wave energy spectrum. The spectral ordinates (or wave spectral density) are given the symbol: $S_\zeta(\omega)$. (This is similar to the power spectral density, PSD, used in electronics and communications analysis.) A typical wave spectrum is shown below:
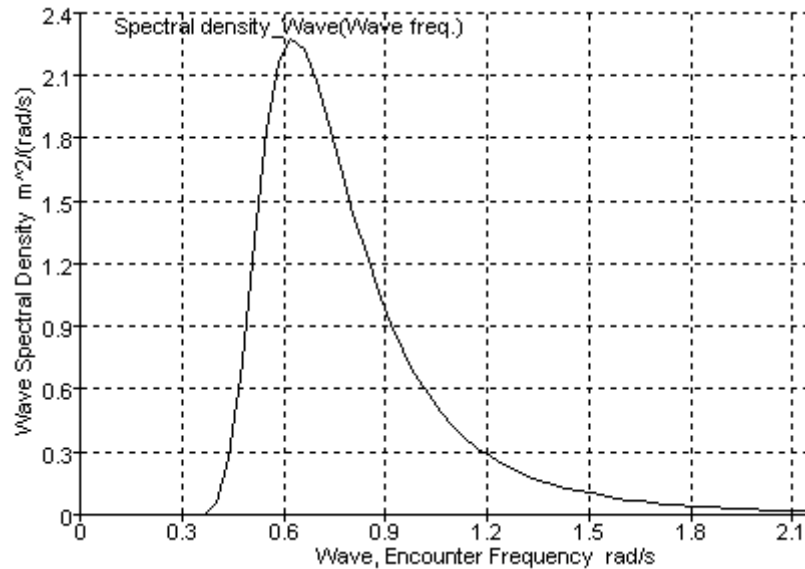
These spectral representations of sea conditions are central to determining the response of a vessel in the seaway. This will be discussed more fully in the following sections and also see: Wave Spectra on page 81 for more in-depth information.

Idealised Spectra

It is often useful to define idealised wave spectra which broadly represent the characteristics of real wave energy spectra. Several such idealised spectra are available in Motions and are described below:

- **Bretschneider or ITTC two parameter spectrum**
- **One parameter Bretschneider**
- **JONSWAP**
- **DNV Spectrum**
- **Pierson Moskowitz**
- **Ochi Hubble**
- **ISSC (SACS only)**

The spectra and their formulations are dealt with in more detail in the Chapter 4 Theoretical Reference on page 79.

Encounter Spectrum

An important concept when calculating vessel motions is that of the encountered wave spectrum. This is a transformation of the wave spectrum which describes the waves encountered by a vessel travelling through the ocean at a certain speed. This is effectively a Doppler shift of the spectrum which smears the spectrum towards the higher frequencies in head seas and towards the lower frequency in following seas.

A full explanation of this effect is given in Chapter 4 Theoretical Reference, section ISSC
See Bretschneider (ITTC 2 parameter) spectrum formulation.
What the Vessel Sees on page 85.

Characterising Vessel Response

This section outlines the method used to describe a vessel's response in a seaway.

Harmonic Response of Damped, Spring, Mass system

For most purposes, it is sufficient to model the vessel as a set of coupled spring, mass, damper systems undergoing simple harmonic motion. This is assumed by Motions and most other seakeeping prediction methods. This method may be successfully applied to the analysis of the vessel's motions provided that these motions are linear and that the principle of superposition holds. These assumptions are valid provided that the vessel is not experiencing extremely severe conditions.

Response Amplitude Operator

The Response Amplitude Operator (RAO), also referred to as a transfer function (this is similar to the response curve of an electronic filter), describes how the response of the vessel varies with frequency. These are normally non-dimensionalised with wave height or wave slope. Typical heave and pitch RAOs are shown below:
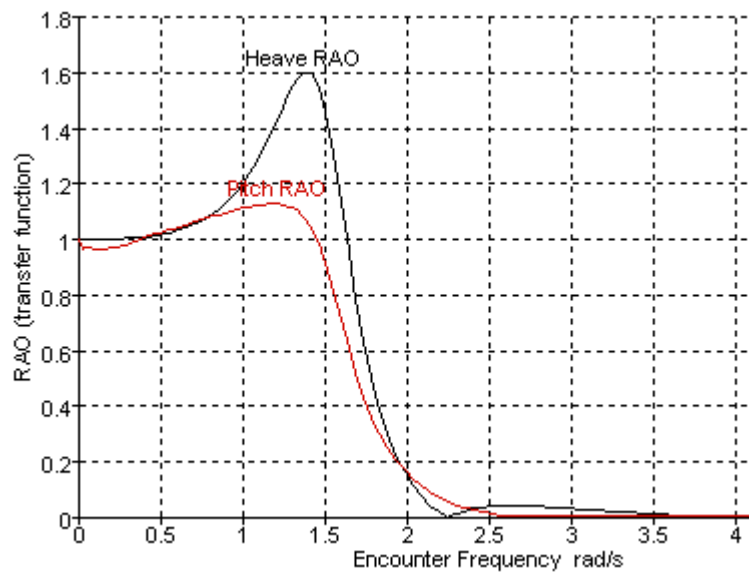
It may be seen that the RAOs tend to unity at low frequency, this is where the vessel simply moves up and down with the wave and acts like a cork. At high frequency, the response tends to zero since the effect of many very short waves cancel out over the length of the vessel. Typically the vessel will also have a peak of greater than unity; this occurs close to the vessels natural period. The peak is due to resonance. An RAO value of greater than unity indicates that the vessel's response is greater than the wave amplitude (or slope).

Calculating Vessel Motions

Assuming linearity, the vessel's RAOs depend only on the vessel's geometry, speed and heading. Thus once the RAOs have been calculated the motion of a vessel in a particular sea state of interest may be calculated.

It is hence possible to obtain a spectrum for a particular vessel motion in a particular sea spectrum:
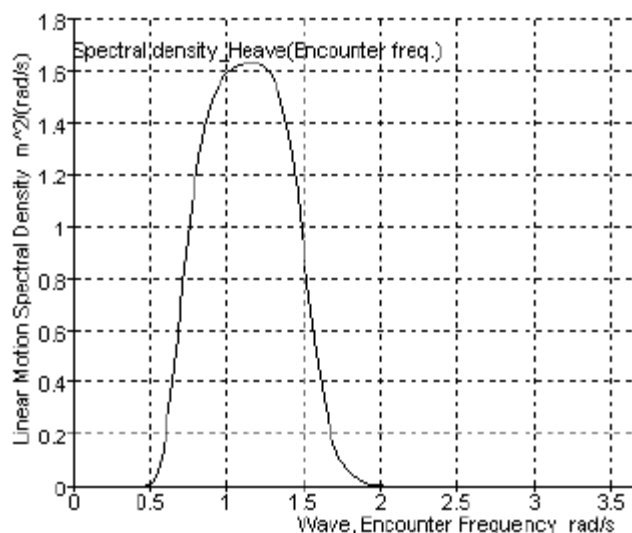
See Chapter 4 Theoretical Reference on page 79 for further details on how these calculations are performed.

Statistical Measures

Often it is important to know certain statistical measures of the motion spectrum. These can be used to determine significant motions, Root Mean Square (RMS) motions as well as other measures such as Motion Sickness Incidence (MSI).

It is also possible to calculate the probability of exceeding certain limiting criteria, such as limiting maximum vertical accelerations, propeller emergence, slamming and so forth.

Motions is able to calculate the significant and RMS motions, velocities and accelerations of the heave, pitch and roll motions at the centre of gravity of the vessel. In addition, the user is able to specify remote locations away from the centre of gravity. Motions calculates the vertical absolute and relative motions as well as the MSI at these locations.

For further information please refer to Statistical Measures and Probability Measures on pages 92 and 94.

Computational Methods

Several numerical methods are available for estimating a vessel's response. Two of the most widely used are Strip Theory and Panel Methods. Motions uses linear strip theory method to predict the vessels heave and pitch response. Roll response is estimated assuming that the vessel behaves as a simple, damped, spring/mass system, and that the added inertia and damping are constant with frequency. The linear strip theory method, originally developed in the 1970s (Salvesen et al. 1970) is useful for many applications; unlike more advanced methods, it is relatively simple to use and not too computationally intense.

The panel method is used when looking at vessel motions in all six degrees of freedom at zero forward speed and is applicable to a wider range of vessel geometries than linear strip theory. Motions uses a 3d first order diffraction/radiation hydrodynamics solver.

The Martec solver (SACS only) is another panel method with the added advantage of a forward speed correction to the greens function so that it is applicable to a large range of speeds than the Panel Method.

Strip Theory

Strip theory is a frequency-domain method. This means that the problem is formulated as a function of frequency. This has many advantages, the main one being that computations are sped up considerably. However, the method, generally becomes limited to computing the linear vessel response.

The vessel is split into a number of transverse sections. Each of these sections is then treated as a two-dimensional section in order to compute its hydrodynamic characteristics. The coefficients for the sections are then integrated along the length of the hull to obtain the global coefficients of the equations of motion of the whole vessel.

Finally the coupled equations of motion are solved.

Further details of Motions's implementation of the Strip Theory method is given in the Appendix A – Strip Theory Formulation on page 105.

Conformal Mapping

Fundamental to strip theory is the calculation of the sections' hydrodynamic properties. There are two commonly used methods: conformal mapping and the Frank Close Fit method. The former is used by Motions and will be discussed at greater length.

Conformal mappings are transformations which map arbitrary shapes in one plane to circles in another plane. One of the most useful is the Lewis mapping. This maps a surprising wide range of ship-like sections to the unit circle, and it is this method that is used by Motions. Lewis mappings use three parameters in the conformal mapping equation, but by adding more parameters, it is possible to map an even wider range of sections. See: Calculation of Mapped Sections on page 37 for more information.

Panel Method

The Motions Panel Method is a zero speed radiation/diffraction solver in the frequency domain. It performs linear analysis of the interaction of surface waves with floating structures. The body is defined by a mesh auto generated on the NURBS surface in the design file (.msd). The mesh size and shape can be controlled by user specified input parameters. The solutions of the velocity potential and source strength are assumed constant over each panel.

The panel method is also is a frequency-domain method. This means that the problem is formulated as a function of frequency. This has many advantages, the main one being that computations are sped up considerably.

Further details of Motions's implementation of the Panel method are given in Appendix C – Panel Method Formulation on page 175.

Martec Solver (SACS only)

The SACS Motions Martec Solver is a radiation/ diffraction solver in the frequency domain. It is based on the a zero speed greens function with forward speed correction. It performs linear analysis of the interaction of surface waves with floating structures. The body is defined by a mesh auto generated on the NURBS surface in the Hull Modeler design file (.msd). The mesh size and shape can be controlled by user specified input parameters. The solutions of the velocity potential and source strength are assumed constant over each panel.

The panel method is also is a frequency-domain method. This means that the problem is formulated as a function of frequency. This has many advantages, the main one being that computations are sped up considerably.

Visualisation

Motions is able to provide a three dimensional view of the vessel in the waves. This can be either a static image or an animation of the vessel travelling through the waves.

Further details are given in:

Results and Reporting to MS-Word

Once the results have been computed, they can be not only viewed in the various tables and graphs within Motions, but also spooled to Microsoft Word (if installed). These reporting options are specified in the File menu.

Chapter 2 Using Motions

You have been introduced to the way in which Motions works and can now go on to learn in detail how to use Motions by following the examples outlined in this chapter.

- **Getting Started**
- **Opening a Model in Motions**
- **Additional Hull Parameters**
- **Environmental Parameters**
- **Completing the Analysis**
- **Viewing the Results**
- **Graphing the Results**
- **Rendered view**
- **Time Simulation and Animation**
- **Saving the Motions Model**
- **Saving the Seakeeping Analysis Results**
- **Limitations and Guidelines**

Getting Started

Once installed with the rest of the PackageMotions may then be started, and will display three windows containing a view of the model, a tabulated results window and a graph of the results.

Windows Registry

Certain preferences used by the Motions program are stored in the Windows registry. It is possible for this data to become corrupted, or you may simply want to revert back to the default configuration. To clear Motions's preferences, start the program with the Shift key depressed. You will be asked if you wish to clear the preferences, click OK.
These preferences are:

- **Colour settings of contours and background**
- **Fonts**
- **Window size and location**
- **Recent files**
- **Units**

Opening a Model in Motions

A Motions model consists of surface information and Motions input data, such as sea environment, speeds etc., and also information on the hull characteristics such as damping factors and moments of inertia.

Opening the Design File in Motions

Choose Open from the File menu. Select the file titled 'Motions Sample.msd' in the 'MAXSURF\Sample Designs' folder and open it. This file contains the surface information of a simple design.

> **Note:**
> The .msd file contains the surface information.

Opening Motions Data

Existing Motions designs have Motions input data already defined and saved in a .skd file. To open existing Motions data, choose "Open Motions Data" from the File menu. Sample Seakeeping data corresponding to the Motions Sample.msd file can be found in the Sample Designs folder: Motions Sample.skd.

The Motions data consists of:

1) Input data window data

　　Locations, Speeds, Headings and Spectra

2) Analysis menu data

　　Measured Hull sections, Vessel type, Mass distribution, Damping factors, Frequency range, Environment, Analysis methods.

> **Note:**
> The .skd file contains all Motions analysis input data.

Choosing Analysis Method

Depending on the required output the user needs to specify the type of analysis method to be used. An overview of the applicability and limitations of each method are given below:

Method	Speed range (Fn)	Motions analysed	Applicable vessels
Strip theory	0.0 ~ 0.7	Heave, roll, pitch	"slender"
Panel method	0.0 ~ 0.1	Surge, sway, heave, Roll, pitch, yaw	all
MOSES strip	0.0 ~ 0.7	Surge, sway, heave, Roll, pitch, yaw	"slender"
MOSES radiation difraction	0.0 ~ 0.1	Surge, sway, heave, Roll, pitch, yaw	all
Martec Solver	0.0 ~ 0.7	Surge, sway, heave, Roll, pitch, yaw	all

The analysis method is chosen by selecting from the Analysis | Analysis Type menu. Depending on which analysis method is chosen a different analysis workflow will be required. As a general rule, once the analysis type has been selected the user should sequentially work through the other input dialogs as listed in the Analysis menu (enter data only in the commands that are enabled).

Strip methods workflow

The following steps must be taken to perform a strip theory analysis (more detailed information on each of the steps is provided in subsequent sections). Each command is accessed from the Analysis menu:

- **Select "Strip Theory" from the "Analysis Type" list.**
- **Select "Measure Hull…" to define the parameters for the conformal mapping.**
- **Select "Specify Vessel Type…" to define the number of hulls of the vessel**
- **Select "Mass distribution…" to define the gyradii and VCG of the vessel.**
- **Select "Damping Factors…" to define the non dimensional heave, pitch and roll damping factors.**
- **Select "Environment…" to specify the water density.**
- **Select "Frequency Range…" to define the number of frequencies at which to calculate the RAOs.**
- **Select "Strip theory method…" to specify the Transom Terms, Added Resistance and Wave Force methods to be used during the strip theory calculations.**

Once the analysis specific settings have been entered the user needs to set the spectral analysis settings. These include the vessel speeds, vessel headings, wave spectrums and locations at which motions are to be evaluated (remote locations). These values are to be entered in the Windows | Inputs windows. As a minimum 1 speed, 1 heading and 1 wave spectrum must be entered. Once the minimum required inputs have been specified the Analysis | Solve Seakeeping Analysis function will become enabled.

Panel methods workflow

The following steps must be taken to perform a Panel Method analysis (more detailed information on each of the steps is provided in subsequent sections). Each command is accessed from the Analysis menu:

- **Select "Panel Method" from the "Analysis Type" list.**
- **Select "Mesh Hull" to define the parameters for meshing the surfaces and to execute the automated meshing process.**
- **Select "Mass distribution" to define the gyradii and VCG of the vessel.**
- **Select "Environment…" to specify the water density.**
- **Select "Frequency Range" to define the number of frequencies at which to calculate the RAOs.**
- **Set the Mooring Stiffness and Damping Matricies as appropriate to define any additional stiffness or damping to be used in the equations of motion.**

Once the analysis specific settings have been entered the user needs to set the spectral analysis settings. These include the vessel headings, wave spectrums and locations at which motions are to be evaluated (remote locations). These values are to be entered in the Windows | Inputs windows. As a minimum 1 heading and 1 wave spectrum must be entered. Once the minimum required inputs have been specified the Analysis | Solve Seakeeping Analysis function will become enabled.

MOSES solver workflow

There are some specific steps which must be followed for analysis with the MOSES solvers

The following steps must be taken to perform a MOSES Solver analysis (more detailed information on each of the steps is provided in subsequent sections). The MOSES solvers are only available from MOSES versions of Motions. Each command is accessed from the Analysis ribbon:

- **Select "MOSES Solver" from the drop down combo in the Options list.**
- **Select "Mesh Model" to define the parameters for meshing the surfaces and to execute the automated meshing process.**
- **Select "Mass distribution" to define the gyradii and VCG of the vessel.**
- **Select "Environment…" to specify the water density.**
- **Select "Frequency Range" to define the number of frequencies at which to calculate the RAOs.**

Once the analysis specific settings have been entered the user needs to set the spectral analysis settings. These include the vessel speeds, headings, wave spectrums and locations at which motions are to be evaluated (remote locations). These values are to be entered in the Windows | Inputs windows. As a minimum 1 speed, 1 heading and 1 wave spectrum must be entered. Once the minimum required inputs have been specified the Analysis | Solve Seakeeping Analysis function will become enabled. It is important that the model be set up correctly for a MOSES analysis. This means the mesh must be complete and closed. The outside arrows for the panels must be in the correct direction. To use the installation automated tools the frame of reference must be set up so that the zero point is on the centreline and the fwd most and bottom most extremity of the vessel. The outside direction of the surface and the frame of reference for the design are set up in Modeler.

Martec Solver workflow

There are some specific steps which must be followed for analysis with the MOSES solvers

The following steps must be taken to perform a Martec Solver analysis (more detailed information on each of the steps is provided in subsequent sections). The Martec Solver is only available from SACS versions of Motions. Each command is accessed from the Analysis ribbon:

- **Select "Martec Solver" from the drop down combo in the Options list.**
- **Select "Mesh Model" to define the parameters for meshing the surfaces and to execute the automated meshing process.**
- **Select "Mass distribution" to define the gyradii and VCG of the vessel.**
- **Select "Environment…" to specify the water density.**
- **Select "Frequency Range" to define the number of frequencies at which to calculate the RAOs.**

Once the analysis specific settings have been entered the user needs to set the spectral analysis settings. These include the vessel speeds, headings, wave spectrums and locations at which motions are to be evaluated (remote locations). These values are to be entered in the Windows | Inputs windows. As a minimum 1 speed, 1 heading and 1 wave spectrum must be entered. Once the minimum required inputs have been specified the Analysis | Solve Seakeeping Analysis function will become enabled.

User input RAO workflow

Alternatively the user may wish to perform a spectral analysis on a set of RAO data that has been calculated/supplied from a 3rd party. In this case the user just needs to open the RAO text file using the File | Import | RAO Text… command. Set the analysis type to "User Input RAOS" and specify the spectral analysis parameters (speeds, headings, wave spectrums and locations) to perform the seakeeping analysis. The text file formats are described in Appendix E.

Vessel Condition

The vessel draft and trim (and for the panel method, heel) may be specified in the
Draft/Trim and Heel dialogs.
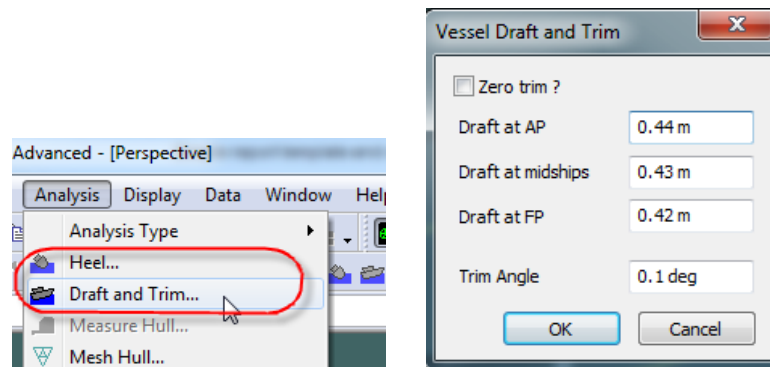
The draughts are specified at the aft and forward perpendiculars as defined by the vessel
Frame of Reference. If the vessel is at zero trim the two draughts may be synchronised
by ticking the Zero trim checkbox. The trim angle may also be specified and this is
shown in the current angle units. In the heel dialog, the heel angle is also specified in the
current angle units.

Vessel Hydrostatics

The vessel hydrostatics at the specified condition may be shown in the Data |
Hydrostatics dialog; the seawater density and VCG are taken from the other Analysis and
Hull settings.

Hydrostatics dialog

Trimmed Display

An option has been added to the Display menu which allows the screen to be in the Ship axis-system (vessel baseline is horizontal) or in the Earth axis-system (waterline is horizontal):
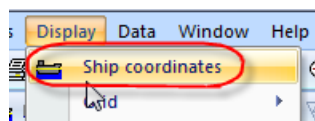
Display in Ship or Earth axis-systems
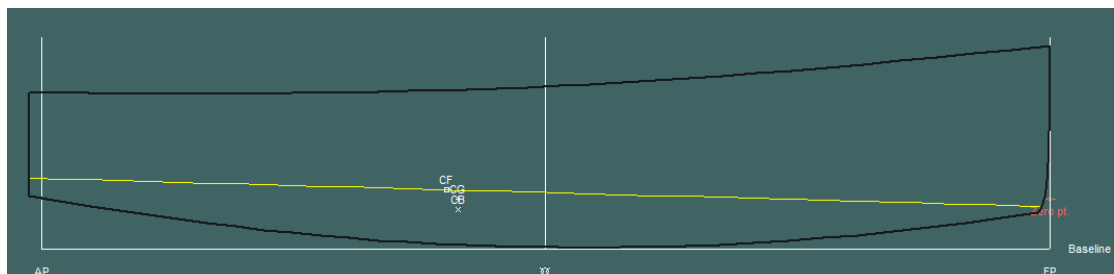
Ship axis-system
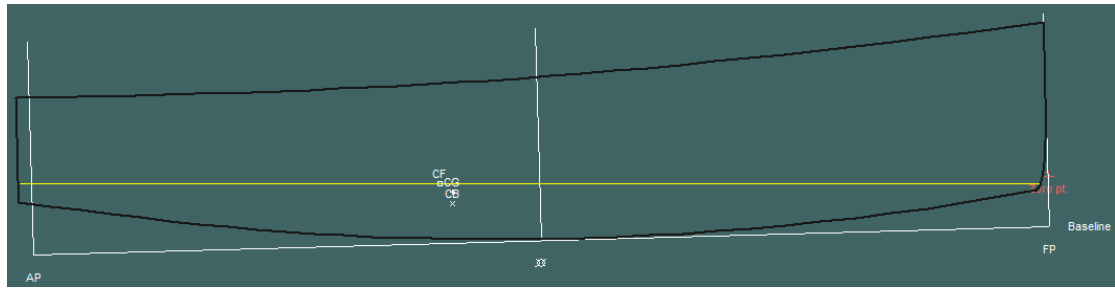
Earth axis-system

Trimmed Analysis

As the vessel is trimmed, the longitudinal centre of gravity is automatically adjusted so that the centre of gravity is vertically aligned with the centre of buoyancy (in the Earth-fixed axis system, i.e. normal to the waterplane).

For the strip theory analysis, the sections are still measured in planes normal to the baseline and these sections are moved vertically to make the waterline horizontal (rather than rotated) this means that there is a slight discrepancy between the actual hull section and the section used to determine the conformal mappings and section hydrodynamics, but since the conformal mapping is only any approximation in any case, this should be sufficiently accurate up to moderate trim.
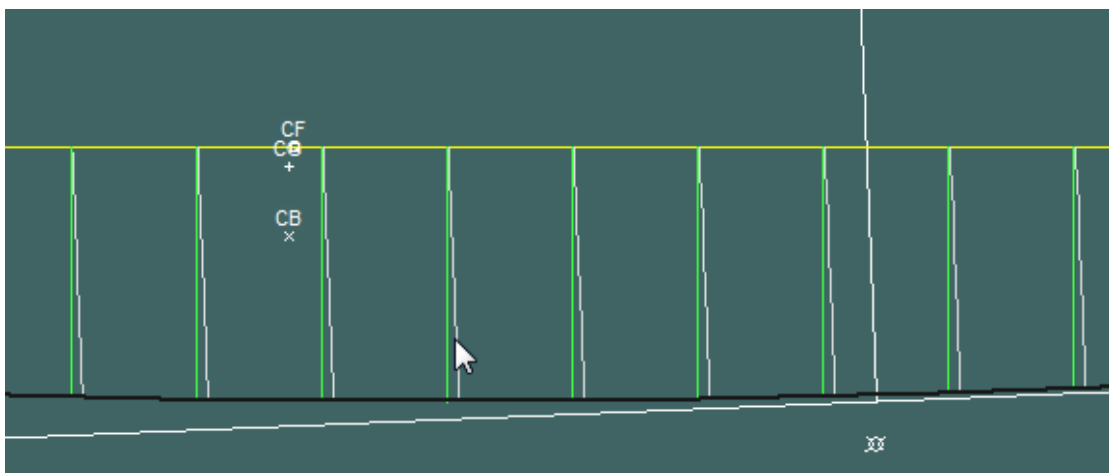
Mapped sections are vertical in the Earth axis-system but generated from vertical Ship axis-system sections

For the panel methods, the mesh is properly rotated to the make the waterline horizontal:
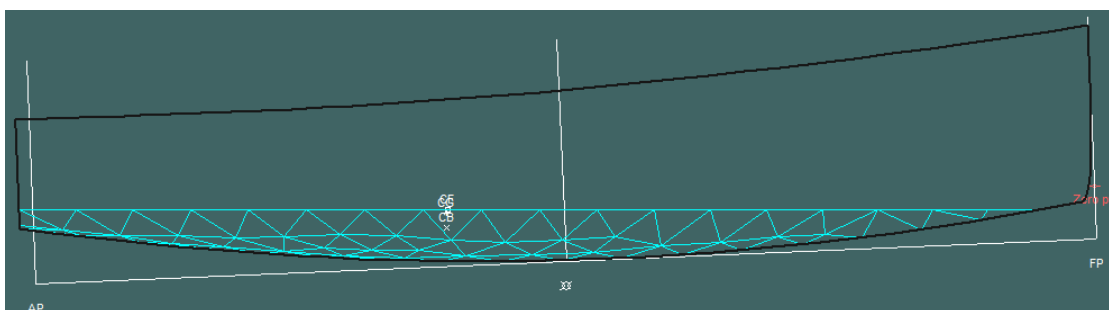
Hull panels are calculated in the Ship axis-system and rotated so that the waterline is horizontal in the Earth axis-system

Trimmed Results

The axes of the linear and angular motions are in the Earth-fixed coordinate system so surge and sway remain in the waterplane and heave is perpendicular to the waterplane. However the vessel gyradii for roll, pitch, and yaw are still specified in the ship axis system and are not corrected for vessel trim.

In calculating the transfer functions an hence motions and accelerations of remote points, the distances of the remote points from the centre of gravity are measured in the Earth-fixed axis system and the directions of these motions are also in the Earth-fixed axis system.

Measure Hull (Strip theory)

Choose Analysis | Measure Hull from the menu. By default all the surfaces will be measured and 11 equally spaced sections will be calculated. This is sufficient for initial calculations but for more accurate predictions, the number of sections should be increased to between 15 and 30.

If the design includes surfaces which are trimmed, then the Trim Surfaces checkbox should be ticked.
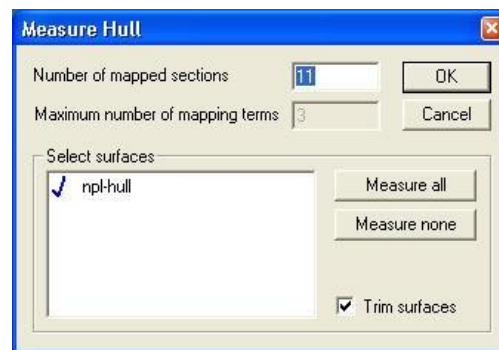
By default sections are mapped with Lewis sections (see Calculation of Mapped Sections on page 37 for further details).. This has the limitation that, for sections which are very wide and/or deep and have a low section area, the mapping may be inaccurate. This may occur for sections which have a skeg, rudder or keel. For sections such as these, it may be necessary to remove the skeg, rudder or keel surfaces from those that are measured. For heave and pitch motions, removing these surfaces is unlikely to have a significant influence on the results. Roll motions use the damping factor specified by the user, which should include the effect of such appendages (see Setting Damping Factors section on page 27).

Motions will automatically choose the best Lewis section that can be fitted to the section. If the sectional area coefficient is too low for the section's depth, Motions will limit the draft of the mapping. Conversely, if the sectional area coefficient is too large, Motions will select the Lewis section with the largest possible sectional area coefficient for the given section breadth to draft ratio.

However, in most circumstances, it is normally advisable to remove appendages from the design, this will result in better modelling of the main hull. It should also be noted that a feature of the conformal mapping is that the mapped section is always vertical where it crosses the horizontal axis and horizontal where it crosses the vertical axis. However, this is not generally a major limitation.

Section through hull

Conformal mapping

cf

qb

zero pt.
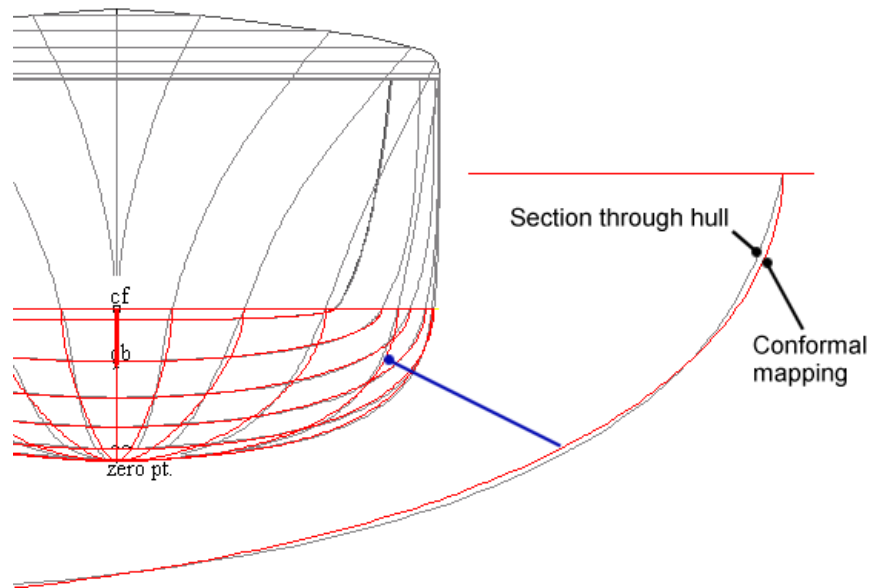
Higher order conformal mappings (Strip theory)

Conformal mappings of up to 15 terms may be used in Motions. The increase in mapping terms can lead to conformal mapping sections to better match the actual hull sections – see below: The picture on the left shows three different sections mapped with three-parameter Lewis mappings, whilst the picture on the right shows conformal mapping using 12 terms to map the same sections. The number of terms is set in the Analysis | Measure Hull dialog. (If Motions is unable to compute a higher order mapping for a section, it will revert to the Lewis Mapping.)
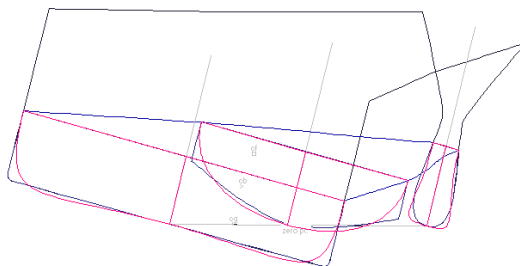
three-parameter Lewis mappings   12 parameter conformal mappings

Measure Hull

Number of mapped sections   11   OK

Maximum number of mapping terms   12   Cancel

Select surfaces
√ Default   Measure all
                         Measure none
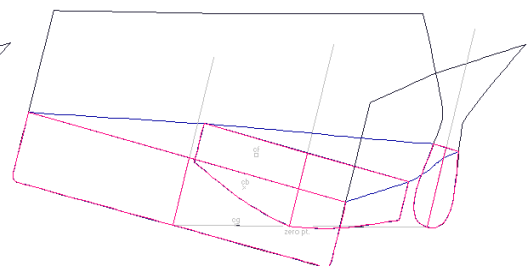
☐ Trim surfaces
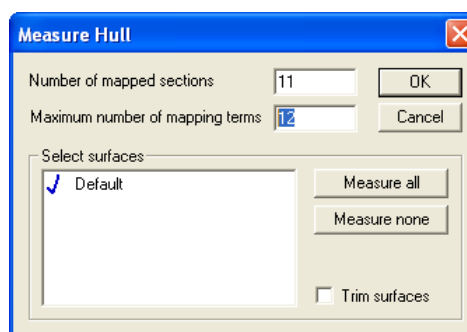
The constraints for the hull model are the same as those for Stability; essentially, any transverse section through the hull must form one or more continuous contours with, at most, one opening in the contour (this is then closed with a straight line to define the water-tight portion of the hull). Motions calculates regularly spaced sections between the fore and aft extents of the waterline. Thus submerged parts of the hull which extend forward or aft of the waterline (e.g. bulbous bows) will not be included.

The Lewis mappings are calculated from the section's properties: draft, waterline beam and cross-sectional area. In some cases the Lewis mapping will be a poor fit to the section, this will occur if: the waterline beam is very narrow for the depth and area (e.g. near a bulbous bow or swath hull); there are multiple contours (e.g. bulbous bow or keel bulb) or where the hull sections are asymmetrical. In this last case, Motions will compute a symmetrical Lewis section having the draft, waterline beam and cross-sectional area of the asymmetrical section.

Occasionally Motions may have problems forming sections at the extremities of the hull; this can happen particularly if there is, say, a very slight wobble in a vertical transom. If this occurs, the following error message will be displayed.
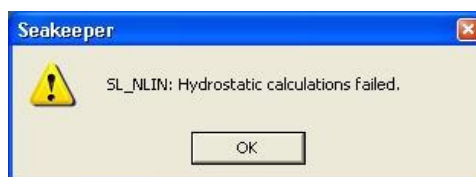
The problem can normally be solved by ensuring that a section can be formed; either by moving the perpendiculars slightly or by adjusting the models surfaces in Modeler.

Generate Analysis Panels (Panel methods)

Choose Analysis | Generate Analysis panels from menu. This menu option will only be enabled with the analysis type set to one of the Panel Methods. The following dialog will appear:
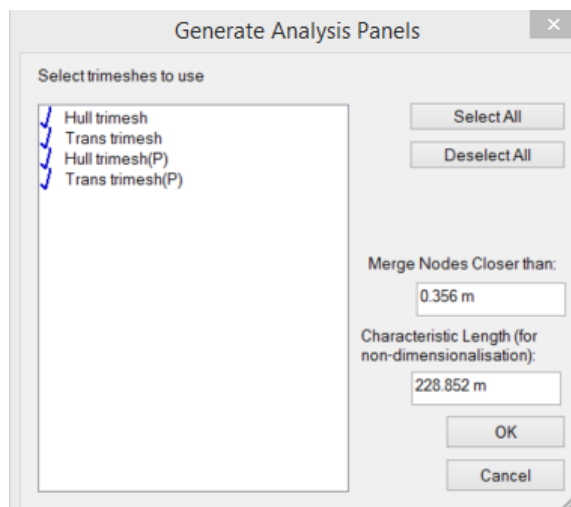
Selecting the meshes to be used for the panel methods

A list of all the trimeshes in the design will appear. Select which trimeshes you want to use to create the panels (if no trimeshes are present in the design they will need to be created in Modeler -or Hull Modeler- first). You can use the "Merge Nodes "tolerance to remove remove excessively small panels from the final mesh. The "Characteristic length" is set to the overall length of the vessel by defualt and it used purely for display of the non-dimensional drift force results.

The analysis panels will be saved in the seakeeper data file (*.skd). Alternatively the panels can be saved to and loaded from an STL file. STL is a universally recognized trimesh file format.

If for some reason the user wanted to manipulate the mesh after it has been automatically generated by the application then this can be done. Firstly save the *.stl file (File | Export | STL mesh). Open the stl file in 3rd party software (or simply in a text editor) and manipulate the verticies until the desired shape is achieved), resave and import back into Motions (File | Import | STL Mesh). Alternatively if the user wanted to bypass the Automatic mesh generator in Motions all together then an STL file from another application may be imported directly.

Additional Hull Parameters (Strip theory)

A number of other hull parameters are required by Motions, these are reviewed in the following text:

Setting the vessel Type (Strip theory)

The Vessel Type dialog in Motions uses the data from the Vessel Type dialog in Modeler whilst allowing the user to make quick changes to the vessel type and, in case of a catamaran, the demihull centreline spacing:
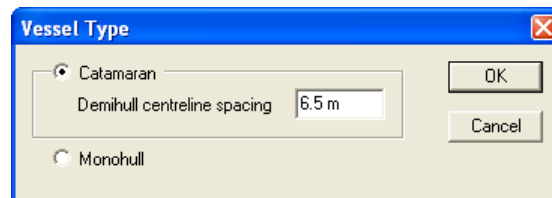
The Demihull centreline spacing is the horizontal distance between both centrelines of each of the demihulls of the catamaran.

The ability to override the vessel type definition in Motions enables you to investigate the effect of demihull separation on seakeeping performance.

Note that changes made in the Vessel Type dialog in Motions will only affect the computations and not the model as drawn. In other words: changing the demihull centreline spacing for a catamaran through the Vessel Type dialog in Motions, will not change the displayed demihull centreline spacing in the view window. If rendered simulations with the correct demihull centreline spacing are required, this model should be generated in Modeler (or Hull Modeler)where the Vessel Type dialog should also be updated. The model can then be loaded into Motions where it should be analysed without making modifications to the Vessel Type dialog.

Catamarans

In head seas, interactions between the demihulls are small and only occur at quite low Froude numbers, for catamarans where the demihull spacing is small. Note that because Motions is a linear theory, the effects of overhangs and non-wall sidedness are ignored, hence results for hulls which have large overhangs or significant flare will be less accurate than for hulls without these features.

The added mass and inertia in roll for catamaran vessels is computed from the heave properties of the vessel. This has been found to provide accurate predictions of the roll response, particularly for catamarans with wider spaced demihulls (S/L above about 0.35). You do not need to specify the roll damping because the roll damping factor for catamarans is computed from the heave damping properties.

When using a single demihull model as a basis for a catamarna in Motions, the demihulls are spaced according to the specified demihull separation. Because of these changes, it is not possible to change the demihull spacing of a full catatamaran model defined in Modeler (or Hull Modeler)

In addition, the demihull spacing is also given in the Summary results table

Motions catamaran modelled as a single demihull in Modeler

Also see:

Asymmetrical sections on page 62.

Setting Mass Distribution (Strip theory and panel methods)

Motions requires the pitch and roll inertias of the vessel. These are input as gyradii in percent of overall length and beam respectively. Typical values are pitch gyradius 25%; roll gyradius 35%–40%. The gyradius, k, is related to the inertia, I, by the equation below:

$$k = \sqrt{\frac{I}{m}}$$
, where
m is the vessel's mass.

The vertical centre of gravity is also to be specified here, this is measured from the user zero point, positive upwards. The VCG is used in the calculation of the roll response. Note that the VCG must be low enough so that the transverse GM is positive. Motions will warn you if the VCG is too high.

Once the RAOs have been calculated, Motions will have calculated the vessel's centre of buoyancy. The centre of gravity will be placed on the centre line, at the same longitudinal position as the centre of buoyancy at the height specified in the dialog. The centres of gravity, buoyancy and floatation will all be displayed in the design views.

Please note that if you open a second design for analysis, the *lengths* of the gyradii will be kept the same, not the *percentages* of length and beam. Hence it is very important to check the gyradii values when you load in a new design, especially if the designs differ considerably in length or beam. If the Motions program is closed and then restarted, the percentages will be reset to their default values of 25% and 40%.

For the SACS and MOSES versions any the inertias of any cargos are automatically included:
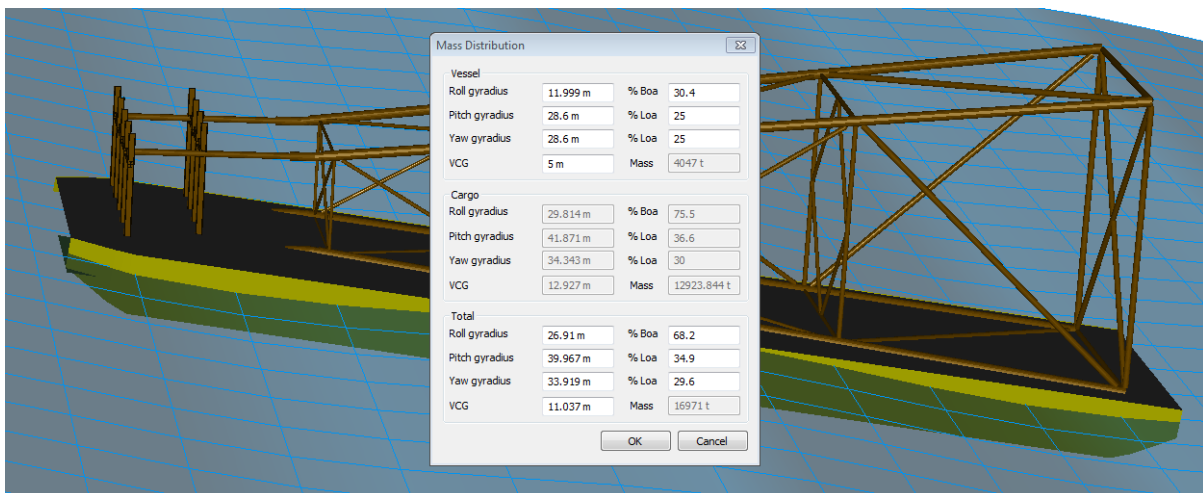
The Cargo VCG and gyradii are automatically loaded from the Cargo datasheet. Changing a value in the Mass distribution dialog will automatically update the other relevant linked fields. For instance if the user changes the Vessel's Pitch gyradius, the %Loa field will get automatically updated, as well as the Total Pitch gyradius and %Loa value. The total values are what are used in the Motions calculations.

Setting Damping Factors (Strip theory)

Heave and Pitch additional damping

An additional damping can be added to the heave and pitch response. The specified non-dimensional damping is assumed to be evenly distributed along the length of the vessel. This is added to the inviscid damping calculated from the oscillating section properties and is applied when the coupled equations of heave and pitch motion are computed.

The heave/pitch damping should probably be left as zero unless you have good reason to change it.
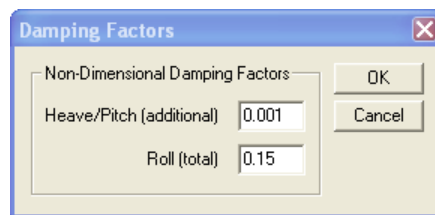
> **Note:**
>
> For catamarans, you do not need to specify the roll damping because the roll damping factor for catamarans is computed from the heave properties.

Roll total damping

The roll response is calculated based on the vessel's hydrostatic properties – which define the roll stiffness, the roll gyradius and the roll damping. Roll damping is almost entirely due to viscous effects, which are not modelled by Motions, hence it is possible for the user to specify the non-dimensional damping (or sometimes called the damping coefficient or damping parameter) factor to be used in the roll model. Typical values for most vessels are between 0.05 and 0.1. Lewis 1989 gives a value of 0.05 for typical ships without roll suppression devices.

Once the calculations have been performed, you can simulate a free roll decay test which can help in selecting the non-dimensional damping to be used.

See: Roll decay simulation on page 52 for more information on roll decay.

Definition of non-dimensional damping

The non-dimensional damping used in Motions follows the formulation used in Lloyd 1989 and Lewis 1989, and is defined as:

$$\eta = \frac{b}{2\sqrt{ca}}$$, where

> b is the dimensional damping;
> c is the stiffness in roll;
> a is the inertia in roll including added inertia effects. Note that this is half the definition used in some texts.

Setting Damping Factors (MOSES solver)

The roll response is calculated based on the vessel's hydrostatic properties – which define the roll stiffness, the roll gyradius and the roll damping. Roll damping is almost entirely due to viscous effects, which are not modelled by MOSES Motions, hence it is possible for the user to specify a Tanaka roll damping factor to be used in the roll model. By default this value is set to 1.0.

Setting Damping Matrix (Panel Method)

When using the panel method (not the Martec or MOSES solvers) the user may specify an additional 6x6 damping matrix:
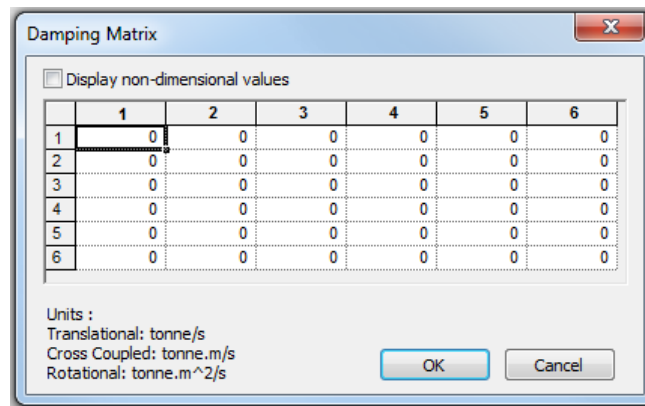
The values input here are dimensional. The top left quadrant contains the translational damping terms, the top right and bottom left quadrants contain the cross coupled damping terms and the bottom right quadrant contains the rotational terms. This matrix is usually used to define additional viscous roll damping not accounted for in the potential flow solvers (B44). The additional damping is added into the damping term of the equations of motion. Viscous damping is very design specific and is usually determined either experimentally or emperically. For more information on calculating viscous damping please refer to the SNAME International Towing Tank Conference Reccommended Procedures document "Numerical Estimation of Roll Damping" (http://ittc.sname.org/). For a free floating body certain terms are normally zero (eg b21, b41 etc) if a significant value is added in one of these cells Motions will warn the user and ask if they would like to proceed.

Setting Stiffness Matrix (Panel Method)

When using the panel method the user may specify an additional 6x6 stiffness matrix:
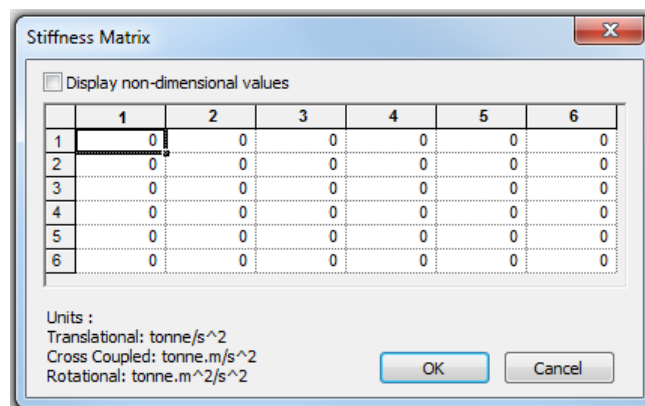
The values input here are dimensional. The top left quadrant contains the translational terms, the top right and bottom left quadrants contain the cross coupled terms and the bottom right quadrant contains the rotational terms. The additional stiffness is added into the stiffness term of the equations of motion. For a free floating body certain terms are normally zero (eg c21, c41 etc) if a significant value is added in one of these cells Motions will warn the user and ask if they would like to proceed.

Specifying Remote Locations

Motions may be used to calculate the motions at the centre of gravity of the vessel and also at specified positions on the hull away from the centre of gravity. These positions are known as remote locations. This may be useful for determining if a slam is likely to occur; what accelerations are likely to be experienced in the bridge or accommodation areas, etc.

Motions will calculate the absolute and relative (to wave surface) vertical motion, velocity and acceleration and MSI at the specified remote locations.

You may specify as many remote locations as you like and they are referred to by name. Remote locations are specified in the Input table:

	Name	Position [m]	Offset [m]	Height [m]	Position from CG	Offset from CG	Height from CG	MII slide friction	MII tip fore/aft.	MII tip side/side
1	Bridge Centre	35.00	0.00	15.00	35.00	0.00	15.00	0.70	0.17	0.25
2	Bridge Port	35.00	-5.00	15.00	35.00	-5.00	15.00	0.70	0.17	0.25
3	Bridge Stbd	35.00	5.00	15.00	35.00	5.00	15.00	0.70	0.17	0.25
4	Fwd Accom Centre	40.00	0.00	10.00	40.00	0.00	10.00	0.70	0.17	0.25
5	Fwd Accom Port	40.00	-5.00	10.00	40.00	-5.00	10.00	0.70	0.17	0.25
6	Fwd Accom Stbd	40.00	5.00	10.00	40.00	5.00	10.00	0.70	0.17	0.25
7	Aft Accom Centre	10.00	0.00	10.00	10.00	0.00	10.00	0.70	0.17	0.25
8	Aft Accom Port	10.00	-5.00	10.00	10.00	-5.00	10.00	0.70	0.17	0.25
9	Aft Accom Stbd	10.00	5.00	10.00	10.00	5.00	10.00	0.70	0.17	0.25
10	Propeller	3.00	0.00	1.00	3.00	0.00	1.00	0.70	0.17	0.25
11	Forefoot	47.00	0.00	0.20	47.00	0.00	0.20	0.70	0.17	0.25
12	Fwd Deck	47.00	0.00	8.80	47.00	0.00	8.80	0.70	0.17	0.25

Locations \ Speeds \ Headings \ Spectra /

The first column is used to specify a label or name for the remote location. The next three columns give the remote location's position relative to the zero point. The next three give the location relative to the vessel's centre of gravity; you may use which ever is most convenient using the following coordinate system: longitudinal, positive forward; transverse, positive to starboard; and vertical, positive up. The centre of gravity will be computed from the vessel's hydrostatics and will be positioned such that its longitudinal and transverse positions coincide with those of the centre of buoyancy. The VCG is specified by the user, see the section on Setting Mass Distribution, above. Note that the vessel's hydrostatics will not be computed until the hull has been measured. To do this, select Measure Hull or Solve Seakeeping Analysis from the Analysis menu.

The last three columns are used to specify the coefficients to be used when calculating motion induced interruptions (MII) at the remote location. Standard sliding and tipping coefficients are:

MII coefficients for different events

Incident	Coefficient
Personnel	
Fore-and-aft tipping	0.17
Side-to-side tipping	0.25
Sliding on dry deck	0.70
Equipment	
Sliding of chair on linoleum floor	0.19
Sliding of helicopters, typical minimum	0.20
Sliding of helicopters, typical maximum	0.80

Source: Standard material requirements for RAN ships and submarines, vol 3, part 6.

MSI after specified exposure

Motions calculates the MSI according to the McCauley *et al*. 1976 formulation which includes an exposure time. This is the *only* place where the MSI exposure time for the remote location is used.

The exposure time can be specified for each remote location (you can define multiple remote locations at the same point on the vessel and give them different exposure times if you wish).

Specification of exposure time for each remote location for the MSI 1976 calculation

See: Calculation of Subjective Magnitude and Motion Sickness Incidence on page 95 for details of the calculations performed.

Remote location display

The remote locations and centre of gravity will be displayed in the Design view as small crosses.
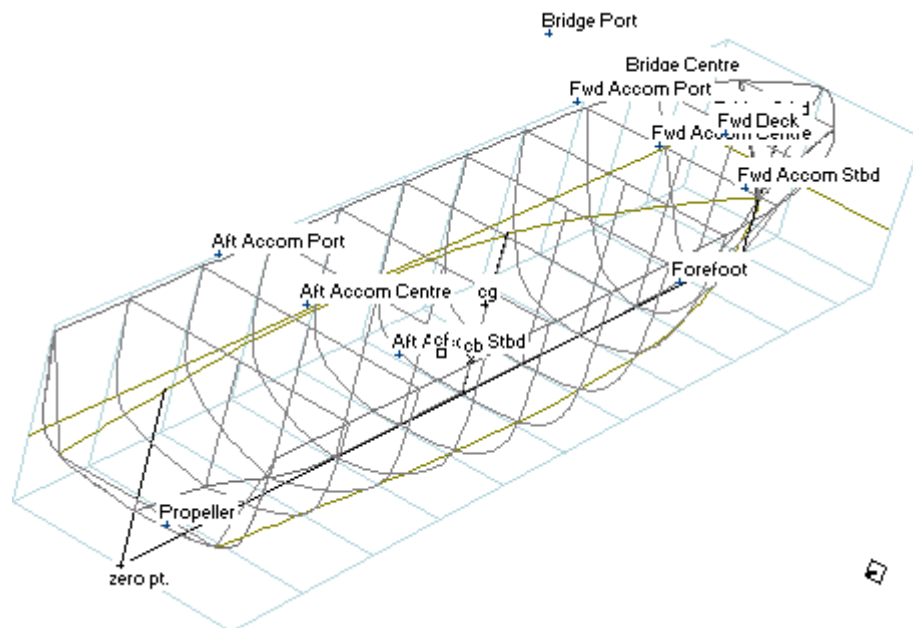
Adding remote locations

Use the Analysis | Add Remote Locations menu item to add remote locations. This will add remote locations below the currently selected row in the Remote Locations table. The number of remote locations inserted is the same as the number of rows selected.

Deleting remote locations

To delete remote locations, select the rows of the remote locations you wish to delete, and then select Delete Remote Locations from the Analysis menu. (You must click in the numbered grey cell on the left to high light the entire row.)

Seakeeping results at remote locations are summarised in the Summary table in the Results window. For each remote location, the absolute and relative (to the wavy surface) motions, velocities and accelerations are calculated:

	Item	m0	units	RMS	units	Significant ampl	units
16	Added resistance	51.464	kN	--		--	
17	Heave motion	0.651	m^2	0.807	m	1.614	m
18	Roll motion	0.00000	rad^2	0.00000	rad	0.00000	rad
19	Pitch motion	0.00178	rad^2	0.04222	rad	0.08444	rad
20	Heave velocity	0.289	m^2/s^2	0.537	m/s	1.075	m/s
21	Roll velocity	0.00000	(rad/s)^2	0.00000	rad/s	0.00000	rad/s
22	Pitch velocity	0.00118	(rad/s)^2	0.03440	rad/s	0.06880	rad/s
23	Heave acceleratio	0.145	m^2/s^4	0.381	m/s^2	0.762	m/s^2
24	Roll acceleration	0.00000	(rad/s/s)^2	0.00000	rad/s/s	0.00000	rad/s/s
25	Pitch acceleration	0.00100	(rad/s/s)^2	0.03155	rad/s/s	0.06310	rad/s/s

Environmental Parameters

Motions calculates the motions of the vessel for a specified sea condition given in terms of a wave spectrum.

Setting Environment Parameters

The water density should be specified in this dialog. At present Motions assumes deep water for all calculations.

Setting Vessel Speeds, Headings and Spectra, and SACS Cargo

The Inputs window allows you to define multiple speeds, headings and spectra for analysis. In MOSES Motions you can also add SACS topsides modules as cargo items.

Use the edit menu to add or delete items (or you can use the keyboard short-cuts of Ctrl+A and the Delete key respectively). New items are added below the currently selected row (multiple rows can be added at once by selecting the number of rows required, these are then added below the lowest selected row). To delete a row or rows, simply select the rows you wish to delete and choose Edit | Delete.

For easier identification, you may give a name to each condition. Headings are given in terms of the relative heading of the waves compared with that of the vessel track (head seas = 180º; following seas = 0º; starboard beam seas = 90º, port beam = 270º etc…).

When entering a vessel speed, the vessel Froude Number should also be considered. The results from Motions are likely to become less reliable at high Froude Numbers since the vertical force due to forward motion is ignored by Motions. Motions has been found to produce useful results at Froude Numbers of up to around 0.6 – 0.7. The Panel method and Martec solvers are only valid for zero forward speed.

The "Analyse" column allows you to select which conditions to compute for the next analysis run.

All the locations, speeds, headings, spectra and other analysis options are saved when you save the Motions data.

If no speeds, headings and spectra are selected for testing it is not possible to perform an analysis and the following warning dialog will be displayed:

Specifying Wave spectra Input

The wave spectrum type may be selected from five different standard spectra. For each spectrum, different input data are required, this is summarised below:

Spectrum:	Characteristic Height	Period	Peak Enhancement Factor	Wind Speed
ITTC or 2 parameter Bretschneider	Required	Required	Fixed 1.0	Not applicable
1 parameter Bretschneider	Required	Specified by method	Fixed 1.0	Not applicable
JONSWAP	Required	Required	Fixed 3.3	Not applicable
DNV	Required	Required	Depends on period and height: 1.0 – 5.0	Not applicable
Pierson Moskowitz	Estimated by Motions	Estimated by Motions	Not applicable	Required
Ochi Hubble	Required (for both spectra)	Required	Required	Not applicable
ISSC	Required	Required	Fixed 1.0	Not applicable

It should be noted that, the ITTC, JONSWAP and DNV spectra are closely related, the only difference being the peak enhancement factor used. The ITTC always has a peak enhancement factor of 1.0, whilst the JONSWAP spectrum has a peak enhancement factor of 3.3. The DNV spectrum has a peak enhancement factor which varies from 1.0 to 5.0 depending on a relationship between period and wave height.

For spectra requiring a wave period to be entered, the user may enter the modal period (T_0), the average period ($T_{ave.}$), or the zero crossing period (T_z). The corresponding values for the other periods and other related values will be estimated by Motions.

The Pierson Moskowitz spectrum is based on wind speed. If this spectrum is selected then the other spectral parameters will be estimated from the wind speed entered.

SACS Cargo Import

In MOSES and SACS, topsides models defined in SACS may be imported into Stability. These are imported (one per row) into the last table of the Input window.

Use the Add Cargo command or Ctrl+A to open the Add Cargo dialog when in the Cargo table of the Input window.

Joints 1 and 2 define the origin of the cargo. The x- and z-values or the cargo origin point are those of Joint 1; the y-value of the cargo origin point is the average of the y-values of Joints 1 and 2. The vector Joint1-Joint 2 now defines the port-direction of the cargo and Joint 3 is used to define the horizontal plane and longitudinal direction.
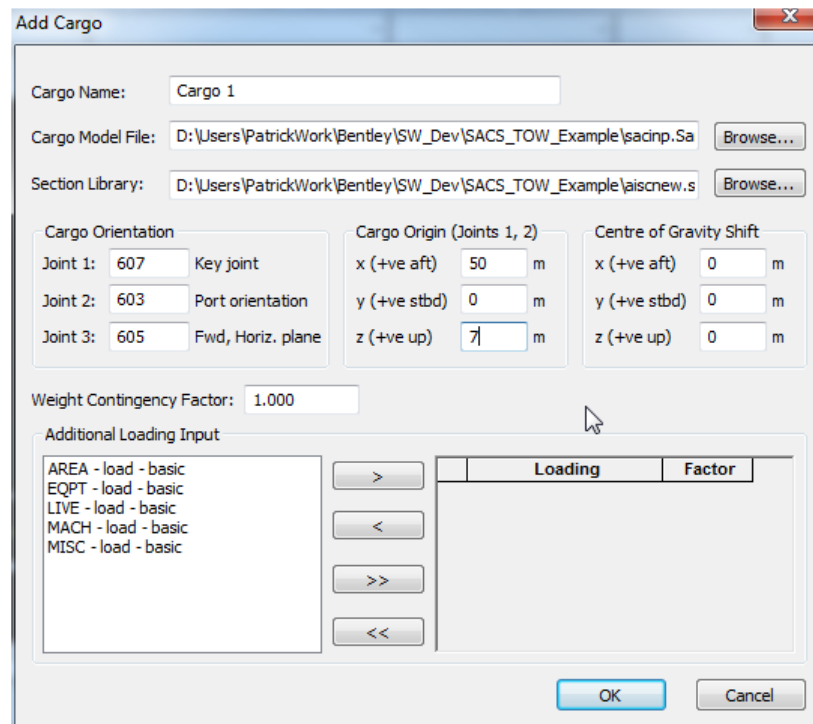
The location of the cargo can be modified in the table entry, but to change the orientation, the cargo must be removed and then added with the new joint references.

	Name	Include	Orientation	Origin x (+ve af	Origin y (+ve st	Origin z (+ve up	CG Shift x (+ve	CG Shift y (+ve	CG Shift z (+ve	Weigh	Cargo Ma	CG x (+ve aft) [	CG
1	Cargo 1	✓	607,603,605	50.000	0.000	7.000	0.000	0.000	0.000	1.000	605.994	44.029	0.2

	Name	Include	Orientation	Origin x (+ve af	Origin y (+ve st	Origin z (+ve up	CG Shift x (+ve	CG Shift y (+ve	CG Shift z (+ve	Weigh	Cargo Ma	CG x (+ve aft) [	CG
1	Cargo 1	✓	607,603,605	50.000	0.000	7.000	0.000	0.000	0.000	1.000	605.994	44.029	0.2

Setting the Frequency Range

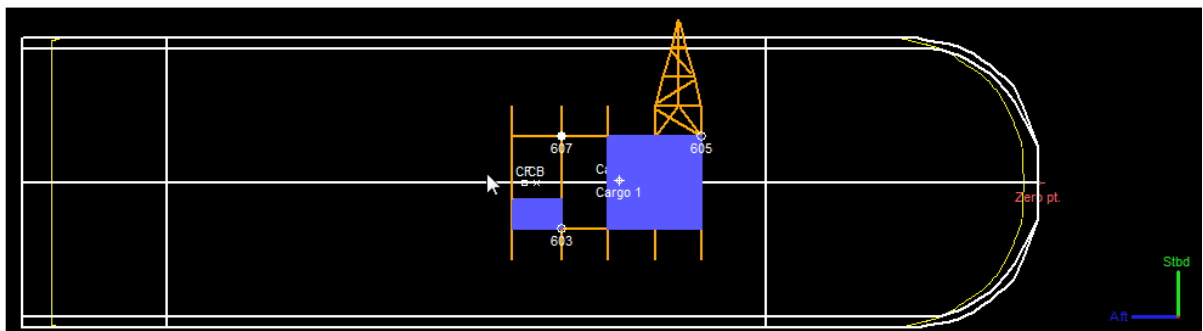Motions can either automatically calculate the range of frequencies over which to predict the vessel's RAOs, or the user can manually enter the range. The number of steps at which the RAOs are calculated may also be specified by the user. A sufficient number of steps should be specified to obtain good definition of the RAO curves, depending on the speed of your computer, you may want to try a smaller number at first and verify that the calculations are preformed correctly then increase the number for the final calculations once all the parameters are set up correctly. If a user defined frequency range is set then the user may choose between inputting the start and end frequencies or the start and end periods.
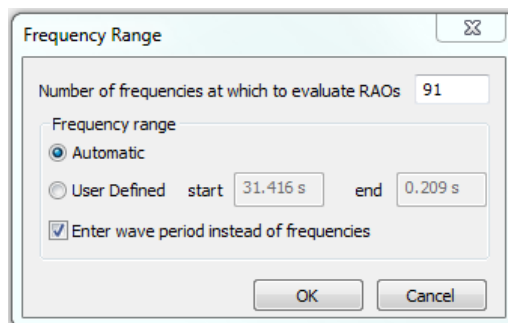
Completing the Analysis

Once you have specified the hull geometry, the hull mass distribution, the roll damping (strip theory) and the environment conditions, you may proceed with the analysis.

Strip theory method

There are various options to control the exact formulation and solution of the strip theory problem. At present it is possible to specify the following options:

Transom terms

According to the underlying strip theory formulation, corrections should be applied to vessels with transom sterns. These corrections have the effect of increasing the heave and pitch damping and thus reducing the maximum response of the vessel. The transom terms also depend on the vessel speed and are greater at high forward speed.

It is possible that the inclusion of transom terms may over predict the damping, particularly for vessels with large transoms at high speeds. It is suggested that for this type of vessel, results be compared for calculations with and without transom terms and that the user use their judgement as to which results are the most plausible.

Added resistance

Four alternative methods are provided for the calculation of added resistance. The different methods will give different results and again the user must use their judgement as to which are the most likely. Added resistance calculations are second order with respect to wave amplitude and are based on the calculated motions. This means that if motions are calculated with an accuracy of approximately 10–15%, the accuracy of the added resistance calculation will be no better than 20–30% (Salvesen 1978).

The method by Salvesen is purported to be more accurate than those of Gerritsma and Beukelman for a wider range of hull shapes; whilst Gerritsma and Beukelman have found their method to be satisfactory for fast cargo ship hull forms (Salvesen 1978).

The Salvesen method is based on calculating the second-order longitudinal wave force acting on the vessel. Theoretically, this method may also be applied to oblique waves.

The two methods by Gerritsma and Beukelman are very similar and are based on estimating the energy radiated by the vessel including the effect of the relative vertical velocity between the vessel and wave. Method I uses encounter frequency in the wave term whereas Method II uses wave frequency. Method II is mathematically rigorous, however, in some cases, Method I has been found to give better results. Strictly speaking, both of these methods apply to head seas only.

Finally the method of calculating added resistance as proposed by Havelock (1942).

The interested user is directed to the original papers: Gerritsma and Beukelman (1967, 1972) and Salvesen (1978) for full details of the methods.

The methods are only applicable to head seas and are calculated only from the heave and pitch motions.

The added resistance calculated is due only to the motion of the vessel in the waves. It does not account for speed loss due to wind; reduction of propeller efficiency or voluntary speed loss in order to reduce motions.
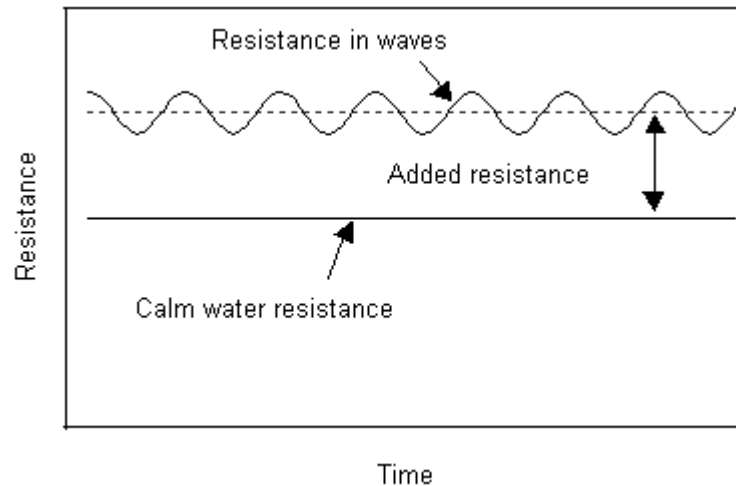
Wave force

Two alternative methods for calculating the wave excitation may be used in Motions:

Head seas approximation: here a simplifying assumption that the vessel is operating in head seas is used, this speeds up the calculations to some degree. This method is exactly valid in head seas and can be applied with reasonable accuracy up to approximately 20° either side of head seas; i.e. $160° < \mu < 200°$

The arbitrary wave heading method does not make this assumption but is somewhat more computationally intensive. This method should be used for off-head seas calculations.
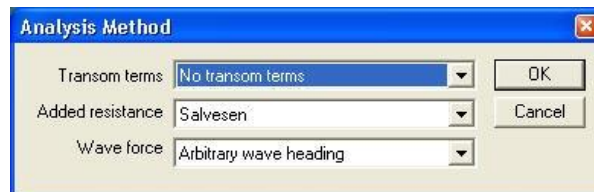
Calculation of Mapped Sections (strip theory)

After the hull has been measured, or at the beginning of the seakeeping analysis, if the hull has not been previously measured, the conformal mappings which are used to approximate the vessel's sections should be computed.

> **Note:**
>
> If you are unfamiliar with the principles and limitations of conformal mapping, or you have an unusual design, then it is a good idea to check the mapped section thoroughly before continuing with the rest of the analysis.
>
> Also see: Choosing Analysis Method and Higher order conformal mappings on page 16.

Motions will first calculate regularly spaced sections along the waterline of the vessel:
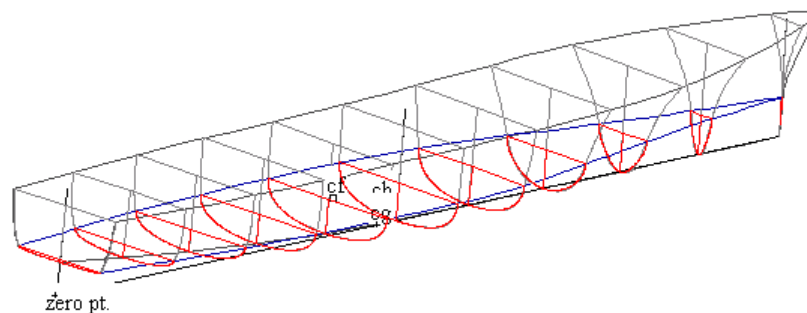
To view the mapped sections, click on the Display Mapped Sections toolbar button.

The mapped sections are used to compute the section hydrodynamic properties. For unusual hull forms, it is advisable to check that the mapped sections are an adequate representation of the hull before proceeding with the more time consuming response and seakeeping calculations. Typical mappings are shown below:

Note that, in general, due to the limitations of the conformal mapping technique, the mapped sections will not be an exact match to the actual hull sections. However, these mapped sections are only used to estimate the hydrodynamic properties of the hull and, in most cases, these will be calculated with sufficient accuracy for the strip theory analysis.

The main limitations of the conformal mapping method are:

- **The mappings always cross the axes at 90°.**
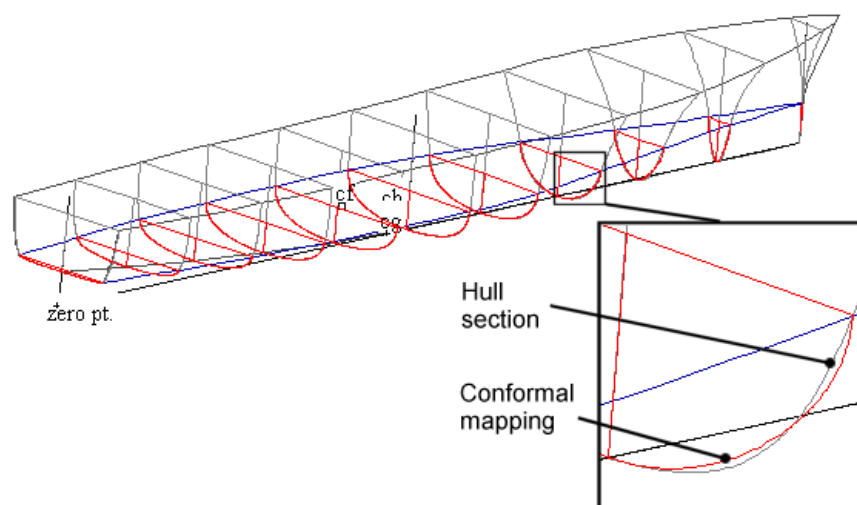- **The sections are continuous (chines will be rounded off).**
- **There are limitations in the range of section breadth to draft ratio and sectional area coefficients that can be accurately mapped.**

Also see Limitations and Guidelines and in particular Asymmetrical sections on page 62.

Solving Seakeeping Analysis

Once you are satisfied with the geometry definition, environmental conditions and specified vessel parameters, you may proceed with calculating the vessel's response; select Solve Seakeeping Analysis from the Analysis menu.

Viewing the Results

The results may be viewed graphically or in tabular format. Also the results may be copied from the tables and pasted into a spreadsheet for further analysis.

The results displayed in the table and plotted in the graphs correspond to the condition selected in the Results toolbar:

To view the results table, select Results from the Window menu.

	Item	m0	units	RMS	units	Significant ampl	units
1	Modal period	9.995	s	--		--	
2	Characteristic wa	4.000	m	--		--	
3	Spectrum type	JONSWAP		--		--	
4	Wave heading	180.0	deg	--		--	
5	Vessel Speed	15.000	kts	--		--	
6	Vessel displacem	197.937	m^3	--		--	
7	Vessel GMt	2.896	m	--		--	
8	Vessel trim	0.0	deg	--		--	
9	Transom method	No transom terms		--		--	
10	Wave force meth	Arbitrary wave h		--		--	
11	Added res. metho	Salvesen		--		--	
12	Pitch gyradius	6.277	m	--		--	
13	Roll gyradius	1.800	m	--		--	
14	Wave spectrum	1.003	m^2	1.001	m	2.003	m
15	Encountered wav	1.003	m^2	1.001	m	2.003	m
16	Added resistance	53.952	kN	--		--	
17	Heave motion	0.857	m^2	0.926	m	1.851	m
18	Roll motion	0.00000	deg^2	0.00000	deg	0.00000	deg
19	Pitch motion	8.10	deg^2	2.85	deg	5.69	deg
20	Heave velocity	1.028	m^2/s^2	1.014	m/s	2.028	m/s
21	Roll velocity	0.00000	(rad/s)^2	0.00000	rad/s	0.00000	rad/s
22	Pitch velocity	0.00542	(rad/s)^2	0.07359	rad/s	0.1472	rad/s
23	Heave acceleratio	1.742	m^2/s^4	1.320	m/s^2	2.640	m/s^2
24	Roll acceleration	0.00000	(rad/s/s)^2	0.00000	rad/s/s	0.00000	rad/s/s)
25	Pitch acceleration	0.01805	(rad/s/s)^2	0.1343	rad/s/s	0.2687	rad/s/s)
26	Remote location1:	0.918	m^2	0.958	m	1.916	m
27	Remote location1:	0.051	m^2	0.225	m	0.450	m
28	Remote location1:	1.161	m^2/s^2	1.078	m/s	2.155	m/s

You may view different sets of results by selecting the different tables of the Results window. The tables are described more fully in the following sections.

Summary Table Results

This table displays m_0, RMS and significant amplitude of the different motion, velocity and acceleration spectra of the vessel for the specified conditions. The mean square, m_0, of the spectrum is the area under the spectrum and gives a measure of the total response of the vessel. The RMS is the square root of the mean square, and for seakeeping calculations, the significant amplitude is twice the RMS value. (The significant height, measured peak to trough, is twice the significant amplitude.)

The mean square of the spectrum is given by:

$$m_0 = \int_0^\infty S(\omega)d\omega \text{ or } m_0 = \int_0^\infty S(\omega_e)d\omega_e$$

where $S(\omega)$ is the density of the spectrum of interest.

The first 13 rows describe the conditions specified for the analysis: wave spectrum modal period; wave spectrum characteristic wave height; spectrum type; wave heading; vessel speed; vessel displacement, vessel trim, analysis method and gyradii.

The next two rows provide a check of the calculations by displaying the integrated wave spectrum values. The first row, labelled Wave spectrum, has been obtained by integrating the wave spectrum as viewed by a stationary observer; the second (Encountered wave spectrum) from integrating the waves as experienced by the moving vessel. Both should give the same answer. Also the significant amplitude should be approximately half the specified characteristic wave height of the spectrum.

If these values do not match, then the rest of the results are in question. You should increase the frequency range and redo the calculations.

The following row gives the added resistance of the vessel in the specified sea conditions. The added resistance is given by:

$$\bar{R}_{aw} = 2\int_0^\infty C_{aw}S_\zeta(\omega_e)\,d\omega_e$$

where $C_{aw} = \dfrac{R_{aw}}{\zeta_0^2}$ is the added resistance coefficient in regular waves, and $S_\zeta(\omega_e)$ is the encountered wave energy spectrum.

The next 9 rows give the vessels integrated heave, roll and pitch motions, velocities and accelerations.

The vertical absolute and relative motions, velocities and accelerations are then given for all the specified remote locations.

The last row for each remote location gives the MII per hour for sliding, fore-and-aft tipping and side-to-side tipping. The table below correlates MII rate to risk. Note that the lateral acceleration used to calculate the MII is that due to roll only.

MII Risk level

Severity	MII Risk level	MII per hour	MII per minute
1	Possible	6	0.1
2	Probable	30	0.5
3	Serious	90	1.5
4	Severe	180	3.0
5	Extreme	300	5.0

Source: Standard material requirements for RAN ships and submarines, vol 3, part 6.

The SM and MSI for 2 hours exposure and the specified exposure time are presented in the Summary results table. The last row for each remote location gives the SM in the second column, then MSI for 2 hours exposure using the O'Hanlon and McCauley 1974 method in the fourth column and in the MSI for the specified exposure period is given in the sixth column.

Subjective magnitude is on a scale of zero to 30:

0 – 5	Moderate
5 – 10	Serious
10 – 15	Severe: necessary to "hang on"
15 – 20	Hazardous
20 – 30	Intolerable

See: Calculation of Subjective Magnitude and Motion Sickness Incidence on page 95 for details of the calculations performed.

The majority of the results calculated in the Summary table may be plotted in the Polar Plot window provide that the response for at least two speeds and a range of headings has been calculated.

MSI – ISO 2631/3 1985 and BS 6841:1987 Results

Although not strictly MSI accelerations integrated over 1/3 octave bins are plotted against acceleration limits defined by these two standards. The likely discomfort can be gauged by how close these acceleration curves approach the defined acceleration limits.

The second table gives the acceleration curves at the specified remote locations. The data for the standard ISO and BSI curves are also given. The MSI acceleration is calculated from the equation below:

$$ MSI(\omega_{e\,centre}) = \int_{\omega_{e1}}^{\omega_{e2}} S_{vert\,accel}(\omega_e)\,d\omega_e $$

where the frequency interval ω_{e1} to ω_{e2} is the 1/3 octave range centred about $\omega_{e\,centre}$ and $S_{vert\,accel}$ is the absolute vertical acceleration at the point of interest on the vessel.
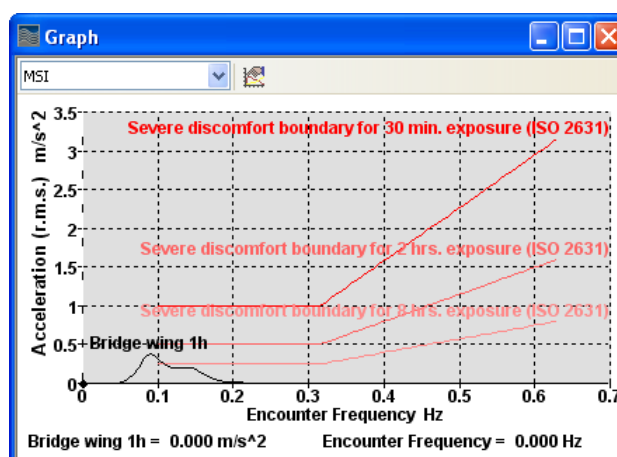
Existing comparison of accelerations compared with ISO and BS standards

CG RAO Results

The next table gives the computed vessel RAOs for the principal degrees of freedom, at the vessel's centre of gravity. These have been non-dimensionalised with wave height for heave and with wave slope for roll and pitch motions. The phase lag of the motions is also given.

Finally the added resistance coefficient in regular waves, $C_{aw} = \dfrac{R_{aw}}{\zeta_0{}^2}$, is given. This value is dimensional, and has units of Force / Length^2. There are several accepted ways of non-dimensionalising added resistance. Gerritsma and Beukelman use the following:

Added resistance coefficient $= \dfrac{R_{aw}}{\zeta_0{}^2 \rho g B^2 / L}$; where B is the vessel beam and L the vessel length.

Remote Location RAO Results

The next table gives the computed longitudinal, lateral and vertical RAOs (non-dimensionalised against wave amplitude) at the currently selected remote location (from the remote location drop down list). The absolute harmonic motion (Xp, Yp, Zp) of a point on the body can be found from:

$$Xp = Xcg - dY \cdot Yaw + dZ \cdot Pitch$$
$$Yp = Ycg - dX \cdot Yaw - dZ \cdot Roll$$
$$Zp = Zcg - dX \cdot Pitch + dY \cdot Roll$$

where angles are assumed small and in Radians. dX, dY, dZ are the linear distances of the point from the centre of gravity at which the motion is being calculated. Xcg, Ycg and Zcg are linear motions at the centre of gravity and Yaw, Pitch and Roll are the angular motions of the vessel. Xp, Yp and Zp are the resulting linear motions at the point of interest. The phase relationship is accounted by the fact that Xcg, Ycg, Zcg, Yaw, Pitch and Roll are all complex numbers (as are the resulting motions Xp, Yp and Zp).

These data are represented as linear transfer functions (RAOs) in this results table. Thus similar equations can be formulated using the transfer functions of the linear displacements at the centre of gravity combined with the transfer functions of the angular motions multiplied by the wave slope for unit wave amplitude ($\omega_0{}^2 / g$).

Please note that the sign on dZ in the above equations assumes a right-handed coordinate system with the vertical axis (z-axis) positive down. (In MAXSURF Motions the vertical direction is upwards so the sign is inverted.)

CG Spectrum Results

This table gives the specified wave spectrum, in wave and encounter frequency domains. The response spectra for the principal degrees of freedom, at the vessel's centre of gravity are also calculated in the encounter frequency domain. For the spectra the horizontal axis is always in rad/s.

The various spectra are calculated as follows:

$$S_x(\omega_e) = \mathrm{RAO}_x{}^2(\omega_e) S_\zeta(\omega_e)$$

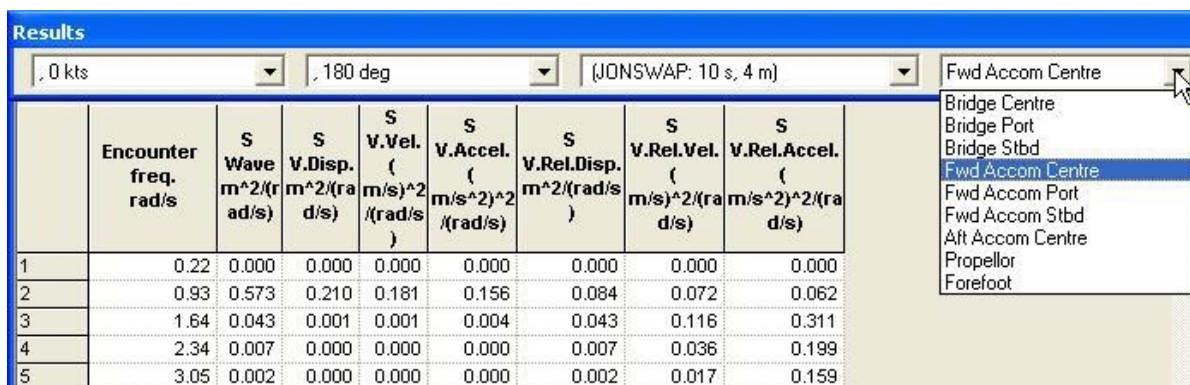The added resistance spectrum is also provided. It is calculated as follows:

$$S_{aw}(\omega_e) = 2C_{aw}(\omega_e)S_\zeta(\omega_e)$$

Encounter freq.	Encountered wave circular frequency
Wave freq.	Wave circular frequency
S Wave	Spectral density of wave spectrum, as experienced by vessel
S Heave	Vessel CG heave spectrum, encounter frequency
S Roll	Vessel roll spectrum, encounter frequency
S Pitch	Vessel pitch spectrum, encounter frequency
S Added resistance	Added resistance, encounter frequency
S Wave(Wave freq.)	Spectral density of wave spectrum, as experienced by stationary observer

Remote Location Spectrum Results

This table repeats the encountered wave spectrum and gives the response spectra at the specified remote location. Absolute and relative vertical displacement, velocity and acceleration spectra are calculated. The vessel is assumed to be a rigid body and the effects of heave, roll and pitch are combined; the relative motions are the absolute motions relative to the local wave surface.

The remote location for which the spectra are displayed may be selected from the pull-down menu in the Results window toolbar:

|	
Encounter freq.	Encountered wave circular frequency
S Wave	Spectral density of wave spectrum, as experienced by vessel
S V.Disp.	Absolute vertical displacement spectrum, encounter frequency
S V.Vel.	Absolute vertical velocity spectrum, encounter frequency
S V.Accel.	Absolute vertical acceleration spectrum, encounter frequency
S V.Rel.Disp.	Relative vertical displacement spectrum, encounter frequency
S V.Rel.Vel.	Relative vertical velocity spectrum, encounter frequency
S V.Rel.Accel.	Relative vertical acceleration spectrum, encounter frequency

Global Hydrodynamic coefficients

These are the integrated coefficients, cross-coupling terms and excitation forces in the equations of motion of the vessel. These are the total values for the complete vessel.

Encounter freq.	Encountered wave circular frequency
Mass + added mass33	Total vessel mass and added mass at specified encounter freq.
Added mass35	Cross-coupling heave added mass due to pitch
Added inertia53	Cross-coupling pitch added inertia due to heave
Inertia + added inertia55	Total vessel inertia and added inertia in pitch
Damping33	Heave inviscid damping
Damping35	Heave/Pitch cross-coupling damping
Damping53	Pitch /Heave cross-coupling damping
Damping55	Pitch inviscid damping
Stiffness33	Heave stiffness (restoring force term)
Stiffness35	Heave/Pitch cross-coupling stiffness
Stiffness53	Pitch / Heave cross-coupling stiffness
Stiffness55	Heave stiffness (restoring force term)
Wave excitation3	Total wave excitation amplitude in heave
Wave excitation Phase3	Heave wave excitation phase
Wave excitation5	Total wave excitation amplitude in pitch
Wave excitation Phase5	Heave wave excitation phase

Sectional Hydrodynamic Coefficient Results (Strip theory)

The added mass, damping, stiffness and wave excitation for each of the sections of the hull at each of the frequencies of interest. For each frequency, these values are integrated along the vessel's length to give the global coefficients in the equations of motion.

The remote location for which the spectra are displayed may be selected from the pull-down menu in the Results window toolbar:
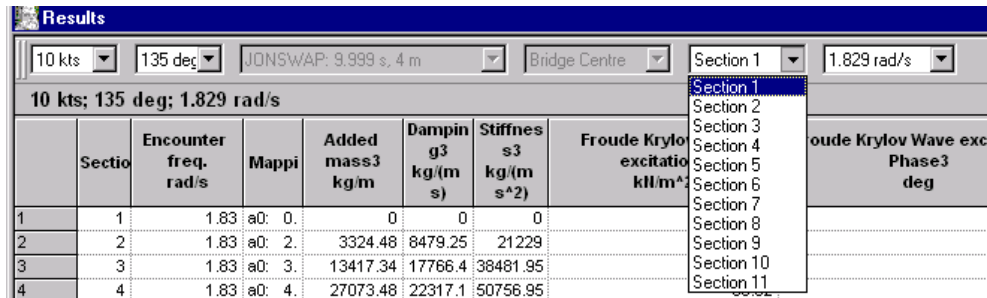
Choosing a section from the pull down menu, will display the variation of terms with frequency for the selected section; similarly, selecting a frequency will display the variation of terms along the length of the vessel for the selected frequency.

Section	Section number from bow
Encounter freq.	Encountered wave circular frequency
Mapping	Terms in Lewis mapping
Added mass3	Section added mass for vertical motion
Damping3	Section inviscid damping for vertical motion
Stiffness3	Section restoring force term for vertical motion
Froude-Krilov Wave excitation3	Amplitude of Froude-Krilov component of wave excitation for vertical motion

Froude-Krilov Wave excitation Phase3	Phase of Froude-Krilov component of wave excitation for vertical motion
Diffraction Wave excitation3	Amplitude of diffraction component of wave excitation for vertical motion
Diffraction Wave excitation Phase3	Phase of diffraction component of wave excitation for vertical motion

Panel Pressures (Panel Method only)

The panel pressures table displays the panel pressure values for the selected frequency. By default values for all the panels are displayed, alternatively the user can specify just to see just the results for the panel of interest (select from the drop down list on the results toolbar).

The magnitude and phase of the pressure are shown. In addition the magnitude of the Incident, Diffraction and Radiation components of the pressure are shown. The non-dimensional values are displayed. The pressure values are non-dimensionalised according to the water density (ρ), gravity(g) and wave amplitude (A):

Pressure'=Pressure/ρgA

Drift Forces

Drift forces and moments are displayed in the Drift Forces table. These results are available from Panel Method, Martec Solver and MOSES Solver. Each row represents the value for a single frequency and heading. The values displayed are non-dimensional.

Drift forces (surge, sway and heave) are non-dimensionalised according to water density (ρ), gravity(g), characteristic length (L) and wave amplitude (A):

Drift force'=Drift force/ρgLA^2

Drift moments (roll, pitch and yaw) are non-dimensionalised according to water density (ρ), gravity(g), characteristic length (L) and wave amplitude (A):

Drift moment'=Drift moment/ρgL^{2}A^2

Characteristic length defaults to the vessel's maximum dimension. The characteristic length can be altered by the user in the Analysis | Mesh Hull… dialog.

> **Note:**
> MOSES Motions drift force is calculated with md_force parameters set to 1 1 0. See MOSES documentation for more information on md_force.

Wave Excitation Forces

Wave excitation forces are displayed in the wave excitation forces table. These results are available from Panel Method, Martec Solver and MOSES Solver. Each row represents the exciting force in each of the 6 degrees of freedom for a frequency and heading. The non-dimensional force magnitude and phase are presented for surge, sway, heave, roll, pitch and yaw respectively.

Wave excitation forces (surge, sway and heave) are non-dimensionalised according to water density (rho), gravity(g), displaced volume (∇), characteristic length (L) and wave amplitude (A):

Wave Force'=Wave Force/(ρ g∇A/L)

Wave excitation moments (roll, pitch and yaw) are non-dimensionalised according to water density (rho), gravity(g), displaced volume (∇) and wave amplitude (A):

Wave Force'=Wave Force/(ρ g∇A)

Characteristic length defaults to the vessel's maximum dimension. The characteristic length can be altered by the user in the Analysis | Mesh Hull… dialog.

Added Mass and Damping

Added mass and damping results are displayed in the Added Mass and Damping table of the results window. The 6x6 Added Mass (A_{ij}) and Damping (B_{ij}) coefficient matricies are presented for each frequency. These results are only available from the Panel Method analysis.

The coefficients are non dimensionalised using water density (ρ), gravity (g), volumetric displacement (∇), wave amplitude (A), wave frequency(ω) and characteristic length (L).

For the added mass coefficients (A_{ij}) the values are non-dimensionalised as follows:
i=1-3 and j=1-3: A_{ij}'=A_{ij}/($\rho\nabla$)
i=1-3 and j=4-6: A_{ij}'=A_{ij}/($\rho\nabla$L)
i=4-6 and j=1-3: A_{ij}'=A_{ij}/($\rho\nabla$L)
i=4-6 and j=4-6: A_{ij}'=A_{ij}/($\rho\nabla$L^2)

For the damping coefficients (B_{ij}) the values are non-dimensionalised as follows:
i=1-3 and j=1-3: B_{ij}'=B_{ij}/($\rho\nabla\omega$)
i=1-3 and j=4-6: B_{ij}'=B_{ij}/($\rho\nabla$L ω)
i=4-6 and j=1-3: B_{ij}'=B_{ij}/($\rho\nabla$L ω)
i=4-6 and j=4-6: B_{ij}'=B_{ij}/($\rho\nabla$L^2 ω)

Graphing the Results

All the above datasheets, except for the integrated values, may be graphed and displayed in the graph window:

- **MSI**
- **RAO**
- **CG Spectra**
- **Remote Spectra**
- **Global Hydrodynamics**
- **Section Hydrodynamics**

To select the desired graph, make the graph window the uppermost window by either clicking in the window or selecting Graph from the bottom of the Window menu.

The desired datasheet may now be selected from the Window menu: select Graph and drag right to display the available graphs.
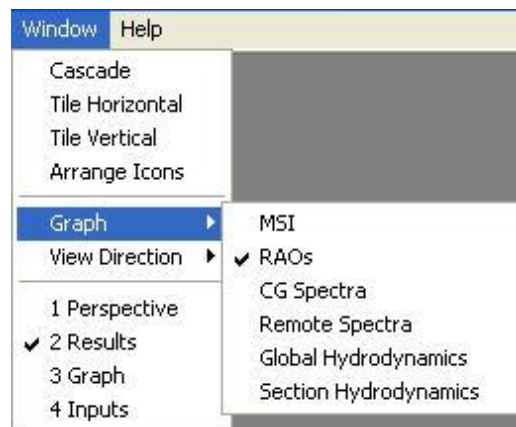

Once a graph has been chosen, the various data sets in the graph may be turned on or off. To do this, select Graph Options from the Display menu. The following dialog will be displayed. If the current graph is of the response spectra, it will not be possible to change the frequency scale.
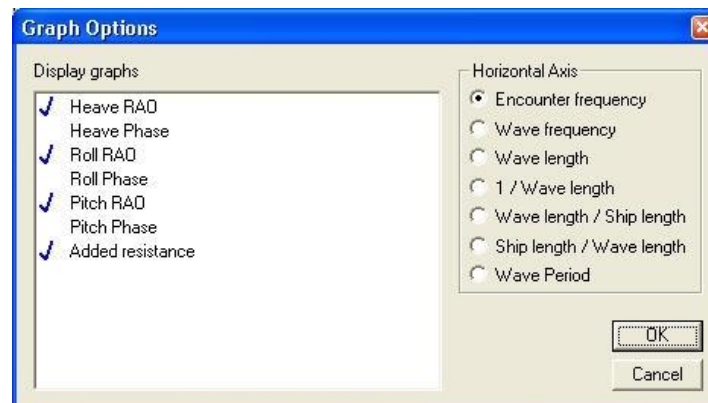
- **Turn on the desired data sets; and**
- **Select the desired horizontal axis format.**

The section data (Sectional forces and Sectional hydrodynamic coefficients) may be plotted and displayed against frequency (for a single section) or against section position (for a single frequency). In addition it is possible to display all the data in the Results datasheet, but in this case the data will not be plotted. To change the format, choose Section Data from the Display menu:
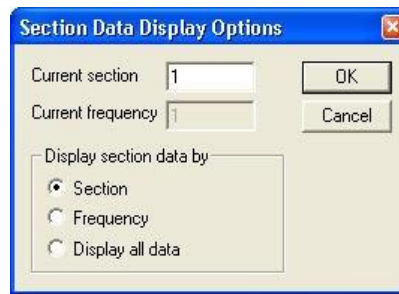
To plot data for a single section, select the Section radio button and type the number of the section for which you want the data in the Current section edit box. To plot the variation along the length of the hull, for a single frequency, select the Frequency radio button and type the frequency of interest in the Current frequency box. To display all the data, select the Display all data – note that, in this case, the data will be displayed in the table only and not in the graph.

Sections start with number 1 at the bow and increment towards the stern. In Plan or Profile views, the station numbers are displayed once the hull has been measured.

Some of the graphing options may be accessed more quickly using the Graph toolbar:

Use the combo box to select which data you would like to graph and the Graph Options button, to edit which lines are shown and the horizontal scale.
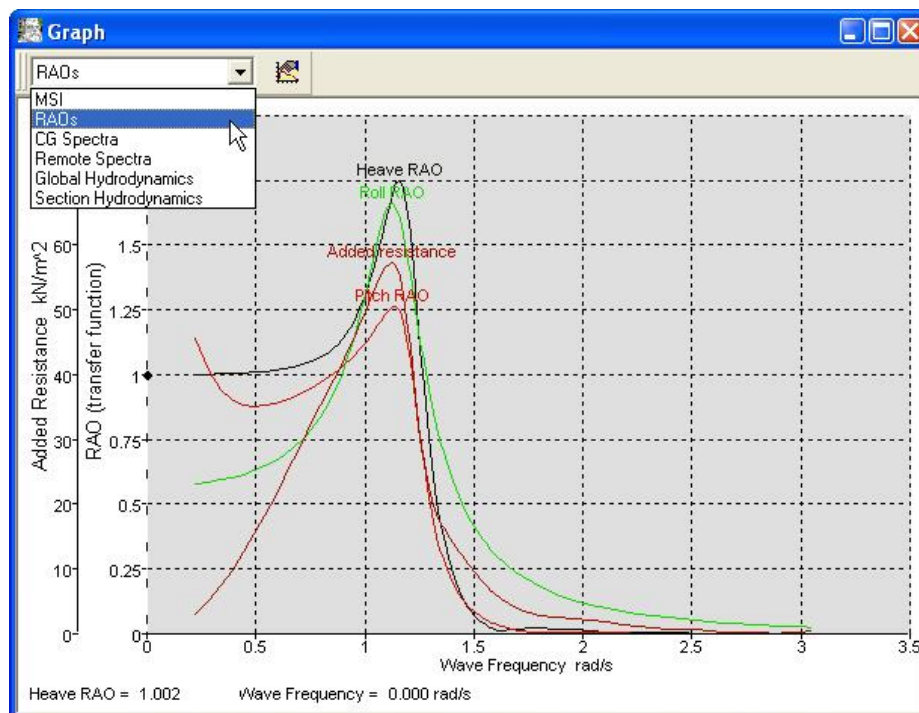

The vertical axis for angular RAOs can be non-dimensionalised either by wave slope (generally preferred by Naval Architects) or by wave amplitude (generally preferred by offshore industry):
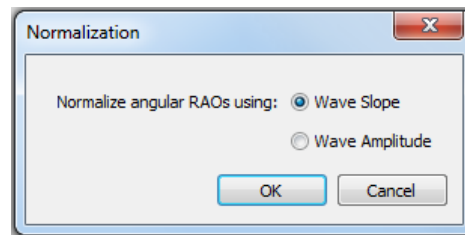
The dialog is opened from the Non-dimensionalisation options from the Settings menu.

Polar Plots

Two polar views of the data are available. Both have angular coordinates representing the wave heading. The first (Polar window) has radial coordinates corresponding to ship speed and the curves are iso-contours of the selected vessel response. The second (Polar Plot window) has radial coordinates corresponding to the selected vessel response and a curve for each of the speeds tested
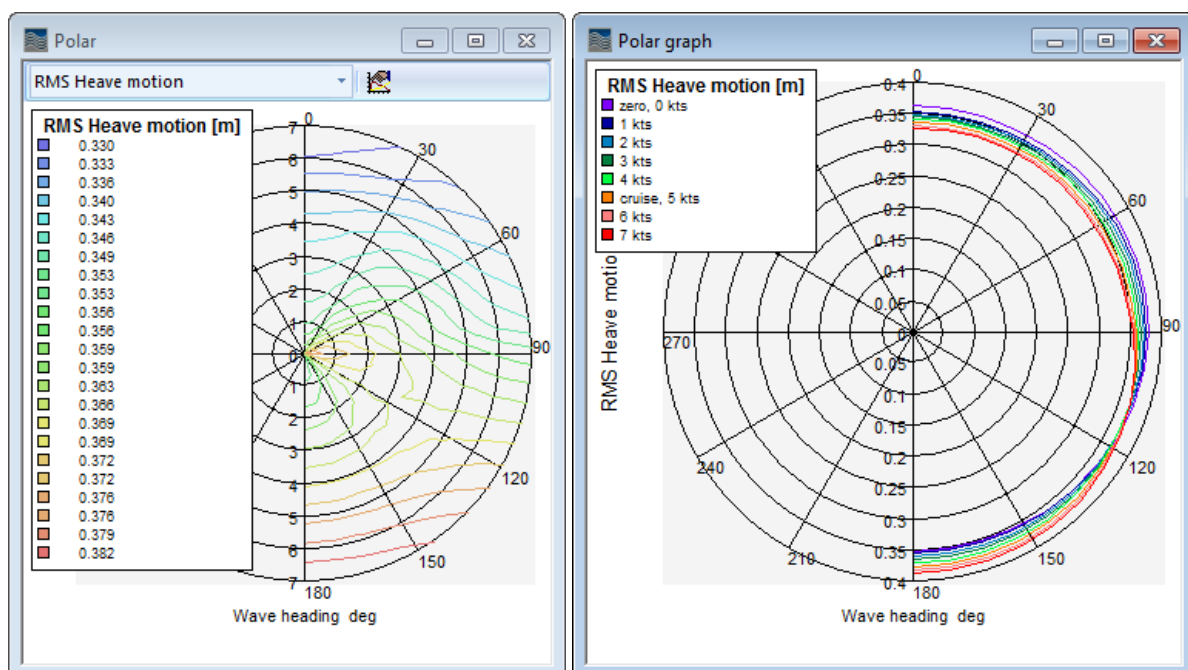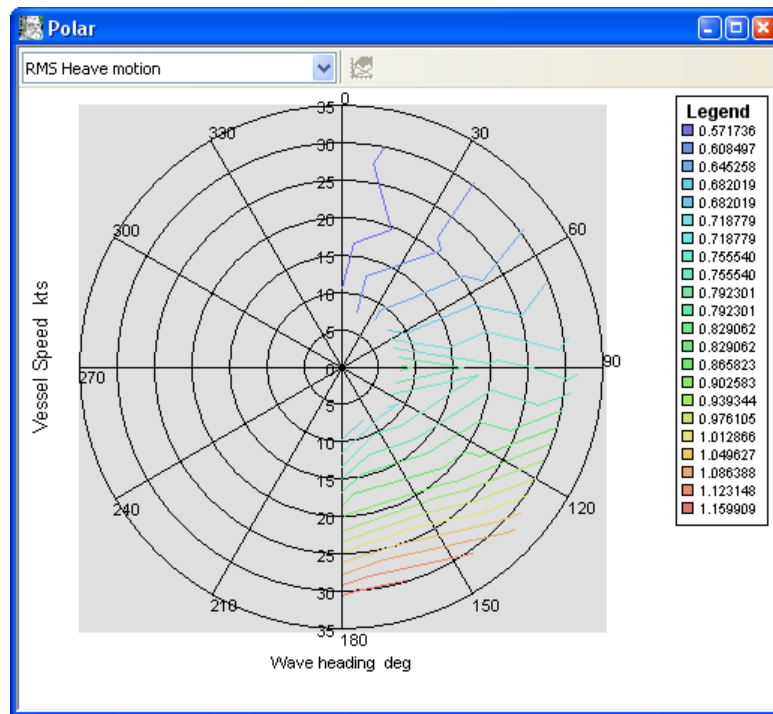

Polar plots of selected result data

Polar Iso-Response Contours

The first Polar Window displays iso-response contours. The global vessel response to be plotted is selected from the drop-down list in the toolbar. At least two different speeds and two different headings (which are separated by less than 180 degrees) must have been analysed. Because the polar plots require the speeds and headings to be ordered, these will be sorted at the start of the analysis.

The spectrum for which the results are plotted is selected from the Results toolbar. The concentric circles correspond to the range of speeds covered and the radial arms the vessel heading; stern waves (0 degrees) is at the top and bow waves (180 degrees) is at the bottom.

Polar Individual Response Curves

Polar Graph window shows the same selected result data plotted on the radial axis (against heading) with one curve for each of the speeds tested.

Rendered view

Motions is able to display a rendered 3D view of the model. If an analysis has been run and the wave surface calculated (Analysis | Calculate Wave Surface…) then the wavey free surface will also be included in the render. In the Perspective view, choose Display | Render, or click on the render button.
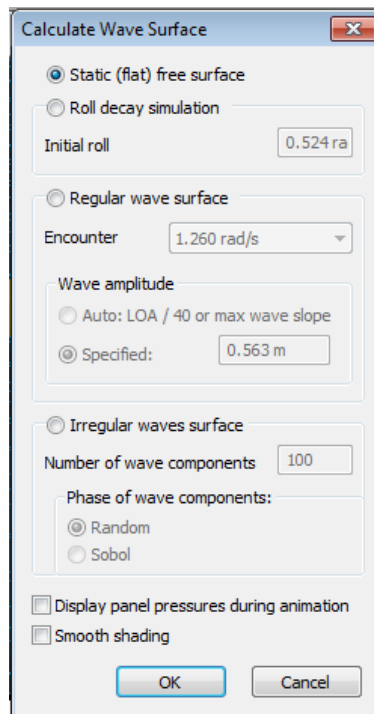
Calculate Wave Surface

If there are a set of valid analysis results open then the Analysis | Calculate Wave Surface function is available. This function will open the following dialog:
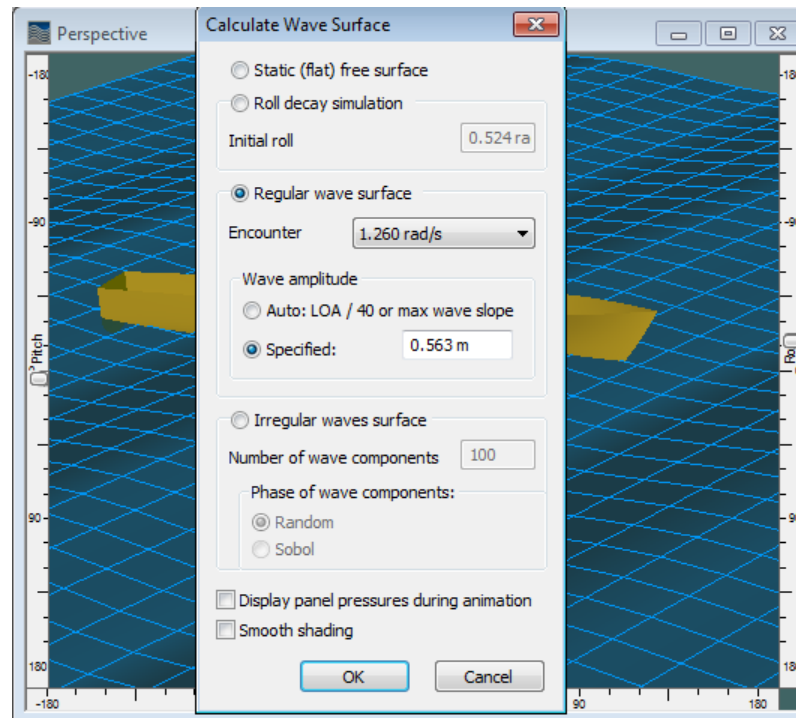
There are three options for the free surface: static (flat) and, once the analysis has been performed, regular and irregular wavy surfaces may be used. At the bottom of the dialog are some options for displaying of panel pressures during an animation in a regular or irregular seaway. If the "Smooth shading" option is shown the pressures will be "smoothed" over the panels, if smooth shading is not checked then the actual calculated pressure for the entire panel is rendered. The panel pressure options are not applicable to the strip theory method.

Static (flat) free surface

With this method selected, no free surface elevation is calculated.

Static (flat) free surface
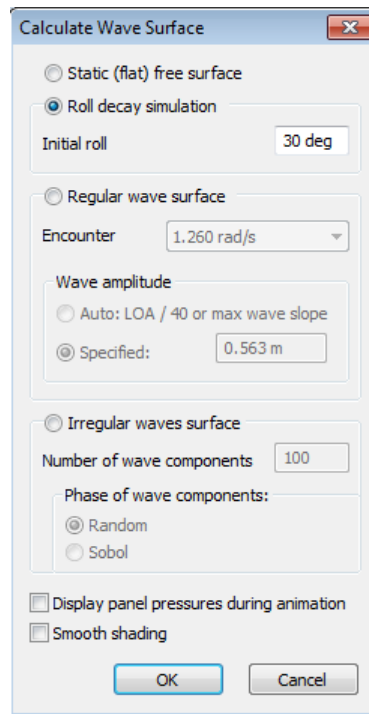
Roll decay simulation

With this method selected, no free surface elevation is calculated, however when you select Animate from the Display menu, a simulation of a free roll decay test will be displayed. This simulates the roll motion of a vessel heeled over to a specified initial roll angle and then released. The initial heel angle is held for 5 seconds, then the vessel is released.

To conduct a roll decay simulation, follow these steps:

- **Make sure you have specified all Seakeeping input data correctly by working your way down the analysis menu**
- **Solve Seakeeping Analysis from the analysis menu**
- **Calculate Wave Surface from the analysis menu**
- **Select Roll decay simulation in the dialog that appears**
- **Specify the initial heel angle**
- **Select Animate from the display menu (make sure you are in the Perspective View)**
- **Click OK**

Note:

Saving the roll decay as a text file and then importing into Excel may help you further analyse the Roll decay simulation data.

Roll decay simulation

Note:
Because the roll motion is considered uncoupled from the heave and pitch motions, no heave or pitch motions are generated.

Regular wave surface

With this method selected, a single frequency and amplitude regular wave pattern is calculated. The wave frequency is one of the discrete wave frequencies for which the motions have been solved. The wave amplitude (half wave height) may be either automatically calculated (LOA / 40 or limited by maximum wave slope for non-breaking waves: $\text{amplitude}_{max} = \lambda/14$ for short wave lengths) or may be specified by the user.
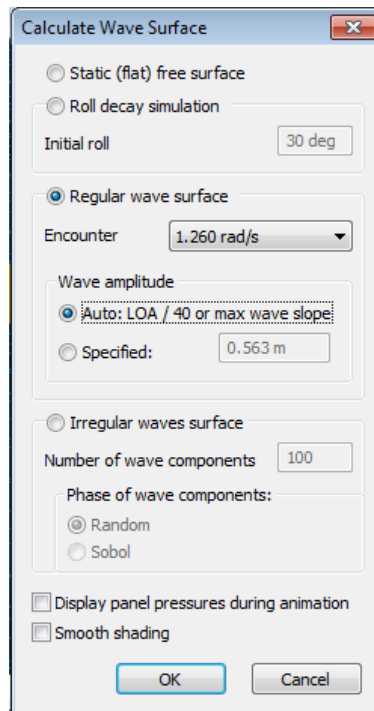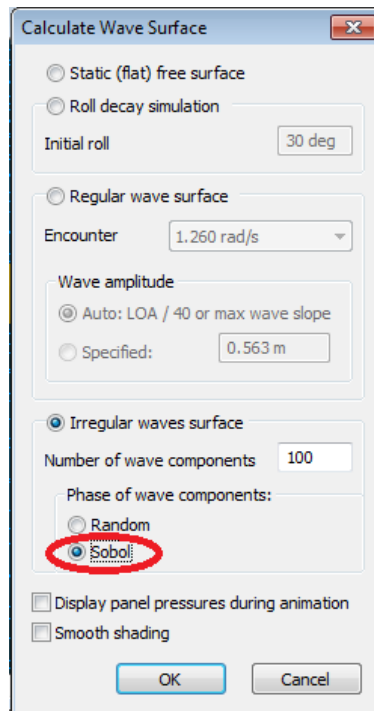
Regular wave surface

Irregular wave surface

The final option allows an irregular wave pattern to be calculated based on the selected wave spectrum. The irregular wave pattern is calculated by superimposing linear regular sine-wave components. The number of wave components to be used can be changed by the user. The wave components are chosen so that the wave energy in each frequency is the same; each component is then given a random phase. If you have a slower computer, it may be necessary to reduce the number of wave components used; increasing the number of wave components will increase the detail with which the wavy surface and vessel motions are calculated.

Either Random or Sobol quasi-random distribution can be used for the wave component phases. The Sobol sequence is quasi-random but will always be the same. This gives a repeatable random sea state which allows the time simulation to be carried out for different models *but* using the same wave-trace. The Random option uses the computer's random function to generate a wave trace that will be different every time.

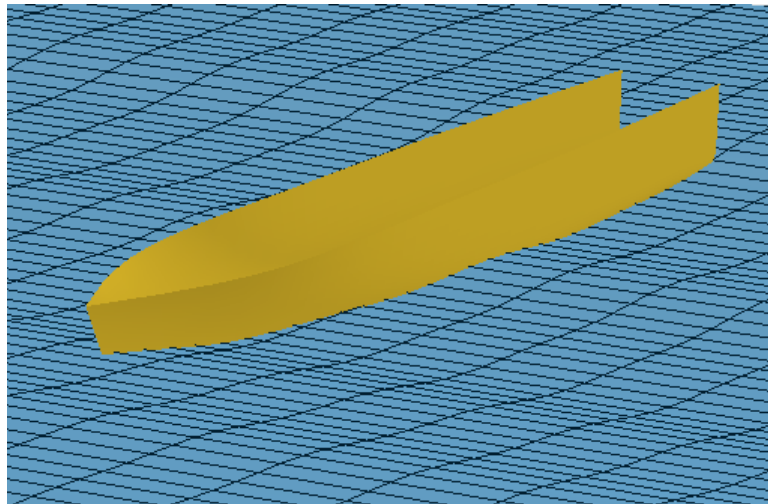Irregular wave surface

Wavy surface display

Once the wavy surface has been calculated it may be displayed. A grid representing the wavy surface may be turned on by selecting the Wave Surface option in the Display | Visibility dialog:
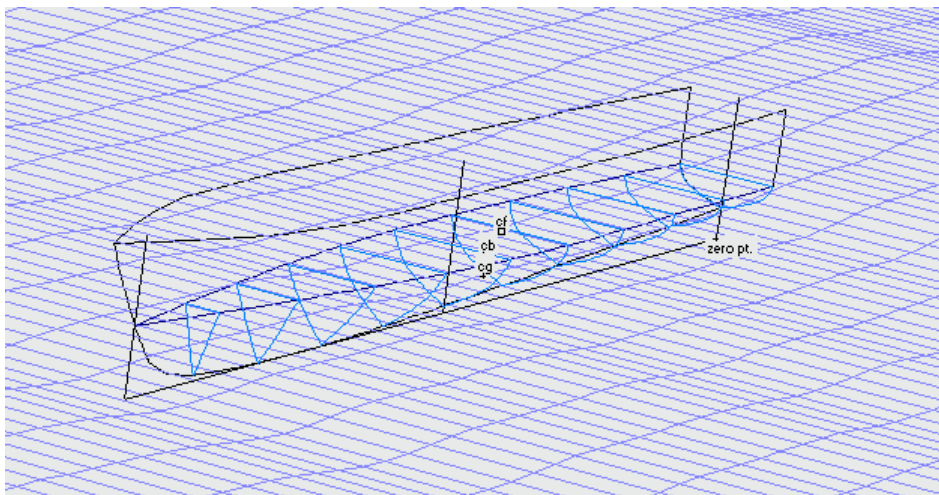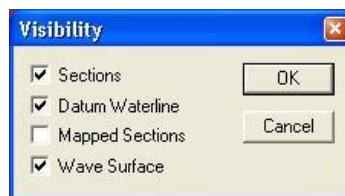
Note
The wavy surface grid uses the Net colour as defined in the View	Colour dialog.

The wavy surface may also be displayed by turning on the rendering (Display | Render).

In either display mode, the wavy surface grid may be turned on or off in the Display | Visibility dialog (as described above) or by using the Wave Grid button in the Contours toolbar: .

Time Simulation and Animation

After calculating the wave surface (see Rendered view on page 50), Motions is able to animate the vessel for the specified initial roll decay angle or in the selected wave pattern. Choose Display | Animate to start the animation and use the Escape (Esc) key to terminate the animation.
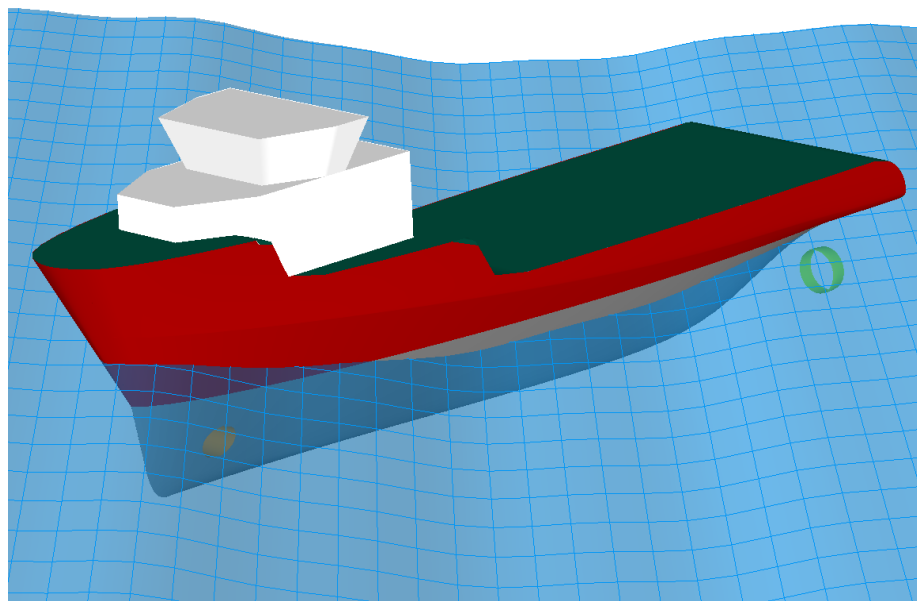
Animation of the vessel motions may be displayed when either a regular or irregular wave surface has been calculated. If you try to animate the model with a flat free-surface, you will get the following error message:
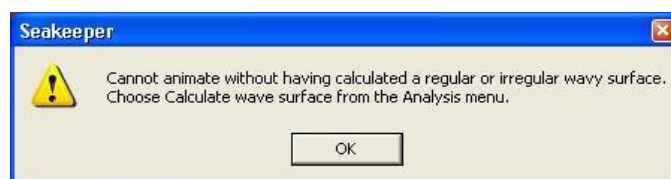
If a wavy surface has been calculated, the animate dialog will be displayed; this gives you the option of saving the time series data to a text file. The text file contains the statistics and time series for wave elevation and heave, roll and pitch motions.
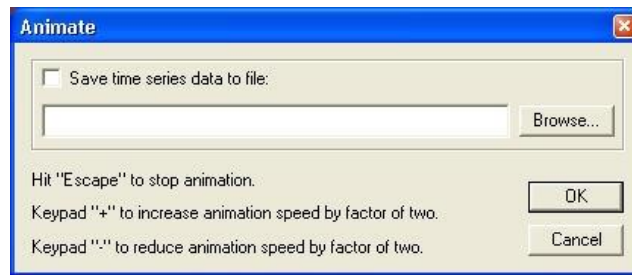
By default, the animation is displayed in real time; the frame rate is adapted so that the animation is in real time. If you have a slow computer or graphics card (hardware accelerated OpenGL graphics cards will generally increase the frame rate) then the frame rate may be too low to provide a reasonable animation. If this is the case you can reduce the attempted play back speed; use the keypad "-" key to slow down the animation which will increase the frame rate, or use the keypad "+" key to speedup the animation which will reduce the frame rate.

The instantaneous time, heave, roll and pitch are displayed in the status bar when the animation takes place:
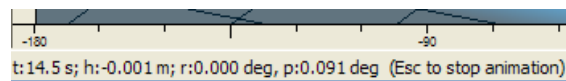
If the panel pressures are being displayed a legend will appear in the top right of the rendered view, displaying which colours correspond with which panel pressure:
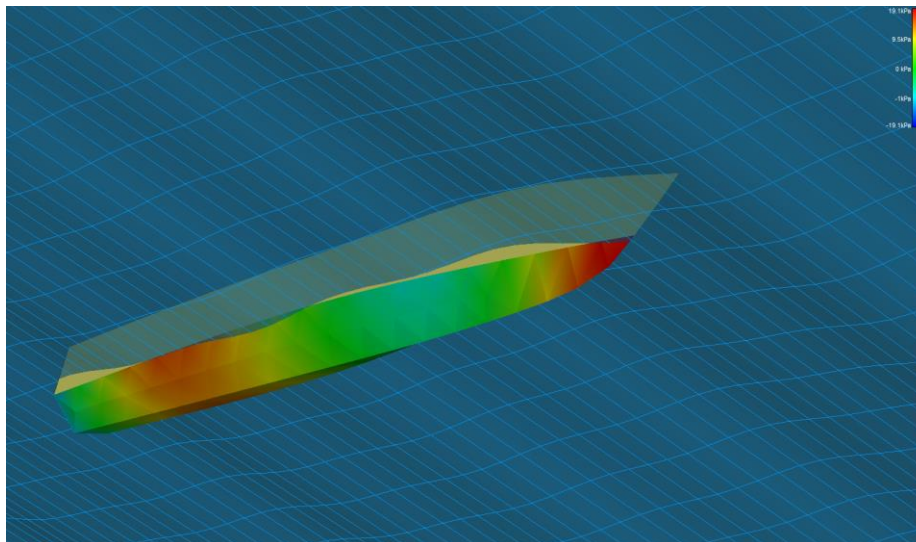
Vessel heading, speed and wave spectrum

The vessel heading, speed and wave spectrum used in the animation is selected from the pull down boxes in the Results toolbar (only results that have actually been evaluated may be chosen):

Saving the Motions Model

After loading the model and preparing and running the Motions analysis, the Motions specific model and input data can be saved into a separate .skd file.

Choose "Save Motions Data" from the file menu. This will save all input data as well as all analysis settings of the Motions model.

Also see:

Saving the Seakeeping Analysis Results

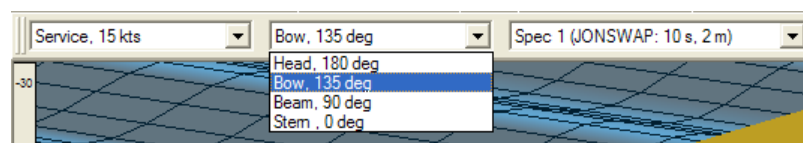You can save each results table separately, or you can save all analysis results, including the Motions input data, in one file.

Saving one Results Table

It is possible to save the results from Motions. Select the table that you wish to save in the Results window, then select "Save *** Results As…" from the file menu:
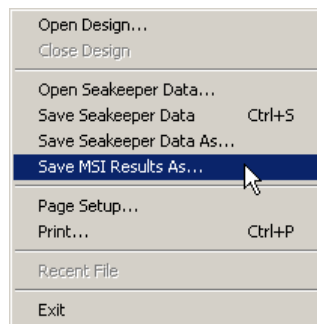
Saving MSI results from the Results window with the MSI table selected.

Motions will step through all the combinations tested and save them in a tab delimited text file. This can take some time, depending on the number of combinations tested; you will see the Results toolbar updating as the different combinations are saved.

Saving and re-opening all Motions results

It is also possible to save and re-open Motions *all* analysis results. To save the results of an analysis choose: File | Save Motions Data As… and select "Motions analysis results (*.skr)" as the file type.
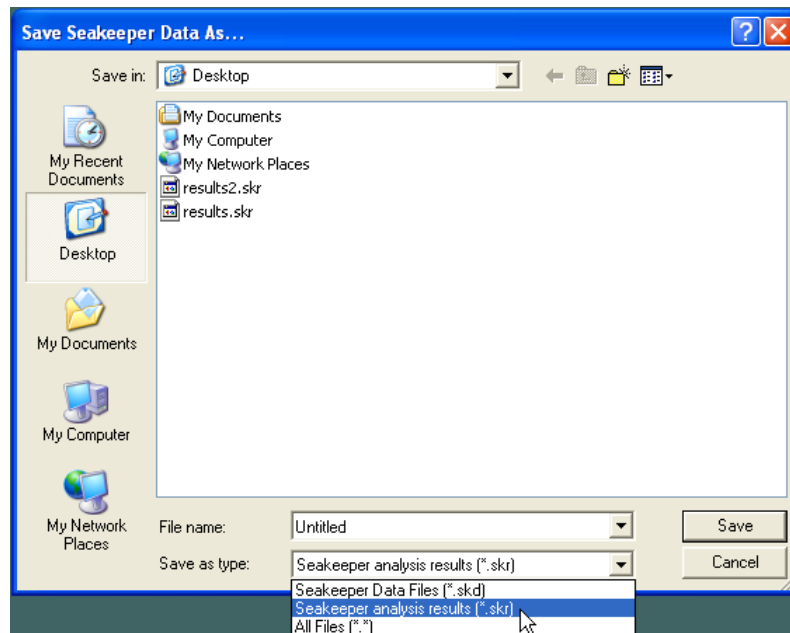
Choose "Motions analysis results" as the file type

Warning!
These files can be quite large, requiring approximately 1.5Mbytes per heading/speed/spectrum combination, depending on the number of remote locations, and frequencies analysed. Thus 5 speeds, 10 headings and 5 spectra would result in a file of about $5 \times 10 \times 5 \times 1.5$Mbytes = 375Mbytes.

To re-open the data at a later date, first open the model, then choose the relevant "Motions analysis results (*.skr)" file from the File | Open Motions Data dialog. Make sure you switch to the Motions analysis results (*.skr) file format in the open dialog.

Results and Reporting to MS-Word

Once the results have been computed, the reporting function will automatically place tables and graphs into a Microsoft Word document (if installed) according to your template document.

In the File menu you will see the Reporting Preferences dialog which enables this option and allows the selection of a report template file.
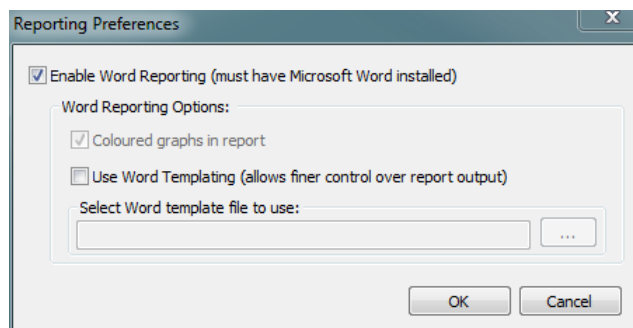
Reporting Preferences dialog

When an analysis has been completed, the results can be sent to the Report with the File | Spool Results to Report command
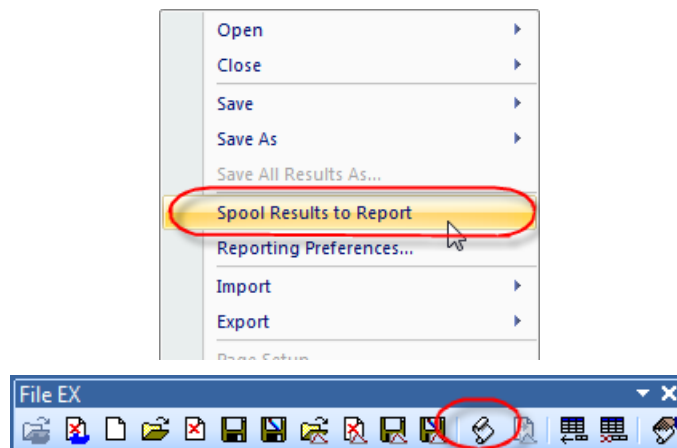
Sending results to the report

Note: Reporting without report template document

If no report template document is specified, then only the summary table and polar graph will be included in the Word document. This is because of the huge volume of data that would be included in the report if all options were "on" by default.

To include other data in the report, you must specify a report template document that includes the keywords for the data and graphs you require.

You can adjust which tables and graphs are included by providing a suitable report template file and adjusting the keywords. All the keyword tags should be within a <Start Results> and <End Results> group:

| Keyword	Function
<Description>	Gives the version of MS Motions, the files being analysed and the analysis method.
<InputTable-Locations>	Input table listing the remote locations of interest
<InputTable-Speeds>	Input table listing the speeds to be tested
<InputTable-Headings>	Input table listing the headings to be tested
<InputTable-Cargos>	Input table listing the cargos (MOSES only)
<InputTable-Spectra>	Input table listing the headings to be tested
<ResultTable-Summary>	Summary results table for each of the speeds, headings and spectra tested
<ResultGraph-Polar>	Polar graph (as currently defined) for each spectrum tested
<ResultTable-RAOs>	RAO results table for each of the speeds and headings tested
<ResultGraph-RAOs>	RAO graph for each of the speeds and headings tested
<ResultTable-RAOsRLoc>	RAO at the remote locations results table for each of the speeds and headings tested
<ResultGraph-RAOsRLoc>	RAO at the remote locations graph for each of the speeds and headings tested
<ResultTable-SpectraCG>	Response spectra tables for the main degrees of freedom for each of the speeds headings and spectra tested
<ResultGraph-SpectraCG>	Response spectra graphs for the main degrees of freedom for each of the speeds headings and spectra tested
<ResultTable-SpectraRLoc>	Response spectra tables for the remote locations for each of the speeds headings spectra and remote locations tested
<ResultGraph-SpectraRLoc>	Response spectra graphs for the remote locations for each of the speeds headings spectra and remote locations tested
<ResultTable-MSI>	Tables of MSI at all remote locations for each of the speeds headings and spectra tested
<ResultGraph-MSI>	Graphs of MSI at all remote locations for each of the speeds headings and spectra tested
< ResultTable-	Table of Strip Theory global hydrodynamic coefficients for each

GlobalHydrodynamicsST>	speed and heading tested
< ResultGraph-GlobalHydrodynamicsST>	Graph of Strip Theory global hydrodynamic coefficients for each speed and heading tested
<ResultTable-SectHydrodynamicsST>	Table of Strip Theory section hydrodynamic coefficients for each speed and heading tested (using current table layout)
<ResultGraph-SectHydrodynamicsST>	Graph of Strip Theory section hydrodynamic coefficients for each speed and heading tested (using current table layout)
<ResultTable-PressuresPM>	Table of Panel Method pressures
<ResultTable-DriftForcesPM>	Table of Panel Method drift forces
<ResultTable-WaveForcesPM>	Table of Panel Method wave excitation forces
<ResultTable-AddedMassDampingPM>	Table of Panel Method added mass and damping
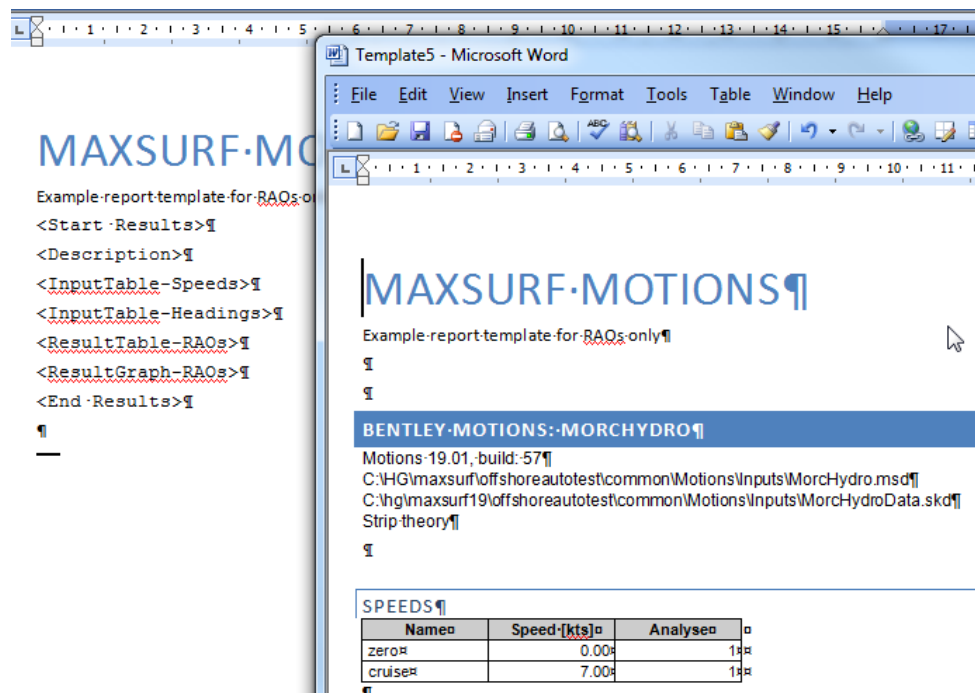
Simple report template and output

Limitations and Guidelines (Strip theory)

The following section describes some of the limitations inherent in the strip theory method and those which are specific to Motions.

Underlying Assumptions

The basic principle behind strip theory is that the hydrodynamic properties of a vessel (that is added mass, damping and stiffness) may be predicted by dividing the vessel into a series of two-dimensional transverse strips, for which these properties may be computed. The global hydrodynamic values for the complete hull are then computed by integrating the two-dimensional values of the strips over the length of the ship. The interested reader is directed to the paper by Salvesen et al. (1970) for a full description of strip theory.

Linear strip theory assumes the vessel's motions are linear and harmonic, in which case the response of the vessel in both pitch and heave, for a given wave frequency and speed, will be proportional to the wave amplitude.

Lloyd (1989) states that linear strip theory makes the following additional assumptions:

- **The fluid is inviscid – viscous damping is ignored, (In fact the damping factor which the user enters for roll should include viscous roll damping, which is the primary source of damping for roll.)**
- **The ship is slender (i.e. the length is much greater than the beam or the draft, and the beam is much less than the wave length).**
- **The hull is rigid so that no flexure of the structure occurs.**
- **The speed is moderate so there is no appreciable planing lift.**
- **The motions are small (or at least linear with wave amplitude).**
- **The ship hull sections are wall-sided.**
- **The water depth is much greater than the wave length so that deep water wave approximations may be applied.**
- **The presence of the hull has no effect on the waves (Froude-Krilov hypothesis.)**

In addition, the principle of superposition is used to compute the response of the vessel to irregular waves. Simply, this means that the contributions to the motions of the individual regular waves making up the irregular sea spectrum may be summed to obtain the total response of the vessel to the irregular seas.

Boyd (1995) outlines possible causes of non-linear behaviour, or effects that may cause strip theory to become inaccurate, such as:

- **Emergence or submergence of the bow or stern.**
- **Flare (non-wall sidedness) of the hull at the waterplane.**
- **Submergence of the bow or stern overhangs in a vessel.**
- **Three dimensional flow effects and flow interaction in a longitudinal direction along the hull including dynamic lift at speed.**
- **Increasing importance of wave diffraction, wave radiation and the Kelvin wave pattern at speed.**

At present, Motions is based upon linear theory, however, in the future, it is intended to include options to account for some of the geometric non-linearities mentioned above.

Asymmetrical sections

Whilst not being able to actually represent asymmetrical hulls in the hydrodynamic model, Motions can approximate asymmetric hulls by correctly measuring the sections' cross-sectional area and waterline beam. This is shown for the asymmetric catamaran demihull below. Here the asymmetrical hull sections are shown in black and the mapped symmetrical sections are shown in red.
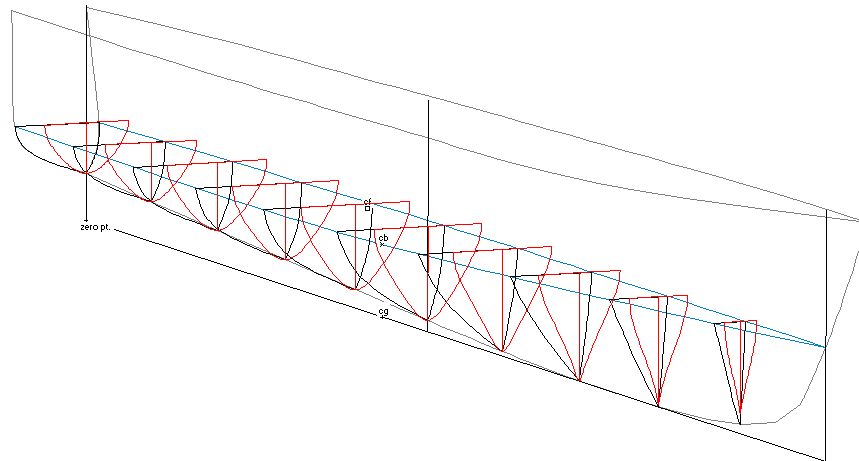
See Calculation of Mapped Sections on page 37 for information on mapping sections.

Vessel Speed

Generally strip theory is applied to vessels operating at low to moderate Froude number. However, experience has shown that reasonable predictions may be made for Froude numbers approaching 0.8, provided that the vessel is quite slender. It should also be remembered that dynamic vertical forces due to forward speed are not included, hence Motions is likely to over-estimate the motions of planing vessels operating in the planing regime.

The Panel method is based on a zero speed Green's function and as such is only valid for the zero forward speed case. A non zero forward speed may be set however the user will get a warning when the analysis is started. Running the Panel method analysis at speeds other than zero will conduct the seakeeping analysis at the modified encounter frequency. This approximation for low speeds (Fn<~0.1) may be a valid approach for some applications.

Wave Heading

At present Motions uses strip theory to predict vertical plane motions. The roll motion is simply estimated assuming a damped spring/mass system. The computations have been found to be most reliable from head seas (180deg.) to bow-quartering seas (135, 215deg.). For these calculations, the head seas approximation for the wave excitation force and moment is normally sufficiently accurate. As the wave heading moves more to beam seas, the arbitrary wave heading method should be used to calculate the wave excitation. However, due to the lack of available data, this formulation has not been rigorously tested.

Strip theory calculations for following seas are less accurate, since the motions become much less linear. Calculations made with Motions (or any frequency domain, strip theory program) for these conditions, should be treated with caution and, if possible, validated by other means.

Because of the non-unique mapping of encounter frequency to wave frequency in following waves (i.e. in some cases it is possible to get the same encounter frequency from three different wave frequencies) MSI calculations in following seas are unreliable.

Chapter 3 Motions Reference

This chapter describes the windows, toolbars and menu commands available in Motions.

- **Windows**
- **Toolbars**
- **Menus**
- **Ribbon**

Windows

Motions uses the graphical view windows to display the Motions model. Please refer to the Modeler (or Hull Modeler) manual for more information on the view windows. Specific Motions windows are described below:

- **Inputs Window**
- **Results Window**
- **Graph Window**
- **Polar Window**
- **Polar Graph Window**
- **Curve of Areas Window**

Inputs Window

This window has four tables:

Locations

This is where the remote locations positions are specified.

Speeds

This is where the vessel speeds are specified.

Headings

This is where the wave headings are specified.

Spectra

This is where the wave spectrum characteristics are specified.

Cargo

This is where you can import SACS topsides models as cargos.

Use the Add and Delete Remote Locations functions in the Analysis menu to add or delete remote locations; when deleting select the entire row of the remote location to be deleted. You may define any number of each variable, place a tick in the "Analyse" column of the rows you wish to include for analysis.

Results Window

This window displays the computed data in tabulated form. The data are arranged on separate data sheets. The following datasheets are available:

Summary

m_0, RMS and significant amplitude of motions, velocities and accelerations of the vessel for the specified sea condition.

MSI

Motion Induced Sickness data for the specified remote locations

RAOs at CG

Vessel RAOs for the principal degrees of freedom, at the vessel's centre of gravity, for the specified speed and heading.

Spectra at CG

Wave and response spectra for the principal degrees of freedom, at the vessel's centre of gravity.

Spectra at remote location

Wave and response spectra for the principal degrees of freedom, at the vessel's centre of gravity.

Global hydrodynamic coefficients

Global coefficients, cross-coupling terms and excitation forces in the equations of motion.

Sectional hydrodynamic coefficients

Added mass, damping, stiffness and wave excitation for each of the sections at each of the frequencies of interest.

Panel Pressures (not applicable to strip theory)

Panel pressure magnitudes and phases for each panel at each of the headings and frequencies of interest. Including incident, diffraction and radiation components.

Drift Forces (not applicable to strip theory)

Drift forces and moments in each degree of freedom for frequencies and headings of interest.

Wave Excitation Forces (not applicable to strip theory)

Wave excitation forces and moments (and phases) in each degree of freedom for frequencies and headings of interest.

Added Mass and Damping (not applicable to strip theory)

Added Mass and wave damping matrix for frequencies and headings of interest.

Depending on which table is selected, you may use the pull-down menus in the Results toolbar to select which data are displayed:


Graph Window

The above data may also be displayed in graphical form. To display the data in graphical format choose Graph from the Window menu or click on the graph button.

The different data sets within the datasheet may be turned on or off by selecting Graph Options from the Display menu. The x-axis scale may also be changed in this dialog.

The Graph window Toolbar allows you to quickly select which graph is displayed and adjust which lines are drawn on the graph:
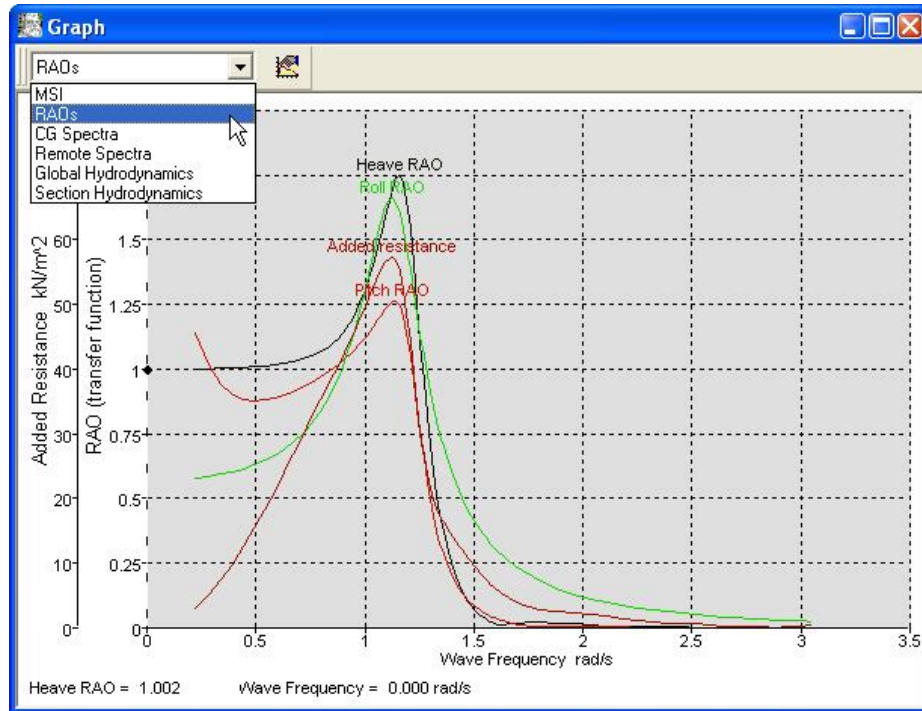
Sectional data may be plotted against frequency or section position. Select Section Hydrodynamics from the drop down menu in the toolbar.

Coordinate data may be viewed by clicking on the curve of interest. The slider may then be dragged left and right; the coordinates are displayed in the bottom left-hand corner of the window.

Also see
> Graphing Results on page 45
> Results Toolbar on page 68
> Graph window Toolbar on page 68

Polar Window

This window shows iso-response contours of a selected statistical measure of the vessel's motions for a selected wave spectrum. These iso-contours are traced through a surface which comes out of the screen and corresponds to the magnitude of the response; the radial axis is the vessel speed and the angular axis the wave heading. See Polar Plots on page 49 for information on the Polar Plot graph.

Polar Graph Window

This window shows individual response contours for each speed of a selected statistical measure of the vessel's motions for a selected wave spectrum. The radial axis is the response value and the angular axis the wave heading. See Polar Plots on page 49 for information on the Polar graph.
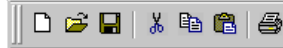
Sectional Area Curve

Shows a plot of the longitudinal variation of immersed cross-sectional area of the vessel at the analysis draft, trim and heel.

Toolbars

Some of the features of Motions can be accessed using the icons on the toolbars. You can hold your mouse over an icon to reveal a pop-up tip of what the icon does.

File Toolbar

The File toolbar contains icons that execute the following commands:
New – Open Design file– Save Motions Input data| Cut - Copy - Paste | Print

View Toolbar

The View toolbar contains icons that execute the following commands:
Zoom – Shrink – Pan – Home View – Rotate – Assembly window (disabled).

The Rotate command is only available in the Perspective window. The Assembly window is not available in Motions.

Analysis Toolbar

The Analysis toolbar contains an icon that starts the analysis: Solve Seakeeping Analysis

Contour Toolbar
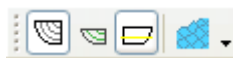
The Contours toolbar contains icons that facilitate the display of surface contours:
Sections – Waterlines – Mapped Sections | Wave Grid

Window Toolbar

The Window toolbar contains icons that make the corresponding window come to the front:
Perspective – Plan – Profile – Body Plan – Input Data – Results Data – Results Graph – Polar Plot – Sectional Area Curve

Results Toolbar
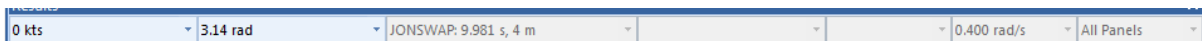
The Results Window toolbar contains icons that allow you to quickly choose which analysis run is displayed. Menus that are irrelevant to the currently active table are greyed out and unavailable for selection:
Speed – Heading – Spectrum – Remote Location – Section – Encounter Frequency – Panel Number

Graph window Toolbar
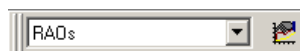
The Window toolbar contains icons that facilitate swapping between the different graph types. Use the Results toolbar to select the data for the desired analysis run and dataset, for example: using the results toolbar you can display the remote spectrum graph for the Bridge Centre for a JONSWAP 10s 4m headseas condition at 15 kts. See the Results Toolbar paragraph above for more information.

Also see Graphing the Results on page 45

Menus

Motions uses the standard set of menu commands for File, Edit and Window operations. It also has a range of menus for inputting the design data, completing the analysis, and control of data and view displays.

- **File Menu**
- **Edit Menu**
- **View Menu**
- **Analysis Menu**
- **Display Menu**
- **Data Menu**
- **Window Menu**
- **Help Menu**
- **Bentley Cloud Services Menu**

File Menu

The File menu contains commands for opening and saving files, and printing.

Open Design

Motions uses a Modeler (or Hull Modeler) design to determine the hull parameters necessary for the seakeeping calculations. Select Open Design to open an existing design.

Open Motions Data (*.skd)

Opens a file containing the main run options and environment conditions. To re-generate the seakeeping data, it is necessary to re-run the analysis. If an *.skdr file is opened when there is no model currently loaded Motions will search for the *.msd file with the same file name and load it.

Open Motions Results (*.skr)

The *.skr file contains all the results information including RAOs and spectral results. If an *.skr file is opened when there is no model currently loaded Motions will search for the *.msd file with the same file name and load it.

Close Design

Select Close Design when you wish to finish with the current design.

Close Motions Data (*.skd)

Closes the file containing the main run options and environment conditions including design mesh, mapped sections and mass distribution.

Close Motions Results (*.skr)

This closes all the results information. The user will be warned that all results will be deleted if not saved.

Save Motions Data (*.skd)

Saves a file containing the main run options and environment conditions.

Save Motions Results (*.skr)

Saves a file containing all the currently loaded results including RAOs and spectral analysis results.

Save Motions Data As (*.skd)

Saves the Motions run options file with a new name.

Save Motions Results As (*.skr)

Saves the Motions results file with a new name

Save Results As

Saves the results data from the selected Results table for all the analysis runs to a tab-delimited text file. Save Summary Results As; Save MSI Results As; Save CG RAOs Results As; Save CG spectra Results As; Save Remote location spectra Results As; Save Global hydrodynamic coeffs. Results As; Save Sectional hydrodynamic coeffs. Results As

Spool Results to Report

This command sends the current results to a report in Microsoft Word (if installed) using the defined template (if selected).

Reporting Preferences

Specifies the preferences for reporting, including the template to be used.

Import RAO text

Imports RAO data generated from an external source directly into Motions RAO data structure. This data can then be used for running a spectral analysis to generate motions without the need for running a strip theory or panel method analysis.

Export bitmap image

Exports a bitmap image from the rendered perspective view.

Export RAO text

Exports RAO data points to text file.

Export SACS TOW

Exports panel methods RAO results data in required format for SACS Tow input files.

Orcaflex RAO text

Exports panel methods RAO results data in required format for input to Orcaflex software.

Orcaflex drift text

Exports panel methods Drift results data in required format for input to Orcaflex software.

Export STL Mesh

If a mesh has been calculated for one of the panel analysis methods (Panel Method, Martec or MOSES) it may be export to an STL file. This can then be opened/altered in a 3rd party application and reloaded back into Motions.

Export Direct X

Exports the current analysis mesh to direct x (*.x) file format

Export Moses

Exports the current analysis mesh to MOSES geometry (*.dat) file format

Page Setup

The Page Setup dialog allows you to change page size and orientation for printing.

Print

Choosing the Print function prints out the contents of the uppermost Motions window.

Exit

Exit will close down Motions and return you to the Desktop.

Edit Menu

The Edit menu contains commands for working in the Results window. In addition, graphs or design views may be copied to the clipboard

Copy

Copy may be used to copy data from the Results table. This may then be pasted into a spreadsheet for further analysis and saved. To copy the cell headings, hold down the Shift key while selecting copy. It may also be used to copy the contents of one of the view windows or graph windows into a picture or metafile that may then be pasted into other programs.

Paste

Paste may be used to paste data into tables that have editable cells (such as the Inputs window).
Copy may be used to copy data from the Results table

Select All

Select all may be used in any of the input or results table to select all the cells

Fill Down

Fill down may be used in any of the editable input tables to fill data from the top cell down.

Add location/speed/heading/spectrum

Used to add locations/speeds/headings/spectrums in the inputs window tables.

Delete location/speed/heading/spectrum

Used to delete locations/speeds/headings/spectrums in the inputs window tables.

View Menu

The View menu contains commands for changing the way in which the drawing and data windows are displayed.

Zoom

The Zoom function allows you to examine the contents of the Perspective, Plan, Body Plan or Profile window in detail by enlarging any particular area to fill the screen.

Shrink

Choosing Shrink will reduce the size of the displayed image in the front window by a factor of two.

Pan

Choosing Pan allows you to move the image around within the front window.

Rotate

Activates the Rotate command, which is a virtual trackball which lets you freely rotate a design in the Perspective view window.

Saved Views

Open the Saved Views dialog where you can add, edit and delete the home and other saved views.

Home View

Choosing Home View will set the image back to its Home View size. Motions starts up with default Home View settings for the view window. However, the Home View may be set at any time by choosing the Set Home View function.

Set Home View

Choosing Set Home View allows you to set the Home View in the View window.
To set the Home View, use Zoom, Shrink, and Pan to arrange the view as you require, then select Set Home View from the View menu.

Colours and Lines

The Colour function allows you to set the colour of lines displayed in the graph and view windows.

Changes to the colours will be saved in the program's preferences and re-loaded when Motions is re-started.

To set the colour of a given line:

- **Select Colour from the View menu.**
- **Select the line type from the scrollable list.**
- **Click in the coloured box on the left.**
- **Select a colour from the colour pallet.**
- **Click OK in the pallet dialog.**

Font

The Font command allows you to set the size and style of text used in the windows.

Toolbar

Toggles the visibility of the toolbars in the popup menu.

Status bar

Toggles the status bar (shown at the bottom of the application) on and off.

Full Screen

Toggles the application between normal and full screen viewing modes.

Analysis Menu

The Analysis menu is used to specify the conditions for the analysis and calculate the vessel's seakeeping characteristics.

Analysis Type

Sets the type of analysis to be conducted. The 5 options are Strip theory, Panel method, User defined RAOs, Martec solver and MOSES solver. Strip theory to be used for longer slender vessels at non zero forward speed, motions only predicted for heave, pitch and roll. The panel methods (Panel Method, Martec solver and MOSES solver) can be used for all vessel shapes at zero forward speed, motions are predicted for all 6 degrees of freedom. The MOSES solver can be used for non-zero fwd speed cases. Depending on which version of Motions is being run some of these analysis options will not be available. If the user has custom RAOs from another analysis package they may be imported via the File Import RAO text option and a spectral analysis can be conducted on the custom RAOs.

Heel

The vessel heel can be set when conducting a panel method analysis. Specifies the static (equilibrium) vessel heel. This may be used for the panel method analysis only (since the strip theory mappings are always port-starboard symmetric).

Draft and Trim

This command opens the draft and trim dialog. The vessels draft and trim can be set from here. The draft and trim can be specified either by enter a draft and midships and an angle of trim, or by entering the draft at the FP anf AP.

Measure Hull (strip theory)

Allows the user to select which surfaces should be measured, the number of sections to use (typically 10–30 sections should be adequate) and whether to trim the surfaces. This should be checked if the design is constructed from trimmed surfaces. The conformal mappings are automatically calculated immediately after the hull has been measured.

Mesh Hull (panel method)

Automatic meshing of NURBS surfaces from the .msd file. Enabled when panel method analysis is chosen. A dialog will appear in which the user can enter parameters for control over the automatic meshing process. An edit box at the bottom of the dialog is for entering the "characteristic length" this value is used for non dimensionalisation purposes only (see results section).

Vessel Type (strip theory)

Allows the user to specify whether the design should be treated as a monohull or a demihull of a catamaran; if the latter is specified, the demihull separation must also be given.

Mass Distribution

Input the pitch and roll gyradii of the hull in percentage of overall length and beam respectively. Typical values are pitch gyradius 25%; roll gyradius 35%–40%.

Damping Factors (strip theory)

The roll response is calculated based on the vessel's hydrostatic properties, the roll gyradius and the roll damping. The value of total non-dimensional roll damping may be input here. See Setting Damping Factors on page 27.

For the Panel method additional damping to the full damping matrix may be specified, however it would be normal practice just to add damping on the leading diagonal terms (not the cross-coupling terms)

Mooring Stiffness

Additional stiffness terms may be added for the full stiffness matrix. This applies to the Panel Method only.

Environment

The water density is specified here.

Frequency Range

The number of frequency steps may be changed here. This specifies the points at which the vessel's RAOs are calculated. For spectral integrations, a much larger number of points are used, however the RAOs used are based on the specified points, extra points are linearly interpolated as required. If a panel method is chosen as the analysis type then the user can enter the frequency range (min and max) over which the analysis is to be run.

Strip Theory Method (strip theory)

There are various options to control the exact formulation and solution of the strip theory problem. Options for controlling the analysis may be specified here.

Solve Seakeeping Analysis

Activates calculation of the seakeeping data (RAOs, response spectra and significant motions).

Calculate RAOs

Calculates only the RAOs – doenst perform the analysis of the ship's response to the various sea-spectra that have been defined.

Calculate Spectra

Calculates the vessel motions in the defined sea-spectra (requires that the RAOs are up to date).

Display Menu

The Display menu contains commands for turning on and off items displayed in the graphics windows.

Ship Coordinates

If turned on, the vessel will be shown with the baseline parallel to the bottom of the screen and the water inclined (Ship coordinate frame). If turned off, the waterline will be horizontal and the ship inclined (Earth coordinate frame).

Grid

Displays grid visibility options.

Visibility

The visibility option allows you to select which hull contours are drawn on the screen at any given time. If conformal mappings have been calculated these may be displayed.

Surface Visibility

Executing this command will bring up the surface visibility dialog. Here you can toggle the visibility of each surface in the design.

Show single section

Toggles displaying a single analysis section at a time.

Outside arrows

Toggles the surface outside arrow visibility. It is important for the panel methods analyses that the outside arrows are pointing in the correct direction. If they need to be altered then this needs to be done in Modeler (or Hull Modeler) and the mesh recalculated in Motions.

Graph Options

Select this dialog to select which data sets from the current datasheet are visible.

Section Data

In some cases data may be displayed by section or frequency (section forces and section hydrodynamics). When either of these sheets are displayed in the Results or the Graph Windows, you may select the display method and current frequency or section from this dialog.

Render

Displays a 3D rendered view of the vessel in regular waves of the selected frequency. The default wave height is based on a L_{OA} / 80 wave but is also limited by the maximum wave slope. The wave amplitude may be set in the Display | Render or Animate dialogs.

Animate

Creates a time-domain animation of the vessel travelling through waves of the specified frequency. Once all the frames have been generated, use the numeric key-pad + and - keys to adjust the speed of the animation. The time step[s], heave, roll and pitch are displayed in the bottom right of the window.

Data Menu

The Data menu contains commands for setting the preferred units. The vessel draft may also be changed.

Units

A variety of metric and imperial units may be used in Motions. This command gives you a dialog box where you may choose your preferred units.

Irrespective of the default units specified the Markers window will accept dimensional data in any units. For Example, if the default units were metres, all of the following input would be accepted and converted accordingly:

3 (interprets as 3.00 metres)	
2.5c	3.25"
33cm	4 inches
328mm	5'4"

650.44mil   11.25feet

6ft 3.1in   5f5I

Coefficients

Options for how the hydrostatic coefficients of form are calculated.

Frame of Reference

Displays the vessel frame of reference (for information only – vessel draft and trim are set in the Analysis menu).

Calculate Hydrostatics

Calculates the hydrostatics of the design at the specified waterline (including heel and trim).

Window Menu

Selecting item in the Window menu brings the appropriate window to the front on the screen.

Cascade

This function stacks up all the currently visible windows into the bounds of the screen in a staggered arrangement.

Tile Horizontal

Tiles the open windows horizontally within the application window.

Tile Vertical

Tiles the open windows vertically within the application window.

Arrange Icons

Arranges the closed windows within the application window.

Restore Default Layout

Restores the default layout of the windows.

View direction

Option of switching to Perspective, Plan, Profile or Body Plan views.

Graph

When the graph window is topmost it is possible to choose the datasheet you wish to display in the graph window.

Inputs

Switches focus to the chosen input window.

Results

Switches focus to the chosen results window.

Help Menu

Provides access to Motions help and other resources.

Motions Help

Display the Motions help manual.

Motions Automation Reference

Display the Motions Automation reference help manual.

Online Support

Provides access to a wide range of support resources available on the internet. You must have internet access to access this functionality.

Service Request Manager

Submit and manage your technical support requests with Bentley Systems. You must have internet access to access this functionality.

Check for Updates

Provides access to our website with the most recent version listed. You must have internet access to access this functionality.

News

Display the Bentley Systems Offshore news feed. You must have internet access to access this functionality.

CONNECT Advisor

Display the Bentley Systems CONNECT Advisor. You must be logged in with the CONNECTION Centre to access this functionality.

Legal Notice

Display the Bentley Systems Legal Notice.

About Motions

Displays information about the current version of Motions you are using and other diagnostic information. Use this to obtain version and diagnostic information when reporting a problem to the Support Staff at Bentley Systems, Incorporated.

Bentley Cloud Services Menu

Provides access to Bentley Cloud Services resources. You must be logged in with the CONNECTION Centre to access this functionality.

Associate Project

Register the current file you are working on with a CONNECTED Project.

Disassociate Project

Unregister the current file you are working on from a CONNECTED Project.

CONNECTION Centre

Display your personal CONNECTION Centre web portal in your web browser.

ProjectWise Projects

Display the CONNECTED Project in your web browser.

Ribbon

Some versions of Motions use a Ribbon interface instead of the Menu bar. The Ribbon is divided into Tabs which contain Groups of related commands. According to what tasks you are performing or which part of the application you are working with you will select the appropriate Tab which will normally contain the majority of relevant commands; the Home Tab contains a broad range of common commands. The other ribbon tabs are arranged broadly according to the object being manipulated.
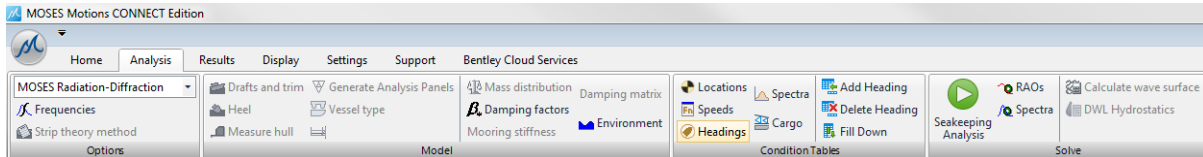
The idea of the Ribbon is to improve productivity, however should you wish to display the menus, this can be done in the Settings Tab, Legacy Group, Toggle menus command.
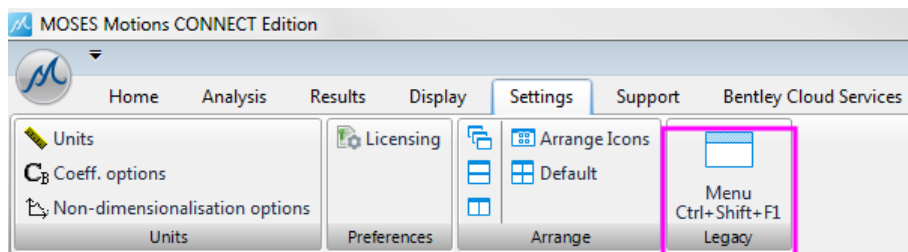
Legacy support: Toggle Menu display

Chapter 4 Theoretical Reference

The chapter contains some additional theoretical background to the Motions program:

- **Seakeeping Theory**
- **Glossary, abbreviations**
- **Nomenclature, symbols and definitions**

- **Bibliography and References**

For more information on Strip Theory, see:

Seakeeping Theory

This Section provides some additional theoretical background to the Motions program.

- **Wave Spectra**
- **ISSC**
- **See Bretschneider (ITTC 2 parameter) spectrum formulation.**
- **What the Vessel Sees**
- **Characterising Vessel Response**
- **Calculating Vessel Motions**
- **Statistical Measures**
- **Probability Measures**
- **Speed Loss**

For descriptions of abbreviations and symbols please refer to and the

Users interested in full details of the strip theory method are referred to and the papers listed in the

Wave Spectra Theory

One of the key concepts in seakeeping analysis is that of wave spectra.

Wave Energy

The average energy, $\overline{E}$, over the wavelength is given by the equation below. It may be seen that the energy is dependent only on the square of the wave amplitude. (This is effectively the mean square of the signal multiplied by a constant – ρg; this draws certain parallels with AC electrical circuits.)

$$\overline{E} = \frac{\rho g \zeta_0^2}{2}$$

Wave Spectra Representation

As has been discussed in the introduction, irregular waves are often described by a spectrum that indicates the amount of wave energy at different wave frequencies. A spectrum is shown by plotting spectral density against frequency; a typical wave spectrum is shown below:
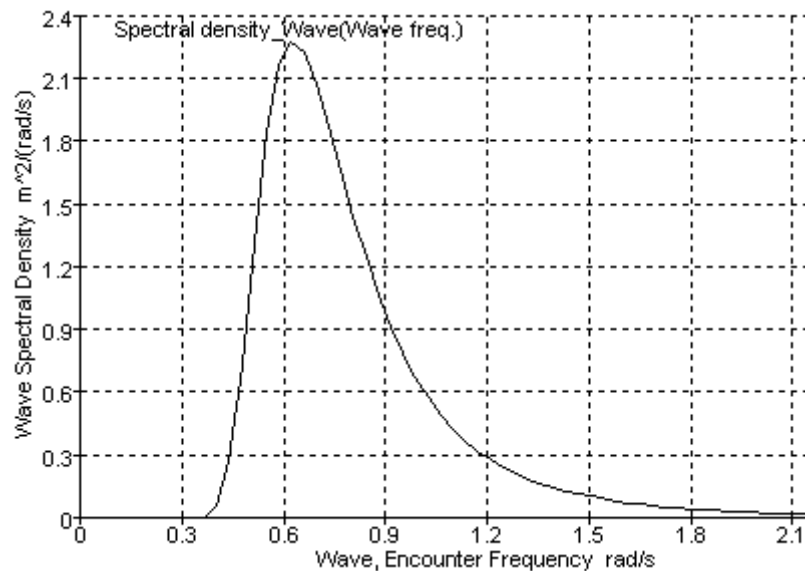
These spectral representations of sea conditions are central to determining the response of a vessel in the seaway. This will be discussed more fully in the following sections.

Spectral Characteristics

There are several spectral characteristics that may be directly related to the time series representation. The key to calculating these characteristics is the spectral moment, m_n. The nth spectral moment is calculated using the equation below; n may take any positive integer value ($n = 0,1,2,\ldots$).

$$m_n = \int_0^\infty \omega^n S_\zeta(\omega)d\omega$$

From these spectral moments it is possible to calculate many of the time series characteristics noted in the Introduction Section.

Of particular importance is the zero[th] spectral moment, m_0, this is equivalent to the area under the wave spectrum curve, which is also the variance of the wave time history.

$$m_0 = \int_0^\infty S_\zeta(\omega)d\omega$$

The RMS or standard deviation, σ_0, is given by: $\sigma_0 = \sqrt{m_0}$

The average period, $\overline{T}$, may be found by calculating the "centre of area" of the energy spectrum; thus the average period is given by:

$$\overline{T} = 2\pi \frac{m_0}{m_1}$$

Note that the modal period, T_0, is the wave period at which the maximum wave energy occurs. For spectra that are defined by continuous mathematical expressions, this may be found by differentiation.

It may be shown that the mean period of the peaks, $\overline{T}_P$, is given by:

$$\overline{T}_P = 2\pi \sqrt{\frac{m_2}{m_4}}$$

and the mean zero crossing period, $\overline{T}_Z$, is given by:

$$\overline{T}_Z = 2\pi \sqrt{\frac{m_0}{m_2}}$$

The bandwidth parameter, ε, may be found using the equation below. If $\varepsilon = 0$ the spectrum is a narrow band spectrum, and if $\varepsilon = 1$ the spectrum is a wide band spectrum. In general it is probably fair to say that most sea spectra are relatively narrow banded. Even if this is simply due to the fact that the very small, high frequency ripples are of no interest in the field of ship motions.

$$\varepsilon = \sqrt{\left(1 - \frac{\overline{T}_P^2}{\overline{T}_Z^2}\right)} = \sqrt{\left(1 - \frac{m_2^2}{m_0 m_4}\right)}$$

Finally, it may be shown that the significant wave height, $\overline{H}_{1/3}$, is dependent on the bandwidth of the spectrum and may be calculated from the equation below:

$$\overline{H}_{1/3} = 4\sqrt{m_0\left(1 - \frac{\varepsilon^2}{2}\right)}$$

In general, it is assumed that wave spectra are narrow banded ($\varepsilon = 0$) and that the significant wave height is given by:

$$\overline{H}_{1/3} = 4\sqrt{m_0}$$

However, if the spectrum is wide band ($\varepsilon = 1$):

$$\overline{H}_{1/3} = 2.83\sqrt{m_0}$$

It is often useful to define idealised wave spectra which broadly represent the characteristics of real wave energy spectra. Several such idealised spectra are available in Motions and are described below:

Note: SI units have been used throughout for the constants in the equations below.

Bretschneider or ITTC two parameter spectrum

The Bretschneider or ITTC two parameter spectrum is defined below:

$$S_{\text{ITTC}\zeta}(\omega) = \frac{A}{\omega^5} \exp\left(\frac{-B}{\omega^4}\right);$$

where:

$$A = 172.75 \frac{\overline{H}_{\text{char}}^2}{\overline{T}^4} \quad \text{and} \quad B = \frac{691}{\overline{T}^4}$$

The two parameters are the characteristic wave height, $\overline{H}_{\text{char}}$, and the average period, $\overline{T}$.

By calculating the various spectral moments it may be shown that:

$$m_0 = \frac{A}{4B}$$

$$\overline{H}_{\text{char}} = 4\sqrt{m_0}$$

$$T_Z = 0.92\overline{T}$$

$$T_P = 0.0$$

$$\varepsilon = 1.0 \text{ (broad band spectrum)}$$

Thus the Bretschneider or ITTC two parameter spectrum is a broad band spectrum and contains all wave frequencies up to infinity. This is why the average period between peaks is zero since there will be infinitesimally small ripples with adjacent peaks. However, in practice the high frequency ripples are neglected and the spectrum will effectively be narrow banded in which case $H_{1/3} \approx 4\sqrt{m_0}$

hence $\overline{H}_{\text{char}} \approx H_{1/3}$.

The modal period may be found by differentiating the wave energy spectrum and finding the maximum (slope = 0), see below:

$$\omega_0 = \sqrt[4]{\frac{4B}{5}} = \frac{4.849}{\overline{T}},$$

where $T_0 = 1.296\overline{T} = 1.41\overline{T}_Z$

One parameter Bretschneider

The one-parameter Bretschneider spectrum is similar to the two parameter spectrum but is defined in terms of wave height only. The spectrum is defined as follows:

$$S_{\text{1Param Bretschneider }\zeta}(\omega) = \frac{A}{\omega^5} \exp\left(\frac{-B}{\omega^4}\right);$$

where: A is the Philp's constant, given by:

$$A = 8.11 \times 10^{-3} g^2 \text{ and } B = \frac{3.11}{H_{1/3}^2}$$

This spectrum may be used when only the significant wave height is known. The associated period is typical of fully developed seas.

JONSWAP

The JONSWAP (JOint North Sea WAve Project) spectrum is often used to describe coastal waters where the fetch is limited. It is based on the ITTC spectrum and defined below. In general both spectra do not contain the same energy for specified significant wave height and characteristic period, however the JONSWAP always has a taller, narrower peak than the ITTC.

$$S_{\text{JONSWAP}\zeta}(\omega) = 0.658 \ S_{\text{ITTC}\zeta}(\omega) \ C(\omega);$$

where:

$$C(\omega) = 3.3 \uparrow \exp\left[\frac{-1}{2\sigma^2}\left(\frac{\omega}{\omega_0} - 1\right)^2\right]$$

with:

$$\sigma = 0.07 \text{ for } \omega < \omega_0; \ \sigma = 0.09 \text{ for } \omega > \omega_0, \text{ and } \omega_0 = \frac{2\pi}{T_0}$$

DNV Spectrum

A more generalised spectrum formulation is used by DNV. Special cases of this spectrum include the Bretschneider spectrum when the peak enhancement factor is 1.0 and the JONSWAP spectrum when the peak enhancement factor is 3.3. The peak enhancement factor, γ, used by DNV is determined from the significant wave height and the modal period:

$$\gamma = 5.0 \quad \text{for} \quad \frac{T_0}{\sqrt{H_{1/3}}} \leq 3.6$$

$$\gamma = \exp\left(5.75 - \frac{1.15 T_0}{\sqrt{H_{1/3}}}\right) \quad \text{for} \quad 3.6 < \frac{T_0}{\sqrt{H_{1/3}}} \leq 5.0$$

$$\gamma = 1.0 \quad \text{for} \quad 5.0 < \frac{T_0}{\sqrt{H_{1/3}}}$$

The spectrum itself is defined as follows:

$$S_{\text{DNV}\zeta}(\omega) = \frac{\alpha}{\omega^5} \exp\left(\frac{-\beta}{\omega^4}\right) \times \gamma \uparrow \exp\left[\frac{-1}{2\sigma^2}\left(\frac{\omega}{\omega_0} - 1\right)^2\right]$$

where:

$$\sigma = 0.07 \text{ for } \omega < \omega_0; \ \sigma = 0.09 \text{ for } \omega > \omega_0, \text{ and } \omega_0 = \frac{2\pi}{T_0}$$

$$\alpha = 5\pi^4 \left(1 - 0.287 \ln(\gamma)\right) \frac{H_{1/3}^2}{T_0^4} \text{ and } \beta = \frac{20\pi^4}{T_0^4}$$

Pierson Moskowitz

The Pierson Moskowitz spectrum may be used to define a spectrum by a nominal wind speed, U_{wind} in m/s at a height of 19.5m above the sea surface.

$$S_{PM\,\zeta}(\omega) = \frac{A}{\omega^5} \exp\left(\frac{-B}{\omega^4}\right);$$

where: A is the Philp's constant, given by:

$$A = 8.11 \times 10^{-3} g^2 \text{ and } B = \frac{0.74 g^4}{U_{wind}^4}$$

Ochi Hubble

The Ochi Hubble spectrum is a formulation that allows the user to specify a two peaked spectrum used to approximate a sea state made of low frequency remotely generated swell combined with high frequency locally generated wind waves. The formulation is given by:

$$S_{OH}(f) = \frac{\pi}{2} \sum_{j=1}^{2} \frac{[4(4\lambda_j + 1)\pi^4 f_{pj}^4]^{\lambda_j}}{\Gamma(\lambda_j)} \frac{h_{sj}^2}{(2\pi f)^{4\lambda_j+1}} \exp\left[-\frac{4\lambda_j + 1}{4}\left(\frac{f_{pj}}{f}\right)^4\right]$$

Where :
f_{pj} represents the modal frequencies of the two component wave trains

λ_j represents the 'peakedness' of the spectrum. This parameter controls the extent to which the spectral energy is concentrated around the modal frequency – larger values give more concentrated component spectra.
h_s represents the the significant wave heights of the component spectra

Graphically a typical spectrum will look like the following:
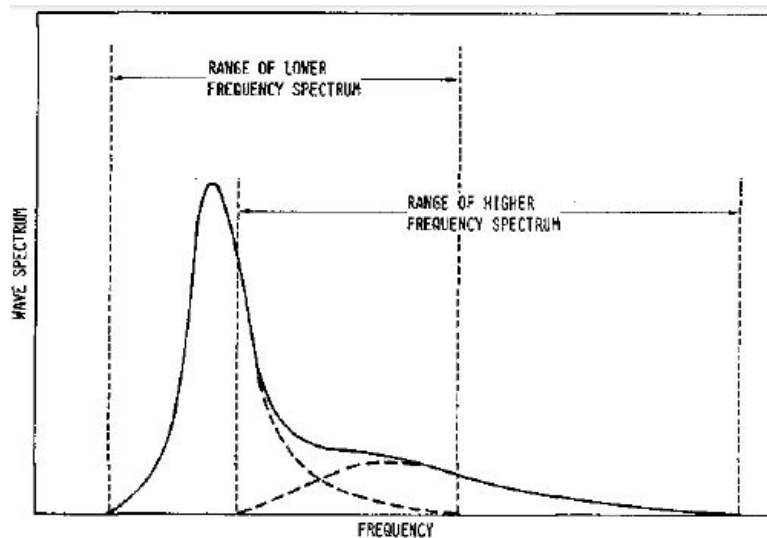
ISSC

See Bretschneider (ITTC 2 parameter) spectrum formulation.

What the Vessel Sees

Due to its forward speed, the vessel experiences a modified wave spectrum, this is explained in the following sections.

Encounter Period

When studying the behaviour of vessels in a sea way it is important to consider the frequency with which the vessel encounters waves (the encounter frequency f_e). The encounter frequency depends on the velocity of the waves, the velocity of the vessel and the heading of the waves in relation to the vessel.

The wave heading, μ, is the angle between the vessel track and the wave direction. Thus following seas correspond to $\mu=0$; since both vessel and waves are travelling in the same direction. Head seas corresponds to $\mu=180°$, and beam seas $\mu=90°$ (starboard) or $270°$ (port).
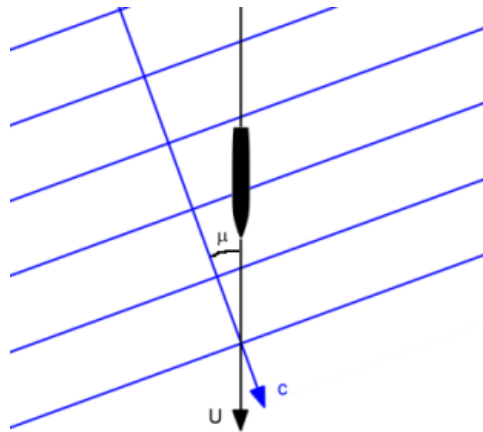
Encounter frequency
The encounter frequency may be calculated by considering the component of vessel velocity in the direction of the waves and subtracting the wave phase velocity c, this is shown visually above. The relative velocity is given by the equation below.

$$\text{relative encounter speed} = c - U\cos\mu$$

Hence the encounter period T_e is given by:

$$T_e = \frac{\lambda}{\text{relative encounter speed}} = \frac{\lambda}{c - U\cos\mu}$$

Recalling that: $\omega = \dfrac{2\pi}{T}$, it follows that the encounter frequency is given by:

$$\omega_e = \frac{2\pi}{\lambda}(c - U\cos\mu)$$

Often the vessel is assumed to be operating in deep water. Thus using the deep water wave speed relationship, the encounter frequency simplifies to:

$$\omega_e = \omega - \frac{\omega^2 U}{g} \cos \mu$$

Encounter Spectrum

An important concept when calculating vessel motions is that of the encountered wave spectrum. This is a transformation of the wave spectrum which describes the waves encountered by a vessel travelling through the ocean at a certain speed. This is effectively a Doppler shift of the spectrum.

Given the relationship between the encounter frequency and the wave frequency (from above), and since the total energy in the ocean must be constant whether experienced by a stationary or moving observer, the transformation of the wave spectrum to the encountered spectrum is shown in the following equation. Examples of the same spectrum encountered at different speeds are shown in the figure below.

for the stationary observer: $m_0 = \int_0^\infty S(\omega)d\omega$, whilst

for the moving observer: $m_0 = \int_0^\infty S(\omega_e)d\omega_e$

Thus: $S(\omega_e)d\omega_e = S(\omega)d\omega$ or $S(\omega_e) = \dfrac{S(\omega)}{\left|\dfrac{d\omega_e}{d\omega}\right|}$

recall that for deep water $\omega_e = \omega - \dfrac{\omega^2 U}{g}\cos\mu$, thus $\dfrac{d\omega_e}{d\omega} = 1 - \dfrac{2\omega U}{g}\cos\mu$.

Hence the spectral ordinates in the encounter frequency domain are given by:

$$S(\omega_e) = \frac{S(\omega)}{\left|1 - \dfrac{2\omega U}{g}\cos\mu\right|}$$
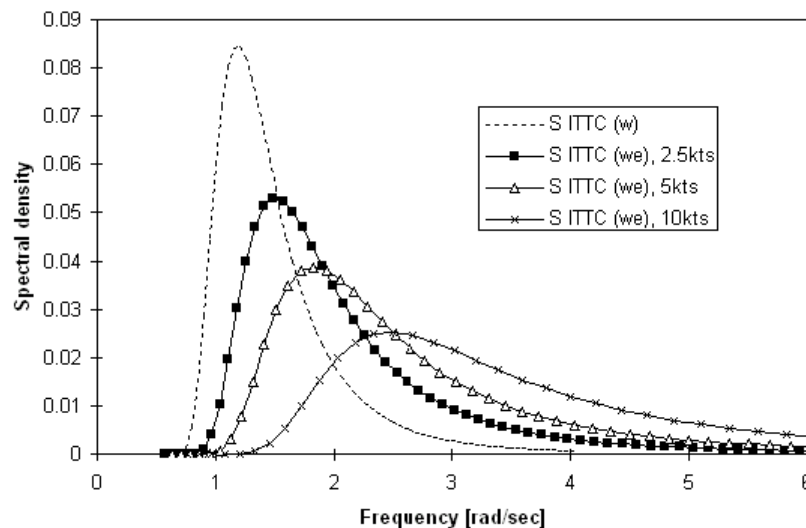
Comparison of encountered wave spectra at different speeds

Characterising Vessel Response

A vessel may be considered as a complex vibratory system with six degrees of freedom; these are often termed the rigid body modes. They can be divided into two groups: The translational or linear motions: surge, sway, heave and the rotational motions roll, pitch and yaw. In addition, if the flexibility of the vessel is significant it may be appropriate to consider the hydroelastic response of the vessel; however this is beyond the scope of Motions.

Harmonic Response of Damped, Spring, Mass System

The vessel's motions may be represented by a second order differential equation, such as that describing a forced spring, mass and damper system.

$$a\ddot{z} + b\dot{z} + cz = F\cos(\omega t)$$

where: a, b, c are the mass, damping and restoring force terms; F is the forcing function at frequency ω.

It may be shown that the solution to the above equation is given by:

$$z = \frac{F}{\sqrt{\left(c - a\omega^2\right)^2 + b^2\omega^2}}\cos(\omega t + \varepsilon)$$

where: ε is the phase lag relative to the forcing function: $\tan\varepsilon = \dfrac{b\omega}{c - a\omega^2}$.

This equation may be re-expressed in terms of the damping ratio, $\beta = \dfrac{b}{2\sqrt{ca}}$, the natural frequency of the system, $\omega_0 = \sqrt{\dfrac{c}{a}}$, and the tuning factor, $\lambda = \dfrac{\omega}{\omega_0}$.

Further, it may be shown that the damped natural frequency is given by

$$\omega_{b0} = \sqrt{\frac{c}{a} - \frac{b^2}{2a^2}}$$

The response is often described by a transfer function or response function, zc/F, given by:

$$\text{RAO} = zc/F = \frac{1}{\sqrt{\left(1 - \lambda^2\right)^2 + 4\beta^2\lambda^2}}$$

The typical response of this type of system is shown below:
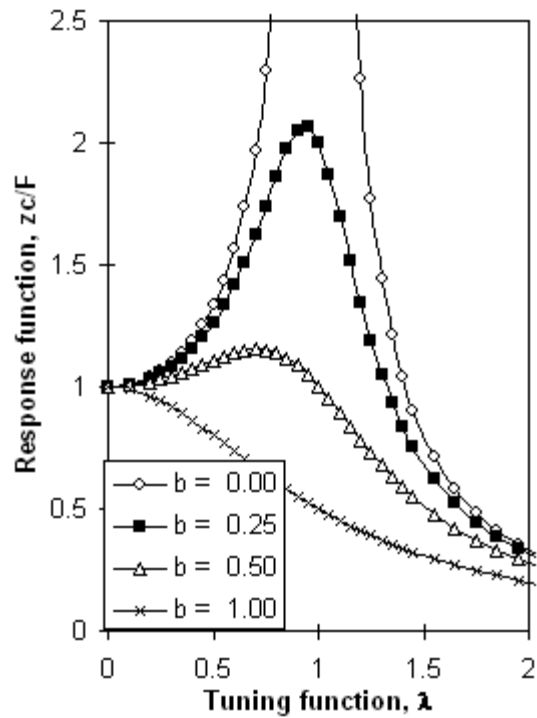
Typical transfer function for different damping factors
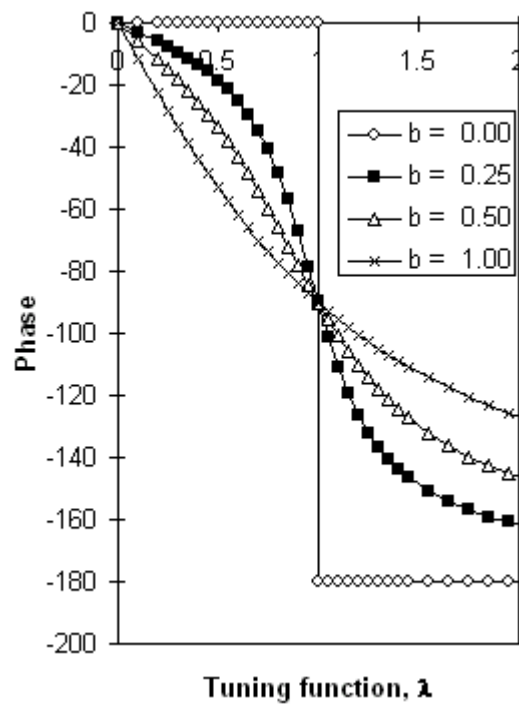
The analysis above describes a single, uncoupled motion. In reality the motions of all six degrees of freedom are coupled to some extent, though it is normal to solve the vertical plane motions independently of the lateral plane motions.

Motions solves the heave and pitch motions coupled together and the roll motion independently.

Response Amplitude Operator

The response amplitude operators (or transfer functions) of a vessel are similar in form to those described above. In general for each motion the mass, damping and stiffness coefficients will be different, thus changing the exact shape of the curve.

It is normal practice to non-dimensionalise the various motions as follows:

Linear motion — wave amplitude; Linear motion transfer function:

$$RAO_z = \frac{z_0}{\zeta_0}$$

Angular motion — wave slope; Angular motion transfer function:

$$RAO_\theta = \frac{\theta_0}{k\zeta_0}$$

Vertical acceleration — wave acceleration; Acceleration transfer function:

$$RAO_a = \frac{a_0}{\omega_e^2 \zeta_0}$$

Calculating Vessel Motions

By assuming that the response function is linear with respect to wave height and that the principle of superposition holds. (The principle of superposition states that the response of a body to a spectrum of waves is the sum of the individual waves).Thus if the linear heave response of the vessel is given by

$$RAO_z(\omega_e) = \frac{z_0(\omega_e)}{\zeta_0(\omega_e)}$$

then it follows that Motion response spectrum, $S_z(\omega_e)$, is given by:

$$S_z(\omega_e) = RAO_z(\omega_e)^2 S_\zeta(\omega_e)$$

where $S_\zeta(\omega_e)$ is the encountered wave energy spectrum.

Note that since the energy spectra relates to the square of the amplitudes, the transfer function (which relates wave amplitude to motion amplitude) must be squared. This process is demonstrated graphically below:

1. Calculate wave spectrum $S_\zeta(\omega)$
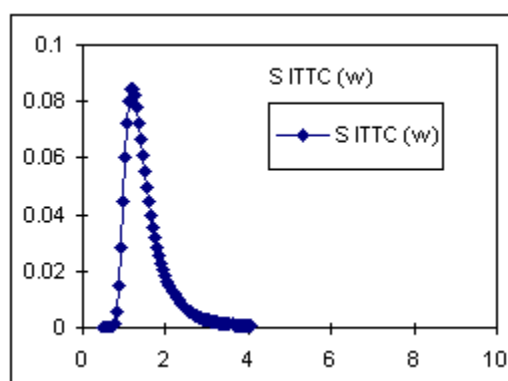
2. obtain Encountered spectrum $S_\zeta(\omega_e)$, as experienced by vessel
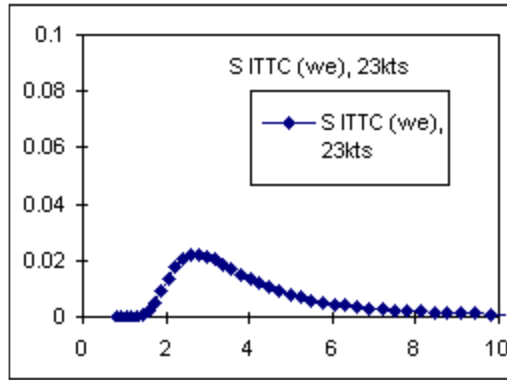
3. obtain Transfer function $\mathrm{RAO}_z(\omega_e)$ and square it
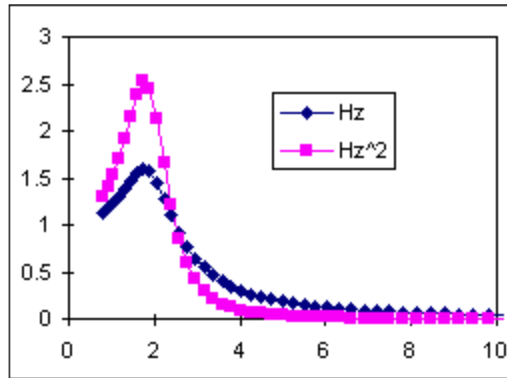
4. multiply the wave spectrum by the square of the transfer function to get the Response spectrum $S_z(\omega_e) = \mathrm{RAO}_z(\omega_e)^2 S_\zeta(\omega_e)$
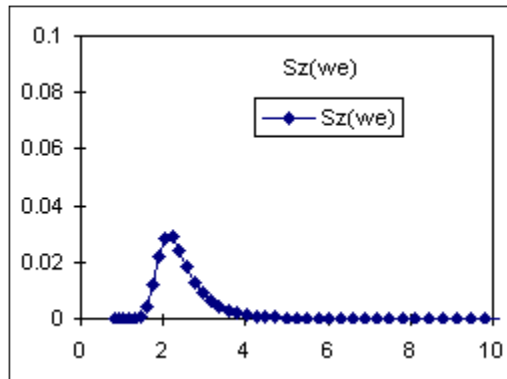
Absolute Vessel Motions

For a given location on the vessel $\mathbf{xb}$, the absolute longitudinal, lateral and vertical motions, $\mathbf{s}$, can be found from the following equation, where x are the complex motions in surge, sway, heave, roll, pitch and yaw:

$$
\begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} x_1 & 0 & 0 & 0 & -x_6 & x_5 \\ 0 & x_2 & 0 & x_6 & 0 & -x_4 \\ 0 & 0 & x_3 & -x_5 & x_4 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ xb_1 \\ xb_2 \\ xb_3 \end{bmatrix}
$$

Statistical Measures

Once the motion energy spectrum has been calculated it is possible to calculate similar statistical values to those calculated for the wave energy spectrum. It should be noted that the units of the motion energy spectrum, $S_z(\omega_e)$, are motion^2 × time. Thus an integration over a frequency interval yields motion^2; recalling that the energy in a wave system is proportional to wave height squared.

The variance of the motion is given by the area under the motion energy spectrum:

$$m_0 = \int S_z(\omega_e)d\omega_e$$

hence the RMS motion is given by $\sqrt{m_0}$. The significant motion amplitude is twice the RMS motion, $2\sqrt{m_0}$, and the significant motion peak-to-peak is twice the amplitude, $4\sqrt{m_0}$ (assuming a narrow band spectrum).

Short-Term statistics

Short-term statistics can be calculated for a single wave spectrum; where the wave spectrum is changing, the probability of occurrence of a given wave spectrum should also be considered, thus producing long-term statistics.

Many short-term statistics can be related back to the variance (m_0) of the spectrum provided that the spectrum is of a known shape. It is generally assumed that wave spectra follow the Rayleigh distribution.

Average 1/nth highest amplitudes

By making this assumption, the mean of the 1/nth highest amplitudes of a spectrum may be calculated by multiplying the RMS ($\sqrt{m_0}$) by the following factors:

n	factor	
1	1.25	Average value
2	1.77	
3	2.00	Significant value
10	2.55	
100	3.34	
1000	3.97	
10000	4.51	

$$\text{factor} = \frac{\overline{\zeta_{1/n}}}{\sqrt{m_0}} = n\sqrt{2}\left\{\frac{\sqrt{\ln(n)}}{n} + \sqrt{\pi}\left(0.5 - erf\left(\sqrt{2\ln(n)}\right)\right)\right\}$$

(Source: *Seakeeping, Ship Behaviour in Rough Weather*. ARJM Lloyd, Note that there are errors in the quoted values in the original publication which have been recomputed here using the equation given)

Thus the average value of the spectrum would be given by 1.25 $\sqrt{m_0}$; the significant value would be given by 2.00 $\sqrt{m_0}$; and the average of the 1/100th highest amplitudes would be given by 3.34 $\sqrt{m_0}$.

The modal or most probable value (wave amplitude with the highest probability of occurrence), again assuming a Rayleigh distribution is equivalent to $\sqrt{m_0}$.

The "most probable largest value" to occur in N wave encounters (again assuming a Rayleigh distribution). May be calculated as follows:

$$R_{\max} = \sqrt{2\ln(N)} \times \sqrt{m_0}$$

where *N* can be expressed by *t* / *T* with *t* = is the time period of interest and *T* is the mean period of the response.

N or t/T	factor
10	2.146
20	2.448
50	2.797
100	3.035
200	3.255
500	3.526
1000	3.717

(Source: *Sea Loads on Ships and Offshore Structures*. OM Faltinsen)

Velocity and Acceleration Spectra

Recalling that the motion is defined by $z = z_0 \sin(\omega_e t + \varepsilon)$, the velocity and acceleration may be found by differentiation:

$$\dot{z} = z_0 \omega_e \cos(\omega_e t + \varepsilon)$$
$$\ddot{z} = z_0 \omega_e^2 \sin(\omega_e t + \varepsilon)$$

Thus the RMS velocity and acceleration are given respectively by $\sqrt{m_2}$ and $\sqrt{m_4}$ of the motion spectrum respectively:

$$u_{zRMS}^2 = \int \omega_e^2 S_z(\omega_e)d\omega_e = m_2$$
$$a_{zRMS}^2 = \int \omega_e^4 S_z(\omega_e)d\omega_e = m_4$$

Motion Frequencies and Periods

Several key frequencies and periods may be calculated for the spectra. Note that the period, $T = 2\pi / \omega$

The average frequency, $\overline{\omega}$, is simply the centre of area of the spectrum:

$$\overline{\omega} = \frac{m_1}{m_0} \quad \text{thus} \quad \overline{T} = 2\pi \frac{m_0}{m_1}$$

It can be shown (Lloyd 1999) that the mean frequency of the peaks, $\overline{\omega}_P$, and the mean frequency of the zero-crossings, $\overline{\omega}_Z$, are given by:

$$\overline{\omega}_P = \sqrt{\frac{m_4}{m_2}} \quad \text{thus} \quad \overline{T}_P = 2\pi\sqrt{\frac{m_2}{m_4}}$$

and

$$\bar{\omega}_Z = \sqrt{\frac{m_2}{m_0}} \text{ thus } \bar{T}_Z = 2\pi\sqrt{\frac{m_0}{m_2}}$$

Probability Measures

One of the primary goals is to be able to compare the ability of different design alternatives to carry out the required mission. We have seen how to compute the motion energy spectrum and the variance of the spectrum. Now we will investigate how to compute the probability of exceeding the defined design criteria. This probability may then be used to guide the selection of the appropriate vessel.

Probability of Exceeding a Critical Value

By assuming that the probability density function of the motions follow that of a Rayleigh distribution, it is possible to evaluate the probability of exceeding some critical value z_{crit} given the variance of the motion energy spectrum, m_{0z}. This is given by:

$$\text{prob}(z > z_{crit}) = \exp\left(\frac{-z_{crit}^2}{2m_{0z}}\right)$$

Occurrence per hour a Critical Value

With the probability of exceeding a certain critical value calculated, as shown above and the average period of the peaks calculated (see above also). It is a simple matter to calculate the number of occurrences per hour of an event, N:

$$N = \frac{3600}{\bar{T}_P} \times \text{prob}(z > z_{crit}) \text{ per hour}$$

Long-Term Statistics

The above equation assumes that the sea state and hence the variance of the motion energy spectrum is constant. These are described as short-term statistics. Over longer periods of time an alternative method is required. If the probability of encountering the seas states is known, long-term statistics may be calculated using the equation below.

$$\text{Combined probability} = \sum_i q_i p_i$$

where q_i is the probability of sea state i occurring, and p_i is the probability of exceeding the criterion in sea state i.

Sea State	Prob. Occurrence	Variance of motion energy spectrum	Prob. exceeding z_{crit}	Combined probability
I	q_I	m_{0I}	p_I	$q_i . p_I$
0-3	0.520	1.80	0.012	0.0062
4-5	0.290	2.17	0.025	0.0073
6-7	0.150	2.46	0.039	0.0059
8-9	0.035	2.81	0.058	0.0020
10-12	0.015	3.06	0.073	0.0011
Sum	**1.000**			**0.0225**

These types of statistics may be used to evaluate the down time over a year or the life cycle of the vessel.

Joint Probability

In the previous section we have touched on the subject of joint probability; the probability of two independent events occurring is the product of the probabilities of the two individual events. Thus the joint probability of two independent events (x,y) is described by:

$$\text{prob}(x, y) = \text{prob}(x).\text{prob}(y)$$

This method may be used to compute the probability of a slam occurring. For a slam to occur two events must occur:

- **The bow must emerge from the water; the relative motion at the bow must be greater than the draft at the bow.**

- **The relative velocity of the bow with respect to the water surface, at the time of impact, must exceed a certain value. This value is referred to as the threshold velocity and is given the symbol u_*. Experiments for a wide variety of hull forms have shown that the threshold velocity is given by:**

$$u_* = \frac{0.0195B}{b}\sqrt{Lg}$$

where B is the total beam of the vessel at amidships; b is the half beam at a station 0.25L from the bow, at a point 0.03B above the keel; L is the length of the vessel.

It is usual to compute the probability of a slam at station 90 to 95% of the length of the vessel.

It may be shown that these two events are statistically independent, hence the joint probability law may be applied and the probability of a slam occurring is the product of the probabilities of the two events occurring.

A summary of events relevant to seakeeping is given below:

Slam	relative vertical bow motion exceeds draft and relative velocity exceeds threshold velocity
deck wetness	relative vertical bow motion exceeds freeboard
propeller emergence	relative vertical motion at propeller exceeds depth of propeller

Calculation of Subjective Magnitude and Motion Sickness Incidence

Motions calculates a number of crew/passenger comfort indices, these are Subjective Magnitude parameter (SM) and Motion Sickness Incidence (MSI).

Subjective Magnitude

The calculation of the Subjective Magnitude follows the method described in Lloyd 1998 pages 306 to 309 and is summarised below.

$$\text{SM} = A\left(\frac{\ddot{s}_{30}}{g}\right)^{1.43}$$

, where A depends on the encounter frequency, ω_e, in rad/s and is given by:

$$A = \left\{1 - \exp\left(-1.65\omega_e^2\right)\right\}\left\{75.6 - 49.6\ln\omega_e + 13.5\left(\ln\omega_e\right)^2\right\};$$

, and the vertical acceleration $\ddot{s}_{30}$ can be evaluated from the vertical motion spectrum:

$$\ddot{s}_{30} = 2\sqrt{m_4}$$

, where m_4 is the 4th spectral moment of the vertical motion spectrum at the location of interest in m/s^2;

g is acceleration due to gravity;

and the average frequency of the motions can be estimated by: $\omega_e = \sqrt{\frac{m_4}{m_2}}$

Subjective magnitude is on a scale of zero to 30:

0 – 5	Moderate
5 – 10	Serious
10 – 15	Severe: necessary to "hang on"
15 – 20	Hazardous
20 – 30	Intolerable

Motion Sickness Incidence

Motion sickness incidence (MSI) is the percentage of subjects who vomit in the specified time that subjects are exposed to the motions. The data were derived from test on healthy, young, males students who were subjected to vertical motions for a period of up to two hours. Thus extrapolation to other demographics or longer durations of exposure can be difficult. There are three different ways in which motion sickness incidence is calculated in Motions.

ISO 2631/3 1985 and BS 6841:1987

Although not strictly MSI accelerations integrated over 1/3 octave bins are plotted against acceleration limits defined by these two standards. The likely discomfort can be gauged by how close these acceleration curves approach the defined acceleration limits.

The MSI acceleration depends on the magnitude of the vertical acceleration at the point of interest on the vessel. This is computed by integrating the power spectrum density (PSD – given the symbol $S_{vert\ accel}$ below) over a one third octave range centred about the frequency of interest:

$$MSI(\omega_{e\ centre}) = \int_{\omega_{e1}}^{\omega_{e2}} S_{vert\ accel}(\omega_e)\, d\omega_e$$

where the frequency interval ω_{e1} to ω_{e2} is the 1/3 octave range centred about $\omega_{e\ centre}$ and $S_{vert\ accel}$ is the absolute vertical acceleration at the point of interest on the vessel.

By varying the centre frequency, a graph of vertical MSI acceleration is built up and this is compared with the standard curves.

MSI after 2 hours exposure

The motion sickness incidence derived from the formulation original 1974 paper of O'Hanlon and McCauley is calculated for a fixed exposure time of 2 hours. Motions evaluates the MSI using the method described in Lloyd 1998, pages 304 to 306.

The MSI is given by Lloyd as:

$$\text{MSI\%} = 100 \times \Phi \left\{ \frac{\log\left(\frac{|\ddot{s}_3|}{g}\right) - \mu_{\text{MSI}}}{0.4} \right\}$$

, where $\Phi(x)$ is the cumulative normal distribution function up to x for a normal distribution with zero mean and unity standard deviation. (see Excel function NORMDIST(x, 0, 1, TRUE).

Lloyd gives the factor $\mu_{\text{MSI}} = -0.819 + 2.32(\log \omega_e)^2$ with ω_e in radians/s this is almost the same as the original equation of O'Hanlon and McCauley 1974:

$$\mu_{\text{MSI}} = 0.654 + 3.697(\log f_e) + 2.32(\log f_e)^2$$

, with f_e in Hz, noting that $\omega_e = 2\pi f_e$. Motions uses the latter. The same frequency as that used in the SM calculations is used, namely: $\omega_e = \sqrt{\frac{m_4}{m_2}}$ and the acceleration, $|\ddot{s}_3|$, is given by $|\ddot{s}_3| = 0.798\sqrt{m_4}$.

MSI after specified exposure

McCauley et al. 1976 reanalysed the original O'Hanlon and McCauley 1974 data and presented a formulation whereby the exposure time could be accounted for. This method is also computed by Motions and is the *only* place where the MSI exposure time for the remote location is used.

The exposure time can be specified for each remote location (you can define multiple remote locations at the same point on the vessel and give them different exposure times if you wish).

The procedure described by Colwel 1994 is used with the frequency and acceleration values given by Lloyd for the 2 hour MSI, above.

$$\text{MSI\%} = 100 \times \Phi(z_a)\Phi(z_t')$$

, where $z_a = 2.128(\log a) - 9.277(\log f_e) - 5.809(\log f_e)^2 - 1.851$;

and $z_t' = 1.134z_a + 1.989(\log t) - 2.904$;

, with the acceleration, in g, given by $a = \frac{|\ddot{s}_3|}{g} = \frac{0.798}{g}\sqrt{m_4}$.
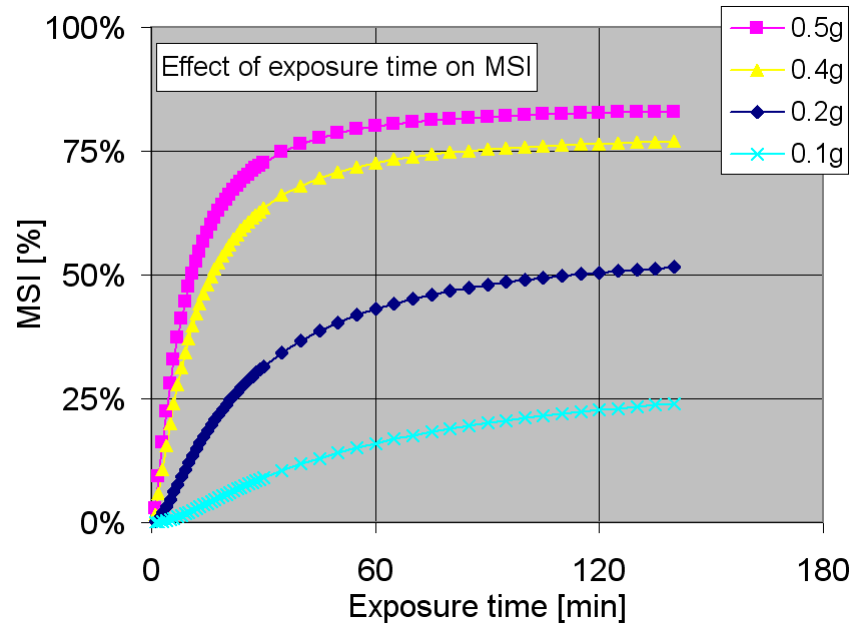
The effect of exposure time on MSI can be seen in the figure above. When the exposure is longer than about two hours, there is relatively little increase in MSI, this is especially true at higher accelerations, i.e. all those victims that are going to vomit have already done so within two hours. As the acceleration decreases, the exposure time required to achieve near-maximum MSI decreases slightly. The frequency of the acceleration in this case was 0.1 Hz which is near the worst case.
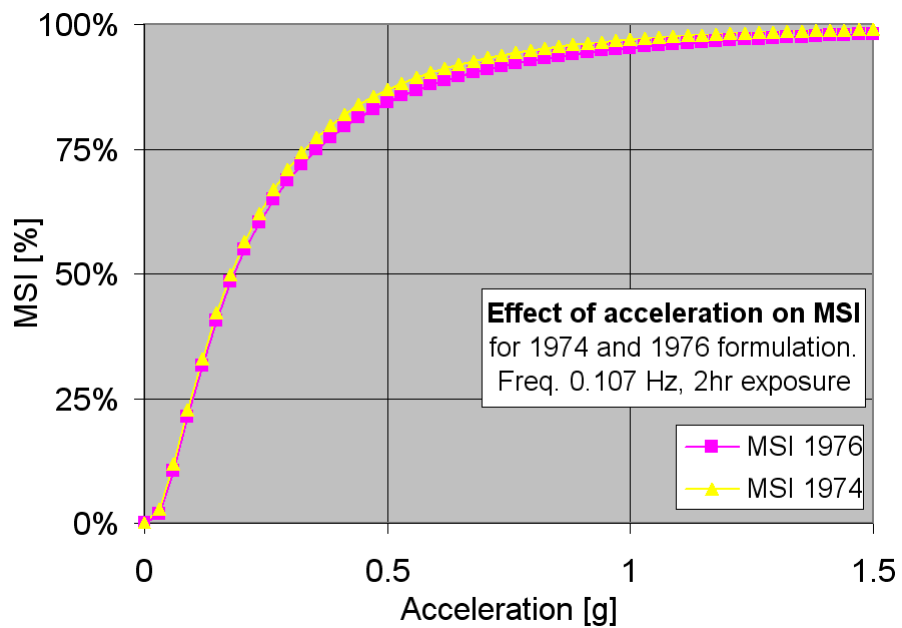
The effect of acceleration on MSI is shown for an exposure period of two hours in the figure above. It can be seen that both formulations give very similar results. The frequency of the acceleration in this case was 0.107 Hz.

> **Note:**
>
> **MSI after 2hours exposure calculated by the two methods:**
>
> The MSI computed for 2hours exposure by the McCauley et al. 1976 / Colwel 1994 method gives a slightly different value to the 2hour exposure MSI values given by O'Hanlon and McCauley 1974 / Lloyd 1998.

Speed Loss

Speed reduction in waves occur because of two factors. These are known as voluntary and involuntary speed loss.

Voluntary speed loss is due to a decision, by the captain, to reduce speed in order to reduce the motions, slams, deck wetness etc. to within acceptable limits. This may also necessitate a change of course.

Involuntary speed loss arises from the fact that a vessel travelling through waves will have a greater resistance due to its motions. Also it is likely that the propeller will be less efficient due to the motions and oscillations of the onset stream. These effects may reduce the vessel's speed by around two to three knots but this may be sufficient to result in substantial financial losses for a merchant ship.

The added resistance is given the symbol R_{aw} and is often expressed as:

$$C_{aw} = \frac{R_{aw}}{\zeta_0^2}$$

where ζ_0 is the wave amplitude.

The total added resistance in irregular waves may be calculated in the same way that the *motion* in irregular waves is calculated — by integrating the contribution of all the individual components, see below. Note that the factor of 2 arises from the fact that we are looking at the amplitude of each sine-wave component $\zeta_0(\omega) = \sqrt{2S_\zeta(\omega)\delta\omega}$, and not the energy associated with that component.

$$\overline{R}_{aw} = 2\int_0^\infty C_{aw}(\omega_e)S_\zeta(\omega_e)d\omega_e$$

Glossary

This glossary describes the key words and abbreviations used in this manual.

AMECRC	Australian Maritime Engineering Co-operative Research Centre.
Datasheet	Group of related data sets, viewed either in tabular or graphical format.
Data set	A set of data which make up a line in a graph or a column in a table.
DNV	Det Norske Veritas – Classification society.
Lewis mapping	A conformal mapping using three parameters. Used to map ship-like sections to unit circle.
MII	Motion induced interruptions.
MSI	Motion sickness incidence.
RAO	Response Amplitude Operator. The response spectrum is calculated by multiplying the wave spectrum by the square of the RAO.
Transfer function	see RAO.

Nomenclature

α	variable in spectrum definition
β	variable in spectrum definition, non-dimensional damping factor
γ	peak enhancement factor
ε	phase
ζ	wave elevation
ζ_0	wave amplitude
$\zeta_{1/3}$	significant wave amplitude
θ	angular motion
λ	wave length
μ	wave heading relative to vessel
ρ	water density
∇	displaced volume of vessel
σ	spectral width parameter
σ_0	standard deviation of wave spectrum (RMS)
ω	wave frequency
ω_e	encounter frequency
A	variable in spectrum definition
a	added mass, linear acceleration
B	vessel beam, variable in spectrum definition
b	Damping
c	wave speed, restoring force (stiffness)
C_{aw}	added resistance coefficient in regular waves
E	Energy
F	excitation force
g	acceleration due to gravity

H	wave height
$H_{1/3}$	significant wave height
H_{char}	Characteristic wave height
k_{xx}	roll radius of gyration
k_{yy}	pitch radius of gyration
I	Inertia
L	length of vessel
m	Mass
m_0	variance of wave spectrum, zeroth spectral moment
m_n	nth spectral moment
p	Probability
q	Probability
R_{aw}	added resistance in regular waves
$\overline{R}_{aw}$	total added resistance in irregular waves
RAO	response amplitude operator
S	spectral energy
s	Absolute motion (s_1, s_2, s_3)
T	wave period
T_0	modal wave period
T_{ave}	average wave period
T_z	zero-crossing wave period
t	Time
tf	transfer function (RAO)
U_{wind}	wind speed
U	ship speed
u_*	threshold velocity
x	Rigid body motions (x_1, x_2, x_3, x_4, x_5, x_6)
xb	Position relative to CG (xb_1, xb_2, xb_3)
z	linear motion
subscripts 1, 2, … 6	These refer to position or motion in longitudinal (1), transverse (2), vertical (3), roll (4), pitch (5), yaw (6) directions

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>   (Original publication including theoretical background for one of the added resistance prediction methods.)

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> (This book has recently been re-published (1998), it provides a through guide to most aspects of Seakeeping. It includes much example data, mainly for naval vessels. It also provides a good introduction to Strip Theory.)

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Appendix A – Strip Theory Formulation

This appendix describes the strip theory formulation and is divided up in:

Calculation of coupled heave and pitch motions

This section describes the strip theory formulation used to compute the coupled heave and pitch motions.

Heave and Pitch Motion of a Vessel in a Seaway

The ship motions of heave, pitch and roll are oscillatory in nature, this is due to the restoring force created by changes in buoyancy involved in these motions. The motions of a ship in response to waves, may be considered as a forced damped-spring-mass system. Motions currently only deals with the coupled motions of pitch and heave. The two relevant equations of motion are for heave:

$$(M + A_{33})\ddot{\eta}_3 + B_{33}\dot{\eta}_3 + C_{33}\eta_3 + A_{35}\ddot{\eta}_5 + B_{35}\dot{\eta}_5 + C_{35}\eta_5 = F_3 e^{i\omega_e t} \qquad (1)$$

and for pitch:

$$(I_5 + A_{55})\ddot{\eta}_5 + B_{55}\dot{\eta}_5 + C_{55}\eta_5 + A_{53}\ddot{\eta}_3 + B_{53}\dot{\eta}_3 + C_{53}\eta_3 = F_5 e^{i\omega_e t} \qquad (2)$$

where the variables are defined as follows:

M	mass of the vessel.
I_5	moment of inertia for pitch.
A_{33}	added mass coefficient for heave due to heave.
A_{55}	added mass coefficient for pitch due to pitch.
A_{35}	added mass coefficient for heave due to pitch.
A_{53}	added mass coefficient for pitch due to heave.
B_{33}	damping coefficient for heave due to heave.
B_{55}	damping coefficient for pitch due to pitch.
B_{35}	damping coefficient for heave due to pitch.
B_{53}	damping coefficient for pitch due to heave.
C_{33}	hydrostatic restoring coefficient for heave due to heave.
C_{55}	hydrostatic restoring coefficient for pitch due to pitch.
C_{35}	hydrostatic restoring coefficient for heave due to pitch.
C_{53}	hydrostatic restoring coefficient for pitch due to heave.
F_3	heave exciting force.
F_5	pitch exciting moment.
η_3	instantaneous heave displacement.
$\dot{\eta}_3$	instantaneous heave velocity.
$\ddot{\eta}_3$	instantaneous heave acceleration.
η_5	instantaneous pitch displacement.
$\dot{\eta}_5$	instantaneous pitch velocity.
$\ddot{\eta}_5$	instantaneous pitch acceleration.

In order to solve these equations it is necessary to obtain the coefficients and excitation force and moment. The procedure used is described in the following sections.

In Motions the damping is calculated using inviscid flow theory. The user has the option of specifying additional heave and pitch damping to allow for damping not calculated by the inviscid flow modelled. The user is able to specify non-dimensional damping β_{33} and β_{55} from which the actual damping is calculated as follows.

$$B_{nn} = 2\beta_{nn}\sqrt{C_{nn}(I_n + A_{nn})} \text{ with n} = 3, 5.$$

These values are then added to the inviscid B_{33} and B_{55}.

Solution of Coupled Heave and Pitch Motions

The solutions to the coupled heave and pitch equations are found using the method described by Bhattacharyya (1978) which is outlined below:

If additional damping for heave and pitch has been specified, then it is applied at this point.

$$\lambda = \frac{\omega_e}{\sqrt{C_{33}/(m + A_{33})}} \text{ this is the tuning factor}$$

The specified non-dimensional damping, β_{33}^+, is assumed to be evenly distributed along the length thus the additional damping terms (given the superscript +) are defined as:

$$B_{33}^+ = 2\lambda\beta_{33}^+\sqrt{C_{33}(m + A_{33})}$$

$$B_{35}^+ = B_{53}^+ = \frac{B_{33}^+ L}{2}$$

$$B_{55}^+ = \frac{B_{33}^+ L^2}{3}$$

$$
\begin{aligned}
P &= C_{33} - (m + A_{33})\omega_e^2 + iB_{33}\omega_e \\
Q &= C_{35} - A_{35}\omega_e^2 + iB_{35}\omega_e \\
R &= C_{53} - A_{53}\omega_e^2 + iB_{53}\omega_e \\
S &= C_{55} - (I_{55} + A_{55})\omega_e^2 + iB_{55}\omega_e
\end{aligned}
\qquad (3)
$$

and

$$Z_3 = \frac{F_5 Q - F_3 S}{QR - PS} = Z_{30}e^{i\varepsilon_3} \text{ , heave response}$$

$$Z_5 = \frac{F_3 R - F_5 P}{QR - PS} = Z_{50}e^{i\varepsilon_5} \text{ , pitch response}$$

$$(4)$$

Global Added Mass and Damping (Strip Theory)

The vertical motions of a vessel (pitch and heave) are most readily calculated by subdividing the vessel into a number of transverse strips and considering the forces on each of the strips. The two dimensional added mass, damping and restoring coefficients are calculated for each strip, and the respective global coefficients are then found by integrating along the length of the hull. It is assumed that the amplitude of oscillation is sufficiently small that the response of the vessel will remain linearly proportional to the amplitude of the waves.

The global added mass and damping are calculated according to the method developed by Salvesen et al. (1970). Two formulations are used: the first ignores the transom terms; whilst these terms are included in the second.

The coefficients in the equations of motion are summarised below, these are the same for both the transom terms and no transom terms versions:

$$A_{33} = \int a_{33}d\xi \qquad (5)$$

$$B_{33} = \int b_{33} d\xi \tag{6}$$

$$C_{33} = \rho g \int b d\xi \tag{7}$$

$$A_{35} = -\int \xi a_{33} d\xi - \frac{U}{\omega_e^2} B_{33} \tag{8}$$

$$B_{35} = -\int \xi b_{33} d\xi + U A_{33} \tag{9}$$

$$C_{35} = C_{53} = -\rho g \int \xi b d\xi \tag{10}$$

$$A_{53} = -\int \xi a_{33} d\xi + \frac{U}{\omega_e^2} B_{33} \tag{11}$$

$$B_{53} = -\int \xi b_{33} d\xi - U\, A_{33} \tag{12}$$

$$A_{55} = \int \xi^2 a_{33} d\xi + \frac{U^2}{\omega_e^2} A_{33} \tag{13}$$

$$B_{55} = \int \xi^2 b_{33} d\xi + \frac{U^2}{\omega_e^2} B_{33} \tag{14}$$

$$C_{55} = \rho g \int \xi^2 b d\xi \tag{15}$$

For the transom terms version, the following terms are added to the coefficients given above:

$$A_{33\,\text{Trans}} = -\frac{U}{\omega_e^2} b_{33}^A \tag{16}$$

$$B_{33\,\text{Trans}} = +U a_{33}^A \tag{17}$$

$$A_{35\,\text{Trans}} = +\frac{U}{\omega_e^2} x^A b_{33}^A - \frac{U^2}{\omega_e^2} a_{33}^A \tag{18}$$

$$B_{35\,\text{Trans}} = -U x^A a_{33}^A - \frac{U^2}{\omega_e^2} b_{33}^A \tag{19}$$

$$A_{53\,\text{Trans}} = +\frac{U}{\omega_e^2} x^A b_{33}^A \tag{20}$$

$$B_{53\,\text{Trans}} = -U x^A a_{33}^A \tag{21}$$

$$A_{55\,\text{Trans}} = -\frac{U}{\omega_e^2}\left(x^A\right)^2 b_{33}^A + \frac{U^2}{\omega_e^2} x^A a_{33}^A \qquad (22)$$

$$B_{55\,\text{Trans}} = +U\left(x^A\right)^2 a_{33}^A + \frac{U^2}{\omega_e^2} x^A b_{33}^A \qquad (23)$$

where the variables are defined as follows:

a_{33}	section added mass.
a_{33}^A	added mass of transom section.
b_{33}	section damping.
b_{33}^A	damping of transom section.
b	section beam.
g	acceleration due to gravity.
U	vessel forward velocity.
x^A	x ordinate of transom (from CoG, negative aft).
ρ	fluid density.
ω_e	wave encounter circular frequency.
ζ	longitudinal distance from LCB.

The integrals are all over the length of the hull.

Wave Excitation Force and Moment

The wave excitation force and moment drive the motions of the vessel. For solutions of the coupled heave and pitch equations of motion, only the global force and moment are required; however, for solution of the wave induced shear force and bending moment, the forces must be divided into the sectional Froude-Krilov and diffraction forces. Because several simplifying assumptions may be made, there are three methods available for the evaluation of the Global Wave Excitation Force and Moment, these are:

Arbitrary wave heading; sectional Froude-Krilov and diffraction forces for arbitrary wave angles.

Head seas approximation; sectional Froude-Krilov and diffraction forces for head seas.

Salvesen et al. (1970); approximation for global force and moment in head seas.

Again, the methods follow the work of Salvesen et al. (1970).

Arbitrary Wave Heading

This method is used to compute the sectional Froude-Krilov and diffraction forces for arbitrary wave angles. Other methods, presented below have additional assumptions that make them simpler and quicker to compute.

Following the work of Salvesen et al. (1970), the global exciting force and moment for arbitrary wave heading are given by:

z
<div align="right">(24)</div>

$$F_5 = -\rho\varsigma_0 \int \left[\xi(f_3 + h_3) + \tfrac{U}{i\omega} h_3 \right] d\xi - \rho\varsigma_0 \tfrac{U}{i\omega} x_A h_3{}^A$$
<div align="right">(25)</div>

where:

f_3 is the sectional Froude-Krilov force.

h_3 is the sectional Diffraction force.

Diffraction Force

The sectional diffraction wave force is given in Equation (26), note that this equation includes the water density, ρ, and the wave amplitude, ς. The depth attenuation exponent in the e^{-kx} term has the opposite sign since Motions sign convention has z $+^{ve}$ down:

$$h_3 = \rho\zeta\omega_0 e^{-ikx\cos\mu} \int_{C_z} (i\hat{z} - \hat{y}\sin\mu) e^{iky\sin\mu} e^{-kz} \phi_{30} dl$$
<div align="right">(26)</div>

Expanding the sine and cosine terms, this may be rewritten as follows:

$$h_3 = \rho\zeta\omega_0 \{\cos(kx\cos\mu) - i\sin(kx\cos\mu)\} \times$$
$$\int_{C_z} (i\hat{z} - \hat{y}\sin\mu)\{\cos(kx\sin\mu) + i\sin(kx\sin\mu)\} e^{-kz} \phi_{30} dl$$
<div align="right">(27)</div>

where:
ω_0	is the wave frequency.
ω_e	is the frequency of the oscillation of the section (encounter frequency).
x,y,z	are the longitudinal position of the section, and transverse and vertical points on the section contour respectively.
$\hat{y}, \hat{z}$	are the outward normal unit vector of the section.
Cs,dl	are the section contour and element of arc along the section.
μ	is the wave heading angle.
ϕ_{30}	is the amplitude of the two dimensional velocity potential of the section in heave.

Further, the time varying velocity potential is given by:

$$\phi_3 = \phi_{30} e^{i\omega_e t} = \frac{g\overline{A}}{\pi\omega_e} \left\{ \cos(\omega_e t) \left[\Phi_c + \sum_{m=1}^{\infty} p_{2m}\phi_{2m} \right] + i\sin(\omega_e t) \left[\Phi_s + \sum_{m=1}^{\infty} q_{2m}\phi_{2m} \right] \right\}$$
<div align="right">(28)</div>

The segment length for the integration is calculated assuming a straight line between integration points. The unit normal vector components are calculated from the slope of the mapped section.

The velocity potential on the surface of the section at p=(y, z), is calculated by combining all the individual terms in the velocity potential as per Equation(28), note that it is the amplitude of the velocity potential that is required.

Froude-Krilov Force

The sectional Froude-Krilov wave force is given in Equation (29), note that this equation includes the water density, ρ, and the wave amplitude, ζ. Again the depth attenuation exponent has the opposite sign since Motions sign convention has z $+^{ve}$ down (for the 2D section data only):

$$f_3 = \rho\zeta g e^{-ikx\cos\mu} \int_{C_s} \hat{z} e^{iky\sin\mu} e^{-kz} dl \qquad (29)$$

Expanding the sine and cosine terms, this may be rewritten as follows:

$$f_3 = \rho\zeta g \{\cos(kx\cos\mu) - i\sin(kx\cos\mu)\} \times$$
$$\int_{C_s} \hat{z} \{\cos(kx\sin\mu) + i\sin(kx\sin\mu)\} e^{-kz} dl \qquad (30)$$

Head Seas Approximation

This method is simplified by assuming that the vessel is in head seas. The sectional Froude-Krilov and diffraction forces are obtained which makes this method suitable for the loads calculations.

The head seas approximation to the sectional Froude-Krilov wave force is given in Equation (31), note that this equation includes the water density, ρ, and the wave amplitude, ζ. This follows the work of Salvesen et al. (1970), Equations STF-32, 33

$$f_3 = \rho\zeta g b e^{ikx} e^{-kds} \qquad (31)$$

Expanding the sine and cosine terms, this may be rewritten as follows:

$$f_3 = \rho\zeta g b \{\cos(kx) + i\sin(kx)\} e^{-kds} \qquad (32)$$

where:

b	is the total section beam.
d	is the section draft.
A_s	is the section area.
$s = \dfrac{A_s}{bd}$	is the section area coefficient. Note that $ds = \dfrac{A_s}{b}$.

Secondly, the head seas approximation to the sectional diffraction wave force is given in Equation (33), note that this equation includes the water density, ρ, and the wave amplitude, ζ:

$$h_3 = -\zeta\omega_0 e^{ikx}\left(\omega_e a_{33} - ib_{33}\right)e^{-kds} \qquad (33)$$

Expanding the sine and cosine terms, this may be rewritten as follows:

$$h_3 = -\zeta\omega_0 \{\cos(kx) + i\sin(kx)\}\left(\omega_e a_{33} - ib_{33}\right)e^{-kds} \qquad (34)$$

where:

a_{33}	is the section added mass in heave.
b_{33}	is the section damping in heave.

Salvesen et al. (1970) Approximation

This method relates the global wave excitation to the sectional added mass and damping coefficients. The sectional wave excitation forces, required for the loads analysis are not computed directly. Both wave heave excitation force and wave pitch excitation moment may be evaluated with or without transom terms. At present, the head seas approximation is used. The excitation can be evaluated using complex notation in order to obtain the magnitude and phase.

It should be noted that these equations are estimates for head seas only. In addition the wave attenuation with depth is approximated by the e^{-kds} term in the expressions below. Again this is a fairly crude approximation valid for 'normal' section shapes only.

$$F_3 = \varsigma_0 \int e^{ik\xi} e^{-kds} \{\rho g b - \omega_0(\omega_e a_{33} - ib_{33})\} d\xi \qquad (35)$$

$$F_5 = -\varsigma_0 \int e^{ik\xi} e^{-kds} \left[\xi\{\rho g b - \omega_0(\omega_e a_{33} - ib_{33})\} - \frac{U}{i\omega_e}\omega_0(\omega_e a_{33} - ib_{33}) \right] d\xi \qquad (36)$$

the transom terms being as follows:

$$F_{3\,Trans} = -\varsigma_0 \frac{U}{i\omega} e^{ik\alpha^A} e^{-kds} \omega_0(\omega_e a_{33}^A - ib_{33}^A) \qquad (37)$$

$$F_{5\,Trans} = +\varsigma_0 \frac{U}{i\omega} e^{ik\alpha^A} e^{-kds} \omega_0 x^A(\omega_e a_{33}^A - ib_{33}^A) \qquad (38)$$

The additional variables are defined as follows:
d   section draft.
s   section area coefficient = Sec. Area / (Sec. Beam x Sec. Draft).
k   wave number.
ω_0 wave circular frequency.

Wave Attenuation (Smith Effect)

The wave depth attenuation term is calculated as:

$$1 - k\int \frac{y(z)}{y(0)} e^{-kz} dz \qquad (39)$$

and the effective wave amplitude ς^* is given by the relationship in Equation (40):

$$\varsigma^* = \varsigma\left[1 - k\int \frac{y(z)}{y(0)} e^{-kz} dz \right] \qquad (40)$$

noting that y(0) is the waterline half beam.

Added Resistance of a Vessel in a Seaway

Four methods have been implemented to compute the added resistance.

Gerritsma and Beukelman

The first two are based on the work of Gerritsma and Beukelman (1972). The added resistance is related to the relative vertical velocity of the vessel compared with the wave surface and the damping coefficient. The difference between the two versions is a small difference in the expression for the relative vertical motion. The general formulation is given in the following equation:

$$R_{AW} = \frac{k}{2\omega_e} \int_L b_{33}^* V_{za}^2 \, \mathrm{d}\xi \tag{41}$$

where $b_{33}^* = b_{33} - U \dfrac{\mathrm{d}a_{33}}{\mathrm{d}\xi}$ is a modified section damping; and V_{za}^2 is the relative vertical velocity, given by Equation (42).

$$V_{za} = \dot{\eta}_3 - \xi \dot{\eta}_5 + U\eta_3 - \dot{\varsigma}^* \tag{42}$$

where ς^* is given by Equation (43).

Note that η_3, η_3 and ς^* are the complex heave, pitch and local relative wave amplitudes, containing both phase and amplitude information.

The two methods vary only in the expression for the derivative of ς^*. Version A uses the expression in Equation (43), whilst Version B uses the expression in Equation (44).

$$\dot{\varsigma}^* = -\omega\varsigma^* \tag{43}$$

$$\dot{\varsigma}^* = -\omega_e\varsigma^* \tag{44}$$

Salvesen

In the third method, described by Salvesen (1978), the added resistance is given by Equation (45).

$$R_{AW} = \frac{ik}{2}\left(\eta_3 \hat{F}_3 + \eta_5 \hat{F}_5\right) + R_7 \tag{45}$$

where

$$\hat{F}_3 = \varsigma \int_L e^{-ik\xi} e^{-kz}\left[c(\xi) - \omega_0\left(\omega_e a_{33}(\xi) - ib_{33}(\xi)\right)\right]\mathrm{d}\xi$$

$$\hat{F}_5 = -\varsigma \int_L e^{-ik\xi} e^{-kz}\left[c(\xi) - \omega_0\left(\xi + \frac{iU}{\omega_e}\right)\left(\omega_e a_{33}(\xi) - ib_{33}(\xi)\right)\right]\mathrm{d}\xi$$

and

$$R_7 = \frac{\varsigma^2 k\omega_0^2}{2\omega_e} \int_L e^{-2kz} b_{33}(\xi)\,\mathrm{d}\xi$$

Note that η_3 and η_3 are the complex heave and pitch amplitudes, containing both phase and amplitude information.

Havelock

Finally the added resistance using a method proposed by Havelock (1942) is given by:

$$R_{AW} = \frac{-k}{2}\left\{ F_3\eta_3 \sin\varepsilon_3 + F_5\eta_5 \sin\varepsilon_5 \right\} \tag{46}$$

Where F_3 and F_5 are the magnitudes of the heave and pitch excitation force and moment; μ_3 and μ_5 are the magnitudes of the heave and pitch motions; and ε_3 and ε_5 are the phase differences of the heave and pitch motions with the corresponding excitation force or moment.

2D Ship Sections

The calculation of the added mass and damping of two dimensional ship sections, is based on the work of Ursell (1949), for a two-dimensional circular cylinder heaving in the free surface. The work of Bishop et al. (1978) has expanded the original approach to include conformal mapping techniques which may be used to map the ship's section to a unit circle centred at the origin, and hence calculate the hydrodynamic coefficients of arbitrary ship sections. The work of Sutherland (1987) is also a useful starting point and rephrases the method of Bishop et al. (1977).

Calculation of Added Mass and Damping of 2D Ship Sections

The conformal mappings are described in greater detail in section Conformal Mapping on page 13, but the general form of the mapping equation is given below in Equation (47).

$$X = y + iz = a_0\left(\zeta + a_1 + \frac{a_2}{\zeta} + \frac{a_3}{\zeta^2} + \frac{a_4}{\zeta^3} + \frac{a_5}{\zeta^4} + \cdots + \frac{a_n}{\zeta^{n-1}} \right) \tag{47}$$

The work of Ursell (1949) develops a formulation for the added mass and damping of a heaving circular cylinder in a free surface. The presence of the free surface gives rise to the frequency dependence of the hydrodynamic coefficients. Ursell used a multipole expansion of the stream function and velocity potential to determine the flow around the cylinder, and hence derive the hydrodynamic coefficients.

The principle steps of the method are given in the following sections.

Calculation of multipole expansion coefficients

As mentioned above, the stream function and velocity potential are expressed as multipole expansions. The coefficients of the multipole expansion, p_{2m} and q_{2m}, are found by applying the appropriate boundary condition at the cylinder surface. This leads to Equation (48).

$$\Psi_c(\theta) - \frac{y(\theta)}{\text{Half beam}} \Psi_c(\pi/2) = \sum_{m=1}^{M} p_{2m} f_{2m}(\theta)$$

$$\Psi_s(\theta) - \frac{y(\theta)}{\text{Half beam}} \Psi_s(\pi/2) = \sum_{m=1}^{M} q_{2m} f_{2m}(\theta) \tag{48}$$

This may be re-arranged and expressed in matrix form:

$$\mathbf{Ax = b} \tag{49}$$

Where the vector $\mathbf{x}$ contains the p_{2m} or q_{2m} terms, the matrix A contains the f_{2m} terms and the vector b contains the Ψ_c or Ψ_s terms.

The terms in Equation (48) are evaluated as follows:

$$y(\theta) = a_0 \left[\sin\theta + a_1 \sin\theta - a_2 \sin 3\theta + a_3 \sin 5\theta + \cdots - (-1)^n a_n \sin(2n-1)\theta \right]$$

$$z(\theta) = a_0 \left[\cos\theta - a_1 \cos\theta + a_2 \cos 3\theta - a_3 \cos 5\theta + \cdots + (-1)^n a_n \cos(2n-1)\theta \right]$$

(50)

$$\Psi_c(\theta) = \pi e^{-kz} \sin ky \qquad\qquad (51)$$

$$\Psi_s(\theta) = -\pi e^{-kz} \cos ky + \int_0^\infty e^{-\upsilon y} \frac{\upsilon \sin \upsilon z + k \cos \upsilon z}{k^2 + \upsilon^2} d\upsilon \qquad (52)$$

The mapped points y, z are obtained by applying the mapping equation at angle θ (Equation(50)).

Since the integral in Equation (52) converges slowly it is evaluated by an alternative method. The method used follows the work of Sutherland and is known as the method of Laguerre-Gauss quadrature. It may be shown that the integral can be evaluated as in Equation (53):

(53)

$$\int_0^\infty e^{-\upsilon y} \frac{\upsilon \sin \upsilon z + k \cos \upsilon z}{k^2 + \upsilon^2} d\upsilon = \sum \frac{w_i}{y} \left[\frac{\dfrac{s_i}{y} \sin\left(\dfrac{s_i z}{y}\right) + k \cos\left(\dfrac{s_i z}{y}\right)}{k^2 + \left(\dfrac{s_i}{y}\right)^2} \right]$$

Where the weighting functions, w_i, and the abscissa, s_i, may be found in standard texts. Finally the f_{2m} terms are calculated for each multipole at each angle according to Equation (54):

$$f_{2m}(\theta) = \frac{y(\theta)}{\text{Half beam}} \psi_{2m}(\pi/2) - \psi_{2m}(\theta) \qquad (54)$$

with ψ_{2m}:

$$\psi_{2m} = \sin 2m\theta + ka_0 \left[\frac{\sin(2m-1)\theta}{2m-1} + \sum_{n=1}^{N} (-1)^{n+1} \frac{(2n-1)a_{2n-1} \sin(2m+2n-1)\theta}{2m+2n-1} \right] \qquad (55)$$

where $a_0, a_1, \ldots a_N$ are the conformal mapping coefficients.

Least squares solution to over-defined set of linear equations

In practice, Equation (49) is solved in a least squares sense: a number of angles, R, are chosen at which the Ψ_c and Ψ_s terms are evaluated from Equations (51) and (52). The number of multipoles, M, is chosen such that M < R, and the f_{2m} terms are evaluated, according to Equation (54), for each of the multipoles at each of the angles. Thus there are more linear equations than unknowns. It may be shown that the least squares solution to this system of equations may be expressed, in matrix form, as in Equation (51). This system may then easily be solved by Gauss elimination or any other matrix solving method, such as Gauss Seidel or SOR (successive over-relaxation).

$$\mathbf{A^T A x = A^T b} \qquad (56)$$

Calculation and integration of pressure functions around contour

Once the coefficients, p_{2m} and q_{2m}, have been found, it is necessary to calculate the terms M_0 and N_0. These are calculated according to Equation (57), where the functions are integrated over one quadrant of the section in the unit circle plane.

$$M_0 = \int_0^{\pi/2} \frac{a_0 M(\theta) W(\theta)}{\text{Half beam}} d\theta$$

$$N_0 = \int_0^{\pi/2} \frac{a_0 N(\theta) W(\theta)}{\text{Half beam}} d\theta \qquad (\,57\,)$$

The terms $M(\theta)$, $N(\theta)$ and $W(\theta)$ are given in Equations (58):

$$M(\theta) = \Phi_s(\theta) + \sum_{m=1}^{M} q_{2m}\phi_{2m}(\theta)$$

$$N(\theta) = \Phi_c(\theta) + \sum_{m=1}^{M} p_{2m}\phi_{2m}(\theta) \qquad (\,58\,)$$

$$W(\theta) = \cos\theta + \sum_{n=1}^{N}(-1)^{n+1}(2n-1)a_{2n-1}\cos(2n-1)\theta$$

The terms Φ_s, Φ_c and ϕ_{2m} are calculated from Equations (59):

$$\Phi_c(\theta) = \pi e^{-kz}\cos ky$$

$$\Phi_s(\theta) = \pi e^{-kz}\sin ky - \int_0^{\infty} e^{-\upsilon y}\frac{\upsilon\cos\upsilon z - k\sin\upsilon z}{k^2 + \upsilon^2}d\upsilon \qquad (\,59\,)$$

$$\phi_{2m} = \cos 2m\theta + ka_0\left[\frac{\cos(2m-1)\theta}{2m-1} + \sum_{n=1}^{N}(-1)^{n+1}\frac{(2n-1)a_{2n-1}\cos(2m+2n-1)\theta}{2m+2n-1}\right]$$

Again the integral in the Φ_s term is evaluated by Laguerre quadrature using Equation (60):

$$\int_0^{\infty} e^{-\upsilon y}\frac{\upsilon\cos\upsilon z + k\sin\upsilon z}{k^2 + \upsilon^2}d\upsilon = \sum \frac{w_i}{y}\left[\frac{\frac{s_i}{y}\cos\left(\frac{s_i z}{y}\right) - k\sin\left(\frac{s_i z}{y}\right)}{k^2 + \left(\frac{s_i}{y}\right)^2}\right] \qquad (\,60\,)$$

Additional terms A and B are calculated from Equation (61):

$$A = \Psi_c(\pi/2) + \sum_{m=1}^{M} p_{2m}\psi_{2m}(\pi/2)$$

$$B = \Psi_s(\pi/2) + \sum_{m=1}^{M} q_{2m}\psi_{2m}(\pi/2) \qquad (\,61\,)$$

Evaluation of hydrodynamic coefficients

Finally, the section added mass, a_{33}, and the section damping, b_{33}, may be calculated from Equations (62) and (63) respectively.

$$a_{33} = \frac{\rho b^2 \left(M_0 B + N_0 A\right)}{2\left(A^2 + B^2\right)} \qquad (62)$$

$$b_{33} = \frac{\rho b^2 \omega \pi^2}{4\left(A^2 + B^2\right)} \qquad (63)$$

Note that in all the above equations k is the wave number.

Checking the solution

A check of the values obtained may be made by equating the energy dissipated by the waves to the work done by the cylinder. This leads to the following relationship:

$$M_0 A - N_0 B = \frac{\pi^2}{2} \qquad (64)$$

Important notes

There are two important points worthy of special attention:

1) The calculations of added mass and damping should be done using the encounter frequency. Further, the wave number should be based on encounter frequency. This is because the radiated waves which cause the damping and contribute to the added mass are generated by the motion of the vessel which is assumed to occur at the encounter frequency and not the natural frequency of the waves causing the excitation.

2) There is an error in the equation for Φ_S quoted by Sutherland (1987) - Equation 4.7 and Bishop and Price (1978) second equation, Appendix I; the bracketed numerator in the integral should be a minus sign for Φ_S, the plus sign should be kept for Ψ_S. The correct equations are given in the original derivation by Ursell (1949) and are also correct in the work of de Jong (1973). Interestingly, the original Sutherland Motions code is correct.

Added Mass at Infinite Frequency

Assuming a symmetric mapping equation is used, the asymptotic value of the section added mass as the exciting frequency approaches infinity is given in Bishop et al. (1978):

$$m_\infty = \frac{\rho \pi a_0^2}{2} \left[(1 - a_1)^2 + \sum_{n=1}^{N} (a_{2n+1})^2 (2n+1) \right] \qquad (65)$$

Non-Dimensional Damping Coefficient

The non-dimensional damping, $\overline{A}^2$ is the ratio of the amplitude of the radiated waves at infinity to the amplitude of the heaving oscillation:

$$\overline{A}^2 = \frac{b_{33} \omega^3}{\rho g^2} \qquad (66)$$

Non-Dimensional Added Mass

The non-dimensional added mass, C_V, is defined in Equation (67), where b is the section beam and a_{33} is the added mass:

$$C_V = \frac{8a_{33}}{\pi\rho b^2}$$

(67)

Non-Dimensional Frequency

The non-dimensional frequency, δ, is defined in Equation (68), where b is the section beam:

$$\delta = \frac{b\omega^2}{2g}$$

(68)

Representation of Ship-Like Sections by Conformal Mapping

Conformal mapping may be used to map an arbitrary ship's section to a unit circle centred at the origin. The solution of the potential flow formulation for a unit circle may then be applied to an arbitrary hull form. The conformal mapping has the general form given below in Equation (69):

$$X = y + iz = a_0\left(\zeta + a_1 + \frac{a_2}{\zeta} + \frac{a_3}{\zeta^2} + \frac{a_4}{\zeta^3} + \frac{a_5}{\zeta^4} + \cdots + \frac{a_n}{\zeta^{n-1}} \right)$$

(69)

Where ζ is a complex number lying on the unit circle. The coordinate system is shown in Figure 1, and in this case $\zeta = ie^{-i\theta}$.
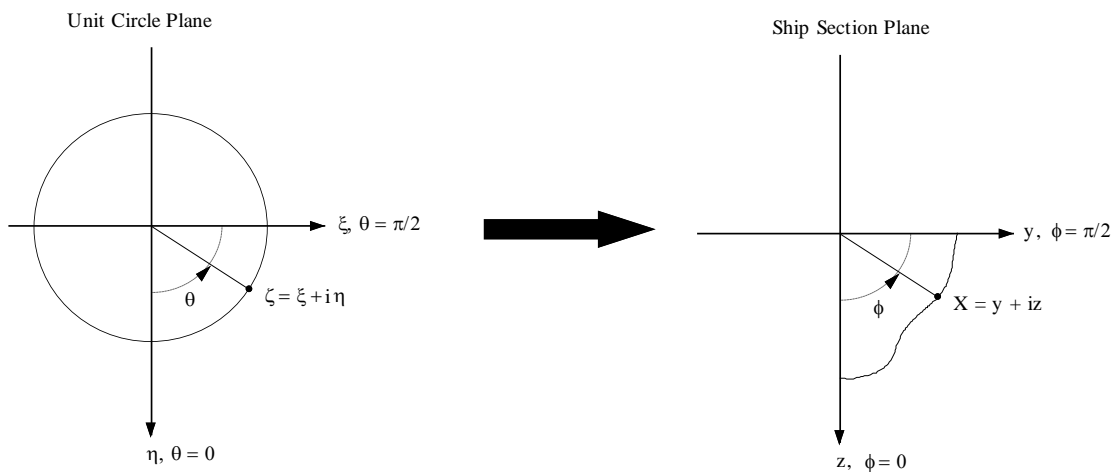
Figure 1: Mapping Coordinate System

The mapping described in Equation (69) will map an arbitrary shape in the X-plane to the unit circle in the ζ-plane. If the entire unit circle is mapped, there is no need for symmetry planes to exist in the y=0 and z=0 axes. This type of mapping is useful for heeled yacht sections or asymmetric catamaran hulls.

A simplified mapping uses only the odd powers of ζ and is mapped over one quarter of the unit circle. This produces a mapping with symmetry planes in both y=0 and x=0 axes. This mapping equation is given in Equation (70) and referred to as the symmetric multi-parameter mapping.

$$X = y + iz = a_0\left(\zeta + \frac{a_1}{\zeta} + \frac{a_2}{\zeta^3} + \cdots + \frac{a_n}{\zeta^{2n-1}}\right) \qquad (\,70\,)$$

It should be noted that several mapping equations may be derived to map ship-like sections to the unit circle. The mapping equations described here follow the work of Ursell (1949), Bishop et al. (1978) and to some extent the work of Sutherland (1987). This is important because of the later derivation and evaluation of the hydrodynamic coefficients.

The symmetric mapping equation described in Equation (70) may be further simplified so that only the first three terms are used. In this case the mapping is known as a Lewis Mapping and the mapped section is a Lewis Section. The Lewis Mapping is given in Equation (71).

$$X = y + iz = a_0\left(\zeta + \frac{a_1}{\zeta} + \frac{a_2}{\zeta^3}\right) \qquad (\,71\,)$$

Where the constants a_0, a_1, and a_2 are calculated from the section water line half beam, section area and section draft. See Equations (72):

$$a_0 = \frac{\text{Draught}}{1 - a_1 + a_2}$$

$$a_1 = (1 + a_2)\frac{H - 1}{H + 1}$$

$$a_2 = \frac{3 - C + \sqrt{9 - 2C}}{C}$$

$$C = 3 + \frac{4\sigma}{\pi} + \left(1 - \frac{4\sigma}{\pi}\right)\left(\frac{H - 1}{H + 1}\right)^2 \qquad (\,72\,)$$

$$\sigma = \frac{\text{Cross section area}}{2 \times \text{Half beam} \times \text{Draught}}$$

$$H = \frac{\text{Half beam}}{\text{Draught}}$$

The mapped point in the ship section plane (y, z) of a point on the unit circle, described by its angle, θ, from the positive η-axis, for the symmetric, multi-parameter mapping, is given in Equations (73):

$$y = a_0\left[\sin\theta + a_1\sin\theta - a_2\sin 3\theta + a_3\sin 5\theta + \cdots - (-1)^n a_n\sin(2n-1)\theta\right]$$
$$z = a_0\left[\cos\theta - a_1\cos\theta + a_2\cos 3\theta - a_3\cos 5\theta + \cdots + (-1)^n a_n\cos(2n-1)\theta\right] \qquad (\,73\,)$$

Derivative of Conformal Mapping

Assuming the symmetrical mapping equations are used (i.e. only the odd ζ terms are used); see Equation (74).

$$X = y + iz = a_0 \left(\zeta + \frac{a_1}{\zeta} + \frac{a_2}{\zeta^3} + \cdots + \frac{a_n}{\zeta^{2n-1}} \right)$$
(74)

The derivatives of the mapping are evaluated according to Equation (75):

$$\frac{dy}{d\theta} = a_0 [\cos\theta + a_1 \cos\theta - 3a_2 \cos 3\theta + 5a_3 \cos 5\theta + \cdots$$
$$- (-1)^n (2n-1) a_n \cos(2n-1)\theta]$$
$$\frac{dz}{d\theta} = -a_0 [\sin\theta - a_1 \sin\theta + 3a_2 \sin 3\theta - 5a_3 \sin 5\theta + \cdots$$
$$+ (-1)^n (2n-1) a_n \sin(2n-1)\theta]$$
(75)

Noting that: $\zeta = i\,e^{-i\theta}$

Shear Force and Bending Moment due to Ship Motion in a Seaway

Equation 64 of Salvesen Tuck and Faltinsen (1970) is reproduced here as Equation (76) and gives the resultant shear force / bending moment of a hull in a seaway as:

$$V_j = I_j - R_j - E_j - D_j$$
(76)

where:

V_j is the resultant shear force or bending moment.
I_j is the inertial component.
R_j is the hydrostatic (restoring) component.
E_j is the wave excitation term.
D_j is the hydrodynamic component.
j is the degree of freedom 1 to 6.

The components are calculated by separate modules. For a specified degree of freedom, each module return the complex coefficients of $e^{i\omega t}$. So we may write V as

$$V = [z_I + z_R + z_E + z_D] e^{i\omega t}$$
$$= z_V e^{i\omega t}$$
$$= r_V e^{i(\omega t + \varphi_V)}$$
(77)

The amplitude and phase of the resultant shear force or bending moment is immediately available from the complex addition of the coefficients of $e^{i\omega t}$.

Inertial Component

This module calculates the vertical shear forces and bending moment components due to the inertial forces.

The section inertial loading in heave, i_3, is given by Equation (78) (Salvesen et al. 1970, Equation 66):

$$i_3 = -m\omega^2 (\eta_3 - \xi\eta_5)$$
(78)

The shear force at section x_0 is calculated by integrating the loading forward of the section of interest. Thus the shear force due to the inertial forces is given by:

$$I_3 = \int_{x_0}^{x_{bow}} i_3 d\xi \qquad (79)$$

The bending moment at section x_0 is given by Salvesen et al. 1970, Equation 68:

$$I_5 = \int_{x_0}^{x_{bow}} i_5 d\xi \qquad (80)$$

where i_5 is given by:

$$i_5(x_0) = (\xi - x_0)(\eta_3 - \xi\eta_5) \qquad (81)$$

$\quad$ m $\quad$ section mass per unit length.
$\quad$ η_3 $\quad$ heave response.
$\quad$ η_5 $\quad$ pitch response.
$\quad$ ω $\quad$ encounter frequency.
$\quad$ ξ $\quad$ longitudinal distance from LCG.
$\quad$ x_0 $\quad$ longitudinal position of section of interest.

The functions in this module are set up to calculate the value of the integrands at all sections and then integrate over the appropriate sections to obtain the shear force and bending moments; integrations are evaluated using the trapezium rule.

The bending moment integrands are dependent on the section, x_0, at which the bending moment is being evaluated and hence need to be re-evaluated for each section.

Hydrostatic (Restoring) Component

This module calculates the vertical shear forces and bending moment components due to the hydrostatic restoring forces.
The section hydrostatic restoring loading in heave, r_3, is given by (Salvesen et al. 1970, Equation 70):

$$r_3 = -\rho g b(\eta_3 - \xi\eta_5) \qquad (82)$$

The shear force at section x_0 is calculated by integrating the loading forward of the section of interest. Thus the shear force due to the hydrostatic restoring force is given by:

$$R_3 = \int_{x_0}^{x_{bow}} r_3 d\xi \qquad (83)$$

The bending moment at section x_0 is given by (Salvesen et al. 1970, Equation 68):

$$R_5 = \int_{x_0}^{x_{bow}} r_5 d\xi \qquad (84)$$

where r_5 is given by:

$$r_5(x_0) = \rho g b(\xi - x_0)(\eta_3 - \xi\eta_5) \qquad (85)$$

$\quad$ b $\quad$ section waterline beam.
$\quad$ η_3 $\quad$ heave response.
$\quad$ η_5 $\quad$ pitch response.
$\quad$ ω $\quad$ encounter frequency.
$\quad$ ξ $\quad$ longitudinal distance from LCG.
$\quad$ x_0 $\quad$ longitudinal position of section of interest.

The functions in this module are set up to calculate the value of the integrands at all sections and then integrate over the appropriate sections to obtain the shear force and bending moments; integrations are evaluated using the trapezium rule. The bending moment integrands are dependent on the section, x_0, at which the bending moment is being evaluated and hence need to be re-evaluated for each section.

Wave Excitation Component

This module calculates the vertical shear forces and bending moment components due to the incident wave excitation forces.

The section wave excitation loading in heave, e_3, is given by (Salvesen et al. 1970, Equation 73):

$$e_3 = f_3 + h_3 \tag{86}$$

Here the Froude-Krilov force, f_3, and the wave diffraction force, h_3, as calculated in the hydrodynamics module, include the wave amplitude and water density factor ζ_ρ.

The shear force at section x_0 is calculated by integrating the loading forward of the section of interest and adding a speed dependent term. Thus the shear force due to the wave excitation is given by:

$$E_3 = \int_{x_0}^{x_{bow}} e_3 d\xi + \left[\frac{U}{i\omega} h_3(x_0) \right] \tag{87}$$

The bending moment at section x0 is given by (Salvesen et al. 1970, Equation 74):

$$E_5 = \int_{x_0}^{x_{bow}} e_5 d\xi \tag{88}$$

where e_5 is given by:

$$e_5(x_0) = (x_0 - \xi)(f_3 + h_3) + \frac{iU}{\omega} h_3 \tag{89}$$

f_3 2D section Froude-Krilov wave force in heave.
h_3 2D section diffraction wave force in heave.
ω encounter frequency.
ξ longitudinal distance from LCG.
x_0 longitudinal position of section of interest.

The functions in this module are set up to calculate the value of the integrands at all sections and then integrate over the appropriate sections to obtain the shear force and bending moments; integrations are evaluated using the trapezium rule.
The bending moment integrands are dependent on the section, x_0, at which the bending moment is being evaluated and hence need to be re-evaluated for each section.

Hydrodynamic Component

This module calculates the vertical shear forces and bending moment components due to hydrodynamic forces.

The section hydrodynamic loading in heave, d3, is given by (Salvesen et al. 1970, Equation 79):

$$d_3 = -\left\{ a_{33}(\ddot{\eta}_3 - \xi\ddot{\eta}_5) + b_{33}(\dot{\eta}_3 - \xi\dot{\eta}_5) - \frac{U}{\omega^2} b_{33}\ddot{\eta}_5 + Ua_{33}\dot{\eta}_5 \right\} \tag{90}$$

On differentiating the heave and pitch accelerations and velocities, Equation (90) may be expressed in terms of the heave and pitch amplitudes:

$$d_3 = -\left\{ -\omega^2 a_{33}(\eta_3 - \xi\eta_5) + i\omega b_{33}(\eta_3 - \xi\eta_5) + Ub_{33}\eta_5 + i\omega U a_{33}\eta_5 \right\} \qquad (91)$$

The shear force at section x_0 is then given by:

$$D_3 = \int_{x_0}^{x_{bow}} d_3 d\xi - \left[iU\omega a_{33}(\eta_3 - x_0\eta_5) + Ub_{33}(\eta_3 - x_0\eta_5) + U^2 a_{33}\eta_5 - \frac{iU^2}{\omega} b_{33}\eta_5 \right] \qquad (92)$$

The bending moment at section x0 is given by (Salvesen et al. 1970, Equation 81):

$$D_5 = \int_{x_0}^{x_{bow}} d_5 d\xi \qquad (93)$$

,with d_5:

$$
\begin{aligned}
d_5(x_0) = (\xi - x_0)\left\{ -\omega^2 a_{33}(\eta_3 - \xi\eta_5) + i\omega b_{33}(\eta_3 - \xi\eta_5) \right\} \\
+ i\omega U a_{33}(\eta_3 - x_0\eta_5) + Ub_{33}(\eta_3 - x_0\eta_5) \\
+ U^2 a_{33}\eta_5 - \frac{iU^2}{\omega} b_{33}\eta_5
\end{aligned}
\qquad (94)
$$

where:

a_{33}	2D section added mass in heave.
b_{33}	2D section damping in heave.
η_3	heave amplitude.
η_5	pitch amplitude.
ω	encounter frequency.
ξ	longitudinal distance from LCG.
x_o	longitudinal position of section of interest.

The functions in this module are set up to calculate the value of the integrands at all sections and then integrate over the appropriate sections to obtain the shear force and bending moments; integrations are evaluated using the trapezium rule.

The bending moment integrands are dependent on the section, x_0, at which the bending moment is being evaluated and hence need to be re-evaluated for each section.

Calculation of uncoupled roll motion

This section describes the formulation used to compute the roll motion.

- **Equation of motion for roll**

Equation of motion for roll

The vessel's roll motion may be represented by a second order differential equation, such as that describing a forced spring, mass and damper system.

$$(I_4 + A_{44})\ddot{\eta}_4 + B_{44}\dot{\eta}_4 + C_{44}\eta_4 = F_4 e^{i\omega_e t} \qquad (95)$$

where the variables are defined as follows:

I_4	moment of inertia for roll.
A_{44}	added inertia coefficient for roll.
B_{44}	damping coefficient for roll.
C_{44}	hydrostatic restoring coefficient for roll.
F_4	roll exciting moment at the encounter frequency ω_e.

η_4	instantaneous roll displacement.
$\dot{\eta}_4$	instantaneous roll velocity.
$\ddot{\eta}_4$	instantaneous roll acceleration.

It may be shown that the solution to the above equation is given by:

$$\eta_4 = \frac{F_4}{\sqrt{\left(C_{44} - (I_4 + A_{44})\omega_e^2\right)^2 + B^2\omega_e^2}} \cos(\omega_e t + \varepsilon)$$

where: ε is the phase lag relative to the forcing function: $\tan\varepsilon = \dfrac{B_{44}\omega_e}{C_{44} - (I_4 + A_{44})\omega_e^2}$

This equation may be re-expressed in terms of the damping ratio,

$$\beta_{44} = \frac{B_{44}}{2\sqrt{C_{44}(I_4 + A_{44})}}, \text{ the natural frequency of the system, } \omega_0 = \sqrt{\frac{C_{44}}{(I_4 + A_{44})}}, \text{ and}$$

the tuning factor, $\lambda = \dfrac{\omega_e}{\omega_0}$.

As an aside, it may be shown (by differentiation of the RAO function) that the damped natural frequency is given by:

$$\omega_{0damped} = \omega_0\sqrt{1 - 2\beta^2}$$

The roll transfer function or response function is then assumed to be given by:

$$\text{RAO}_{\text{Roll}} = \frac{\eta_4 C_{44}}{F_4} = \frac{1}{\sqrt{\left(1 - \lambda^2\right)^2 + 4\beta_{44}^2\lambda^2}}$$

Strictly speaking this is the roll motion transfer function with regard to wave force and not wave slope, however, the two are assumed to be the same.

The RAO is then modified for wave heading and apparent wave slope so that the RAO at off head seas is given by:

$$\text{RAO}_{\text{Roll}}(\mu) = \text{RAO}_{\text{Roll}}\sin(\mu)$$

thus the roll RAO is zero in head and following seas and has a maximum in beam seas.

In Motions the required parameters are determined as follows:

I_4	mass inertia of vessel in roll $I_4 = k_{xx}^2\nabla\rho$, k_{xx} input by user
A_{44}	added inertia coefficient for roll $A_{44} = 0.3I_4$, this is an average of values from Vugts (1968) and Lloyd (1998)
β_{44}	Non-dimensional damping coefficient for roll, input by user.
C_{44}	hydrostatic restoring coefficient for roll $C_{44} = GM_t\nabla\rho g$, VCG input by user

Roll free-decay test

If experimental facilities are available, the roll damping can be obtained from a free-decay test of the roll motions. The vessel is heeled over to one side and released, the roll amplitude is measured and plotted against time. The figure below shows the theoretical free-decay of two vessels with different damping coefficients.
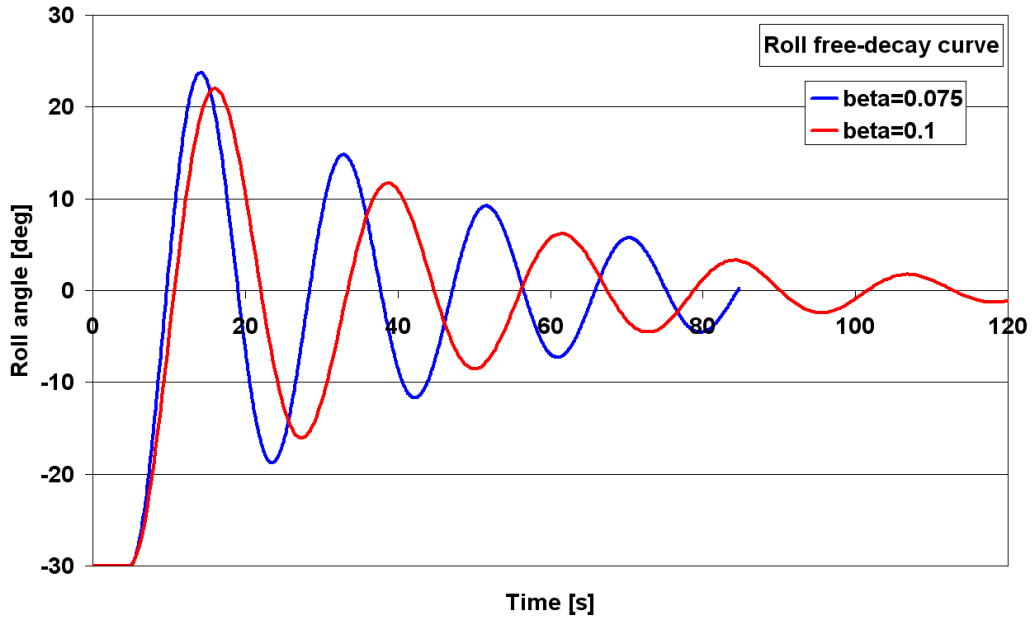
Free-decay time series for two vessels released from an initial heel angle of 30 degrees

By plotting the value of one peak against the value of the next peak (the same can also be done for the troughs to obtain more data), the roll damping can be derived.

Peak/trough amplitudes for beta 0.075 vessel

beta0.075	peak/trough i	peak/trough i+1
trough 1	30.000	18.755
peak 1	23.752	14.808
trough 2	18.755	11.692
peak 2	14.808	9.231
trough 3	11.692	7.289
peak 3	9.231	5.754
trough 4	7.289	4.544
peak 4	5.754	
trough 5	4.544	

Peak/trough amplitudes for beta 0.1 vessel

beta0.1	peak/trough i	peak/trough i+1
trough 1	30.000	16.001
peak 1	21.745	11.646
trough 2	16.001	8.519
peak 2	11.646	6.183
trough 3	8.519	4.518
peak 3	6.183	3.285
trough 4	4.518	2.405
peak 4	3.285	1.747
trough 5	2.405	1.279
peak 5	1.747	
trough 6	1.279	
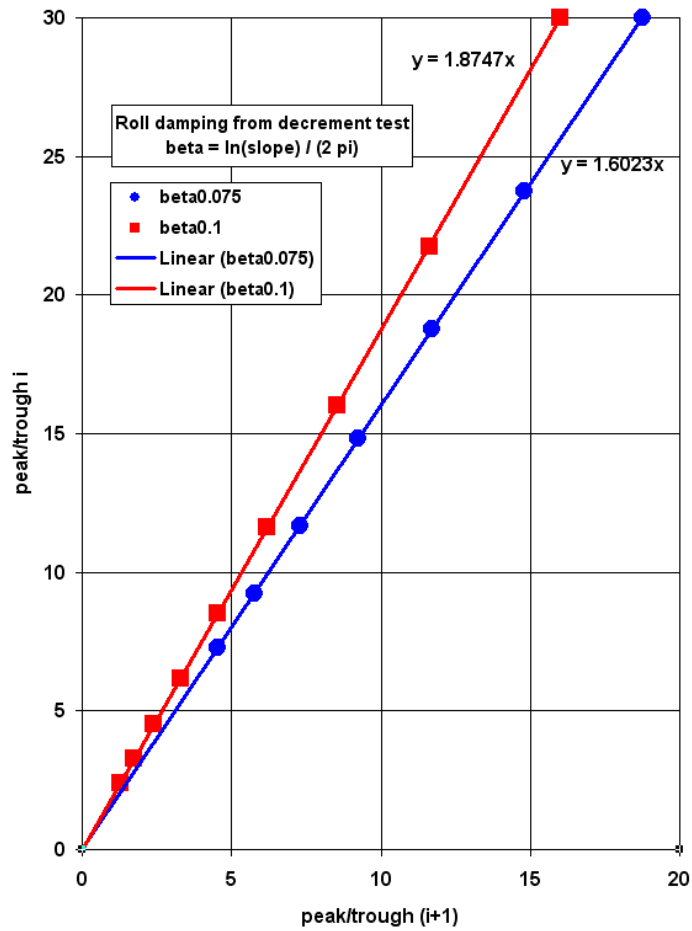
Plot of peak amplitude against peak amplitude of next peak. In this example, data for both peaks and troughs have been plotted

The non-dimensional roll damping parameter used in Motions, β_{44}, is given by:

$$\beta_{44} = \frac{\ln(\text{slope})}{2\pi}$$

Thus for the beta0.075 vessel, the slope is 1.6023, giving a damping of 0.075 (as expected); similarly for the beta0.1 vessel, the slope is 1.8747 giving a damping of 0.100.

The free-decay roll test can be simulated in Motions by choosing Roll decay simulation option in the Analysis | Calculate Wave Surface dialog, then choosing Display | Animate and saving the time-series to a file:
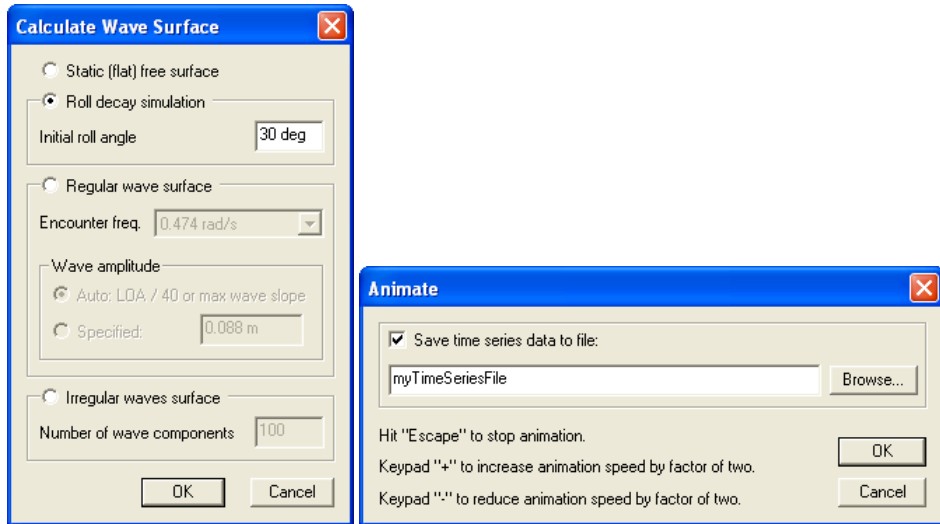
Simulation of free-decay roll test in Motions

Appendix B – Strip Theory Validation

Motions has been validated against various sources of public data.

- **Motions Validation 1 – NPL Round Bilge series**
- **Motions Validation 2 – Series 60**
- **Motions Validation 3 – SA van der Stel**

The numerical calculation routines and algorithms for the two dimensional section hydrodynamic coefficients have been validated against calculations for standard sections, e.g. Bishop and Price (1979).

In addition the heave and pitch response, predicted by Motions, has been validated against several sources of experimental data from both model scale and full size vessels: e.g. Molland et al. (1995) and data obtained by AMECRC for various full scale vessels and model yachts tests.

Motions Validation 1 – NPL Round Bilge series

Use of Motions for predicting the heave and pitch response of high-speed catamaran and monohull hullforms based on NPL Round Bilge series.

Introduction

Motions uses Strip Theory (Salvesen et al. 1970) to predict the heave and pitch response of a vessel in a seaway. Conformal mapping methods and inviscid flow calculations are used to compute the section added mass and damping.

Calculations are presented for a standard series of catamaran forms tested at the University of Southampton (Molland et al. 1995, Couser 1996). The results from Motions are compared with those from towing tank measurements for three Length:Displacement ratios, two demihull separations (and single demihull in isolation) for three different speeds. The hullforms tested were based on the NPL round bilge series (Bailey 1976) see Figure 1; the hulls having been stretched in the longitudinal direction to give suitable Length:Displacement ratios. A summary of the test conditions is given below:

Model principal particulars

Name	Hullform	$L/\nabla^{1/3}$	L/B	B/T	L_{BP}
4b	NPL r/b	7.4	9	2	1.6m
5b	NPL r/b	8.5	11	2	1.6m
6b	NPL r/b	9.5	13	2	2.1m

Separations tested: S/L = 0.2, 0.4, Monohull
Speeds tested: F_n = 0.20, 0.53, 0.80.

Tank Dimensions

Length	60.0m
Breadth	3.7m
Water depth	1.85m
Max. carriage speed	4.6m/s

All tests were conducted in head seas.

Analysis Procedure

The Motions analysis was carried out using 41 equally spaced sections to define the hull and 3 parameter Lewis mappings were used to compute the section hydrodynamic properties. In all cases the basic inviscid Salvesen et al. (1970) Strip Theory formulation was used with no additional damping terms; for the high Froude numbers, the analysis was also performed including the Salvesen et al. (1970) transom stern correction.

The computed RAOs (magnitude and phase) are compared with those from the tank tests in regular waves. In all cases, heave has been non-dimensionalised against wave height and pitch against wave slope. The phase lag between the motion and the wave at the centre of gravity is given in degrees.

A comparison of the hullform and the mapped Lewis sections is given in Figure 1. It may be seen that the mappings give a reasonable definition of the actual surface, though there is scope for improvement of the mapping by using more mapping terms.

The motions of the catamaran have been predicted by computing the motions of a single demihull in isolation. For all but the lowest speeds the interaction between the demihulls is small, particularly for the wider spacing of , Separation:Length ratio, S/L = 0.4.

Results

The results are presented in Figures 2 to 10. For each of the figures, Figure a is the heave RAO; Figure b is the heave phase; Figure c the pitch RAO and Figure d the pitch phase.

The Motions results are for a single demihull in isolation. These have been compared with the towing tank data for the two catamaran configurations and the demihull tested in isolation (monohull). It may be seen that for all but the lowest speeds, the interaction between the two catamaran demihulls is small.

Slow Speed

Figures 2 to 4 give the results for model 4b; Figures 5 to 7 those for model 5b and Figures 8 to 10 those for the most slender model, 6b.

For the slowest Froude number ($F_n = 0.20$), Figures 2, 5 and 8, the Motions results show good correlation with the monohull model tests, for all the Length:Displacement ratios. The interaction between the demihulls, which may be observed as a secondary resonant peak is not computed since the Motions model is for a single demihull in isolation. However, these secondary resonant peaks diminish rapidly with increasing Length:Displacement ratio (going from model 4b to 6b); and excellent correlation is found for model 6b for both catamaran configurations.

Moderate Speed

At moderate Froude number, Motions results have been computed with and without additional transom terms. These terms significantly increase the damping, particularly for pitch. The terms are suggested by Salvesen et al. (1970) because the Strip Theory formulation assumes that the sectional added mass and damping tend to zero at the vessel bow and stern. However, for high-speed semi-displacement forms which have transom areas which are often a large proportion of the maximum cross sectional area, it is felt, that the transom terms provide an excessive amount of extra damping and some judgement, on the part of the designer, is required to determine whether transom terms should be included or not.

For heave at Froude number = 0.53, Figures 3, 6 and 9, Motions predictions including transom terms are closest to the experiment results. However, for pitch, results without transom terms are more accurate. In all cases the Motions predictions show good correlation with the experiment results for the two catamaran configurations. The experimental results for the demihull in isolation (monohull) are slightly reduced when compared with those of the two catamaran configurations.

Motions tends to over predict the frequency at which the resonant peak occurs, and this is more so for the predictions of pitch response. As might be expected, the discrepancies between predictions and experiments reduce for the more slender models (going from model 4b to 6b). However, in general the predictions at $F_n = 0.53$ are quite reasonable.

High Speed

At the highest Froude number tested ($F_n = 0.80$), Figures 4, 7 and 10, Motions predictions start to over predict the heave response if transom terms are not included. If transom terms are included good estimates of the peak response are achieved. However, the difference between the predicted and measured resonant frequencies is increased when compared with the results at slower Froude numbers.

Predictions of the pitch response with and without transom terms show significant differences, and this is due to the velocity term in the transom terms. Again, the magnitude of the resonant peak is relatively well predicted when transom terms are included but the frequency at which the peak occurs is poorly predicted.

Conclusions

Motions predictions for the heave and pitch response to regular head waves have been compared with towing tank results for a number of geometrically similar hullforms of differing length: displacement ratio at three different speeds; $F_n = 0.20$, 0.53, 0.80. For the hullforms tested (based on the NPL round bilge series, Bailey 1976) Motions was able to adequately predict the heave and pitch response up to a Froude number of 0.53; predictions at higher speeds suffered from poor prediction of the resonant frequency, although the magnitude of the resonant peak was reasonably well calculated.

Catamaran results were obtained from Motions by modelling a single demihull in isolation. Experiments show that the differences between the catamaran and isolated demihull configurations are most pronounced at slow Froude numbers ($F_n = 0.20$) and for catamarans with closely spaced demihulls and small length: displacement ratios.

References

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Couser, P.R. 1996. *An investigation into the performance of high-speed catamarans in calm water and waves*. PhD thesis, University of Southampton, March.

Molland, A.F., Wellicome, J.F. and Couser, P.R. 1995. *Experimental measurements of the seakeeping characteristics of fast displacement catamarans in long-crested head-seas*. University of Southampton, Ship Science Report No. 89, December.

Salvesen, N., Tuck, O.E. and Faltinsen, O., 1970. Ship motions and sea loads. *Transactions, Society of Naval Architects and Marine Engineers*, No. 78, pp250-287.

Figures

Figure 1: NPL hullform with Lewis sections used for seakeeping analysis

Figure 2b: Heave phase: Seakeeper and model tests - 4b, Fn = 0.2

Figure 2c: Pitch response: Seakeeper and model tests – 4b, Fn = 0.2

Figure 2d Pitch phase: Seakeeper and model tests – 4b, Fn = 0.2

Figure 3a: Heave response: Seakeeper and modeltests - 4b, Fn = 0.53

Figure 3b: Heave phase: Seakeeper and model tests – 4b, Fn = 0.53

Figure 3c: Pitch response: Seakeeper and model tests – 4b, Fn = 0.53

Figure 3d: Pitch phase: Seakeeper and model tests - 4b, Fn = 0.53

Figure 4a: Heave response: Seakeeper and model tests – 4b, Fn = 0.8

Figure 4b: Heave phase: Seakeeper and model tests - 4b, Fn = 0.8

Figure 4c: Pitch response: Seakeeper and model tests - 4b, Fn = 0.8

Figure 4d: Pitch phase: Seakeeper and model tests – 4b, Fn = 0.8

Figure 5a: Heave response: Seakeeper and model tests – 5b, Fn = 0.2

Figure 5b: Heave phase: Seakeeper and model tests - 5b, Fn = 0.2

Figure 5c: Pitch response: Seakeeper and model tests - 5b, Fn = 0.2

Figure 5d: Pitch phase: Seakeeper and model tests – 5b, Fn = 0.2

Figure 6a: Heave response: Seakeeper and model tests – 5b, Fn = 0.53

Figure 6b: Heave phase: Seakeeper and model tests – 5b, Fn = 0.53

Figure 6c: Pitch response: Seakeeper and model tests – 5b, Fn = 0.53

Figure 6d: Pitch phase: Seakeeper and model tests – 5b, Fn = 0.53

Figure 7a: Heave response: Seakeeper and model tests – 5b, Fn = 0.8

Figure 7b: Heave phase Seakeeper and model tests - 5b, Fn = 0.8

Figure 7c: Pitch response: Seakeeper and model tests - 5b, Fn = 0.8

Figure 7d: Pitch phase: Seakeeper and model tests – 5b, Fn = 0.8

Figure 8a: Heave response: Seakeeper and model tests – 6b. Fn = 0.2

Figure 8b: Heave phase: Seakeeper and model tests – 6b, Fn = 0.2

Figure 8c: Pitch response: Seakeeper and model tests – 6b, Fn = 0.2

Figure 8d Pitch phase: Seakeeper and model tests – 6b, Fn = 0.2

Figure 9a: Heave response: Seakeeper and model tests – 6b, Fn = 0.53

Figure 9b: Heave phase: Seakeeper and model tests – 6b, Fn = 0.53

Figure 9c: Pitch response: Seakeeper and model tests – 6b. Fn = 0.53

Figure 9d: Pitch phase: Seakeeper and model tests – 6b, Fn = 0.53

Figure 10a: Heave response: Seakeeper and model tests – 6b, Fn = 0.8

Figure 10b: Heave phase: Seakeeper and model tests – 6b, Fn = 0.8

Figure 10c: Pitch response: Seakeeper and model tests – 6b, Fn = 0.8

Figure 10d: Pitch phase: Seakeeper and model tests - 6b, Fn = 0.8

Motions Validation 2 – Series 60

Use of Motions for predicting the heave and pitch response of the Series 60 C_B=0.7 hullform – Comparison with SWAN and experiments.

Introduction

Motions uses Strip Theory (Salvesen et al. 1970) to predict the heave and pitch response of a vessel in a seaway. Conformal mapping methods and inviscid flow calculations are used to compute the section added mass and damping.

Calculations are presented for the standard Series 60, C_B=0.7, hullform. This vessel has been the subject of many theoretical and experimental investigations over a number of years. The results from Motions are compared with those from towing tank measurements (Gerritsma 1960; Gerritsma and Beukelman 1966) and also for a non-linear time domain program developed at MIT – SWAN (Sclavounos et al. 1997). Results for two speeds, F_n = 0.2 and 0.3, are presented.

Hullform principal particulars

Hullform	L/∇1/3	L/B	B/T	LBP
Series 60, CB=0.7	5.6	7.0	2.5	101.7m

Speeds tested: F_n = 0.2, 0.3. All tests were done in head seas. The model tested by Gerritsma (1960) was 2.438m in length.

Analysis Procedure

The Motions analysis was carried out using 41 equally spaced sections to define the hull and 3 parameter Lewis mappings were used to compute the section hydrodynamic properties. In all cases the basic inviscid Salvesen et al. (1970) Strip Theory formulation was used with no additional damping terms; transom terms were not included. The hullform is shown in Figure 1.

Results

Froude number = 0.2

The heave response at $F_n = 0.2$ is given in Figure 4a and the pitch response in Figure 4b. The RAOs are plotted against wave frequency for the 101.7m vessel, which was tested in Motions and SWAN (Scalvounos et al. 1997). In these graphs the solid line is the prediction made by Motions, the solid circles those by SWAN and the results of experiments are shown by crosses. For the experimental results error bars of +/- 2.5% were added, scatter of this order was noted in the results published in Gerritsma (1960).

Motions is not as accurate as SWAN, however, it successfully predicts the location of the resonant peaks in heave and pitch. The amplitude of the heave peak is over predicted by around 15%. In the case of pitch, Motions under predict the peak by approximately 15% whilst SWAN over predict by about 8%.

Froude number = 0.3

Results for heave and pitch are presented in Figure 4c and 4d respectively. At the higher Froude number, Motions accurately predicts the position of the heave resonant peak but appears to significantly over predict the magnitude of the peak. Due to insufficient experimental data, it is difficult to determine the exact magnitude of the response peak. The problem is even more acute with the SWAN results. A major advantage with Motions is that it is able to predict the response at 100 frequencies in a matter of seconds; the same calculations would take over 8 hours of CPU time for SWAN or several days in a towing tank. Thus Motions is valuable in determining the position of the resonant peak. The magnitude of the peak may then be found with more sophisticated methods such as SWAN.

The pitch results are more difficult to compare due to lack of experimental data. However it appears that Motions does a reasonable job of predicting the pitch response, though the resonant frequency may have been over predicted slightly. SWAN appears to significantly over predict the magnitude of the resonant peak. The SWAN data was found in the SWAN 1 Vs 3.0 user manual (1999).

Conclusions

Motions was able to produce reasonable predictions of the response of the Series 60, $C_B = 0.7$, hullform at slow speed, $F_n = 0.2$. The heave and pitch resonant frequencies were well predicted, though the magnitude of the heave response was over predicted and that of the pitch response under predicted.

At higher speed, $F_n = 0.3$, the over prediction of heave response was greater, though the accuracy of the pitch prediction seemed to improve. Again the resonant frequencies were well predicted. In fact the Motions results appeared to be significantly better than those of SWAN for the pitch response.

The speed advantage of Motions was highlighted. A full frequency range may be analysed and the RAO defined with some accuracy by Motions in a matter of seconds. A similar number of calculations in SWAN would require 8 hours CPU time.

References

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Salvesen, N., Tuck, O.E. and Faltinsen, O., 1970. Ship motions and sea loads. *Transactions, Society of Naval Architects and Marine Engineers*, No. 78, pp 250-287.

Sclavounos, P.D., Kring, D.C., Huang, Y., Mantzaris, D.A., Kim, S. and Kim, Y. 1997. A computational method as an advanced tool of ship hydrodynamic design. *Transactions, Society of Naval Architects and Marine Engineers*, Vol. 105, pp. 375-397.

SWAN 1 Vs 3.0 user manual, 1999. http://www.bmarc.com
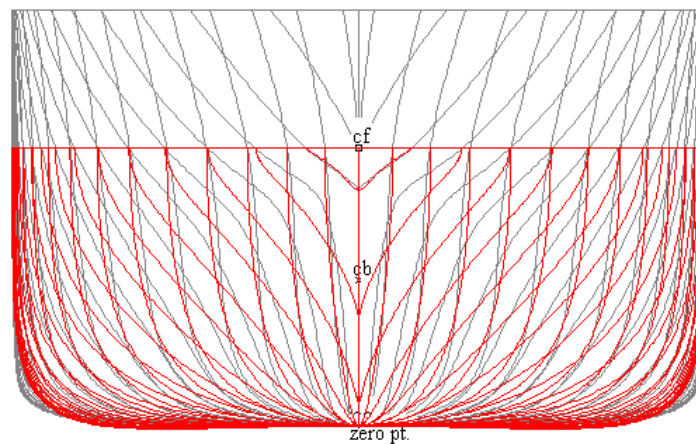
Figures

Figure 1: Series 60 hullform with Lewis sections used for seakeeping analysis
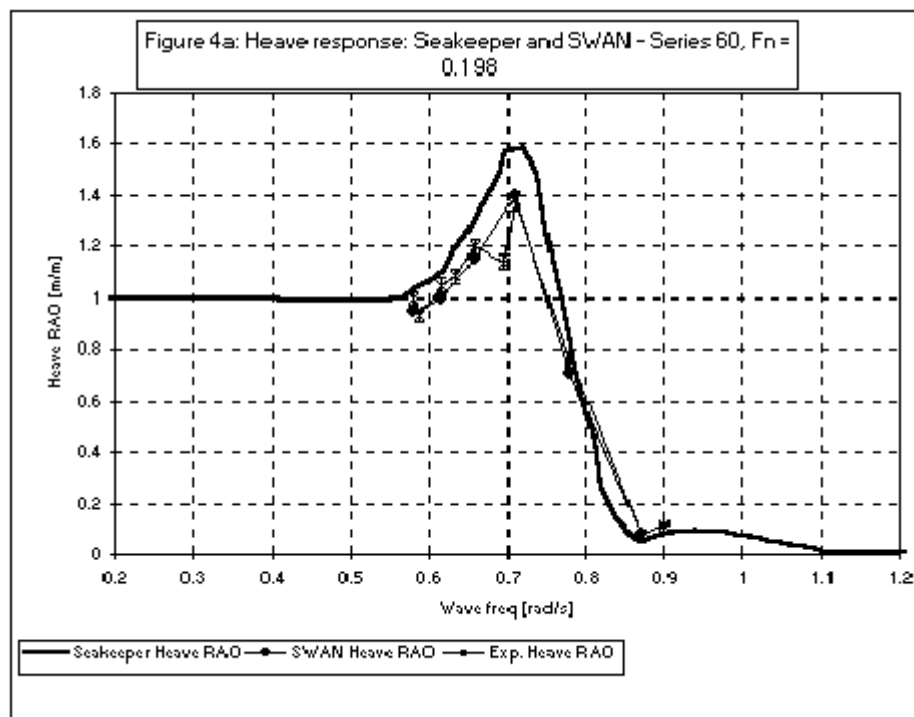
Heave RAO: Motions, SWAN, Experiment. Fn = 0.198
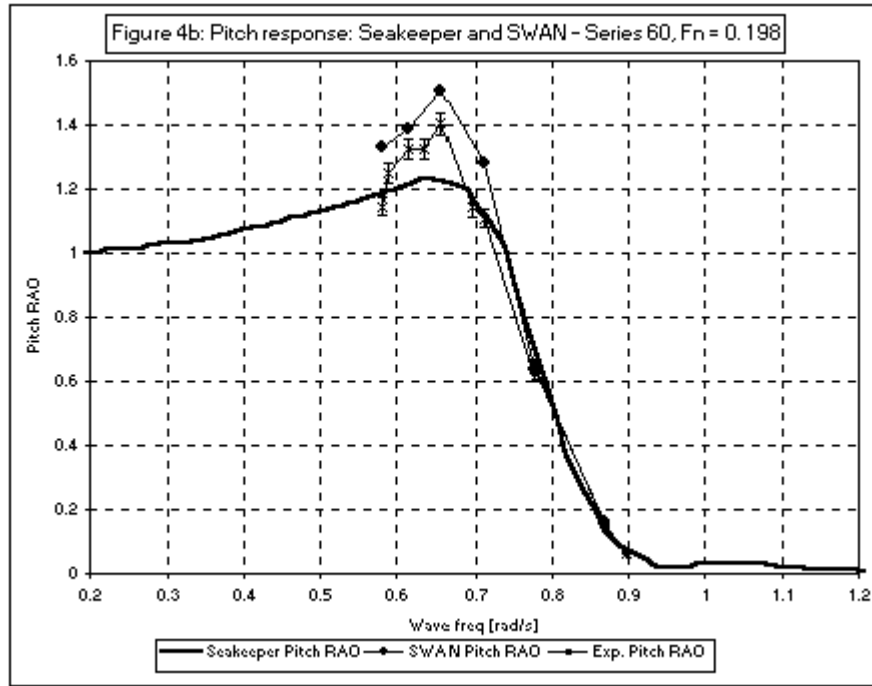
Pitch RAO: Motions, SWAN, Experiment. Fn = 0.198

Heave RAO: Motions, SWAN, Experiment. Fn = 0.297
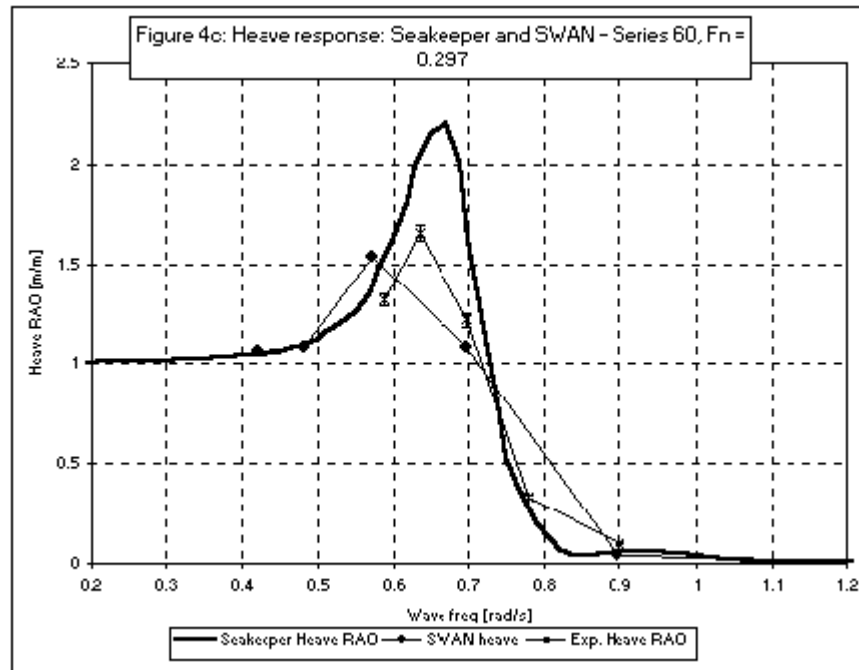
Figure 4d: Pitch response: Seakeeper and SWAN - Series 60, Fn = 0.297
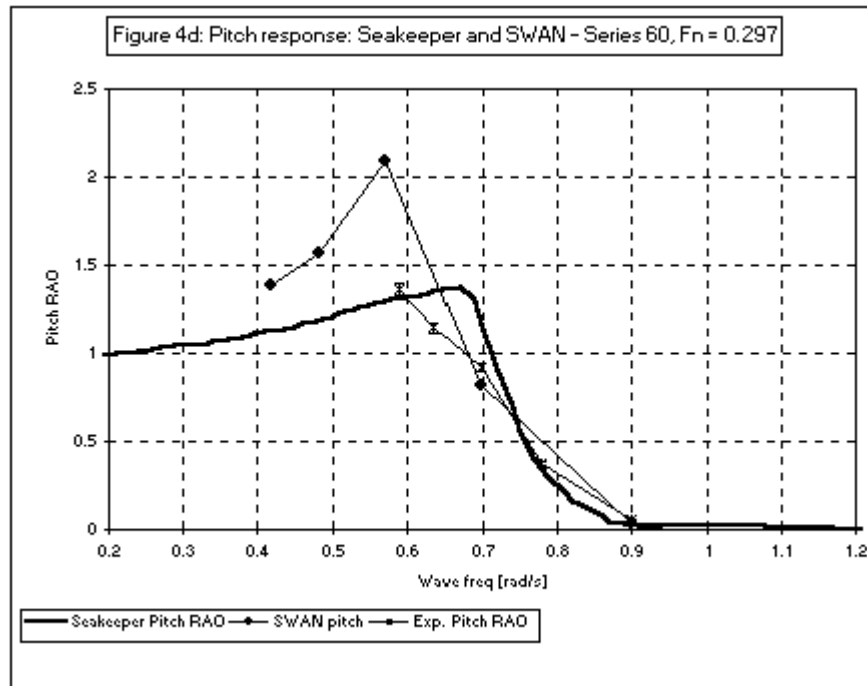
Pitch RAO: Motions, SWAN, Experiment. Fn = 0.297

Motions Validation 3 – SA van der Stel

The results from Motions for added resistance, heave and pitch have been compared with results from tank tests and another strip theory program in head seas.

Hullform and test conditions

The hullform tested is that of the SA van der Stel, a fast freight vessel. The body plan is shown below:

JOURNEE.050: Fast Freighter, 152.50 x 22.82 x 9.14 (9.14) meter.
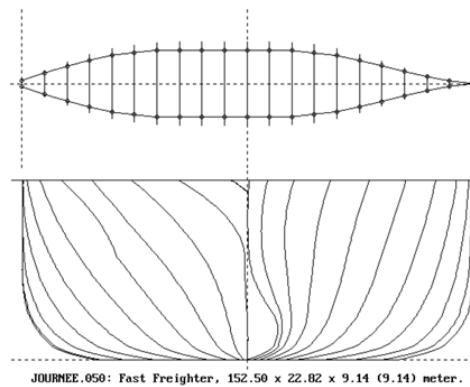
SA van der Stel Body plan

The vessel has the following dimensions:

		Full load
LengthPP	Lpp	152.50m
Breadth	B	22.80m
Amidships draught	d	9.14m
Trim by sten	trim	0.00m
Block coefficient	Cb	0.563
Centre above base	KG	9.14m
Radius gyration pitch	kyy	0.22 % Lpp
Number of sections used for seakeeping analysis		21
Number of conformal mapping parameters per section		8
Water density		1000.00kg/m^3
Acceleration due to gravity	g	9.80665m/s^2

The mapped sections (green) can be seen superimposed on the hull sections below:
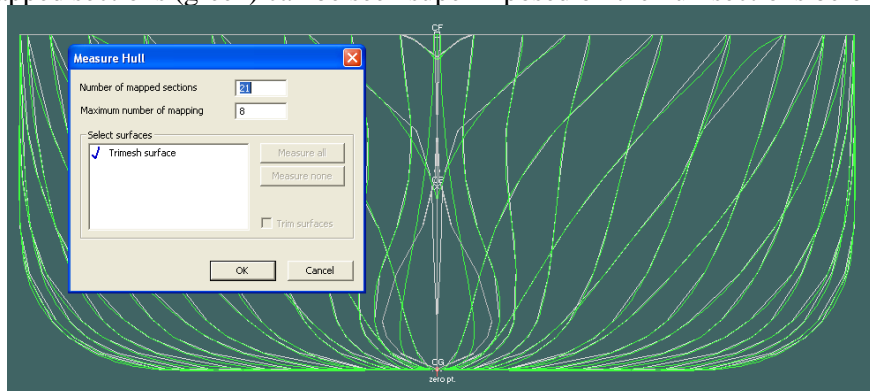
Conformal mapping sections (green) used in Motions

The vessel was tested at four Froude numbers. No transom terms were used in the Motions analysis.

Fn	v m/s	v kts
0.15	5.80	11.28
0.20	7.73	15.04
0.25	9.67	18.79
0.30	11.60	22.55

Results

The data were obtained from results published by Journee 2001. Results are given for two sets of experimental results (circles) and two methods in another strip theory implementation (grey lines). Motions v13 has three methods available for estimating added resistance. One by Salvesen and two by Geritsma and Beukelman (they offer two slightly different formulations). A third method by Havelock is being implemented and is included for comparison.
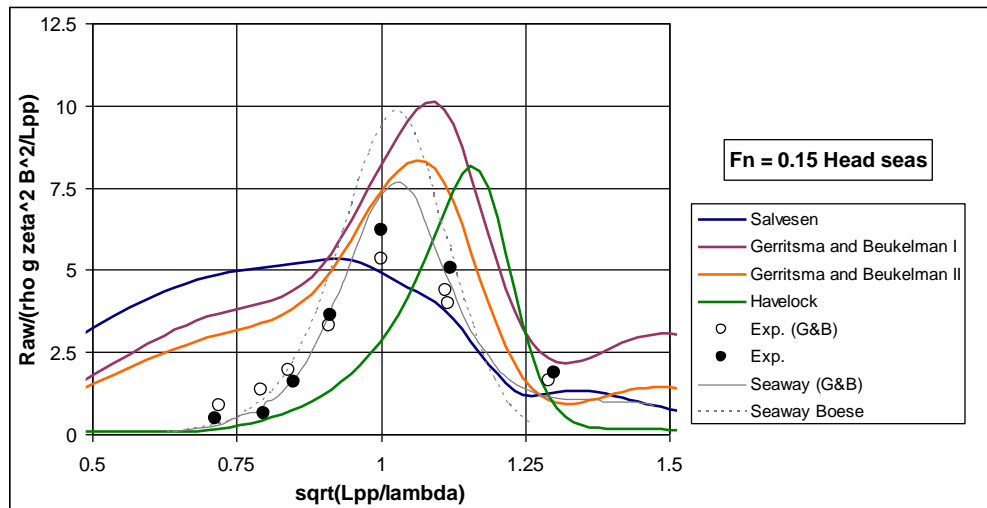
Added resistance for SA van der Stel; Fn = 0.15; Head seas
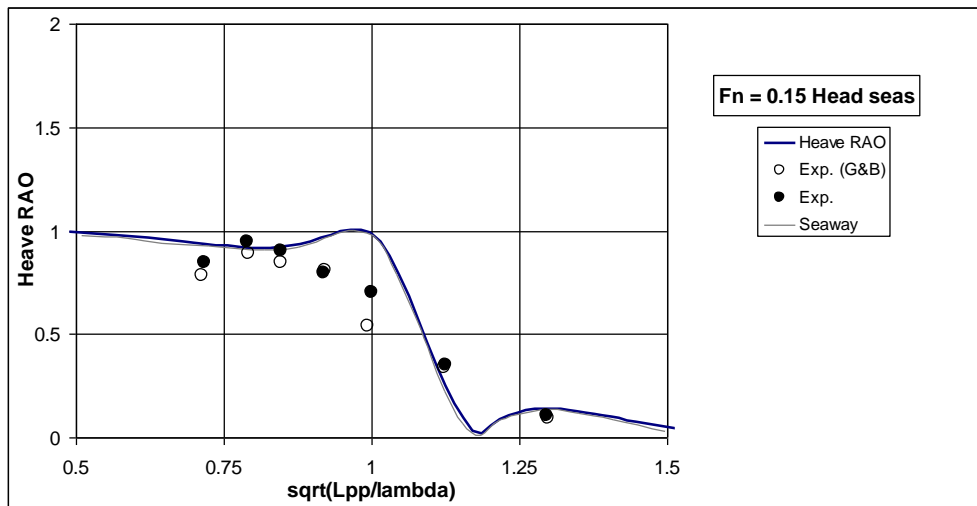
Heave response for SA van der Stel; Fn = 0.15; Head seas
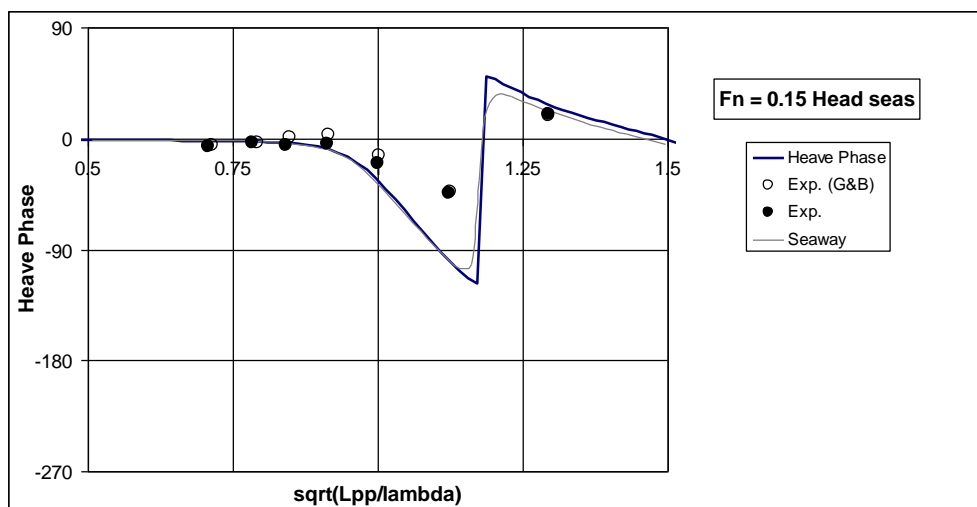
Heave phase for SA van der Stel; Fn = 0.15; Head seas
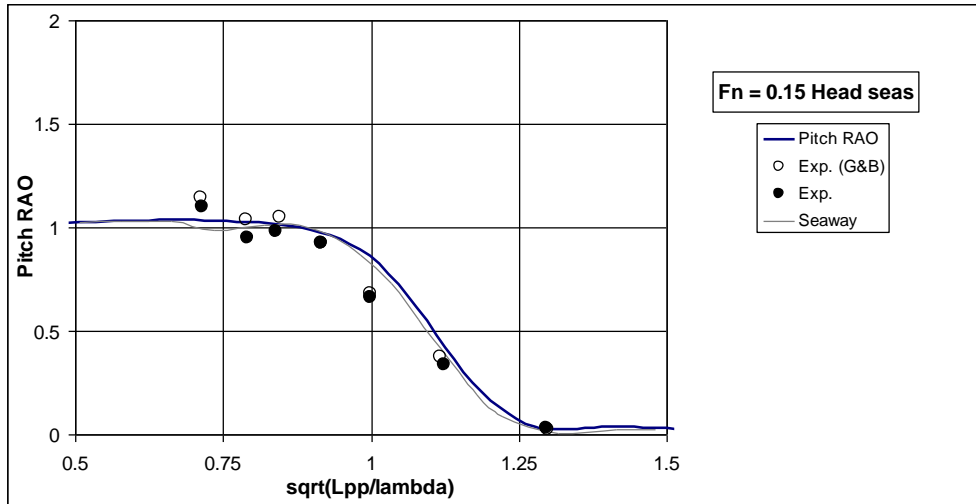
Pitch response for SA van der Stel; Fn = 0.15; Head seas
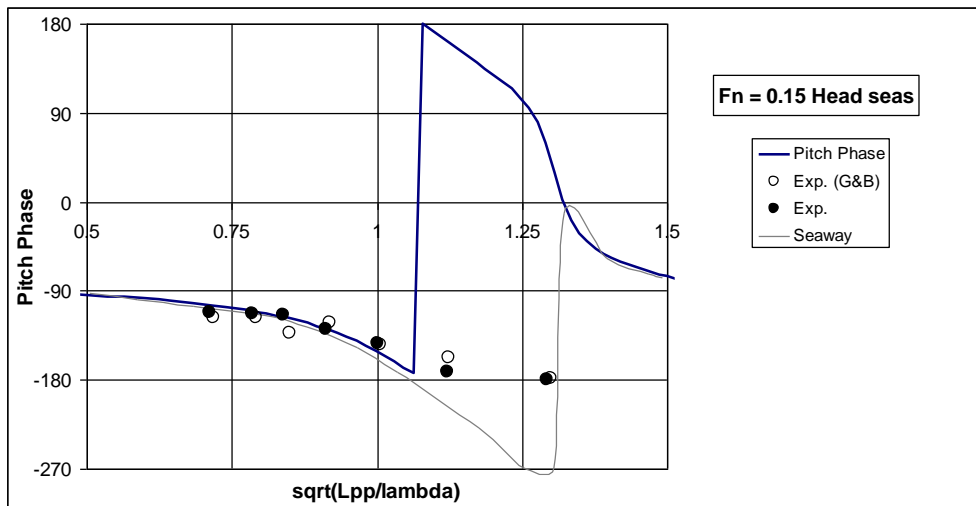
Pitch phasee for SA van der Stel; Fn = 0.15; Head seas

Fn = 0.20
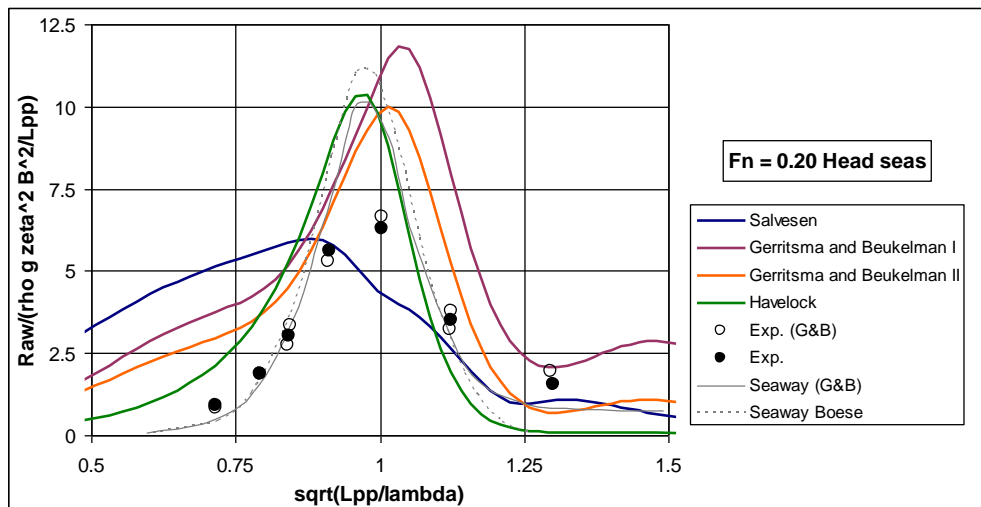
Added resistance for SA van der Stel; Fn = 0.20; Head seas

Fn = 0.25
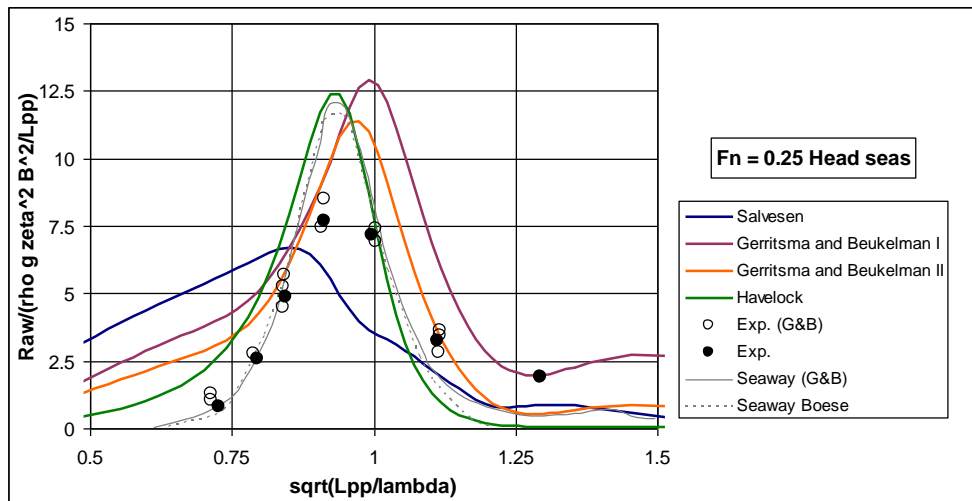
Added resistance for SA van der Stel; Fn = 0.25; Head seas

Fn = 0.30
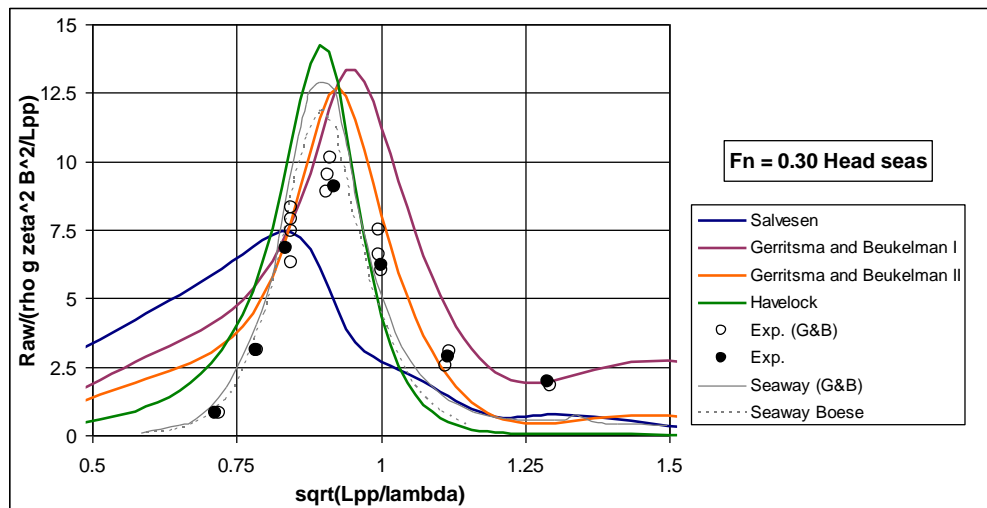
Added resistance for SA van der Stel; Fn = 0.30; Head seas
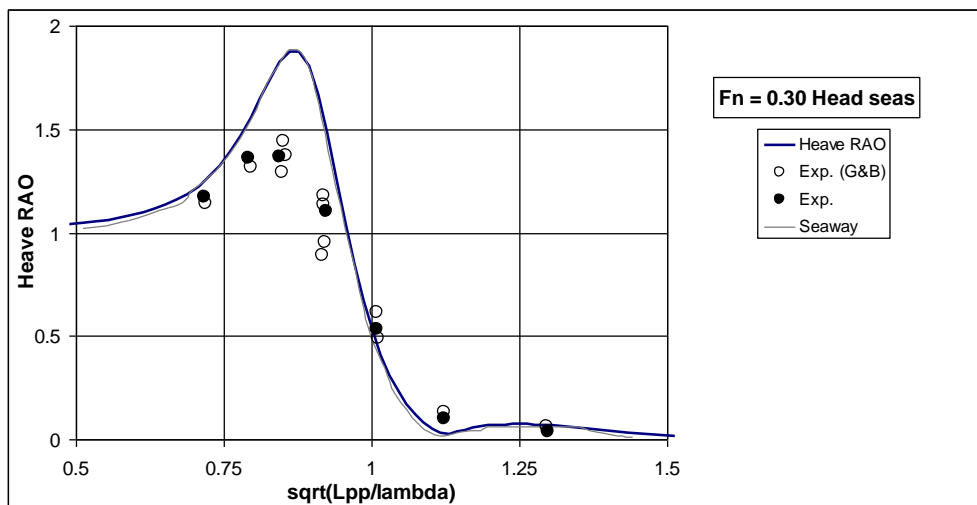
Heave response for SA van der Stel; Fn = 0.30; Head seas
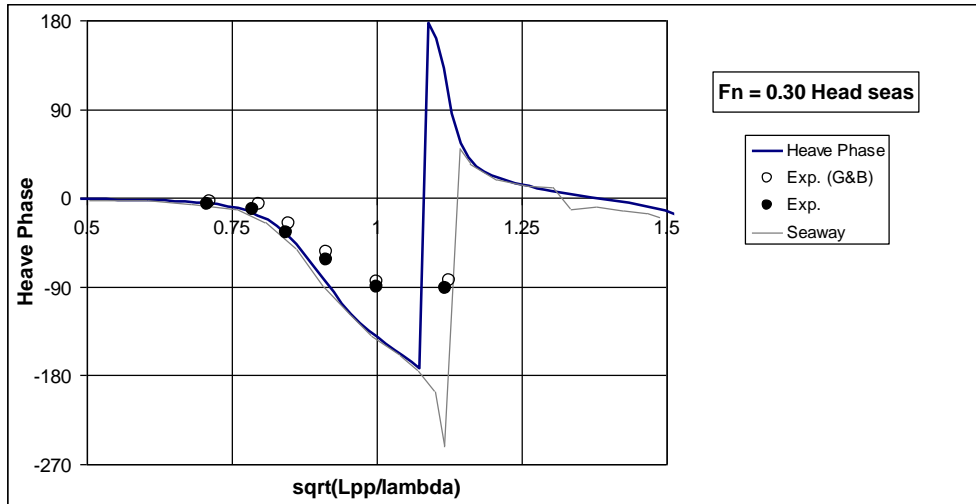
Heave phase for SA van der Stel; Fn = 0.30; Head seas
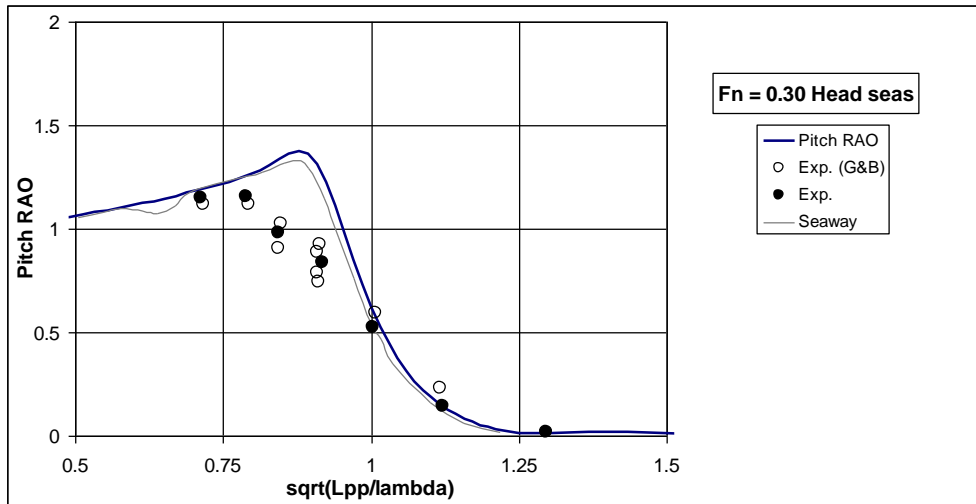
Pitch response for SA van der Stel; Fn = 0.30; Head seas
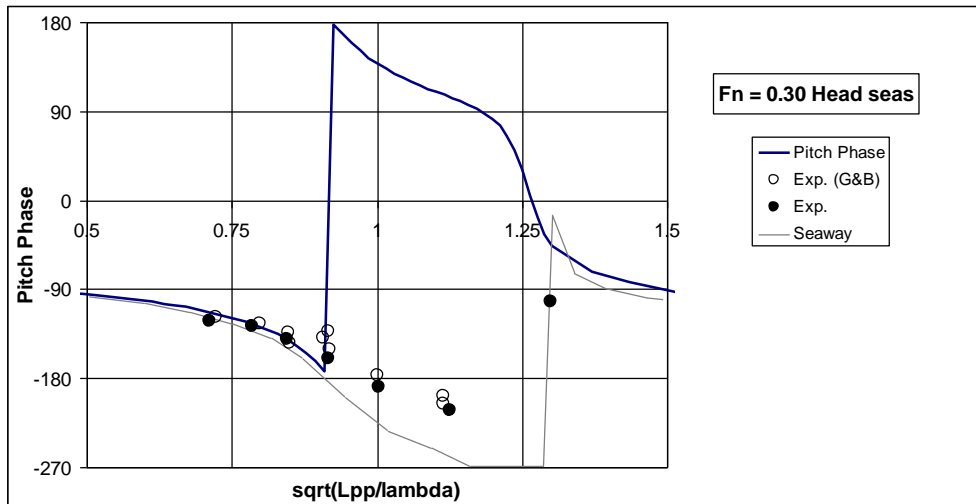
Pitch phasee for SA van der Stel; Fn = 0.30; Head seas

Conclusions

All results show the characteristic peak with frequency for the added resistance response. The Gerritsma and Beukelman methods show some differences but reproduce the experimental data reasonably well – as does the Havelock method. The Salvesen method does not reproduce the experimental results with as much accuracy as the other methods for this hullform.

It should be noted that there is considerable scatter in the experimental results and that the peaks of the response are not well captured by the experiments in all cases, due to insufficient data. It has been the author's experience that it is extremely difficult to measure added resistance with high accuracy in the towing tank. To the author's knowledge no accurate full scale added resistance measurements have been reported in the scientific literature for investigation of scaling effects etc. For these reasons it is felt that the added resistance methods available in Motions produce results of sufficient quality for comparative hullform analysis in the early design stage.

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Jasna Prpić-Oršić, Radoslav Nabergoj and Giorgio Trincas 2008: The methods of added resistance estimation for ships in a seaway, 18th Symposium on Theory and Practice of Shipbuilding

F. Pe´rez Arribas 2007: Some methods to obtain the added resistance of a ship advancing in waves, Ocean Engineering 34 946–955

Motions Validation 4 – USS Arleigh Burke (DDG-51)

Introduction

The USS Arleigh Burke (DDG-51) is the lead ship of the Arleigh Burke-class guided missile destroyers built by Bath Iron Works and launched on 16 September 1989. Towing-tank experiments of coupled pitch and heave motions for the vessel model advancing in regular head waves have been measured and published by IIHR—Hydroscience & Engineering at The University of Iowa (Irvine et al. 2008). These baseline data have been compared with numerical predictions from MAXSURF Motions.

Figure 1: USS ARLEIGH BURKE (DDG-51) underway in rough seas
https://commons.wikimedia.org/wiki/File:Arleigh_Burke_DDG-51.jpg

Table 1: Principal dimensions of the model and full-scale vessel

	Model: UoI 5512		Full scale	
Scale factor	46.6	:	1	
Length PP	3.048	m	142.03	m
Length WL	3.050	m	142.15	m
Bean WL	0.410	m	19.12	m
Draft	0.135	m	6.31	m
Displacement – volume	0.0866	m^3	8762.4	m^3
Displacement – mass (sw)	88.754	kg	8981464	kg

Numerical models

The NURBS surface model in IGES format provided with the baseline data was used for the analysis in MAXSURF Motions. The bare hull only was used for the analysis (no appendages other than the faired sonar dome near the vessel bow were included for the analysis).

MAXSURF Motions is able to compute coupled heave and pitch motions for a vessel with forward speed using a strip-theory approach and the coupled 6 degrees of freedom motions for the vessel at zero speed, using a radiation-diffraction panel method. Both methods are inviscid, linear and compute the motions in the frequency domain. The baseline data available were for head seas only; heave and pitch motions are dominated by inviscid phenomena and are well suited to solution by potential flow physical models, such as the methods available in MAXSURF Motions.

For the pitch analysis, a pitch gyradius of 0.821m was assumed (25% LOA of 3.284m).

Strip theory

A classical strip-theory method was used to calculate the coupled heave and pitch motions. The vessel was divided into 41 equally-spaced strips. Conformal mapping was used to compute the 2D added-mass and damping of each strip. The mappings used 15 mapping terms and provided a better representation of the vessel sections than the more typical Lewis mappings using 3 terms, especially in the bow area where the sonar dome is located.
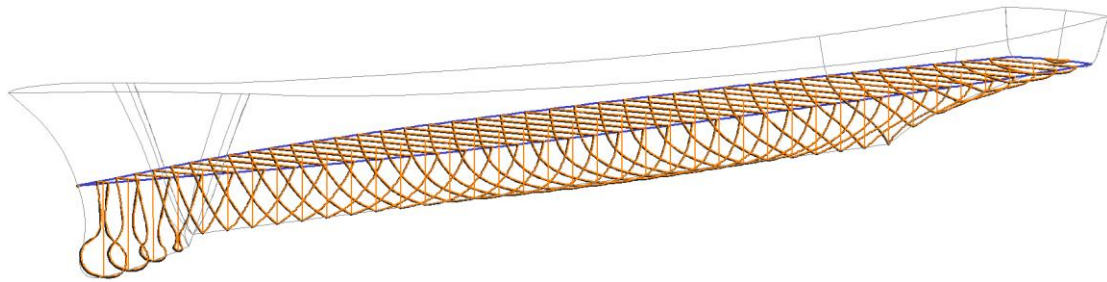
Figure 2: Strip model for analysis (41 strips)
hull sections (black) and conformal mapping contours (orange)
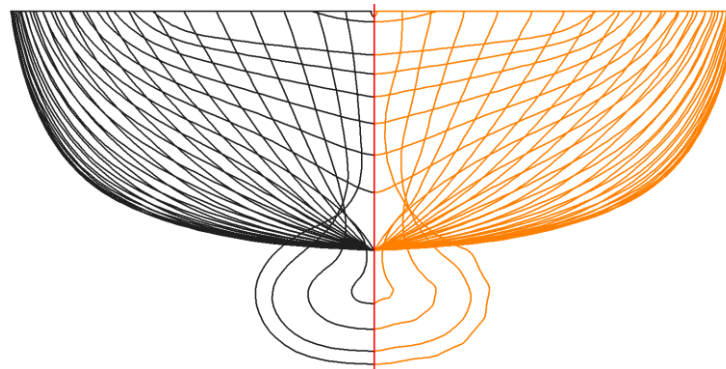
Figure 3: Hull sections (left, black) and conformal mapping contours (right, orange)

Panel Method

A model with 568 triangular panels was used for the radiation diffraction analysis in MAXSURF Motions. The panel method was used only for the zero-speed case.
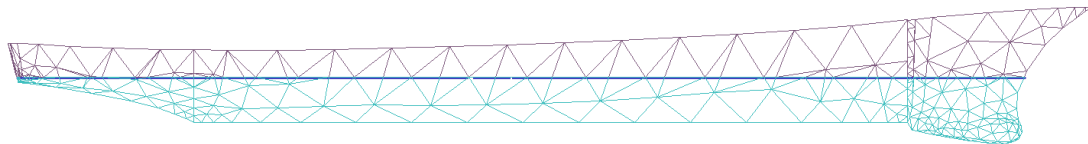
Figure 4: Panel arrangement for radiation-diffraction analysis (568 panels)

Baseline data

Baseline data and model geometry was taken from Irvine et al. (2008).

The baseline data included towing tank model test data as well as numerical simulation using SMP. Experiments were carried out in regular waves and repeated for three different wave steepnesses. Data were given for head seas at five speeds, see Table 2 below.

Table 2: Speeds tested

Full scale speed [kts]	Froude Number	Model speed [m/s]
0	0.000	0.00
14	0.19	1.05
20	0.28	1.51
25	0.34	1.88
30	0.41	2.26

Results

The results are presented in Figures 5 to 14. These show the heave and pitch RAOs (Response Amplitude Operators, or Transfer functions). Separate figures show the magnitude and phase of the RAOs at each of the speeds tested.

In each figure, the three sets of results from towing tank experiments are shown as black, filled diamonds, dots and triangles; these correspond to the three different wave steepnesses used for the experiments. For direct comparison with other numerical models the SMP (Ship Motions Program) results, also published by Irvine et al (2008), are included (blue crosses).

The MAXSURF Motions results are shown as brown circles. The filled circles show the strip theory results and have been computed for each speed; the open circles show the panel method results and have been computed for the zero-speed condition only.

Zero speed

0kts –Heave
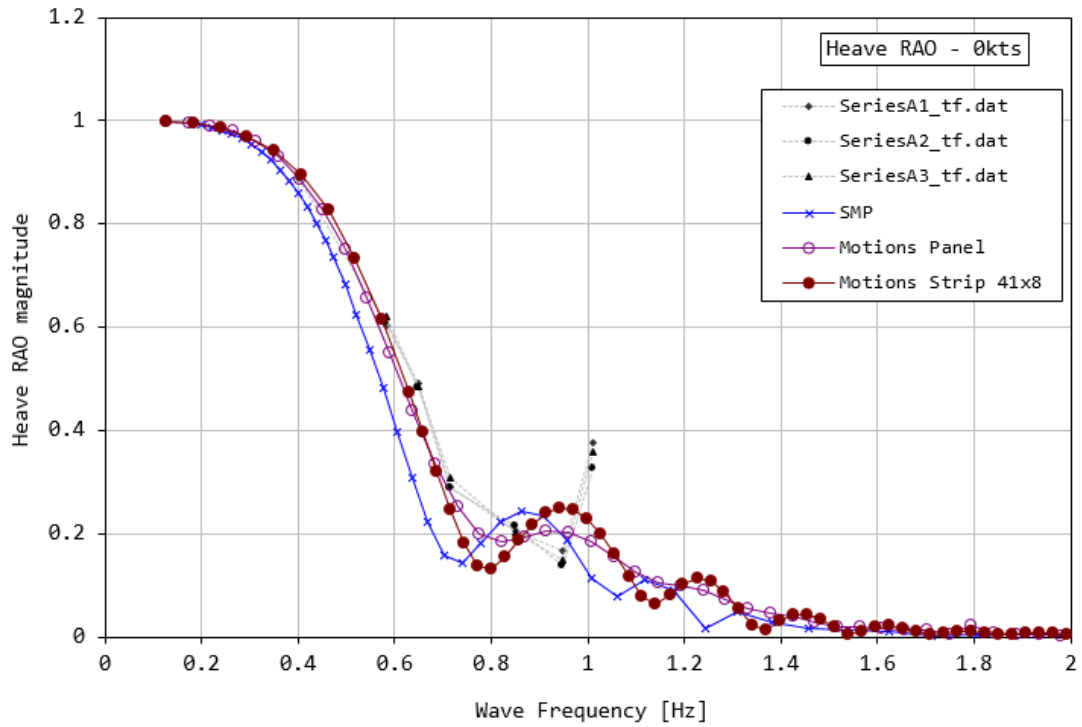
Figure 5a: Heave RAO magnitude, zero speed
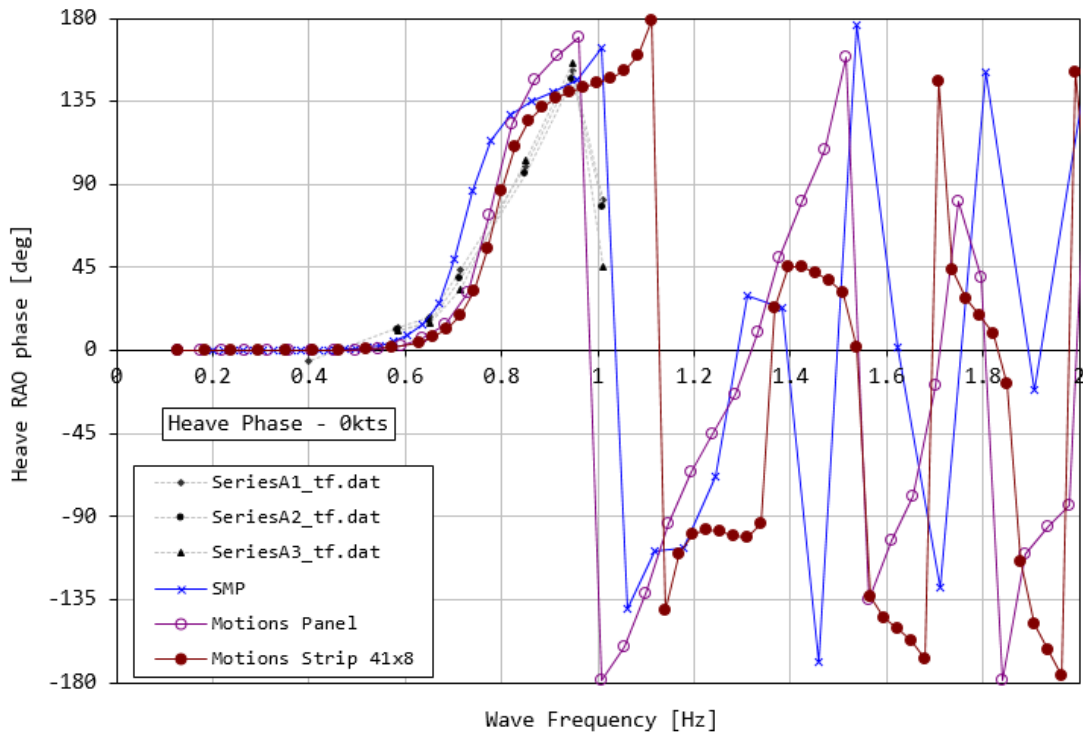
Figure 5b: Heave RAO phase, zero speed
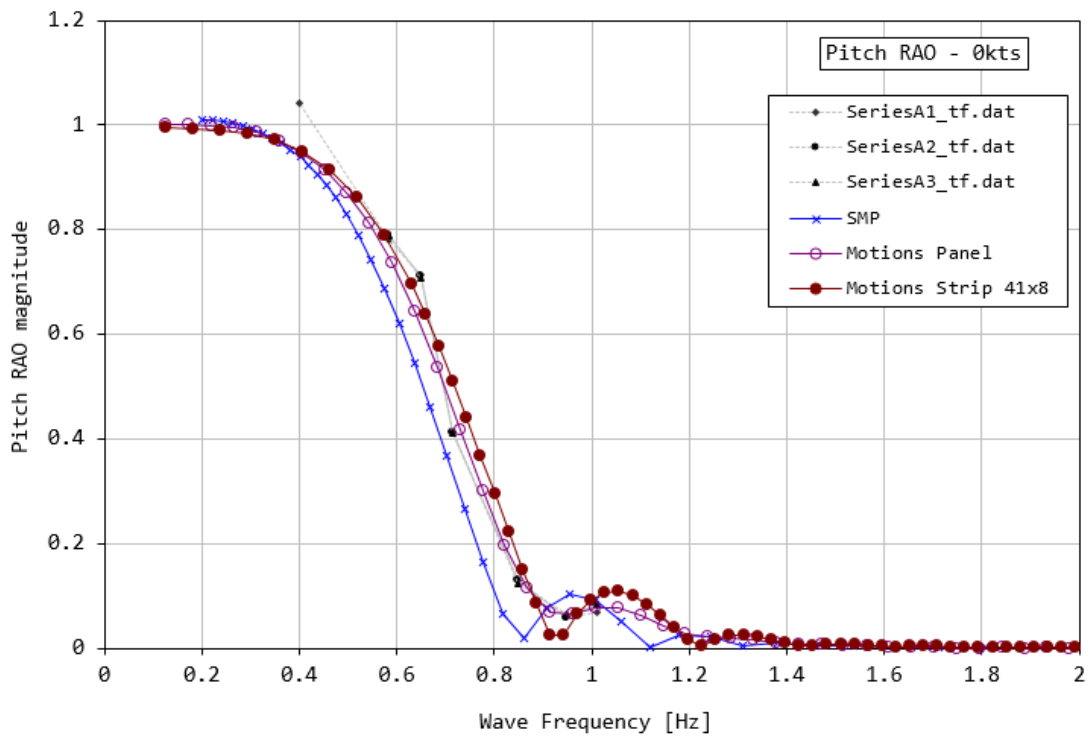
0kts –Pitch

Figure 6a: Pitch RAO magnitude, zero speed
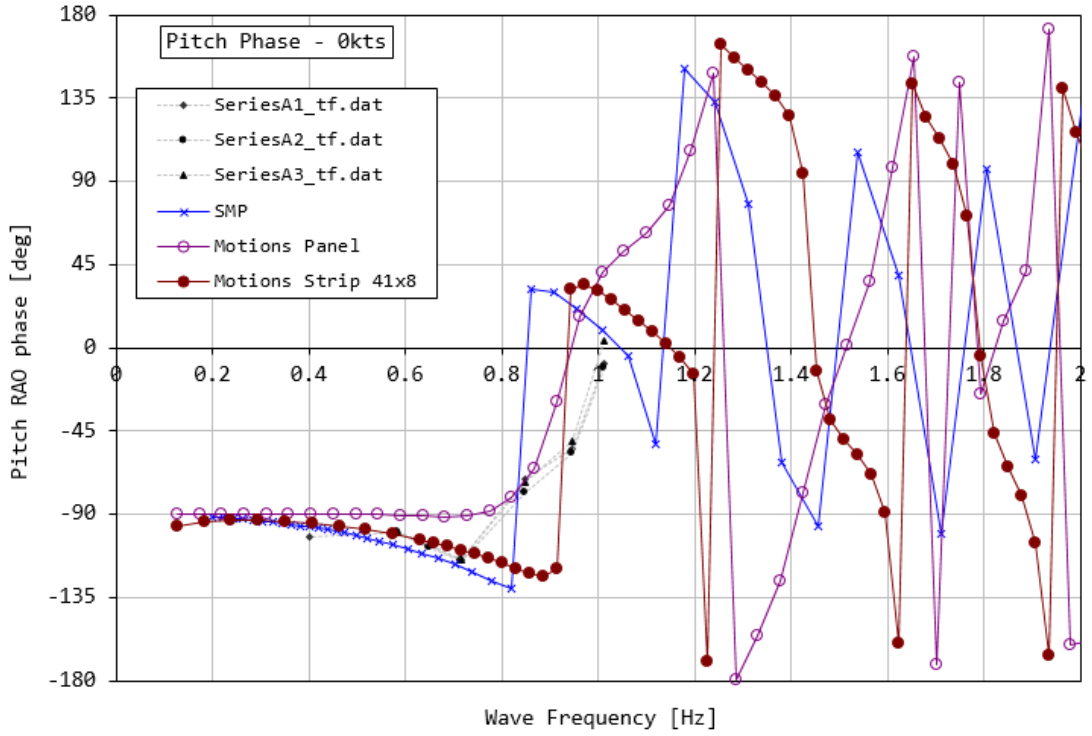
Figure 6b: Pitch RAO phase, zero speed

14kts; Froude No. 0.19

14kts –Heave
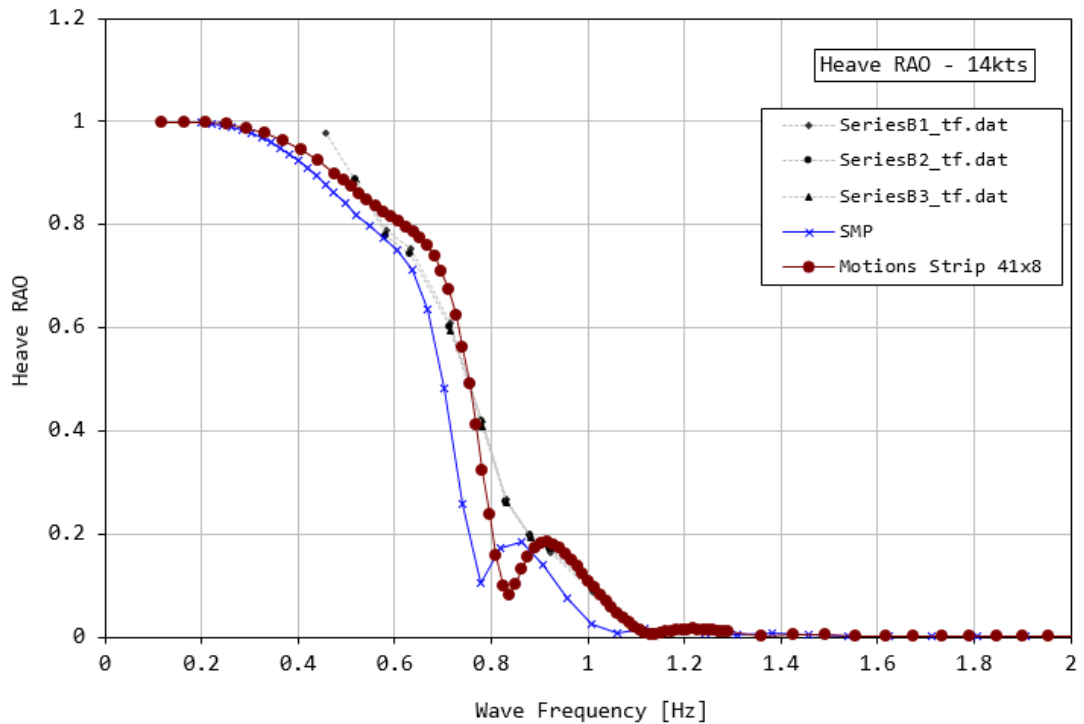
Figure 7a: Heave RAO magnitude, Froude No. 0.19
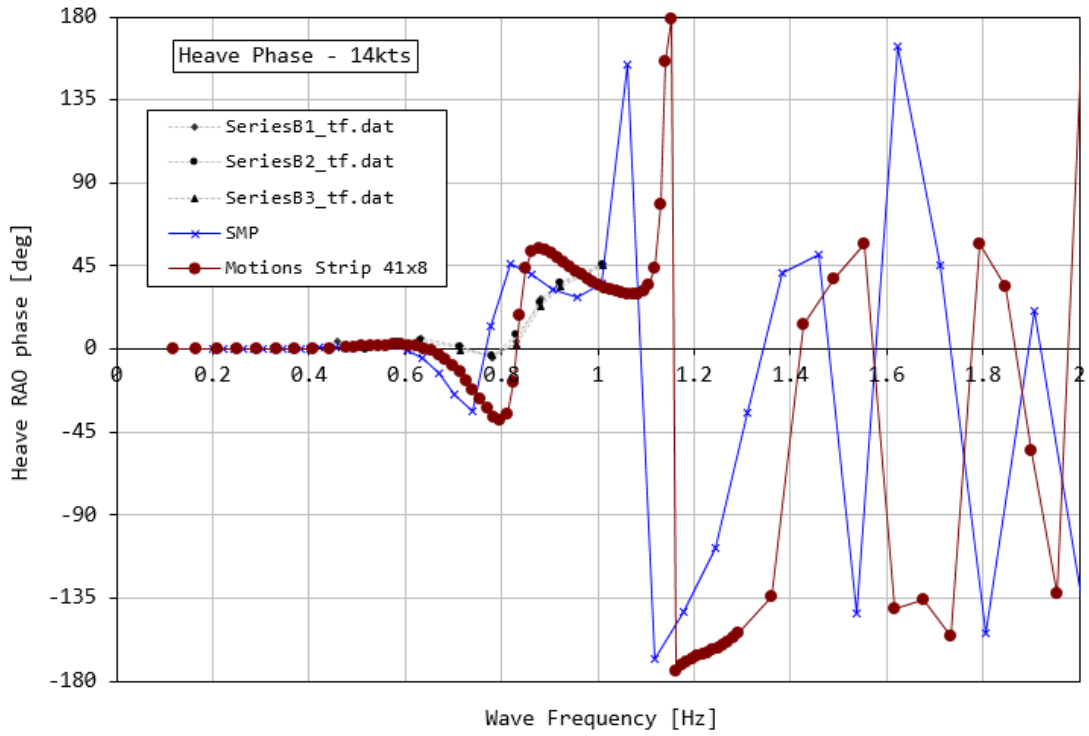
Figure 7b: Heave RAO phase, Froude No. 0.19

14kts -Pitch
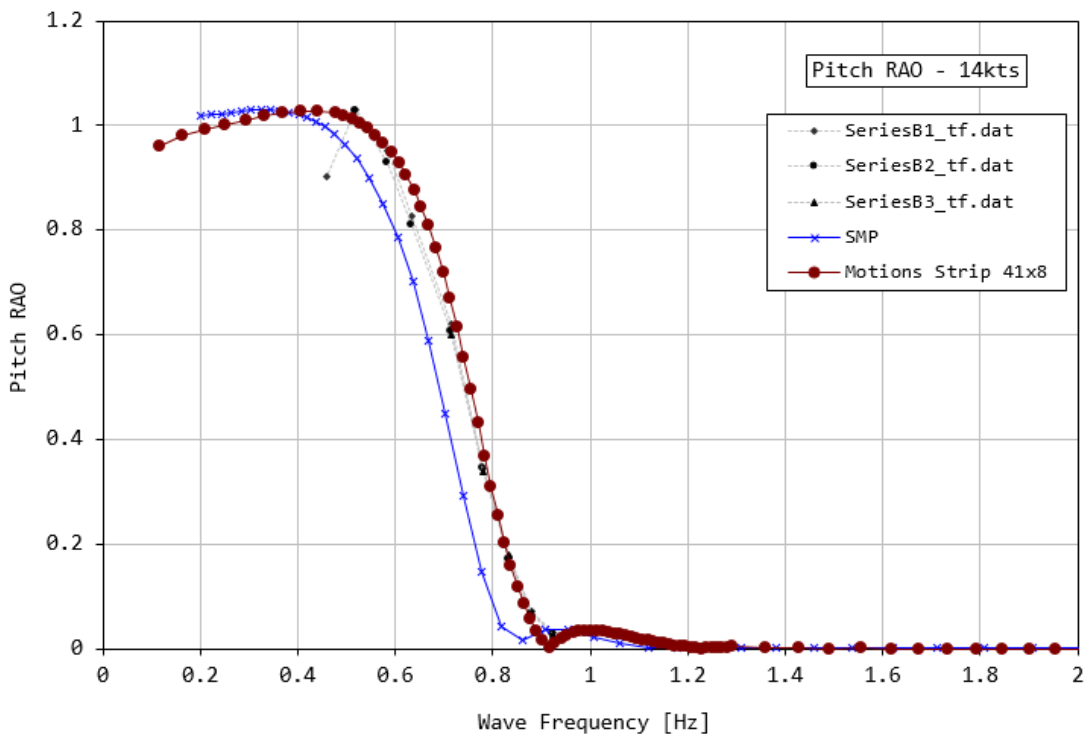
Figure 8a: Pitch RAO magnitude, Froude No. 0.19
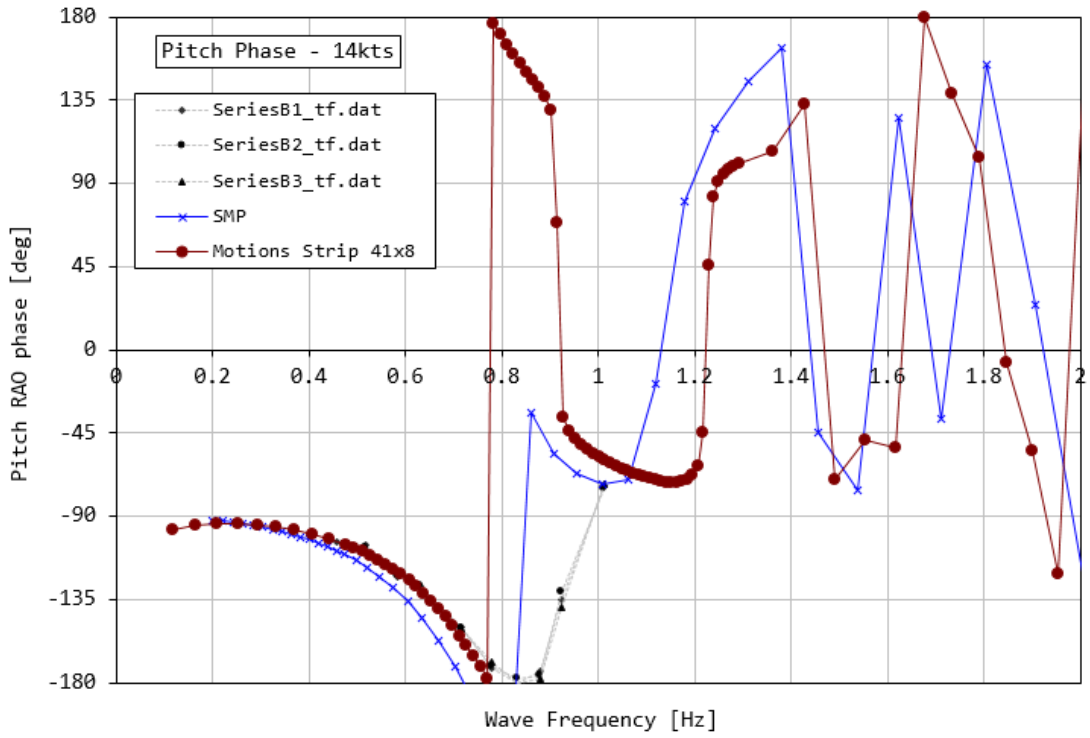
Figure 8b: Pitch RAO phase, Froude No. 0.19

20kts; Froude No. 0.28

20kts –Heave
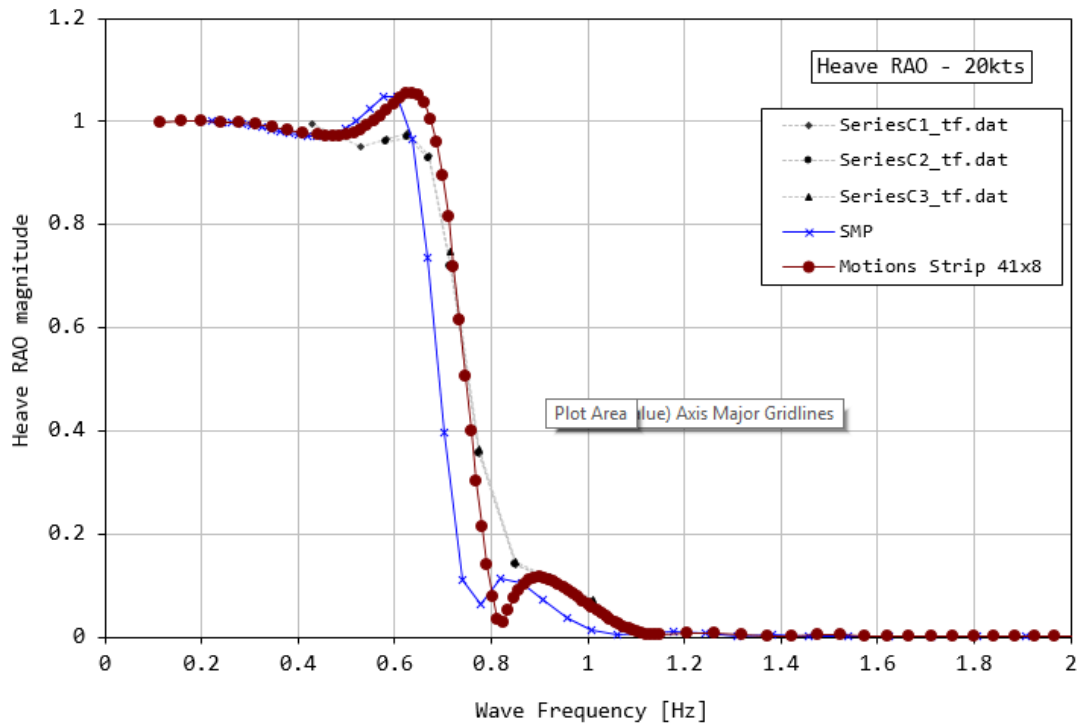
Figure 9a: Heave RAO magnitude, Froude No. 0.28
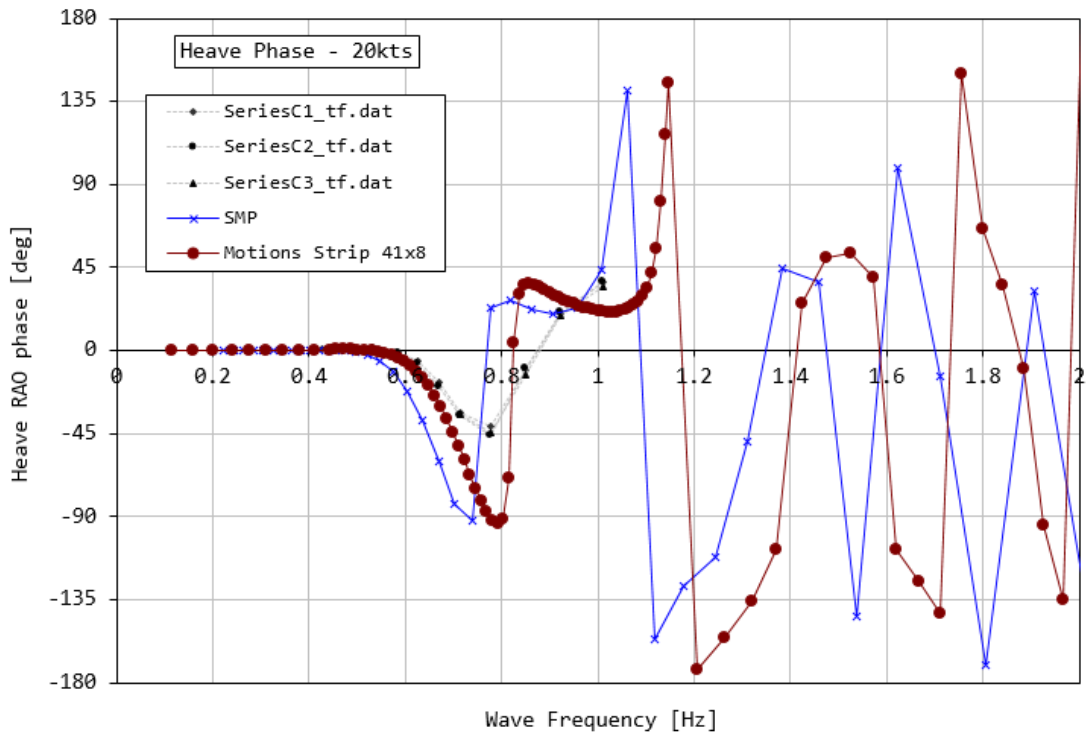
Figure 9b: Heave RAO phase, Froude No. 0.28

20kts –Pitch
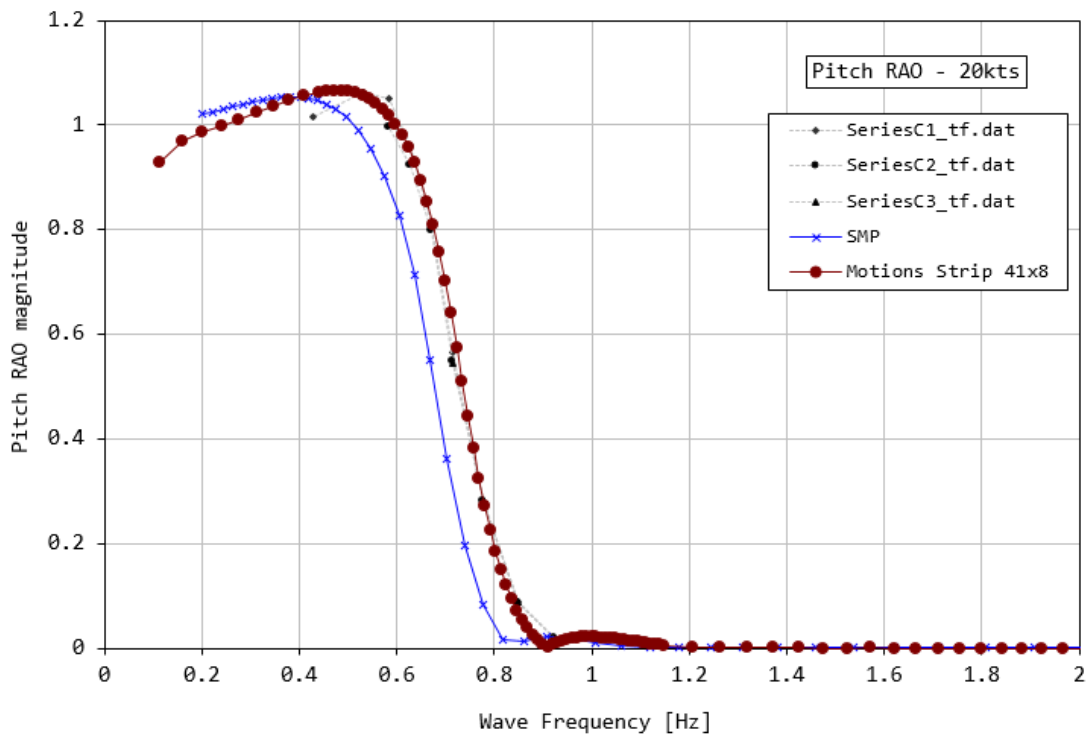
Figure 10a: Pitch RAO magnitude, Froude No. 0.28
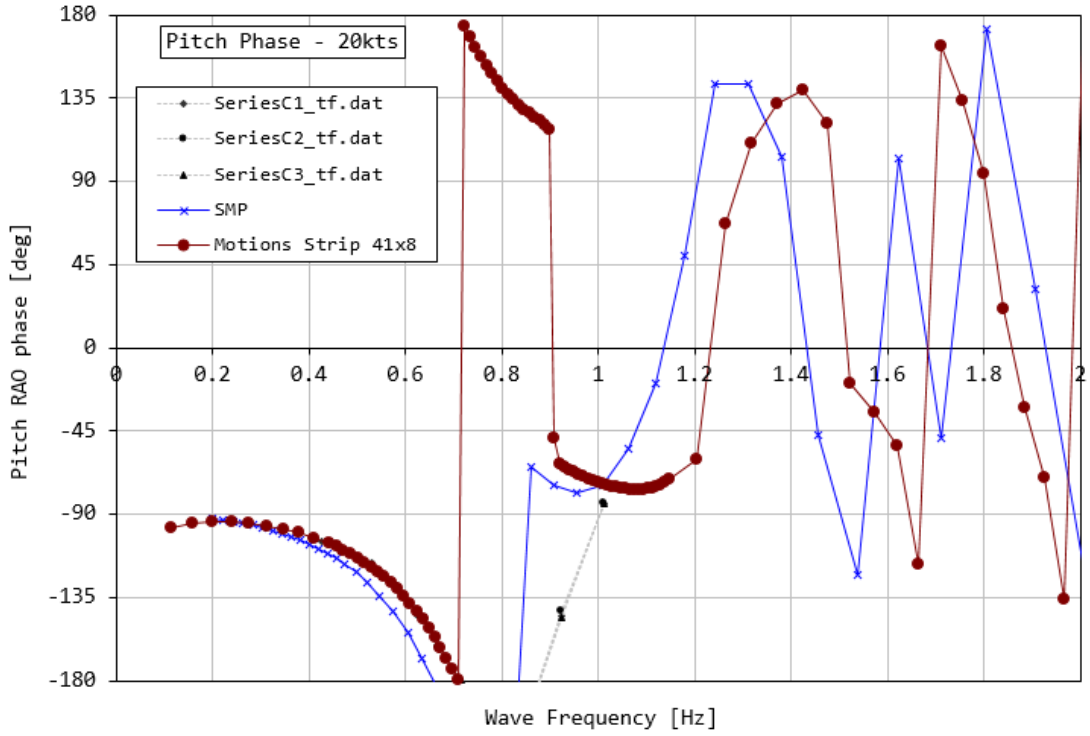
Figure 10b: Pitch RAO phase, Froude No. 0.28

25kts; Froude No. 0.34

25kts –Heave
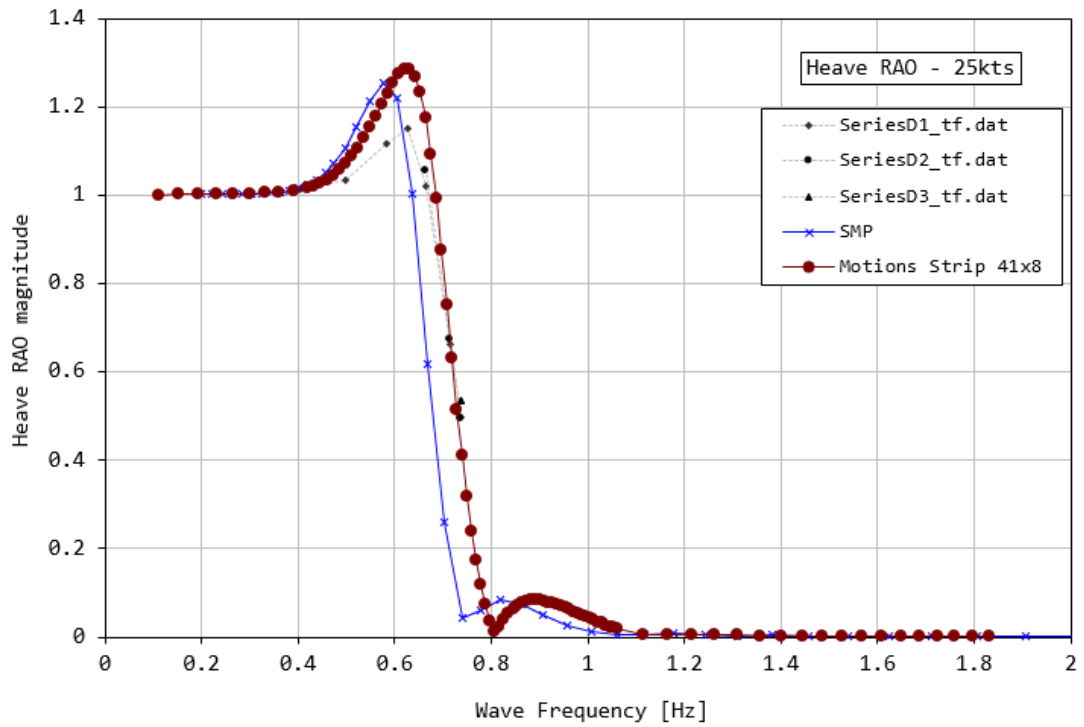
Figure 11a: Heave RAO magnitude, Froude No. 0.34
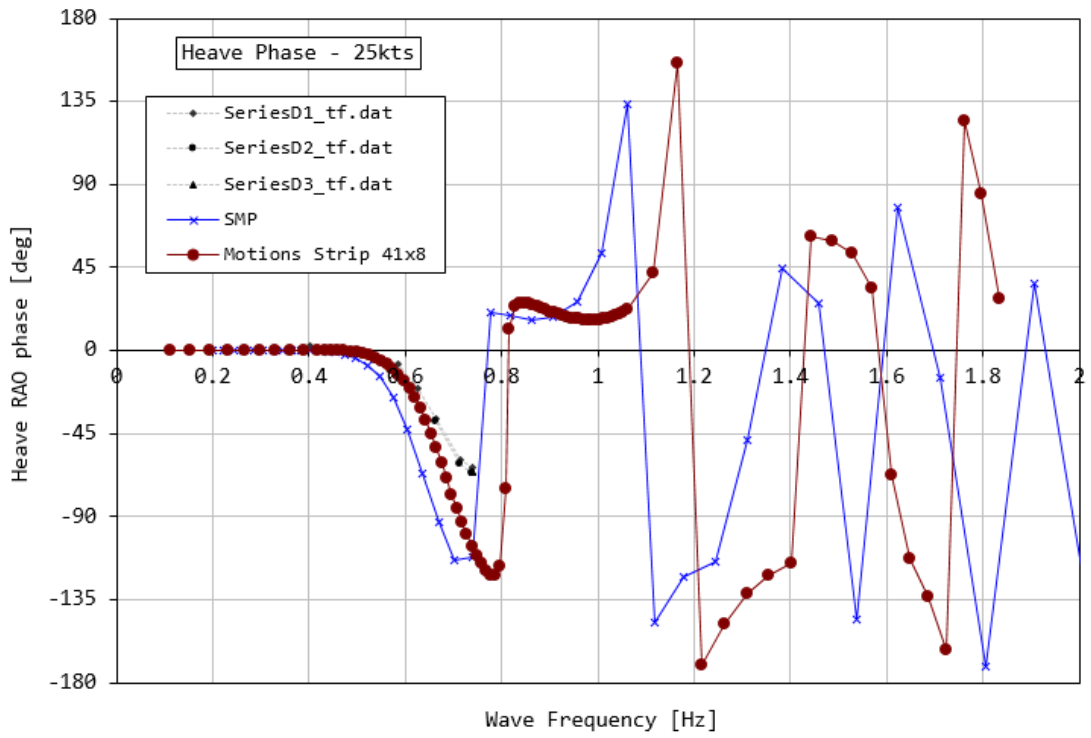
Figure 11b: Heave RAO phase, Froude No. 0.34

25kts –Pitch
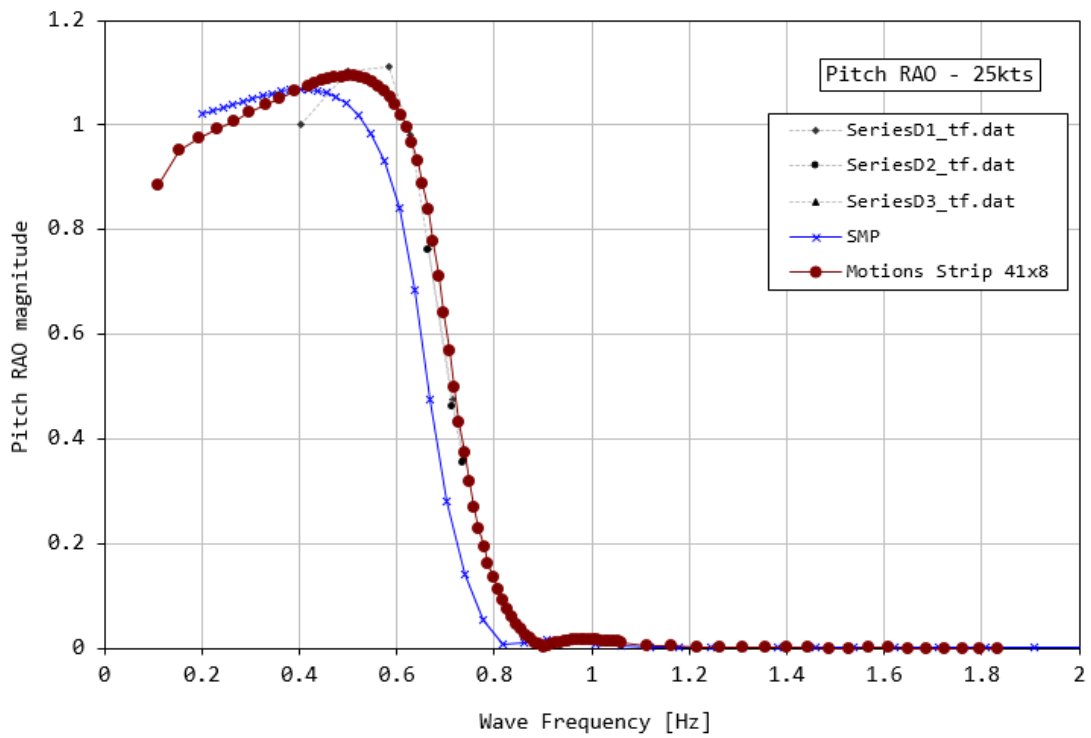
Figure 12a: Pitch RAO magnitude, Froude No. 0.34
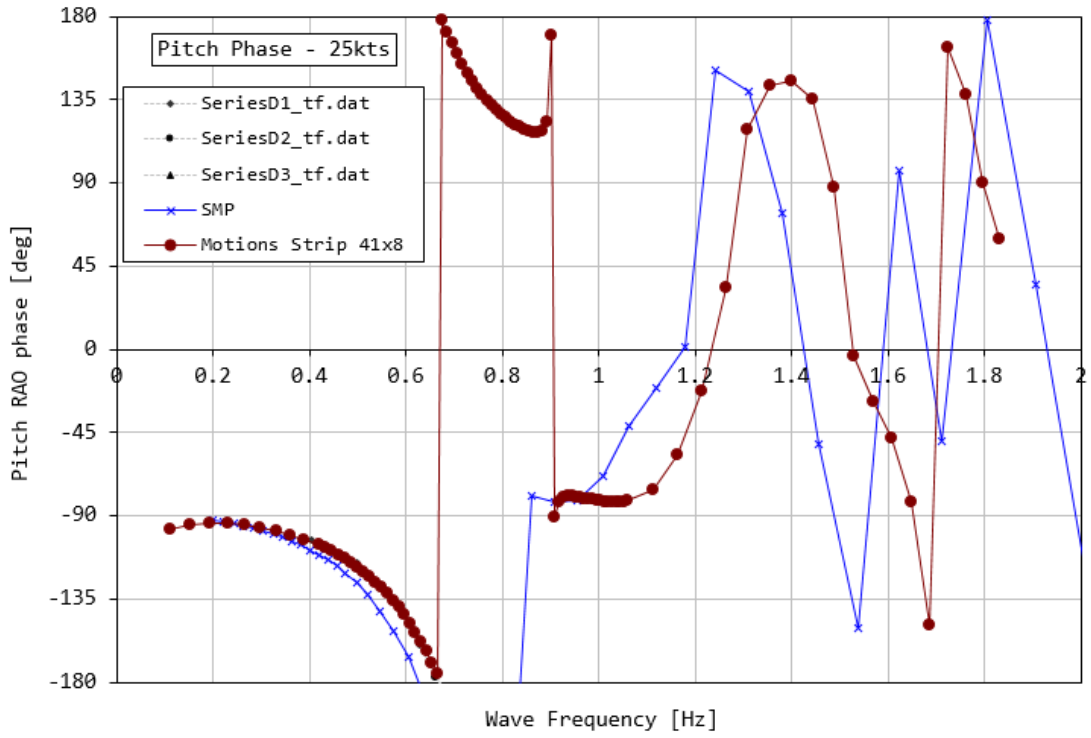
Figure 12b: Pitch RAO phase, Froude No. 0.34

30kts; Froude no. 0.41

30kts –Heave
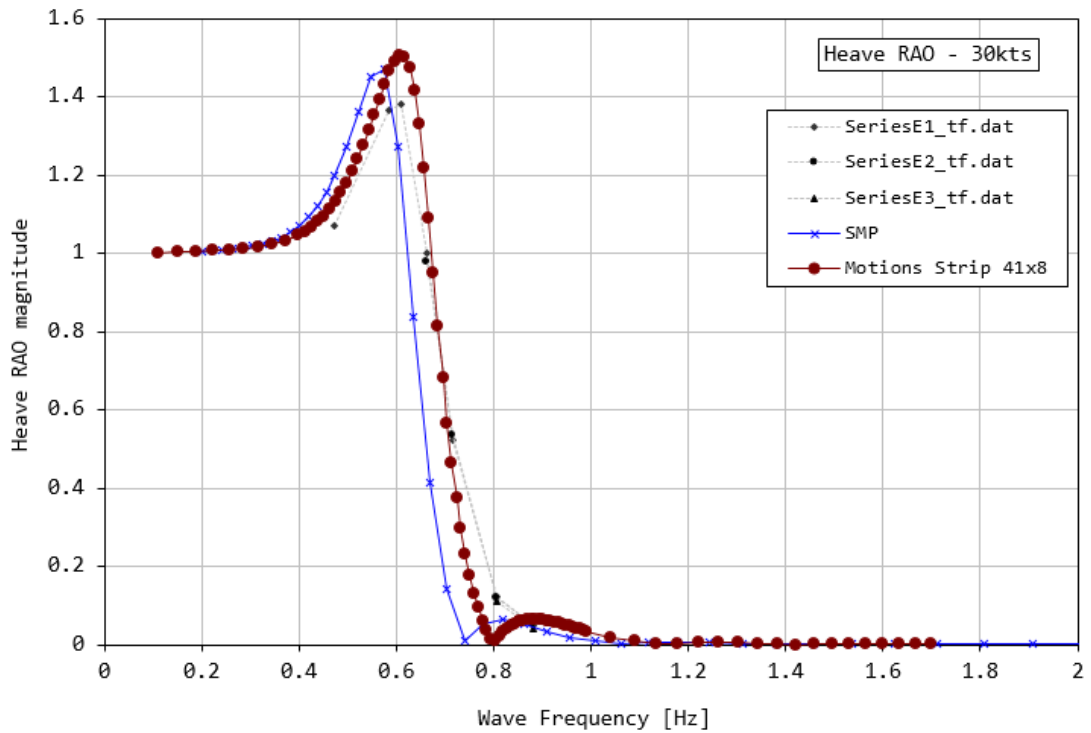
Figure 13a: Heave RAO magnitude, Froude No. 0.41
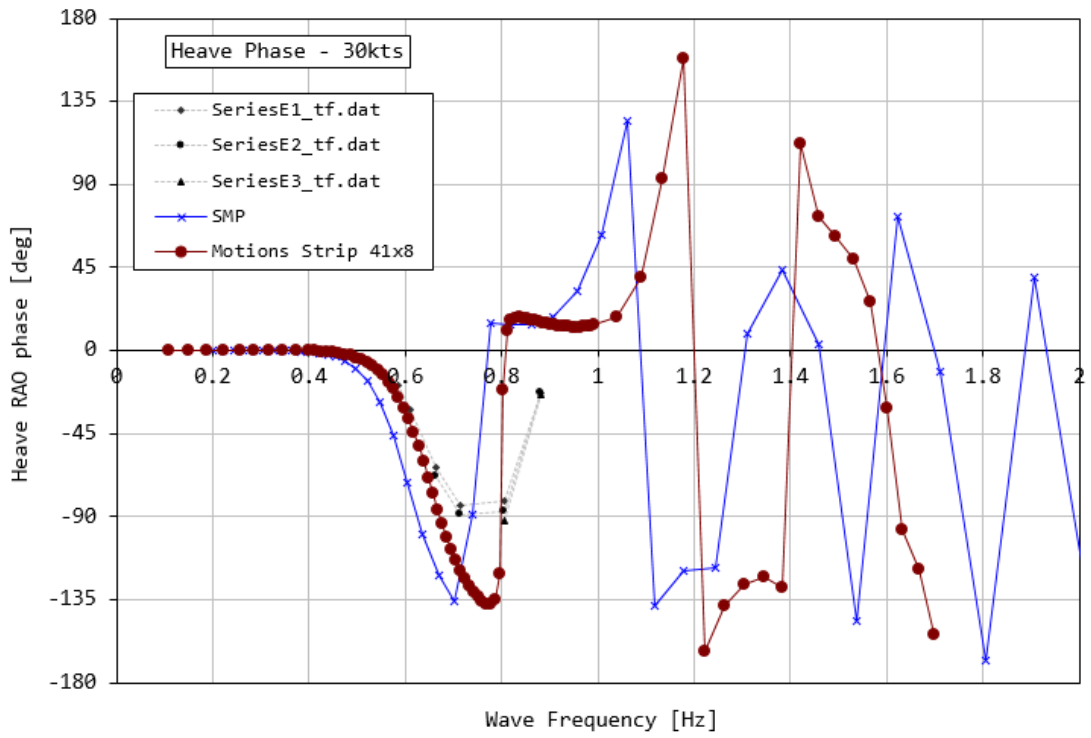
Figure 13b: Heave RAO phase, Froude No. 0.41

30kts –Pitch
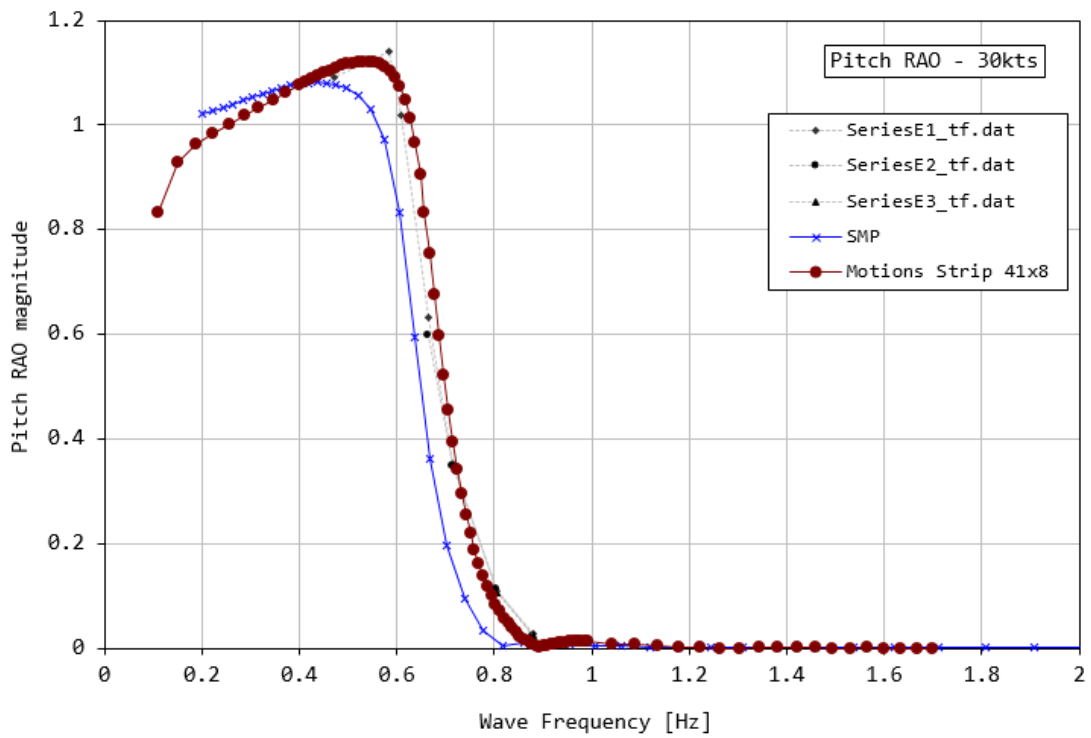
Figure 14a: Pitch RAO magnitude, Froude No. 0.41
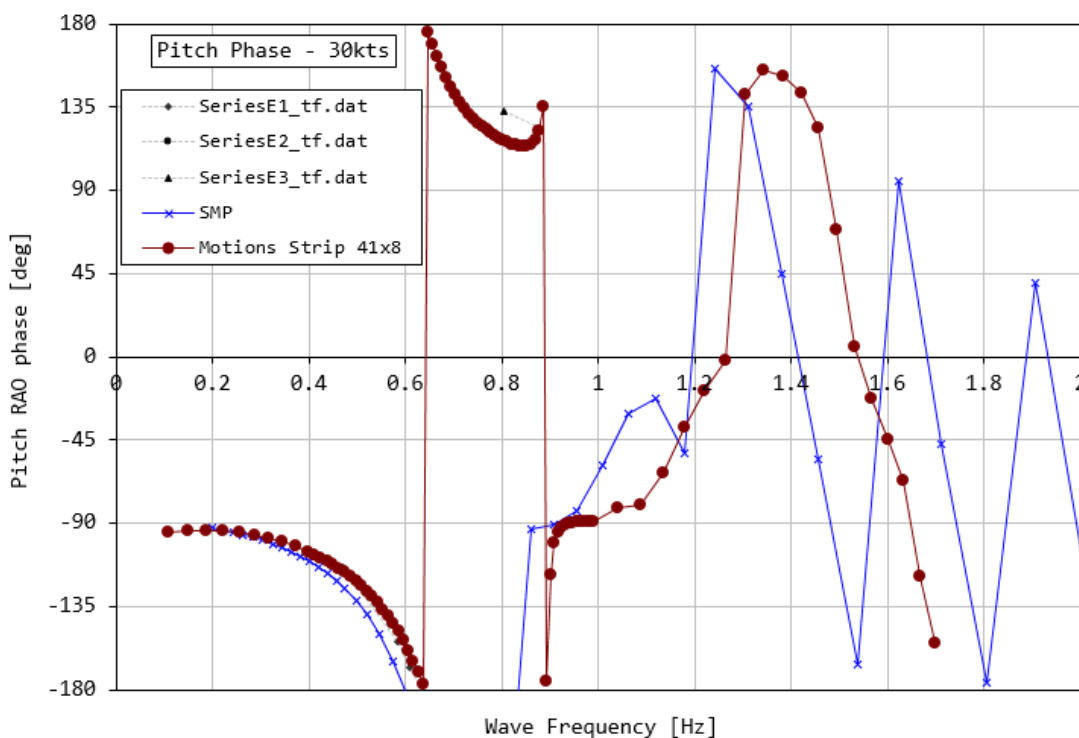
Figure 14b: Pitch RAO phase, Froude No. 0.41

Conclusions

In general terms both SMP and MAXSURF motions match the results from experiments with reasonable degree of fidelity.

For all speeds, MAXSURF Motions gives a good prediction of the frequency at which first resonant peak occurs. There is a small, yet systematic, difference when compared with the SMP results. This could be due to a different pitch gyradius and/or vessel mass used in each analysis. These observations are true for both heave and pitch motions.

At higher forward speeds the magnitude of the heave resonant peak is slightly overestimated, whilst the magnitude of the pitch resonant peak is well predicted at all speeds.

Regarding the magnitudes of the first resonant peak for both heave and pitch motions, the MAXSURF Motions results are slightly greater than those calculated by SMP.

The heave and pitch phase is also reasonably well predicted.

At zero speed, the differences between the MAXSURF Motions panel method and strip theory results are quite small.

References

Irvine, M., Longo, J. and Stern, F., "Pitch and Heave Tests and Uncertainty Assessment for a Surface Combatant in Regular Head Waves" Journal Ship Research, Vol. 52, No. 2, June 2008, pp. 146-163. The full results and IGES model of the vessel may be found at the following URL: http://www.iihr.uiowa.edu/shiphydro/efd-data/5512-pitch-and-heave/ (July 2018)

Naval Vessle Register: USS Arleigh Burke
http://www.nvr.navy.mil/NVR/SHIPDETAILS/SHIPSDETAIL_DDG_51_1204.HTML
(July 2018)

Wikipedia: USS Arleigh Burke
https://en.wikipedia.org/wiki/USS_Arleigh_Burke (July 2018)
https://en.wikipedia.org/wiki/Arleigh_Burke-class_destroyer (July 2018)

Techet, A.H., "13.012 Hydrodynamics for Ocean Engineers Lab#4", November 2004
http://web.mit.edu/13.012/labs_DATA/Lab111204.pdf (July 2018)

Appendix C – Panel Method Formulation

Linear Potential Theory

Coordinates and Boundary Value Problem

For the mathematical descriptions of fluid and body motions, a body fixed coordinates system is adopted. By introducing the centre of rotation, the body motions are able to be described with respect to a translated coordinates system.
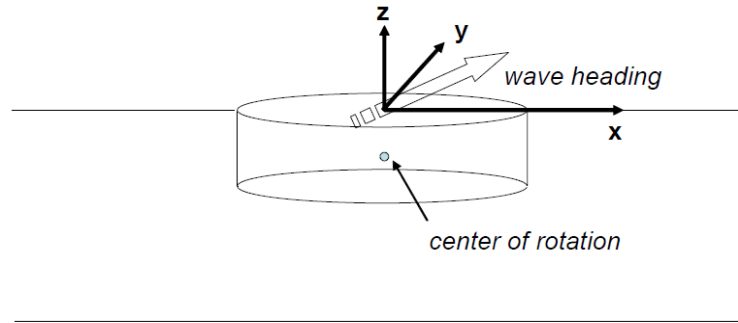
Body Fixed Coordinates and Centre of Rotation

The wave height and steepness are also assumed to be small so that the linear wave theory may be used. The fluid is assumed to be incompressible and inviscid, and the flow be irrotational. Thus the flow field can be described by the gradient of a velocity potential, which is governed by the Laplace equation and at the same time it should satisfy the proper boundary conditions. For harmonic motions, the velocity potential may be written as follows:

$$\Phi(\mathbf{x};t) = Re[\phi(\mathbf{x})e^{-i\omega t}]$$

Governing Equation: $\nabla^2\phi(\mathbf{x}) = 0 \quad for \quad \mathbf{x} \in \Omega$

Free Surface Boundary Condition: $-\omega^2\phi + g\dfrac{\partial\phi}{\partial z} = 0 \quad on \quad z = 0$

Bottom Boundary Condition: $\dfrac{\partial\phi}{\partial z} = 0 \quad on \quad z = -h$

Body Boundary Condition: $\dfrac{\partial\phi}{\partial n} = U_n \quad on \quad S$

Within the framework of linear theory, the velocity potential ϕ is subdivided into the incident potential ϕ_I, the diffraction potential ϕ_D, and the radiation potential ϕ_j.

$$\phi = \phi_I + \phi_D + \sum_{j=1}^{6} x_j\phi_j$$

Where x_j is the complex amplitude of the j-th mode of the motion.

$$\phi_I = -\frac{\omega}{k}\frac{\cosh k(z+h)}{\sinh kh}e^{ik(x\cos\beta + y\sin\beta)}$$

The velocity potential of the incident wave is defined by

$$\phi_I = -\frac{\omega}{k}\frac{\cosh k(z+h)}{\sinh kh}e^{ik(x\cos\beta + y\sin\beta)}$$

The body boundary conditions for the diffraction and radiation velocity are as follows:

$$\frac{\partial \phi_D}{\partial n} = -\frac{\partial \phi_I}{\partial n} \quad on \ S$$

$$\frac{\partial \phi_j}{\partial n} = -i\omega n_j \quad on \ S$$

where n_j is the generalized normal vector.

Boundary Integral Equation

If the Green theorem is applied, the velocity potential turns out to be a solution of the following Fredholm integral equation of the second kind:

$$\alpha \phi(\mathbf{x}) + \int_S \phi(\xi) \frac{\partial G(\mathbf{x};\xi)}{\partial \mathbf{n}} dS = \int_S \frac{\partial \phi(\xi)}{\partial \mathbf{n}} G(\mathbf{x};\xi) dS$$

where n is the unit normal vector on the body surface pointing toward fluid, α is the solid angle, and S is the boundary surface of vessels. $G(\mathbf{x};\xi)$ is the wave Green function(Wehausen and Laitone, 1960). This integral equation is used in the solver.

There is another integral equation for the velocity potential, which is called source distribution method.

$$\phi(\mathbf{x}) = \int_S \sigma(\xi) G(\mathbf{x};\xi) dS$$

Equations of Motion

Here we assume that the body is rigid and in a state of stable equilibrium when in calm water. By considering the hydrodynamic forces acting on the body, the equations of motion are obtained in the form

$$\sum_{j=1}^{6} x_j \left[-\omega^2 (M_{ij} + A_{ij}) - i\omega (B_{ij} + B_{ij}^v) + (C_{ij} + K_{ij}) \right] = F_i$$

where B_{ij}^v is the viscous damping matrix. C_{ij} and K_{ij} are hydrostatic restoring and external mooring stiffness matrix, respectively.

Mean Drift Forces and Moments

The mean drift forces and moments are evaluated based on direct pressure integration method. Pinkster and Oortmerssen (1997) derived second order drift force and moments acting on the floating body as follows:

$$\mathbf{F}^{(2)} = \int_{WL} \frac{1}{2} \rho g \zeta_r^{(1)^2} \mathbf{n} dl$$

$$- \iint_S \frac{1}{2} \rho \nabla \Phi^{(1)} \bullet \nabla \Phi^{(1)} \mathbf{n} dS$$

$$- \iint_S \rho X^{(1)} \bullet \nabla \Phi_t^{(1)} \mathbf{n} dS$$

$$+ \mathbf{\alpha}^{(1)} \times M \ddot{\mathbf{X}}_0^{(1)}$$

$$- \iint_S \rho \nabla \Phi_t^{(2)} \mathbf{n} dS$$

$$\mathbf{M}^{(2)} = \int_{WL} \frac{1}{2} \rho g \zeta_r^{(1)^2} (\mathbf{x} \times \mathbf{n}) dl$$

$$- \iint_S \frac{1}{2} \rho \nabla \Phi^{(1)} \bullet \nabla \Phi^{(1)} (\mathbf{x} \times \mathbf{n}) dS$$

$$- \iint_S \rho X^{(1)} \bullet \nabla \Phi_t^{(1)} (\mathbf{x} \times \mathbf{n}) dS$$

$$+ \mathbf{\alpha}^{(1)} \times I \ddot{\mathbf{\alpha}}^{(1)}$$

$$- \iint_S \rho \nabla \Phi_t^{(2)} (\mathbf{x} \times \mathbf{n}) dS$$

where ς_r is the relative wave elevation along the waterline, and $\boldsymbol{\alpha}^{(1)}$ the oscillatory first order angular motion vector. Fifth terms above equations are the contribution of the second order potential to the drift forces and moments. They will not contribute to the mean drift forces and moments.

Appendix D – Panel Method Validation

Validation with experiment

"Catamaran Motions in Beam and Oblique Seas" - Thomas, Doctors, Couser and Hackett - Ninth International Conference on Fast Sea Transportation, FAST2007, Shanghai, China, September 2007.
All experiments and analysis conducted at zero fwd speed.
s/l = demihull spacing/length overall
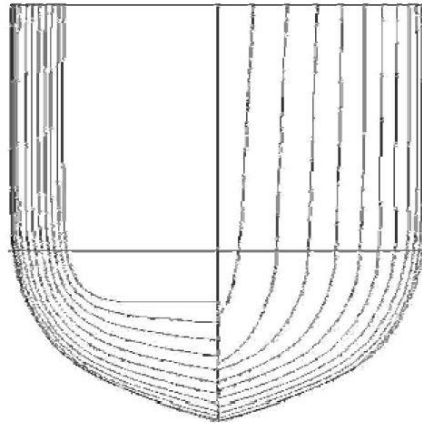
Demi-hull body plan

Motions

Heading = 90 deg, s/l=0.4

Heading = 90 deg, s/l=0.2

Heading = 90 deg, s/l=0.4
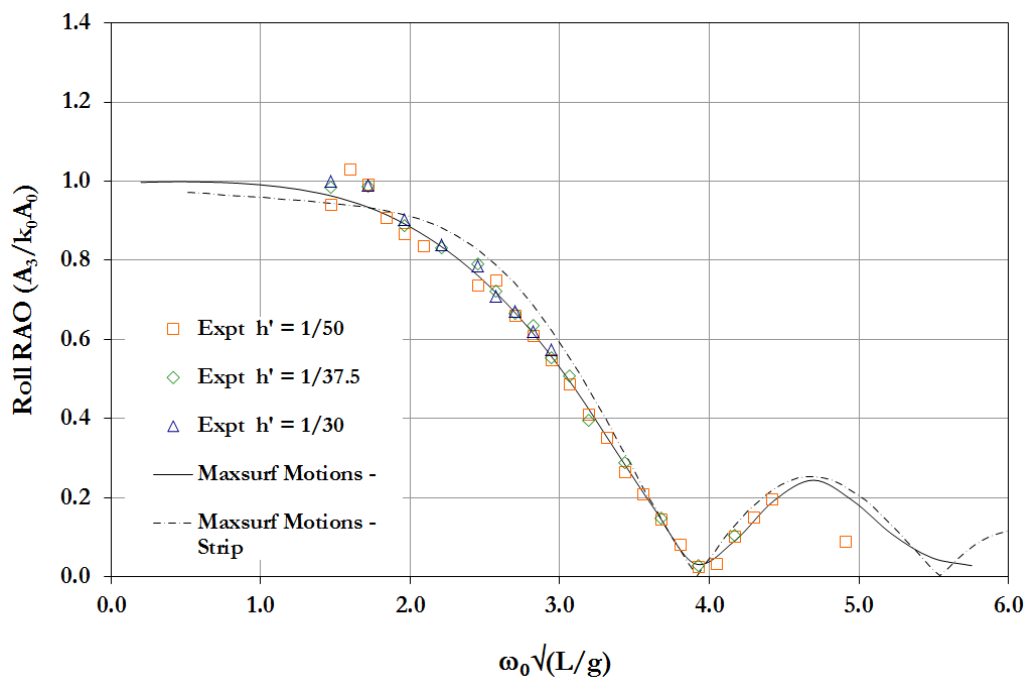
Heading = 90 deg, s/l=0.2

Heading = 90 deg, s/l=0.4

Heading = 90 deg, s/l=0.2
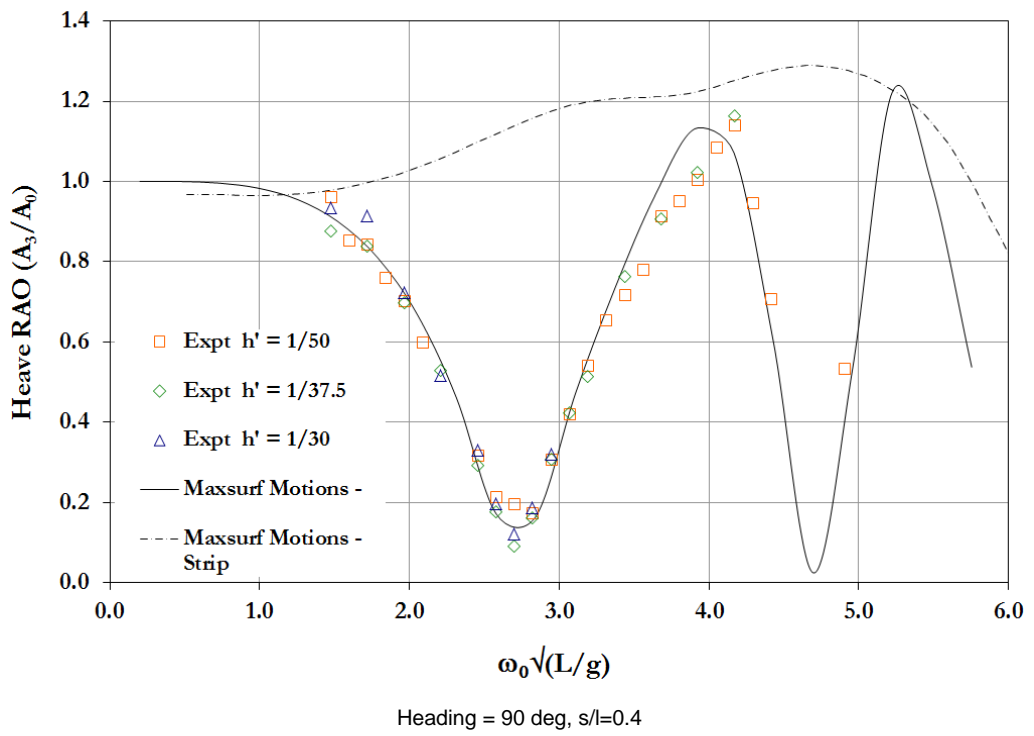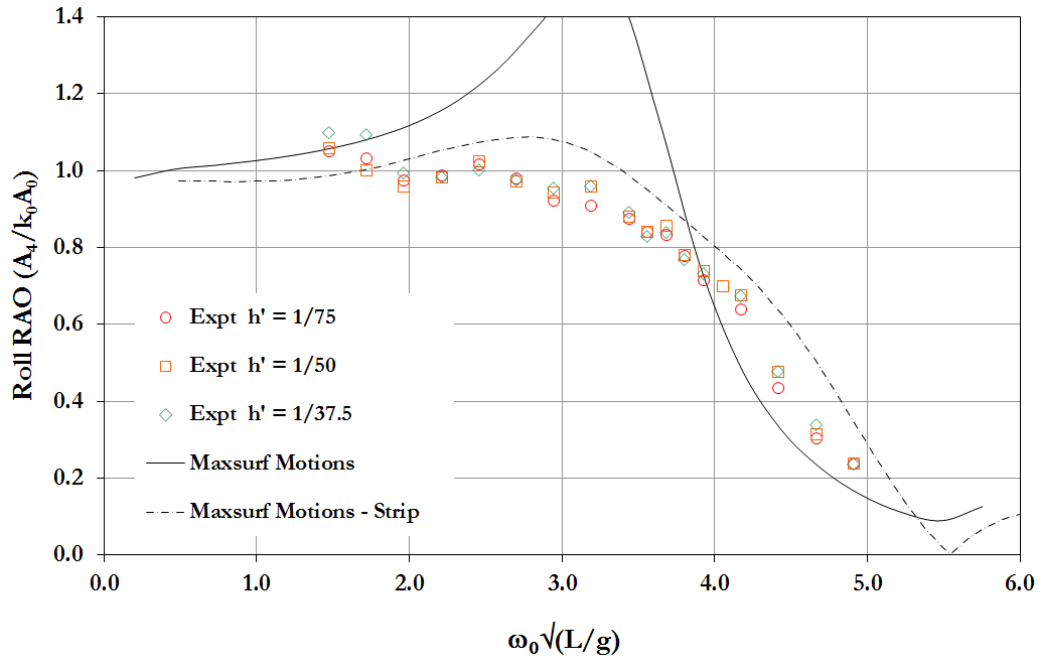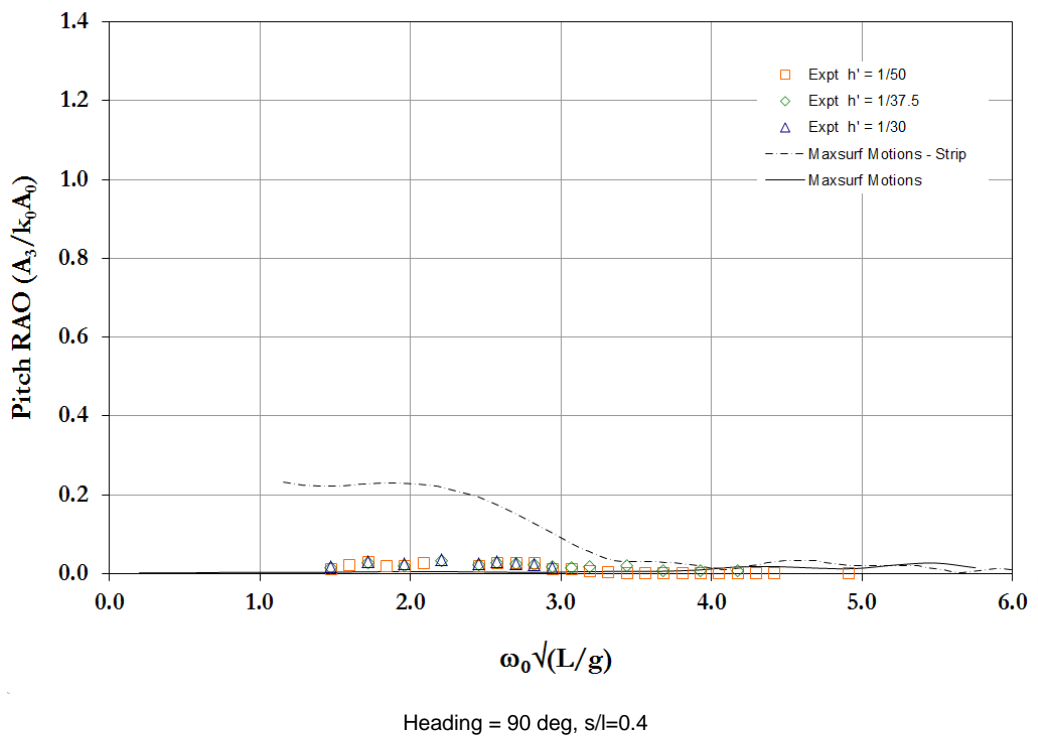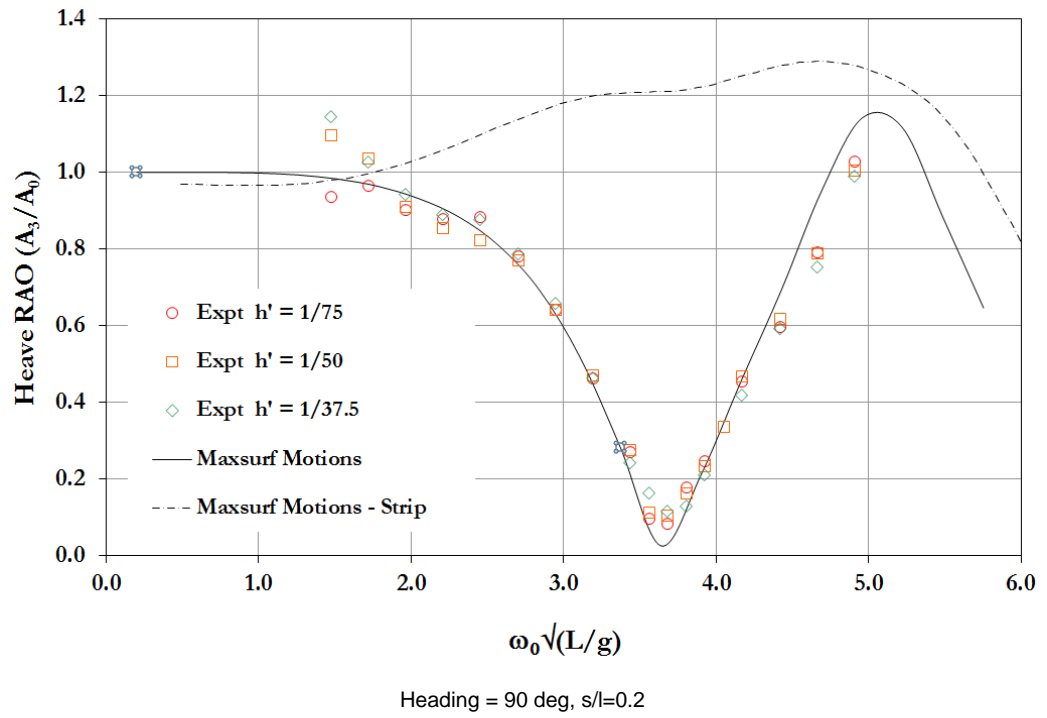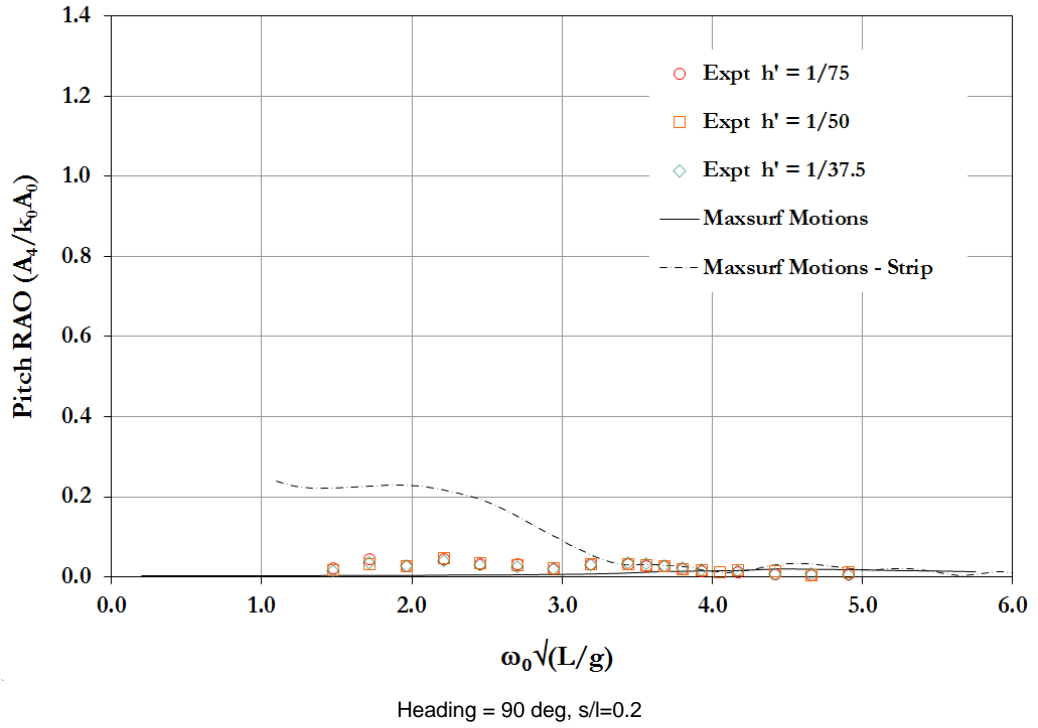
Roll validation with experiment

Roll RAO
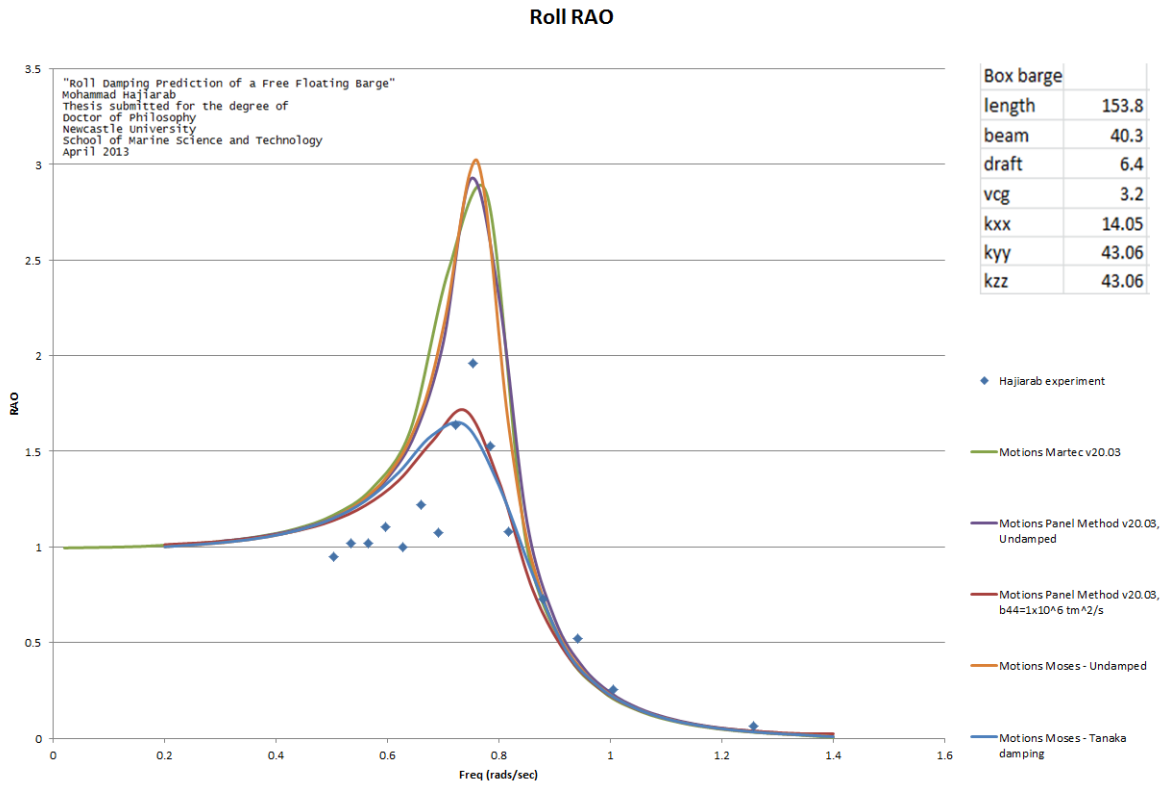
Verification against WAMIT

WAMIT (WaveAnalysisMIT) is a commercially available software package for computing wave loads and motions of offshore structures in waves. WAMIT solves the velocity potential by means of boundary integral equation method (panel method). It first version was launched in 1987. Following are results of motions analysis conducted on a box shaped barge using the Motions panel method compared with the WAMIT panel method.

Analysis parameters:
Geometry : Length = 200m, Beam = 40m, Draft = 28m.
CoG = Centre of Waterplane
Pitch gyradius = 60m
Yaw gyradius = 60m
Roll gyradius = 20m
Characteristic length (L) = 61m.
Characteristic wave amplitude (A) = 1m
Zero Fwd Speed.
Heading = 180 deg (head seas).

Motions
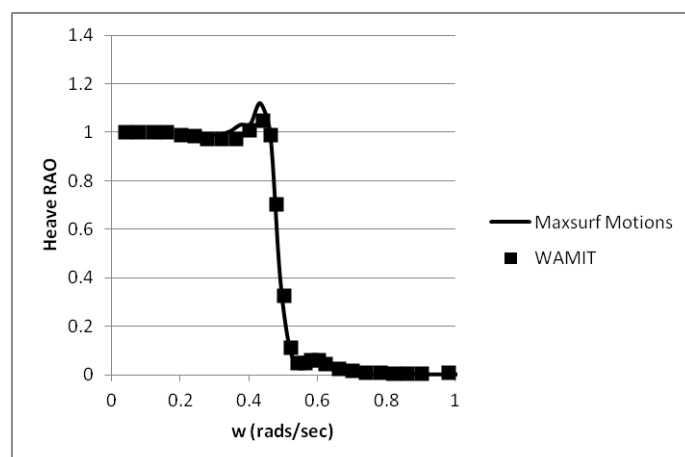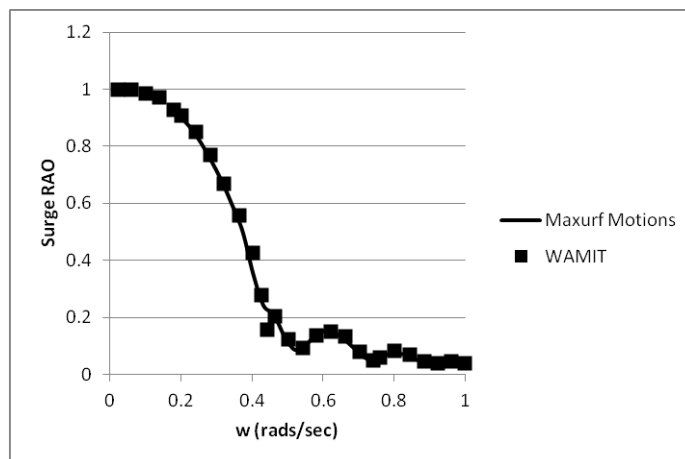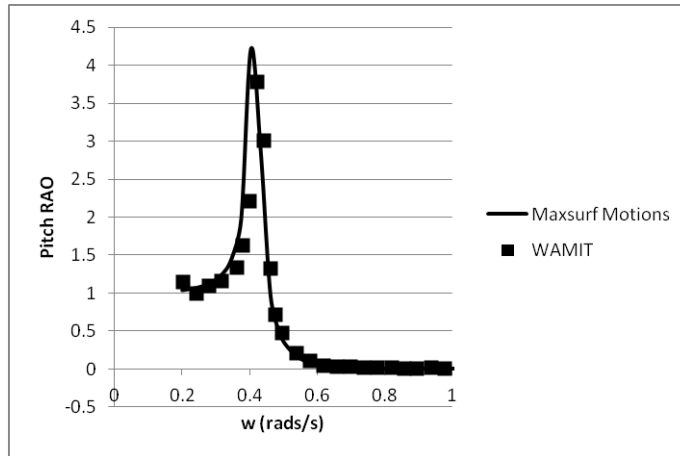
Wave forces
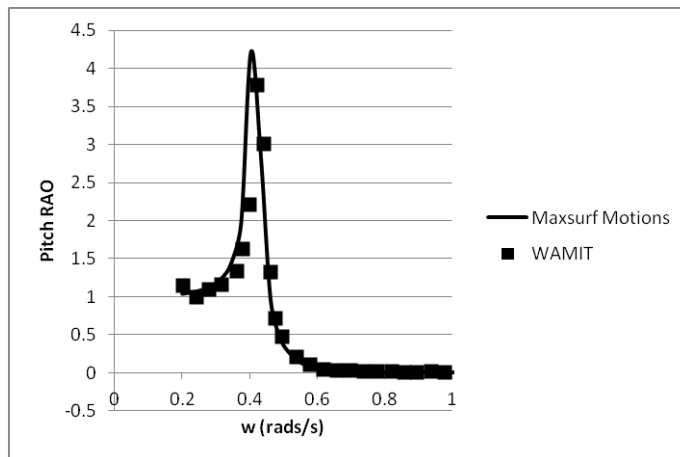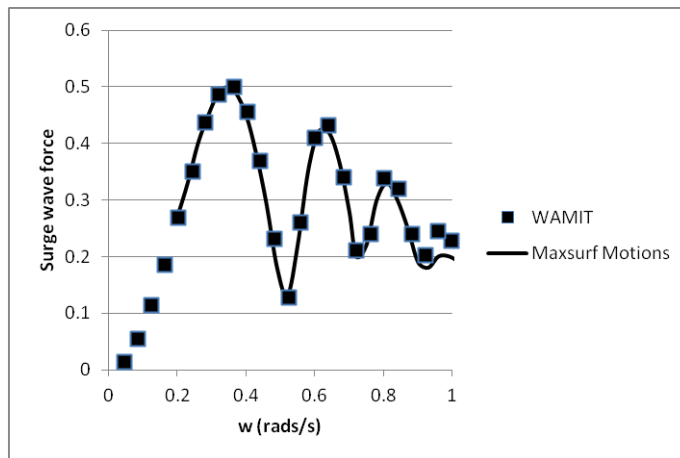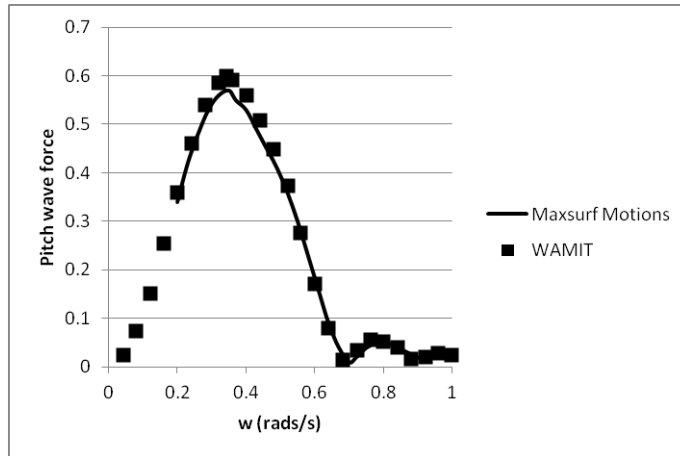
Added Mass
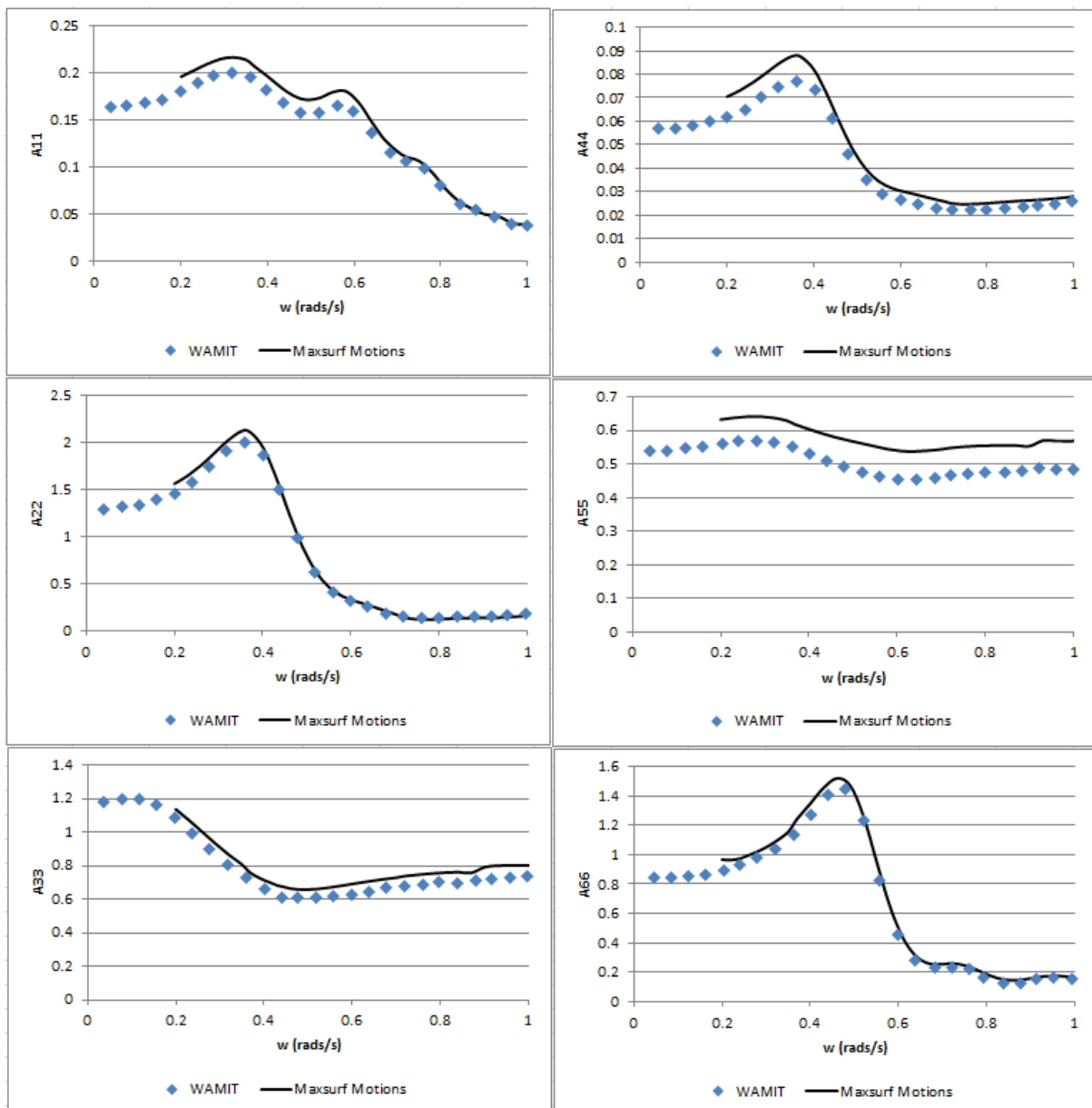
Damping
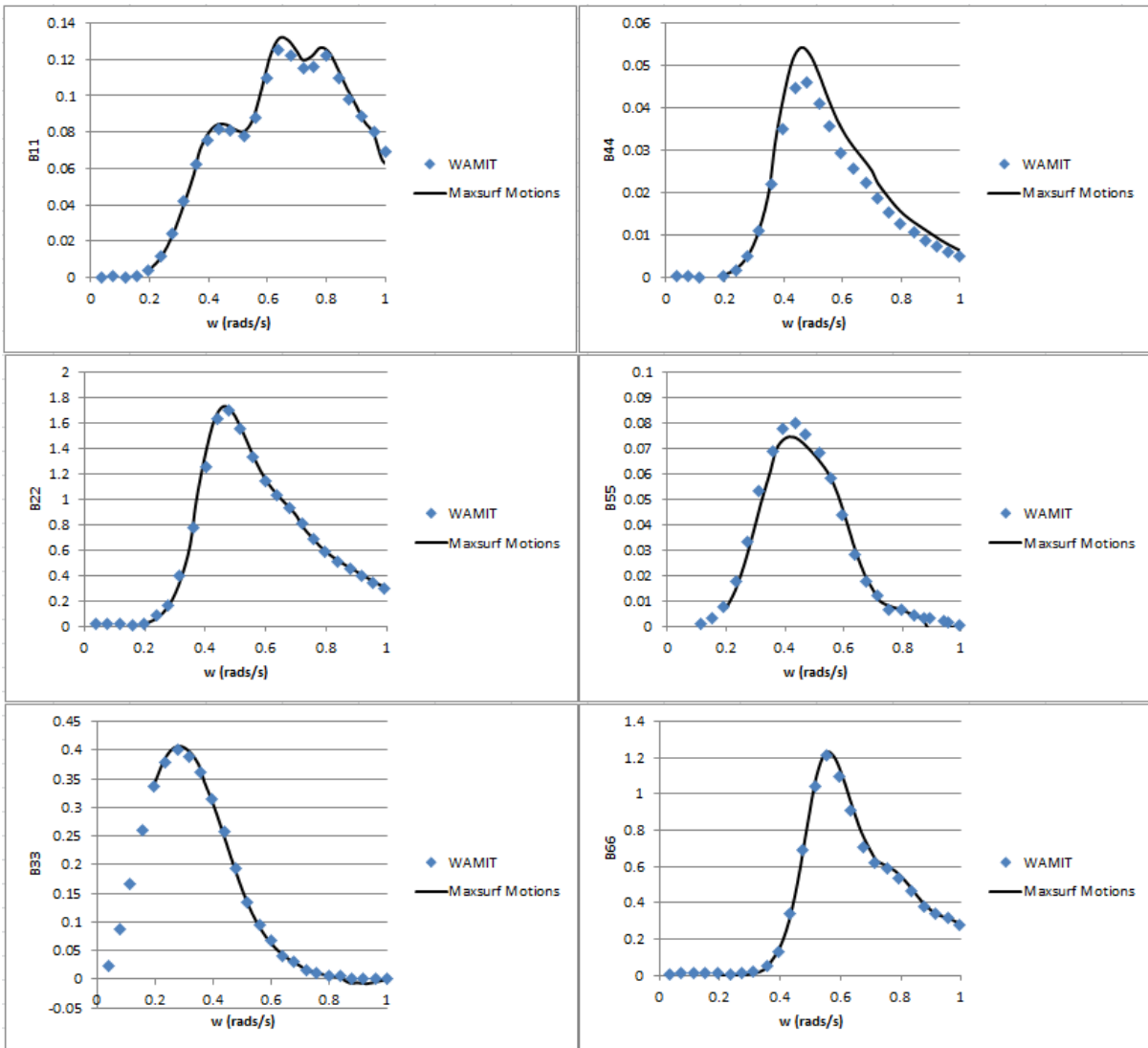
Drift Forces
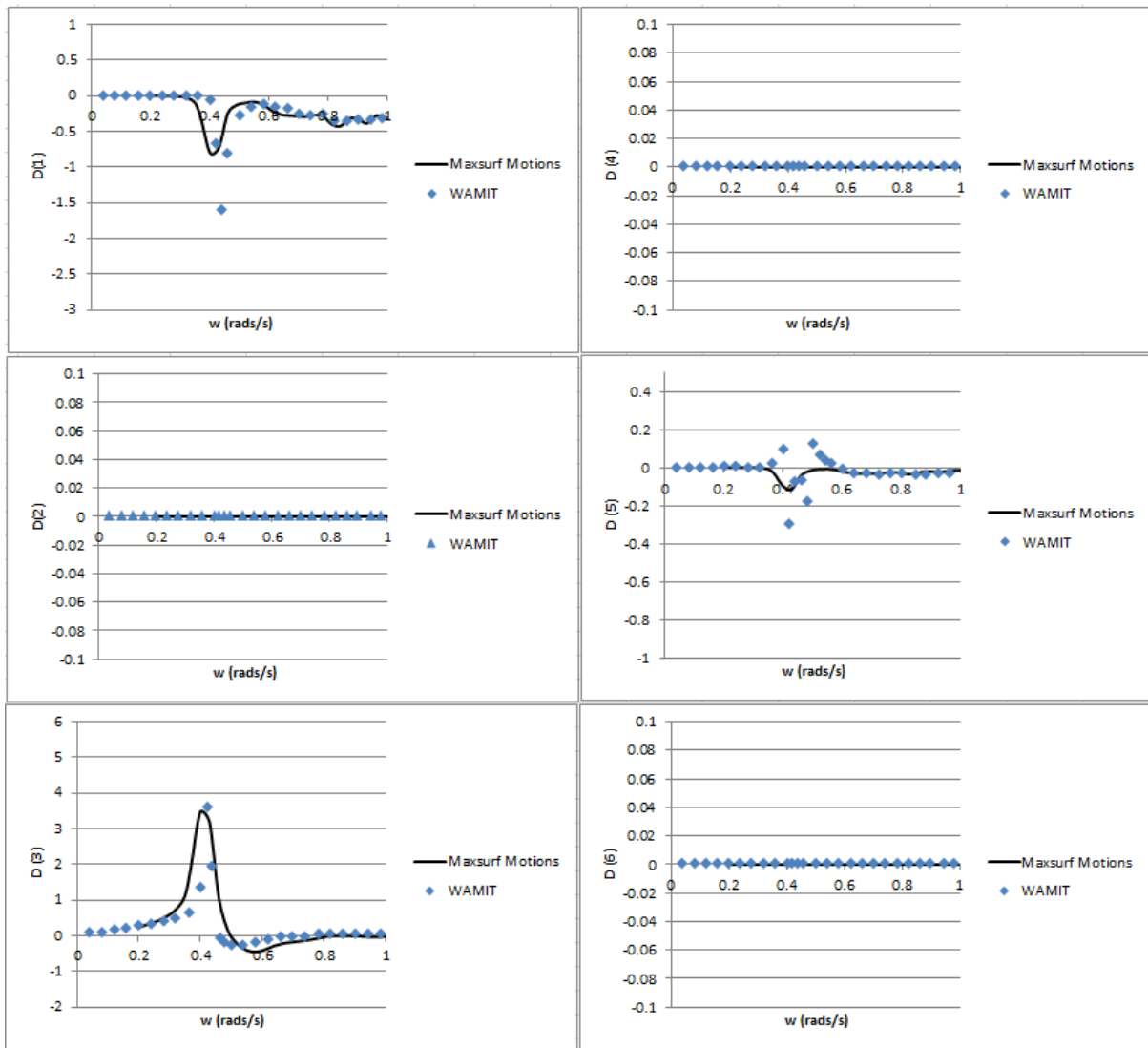
Verification against AQWA

AQWA is a commercially available software package for computing wave loads and motions of offshore structures in waves. Seakeeping tests to determine the motion transfer functions for the Incat 112 were conducted in the Australian Maritime College's model test basin.
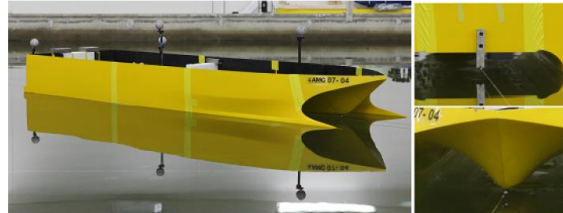
Figure 1: Model in MTB

Model			Head Sea (180 degress)	
			waveheights:	30mm, 60mm, 90mm
Length over all	2.5 m		wavefrequencies:	0.4 hz to 1.4 hz
Displacement	27.2 kg		wavelength:	6.4 m to 0.8 m
Draft	0.076 m		**Beam Sea (90 degress)**	
Demihull Spac.	0.549 m		waveheights:	30mm, 60mm, 90mm
VCG	0.167 m		wavefrequencies:	0.3 hz to 1.7 hz
LCG	0.942 m		wavelength:	8.9 m to 0.54 m
Roll gyradius	0.236 m		**Quarter Bow (135 degress)**	
Pitch gyradius	0.604 m		waveheights:	30mm, 60mm, 90mm
			wavefrequencies:	0.3 hz 1.4 hz
			wavelength:	8.9 m to 0.8 m
(a) Model Details			(b) Test Conditions	

Model Bodyplan

QuarterBow (heading =135deg)

Beam Seas (heading=90deg)
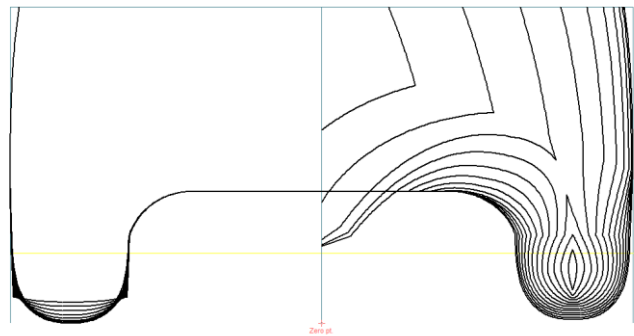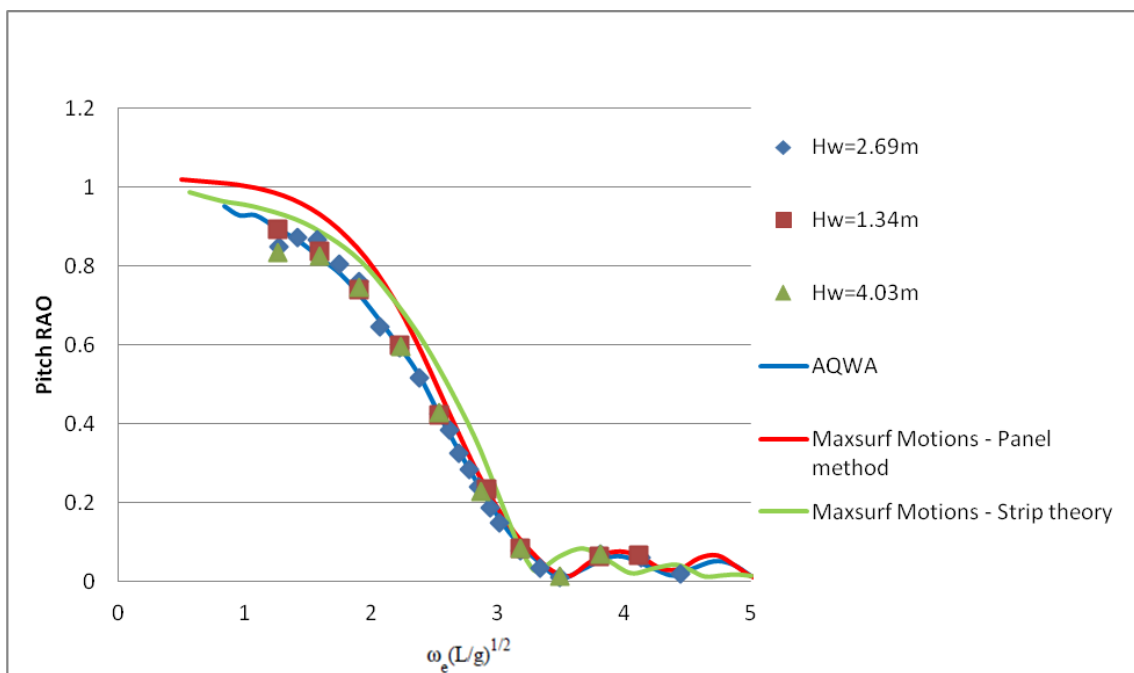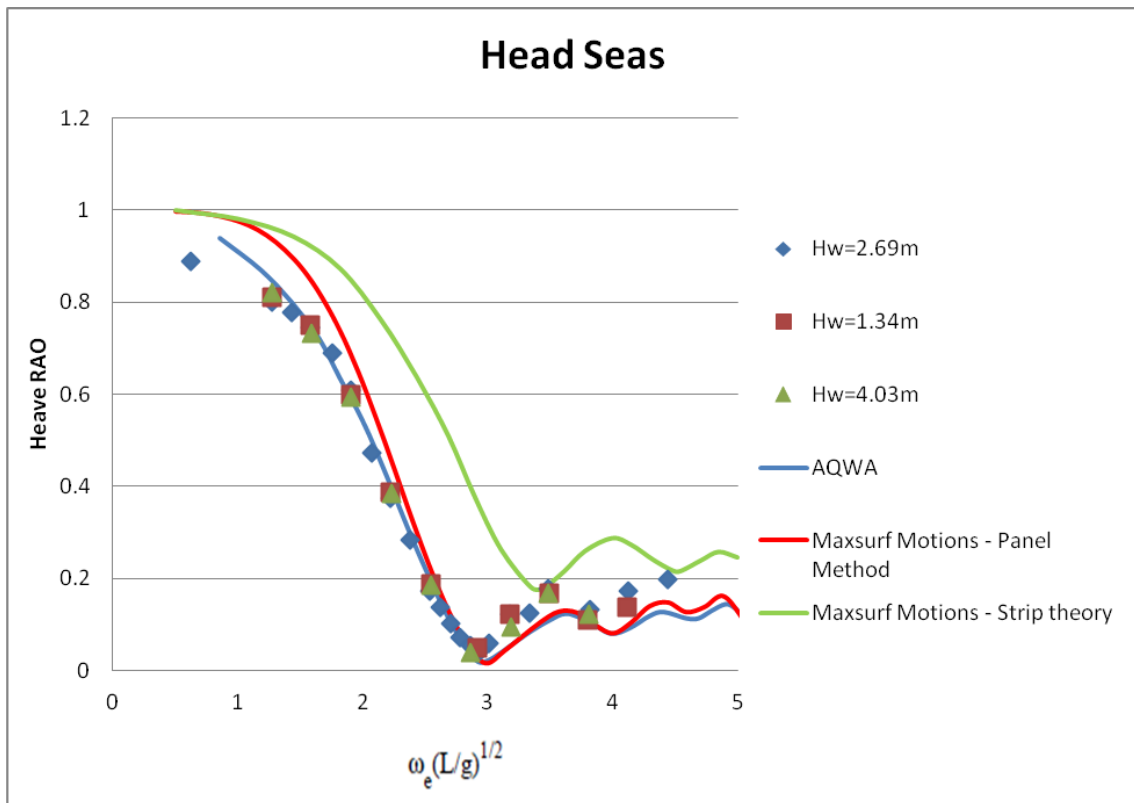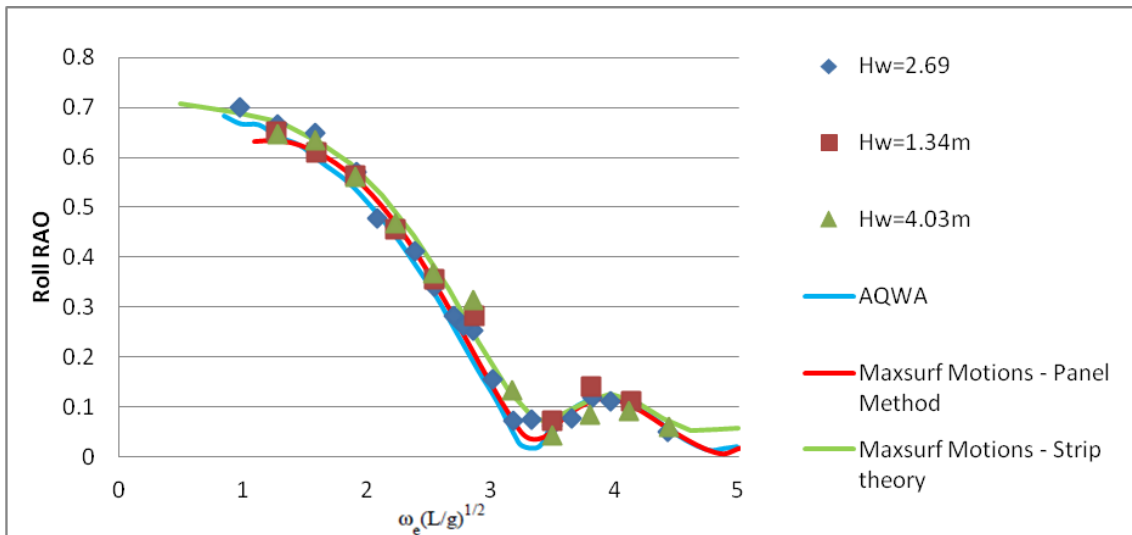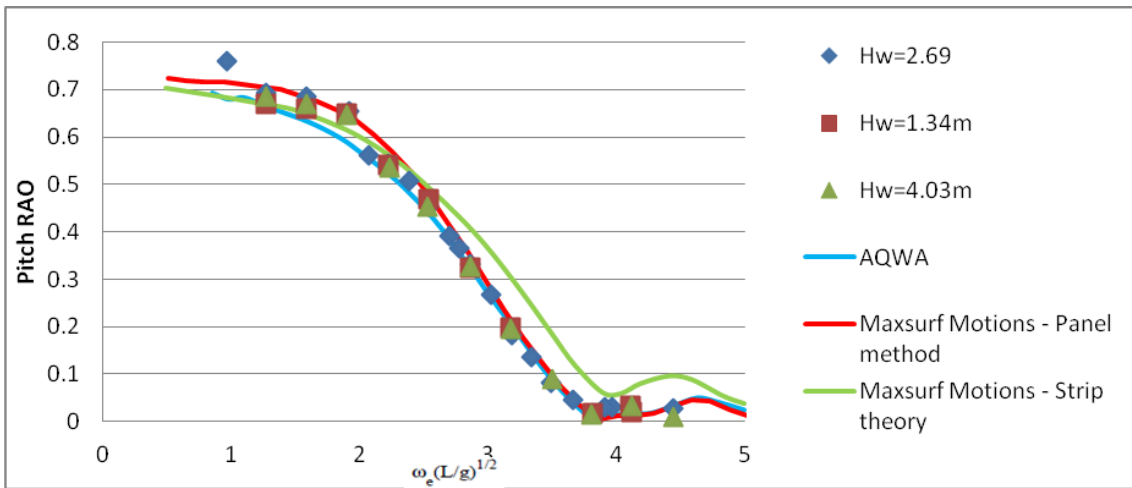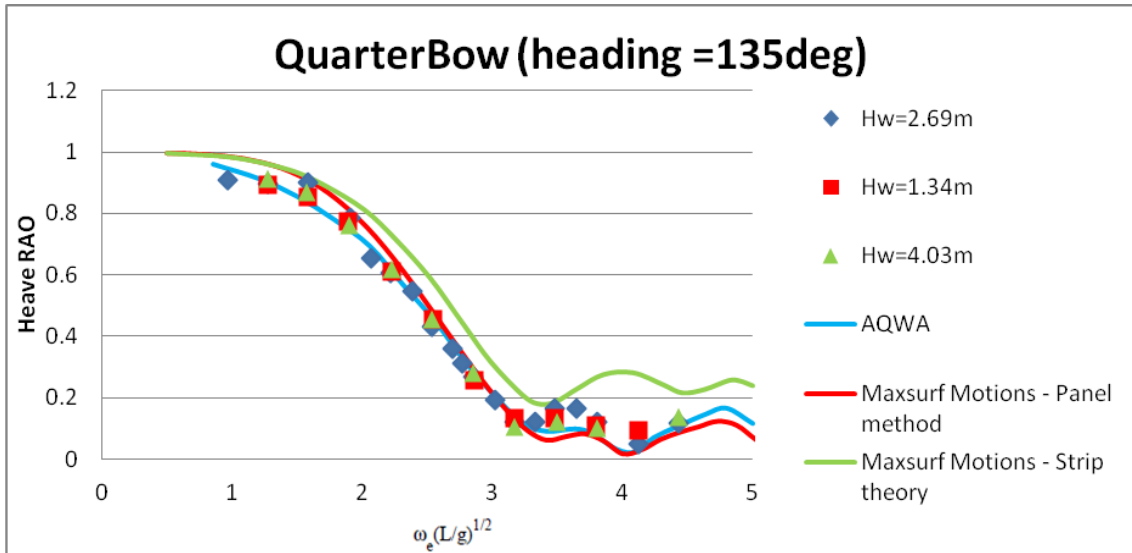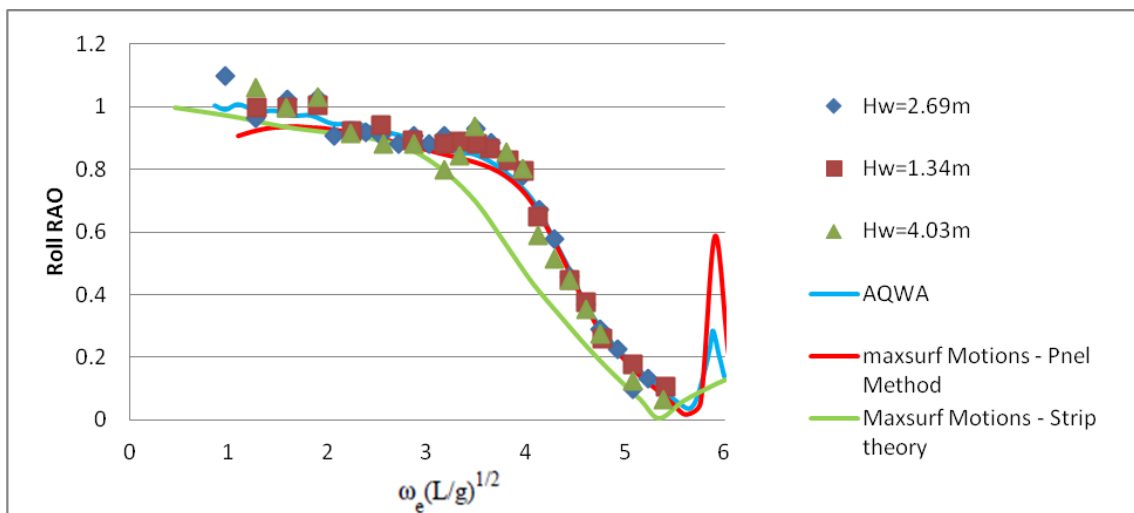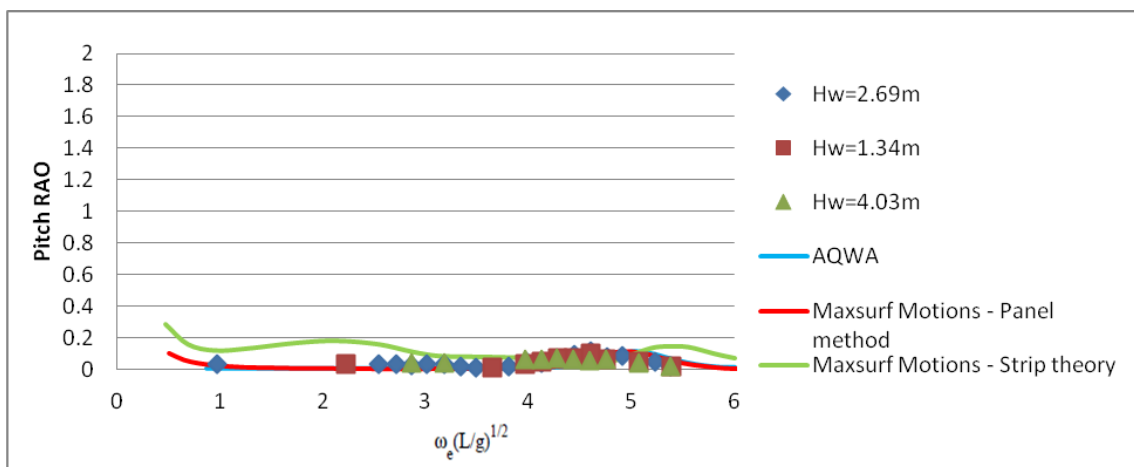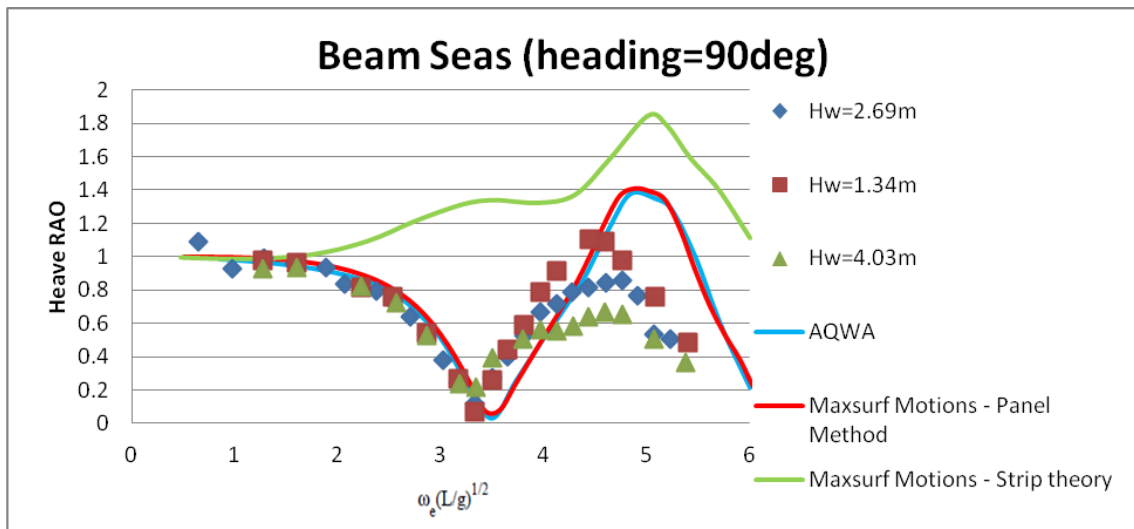
Appendix E – RAO Text File Format

Motions is able to load RAOs defined in a text file. Two file formats are supported and these are described below.

Multiple Condition (Speed, Heading)

RAOs can be imported for a range of speeds and headings, generally it is expected that the full matrix of RAOs will be available ie for each speed the RAO table will be defined for each speed.

The file format is made up of blocks which are started with a block title line with the block name specified within square brackets and the block is terminated with a line containing '[end]'. Within the block a range of parameters may be defined. For example:

```
[Units]
LengthUnits       =       m
SpeedUnits =      kn
TimeUnits   =     s
FreqUnits   =     rad/s
AngleUnits =      deg
[end]
```

Header block (not required)

This is not really used, but can be used to specify the file version number

```
[Header]
Version    =      1.0
[end]
```

Units block (not required)

This can be used to specify the units used in the file; if this section is missing, then the current units as set in the Motions application will be used.

```
[Units]
LengthUnits       =       m
SpeedUnits =      kn
TimeUnits   =     s
FreqUnits   =     rad/s
AngleUnits =      deg
[end]
```

Speeds block

This specifies the speeds included in the file. Speeds are listed sequentially (but may subsequently be sorted once loaded into Motions). For each speed you must at least specify the value (you can also specify a name label, if this is the case, it should be specified *before* the speed value. The following example will specify two speeds, the first labelled "anchor" and be 0.0kn; the second will be labelled "cruise" and have a value of 7 kn.

```
[Speeds]
Name=      anchor
Value      =      0 kn
Name=      cruise
Value      =      7 kn
[end]
```

Headings block

Headings are specified in a very similar way to the speeds, with the optional name label preceding the value:

```
[Headings]
Name=      Beam
Value      =      100 deg
Name=      Bow
Value      =      135 deg
Name=      Head
Value      =      170 deg
[end]
```

Columns block

The columns block is used to specify which columns are included in the RAO tables and which order they appear. Each RAO must be specified by the *wave* frequency. It is not necessary to include all degrees of freedom. If a degree of freedom is included, it is expected that both the magnitude and phase of the RAO will be specified. The RAOs for the angular motions should be non-dimensionalised by wave slope. The following example would indicate that Heave, Roll and Pitch RAOs are to be read. You should have the same format for each RAO table. Note that the columns should be listed on a single line

```
[Columns]
Types = Omega0 HeaveMag HeavePhase RollMag RollPhase PitchMag PitchPhase
[end]
```

The other degrees of freedom follow the same nomenclature

```
SurgeMag SurgePhase SwayMag SwayPhase
```

etc …

RAOset block

The RAOset block provides the actual RAO table, and the columns should match those defined in the preceding Column block.
Alos required are the speed and heading for which the RAOset belongs to. What is important here is the value for the Speed and the Heading (though the Name labels may also be specified to aid readability, although they are ignored when reading the file)

```
[RAOset]
SpeedName   =      anchor
SpeedValue =      0 kn
HeadingName        =      Beam
HeadingValue       =      100 deg
0.400 0.99470 0.159 1.00404 88.780 0.20362 -136.745
0.542 0.99233 0.302 1.02073 88.319 0.13654 -115.042
…
25.000 0.00052 61.032 0.01318 -88.999 0.00002 48.429
[end]
```

For an example single condition RAO text file please refer to the following file which is installed along with the other MAXSURF sample files:

```
Sample Designs\Motions Sample RAOmultiple.txt
```

Single Condition (Speed, Heading)

The older RAO import supports the import of RAOs for a single heading and speed, this is still left in Motions for legacy support. The *wave* periods, at which the RAOs are defined, must be the same for each degree of freedom mode included in the file.

For successful import of the RAOs in text format the file must be structured as follows:

```
Description line [text]
HeadingAngle[deg] Speed[m/s]
Heading description [text]
Speed description [text]
WavePeriod[s]     Heading[deg] DOF[1-6]      RAO    Phase[deg]
```

For Example:

```
Title text
180  3.601
Head
cruise
17.75 180 3 0.998 -0.083
12.261  180 3 1 -0.114
9.541 180 3 1.001 -0.137
```

Example of format of RAO text file since version 22.03 (MOSES 11.03)

The older version described below is also supported in the current version.

Versions prior to 22.03 (MOSES 11.03) required that the data be space-delimited (not tab delimited; although additional white-space between data was tolerated). In addition the second line with heading and Speed required a more complex format:

```
Description line [text]
'Heading = ' HeadingAngle 'deg Speed = ' Speed 'm/s'
Heading description [text]
Speed description [text]
WavePeriod[s]    Heading[deg] DOF[1-6]      RAO    Phase[deg]
```

For Example:

```
Title text
Heading = 180.000 deg   Speed = 3.601 m/s
Head
cruise
17.750  180   3   0.998   -0.083
12.261  180   3   1.000   -0.114
9.541   180   3   1.001   -0.137
7.906   180   3   1.003   -0.175
```

Example of older format of RAO text file prior to version 22.03 (MOSES 11.03)

For an example single condition RAO text file please refer to the following file which is installed along with the other MAXSURF sample files:
```
Sample Designs\Motions Sample RAOsingle.txt
```

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