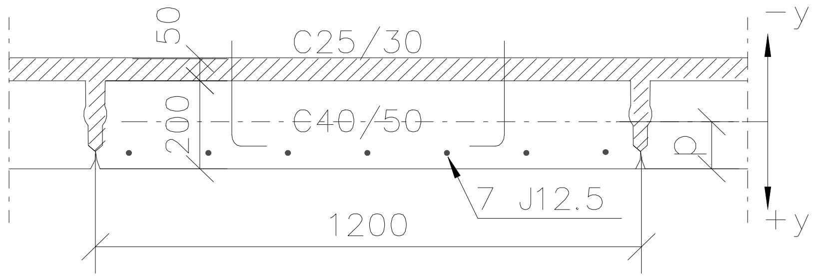
Concrete -concrete - composite structures
Example: Precast plank slab + topping



1. Flexural stiffness

The structure: 200*1200 mm prestressed plank with topping 100 mm

Reinforcement of the precast plank: Prestressing strands 7 d 12,5 mm $A_{p1} := 93 \cdot mm^2$ $n_p := 7$

1.1 Precast plank

Concrete grade C 40/50:

Characteristic compressive strength $f_{ck1} := 40 \cdot MPa$

Average compressive strength $f_{cm1} := f_{ck1} + 8 \cdot MPa$ $f_{cm1} = 48\,MPa$

Elastic modulus $E_{cm1} := 22000 \cdot \left(\frac{f_{cm1}}{10 \cdot MPa}\right)^{0.3} \cdot MPa$ $E_{cm1} = 35220\,MPa$

Average tensile strength $f_{ctm1} := 0.3 \cdot \left(\frac{f_{ck1}}{MPa}\right)^{\frac{2}{3}} \cdot MPa$ $f_{ctm1} = 3.509\,MPa$

Depth $h_1 := 200 \cdot mm$

Width $b := 1200 \cdot mm$

1.1.1 Prestressed strands

Reinforcement of the precast plank: Prestressed strands 7 ϕ_{p7} 12,5 mm $A_{p1} := 93 \cdot mm^2$ $n_p := 7$

Distance of the strands from the bottom side $c_p := 35 \cdot mm$

Elastic modulus $E_p := 195000 \cdot MPa$

Reinforcement area $A_p := n_p \cdot A_{p1}$ $A_p = 651\,mm^2$

First moment of area about the bottom fibre $S_p := A_p \cdot c_p$ $S_p = 2.279 \times 10^4\,mm^3$

Second moment of area $I_p := 0 \cdot mm^4$

Stiffnesses

Axial stiffness $E_p \cdot A_p = 126.945\,MN$

First moment $E_p \cdot S_p = 4.443\,MNm$

Flexural stiffness $E_p \cdot I_p = 0\,MN \cdot m^2$

<u>1.1.2 Concrete</u>

Area of concrete cross-section (excluded the area at the point of the reinforcement)

$A_{c1} := b \cdot h_1 - A_p$ $\quad A_{c1} = 239349\,mm^2$

First moment of area about the bottom fibre $\quad S_{c1} := A_{c1} \cdot \frac{h_1}{2} - S_p \quad S_{c1} = 23.912 \times 10^6\,mm^3$

Centroid from the bottom fibre $\quad p_{c1} := \frac{S_{c1}}{A_{c1}} \quad p_{c1} = 99.905\,mm$

Second moment of area about the centroid

$$I_{c1} := \frac{b \cdot h_1^{\,3}}{12} + b \cdot h_1 \cdot \left(\frac{h_1}{2} - p_{c1}\right)^2 + A_p \cdot \left(p_{c1} - c_p\right)^2 \qquad I_{c1} = 802.745 \times 10^6\,mm^4$$

Stiffnesses:

Axial striffness $\quad E_{cm1} \cdot A_{c1} = 8429.982\,MN$

First moment of afrea $\quad E_{cm1} \cdot S_{c1} = 842.196\,MNm$

Flexural stiffness $\quad E_{cm1} \cdot I_{c1} = 28.273\,MN \cdot m^2$

<u>1.1.3. Stiffness values of the precast plank</u>

Axial stiffness $\quad EA_1 := E_{cm1} \cdot A_{c1} + E_p \cdot A_p \quad EA_1 = 8.557 \times 10^3\,MN$

First moment of area $\quad ES_1 := E_{cm1} \cdot S_{c1} + E_p \cdot S_p \quad ES_1 = 846.639\,MNm$

Centroid from the bottom fibre $\quad p_1 := \frac{ES_1}{EA_1} \quad p_1 = 98.942\,mm$

Flexural stiffness about <u>the centroidal axis</u>:

$$EI_1 := E_{cm1} \cdot I_{c1} + E_{cm1} \cdot A_{c1} \cdot \left(p_1 - p_{c1}\right)^2 + E_p \cdot A_p \cdot \left(p_1 - c_p\right)^2 \qquad EI_1 = 28.79988\,MN \cdot m^2$$

1.2 Composite structure

1.2.1 Topping

Concrete grade C25/30

Characteristic compressive strength $f_{ck2} := 25 \cdot MPa$

Average compressive strength $f_{cm2} := f_{ck2} + 8 \cdot MPa$ $f_{cm2} = 33\,MPa$

Elastic modulus $E_{cm2} := 22000 \cdot \left(\frac{f_{cm2}}{10 \cdot MPa}\right)^{0.3} \cdot MPa$ $E_{cm2} = 31476\,MPa$

Depth $h_2 := 50 \cdot mm$

Width $b := 1200 \cdot mm$

Area of cross-section $A_2 := b \cdot h_2$ $A_2 = 60000\,mm^2$

Centroid from the bottom fibre of the precast plank $p_2 := h_1 + \frac{h_2}{2}$ $p_2 = 225\,mm$

First moment of area about the bottom fibre of the precast plank:

$S_2 := A_2 \cdot p_2$ $S_2 = 13.5 \times 10^6\,mm^3$

Second moment of area about the centroid of the topping

$I_2 := \frac{b \cdot h_2^3}{12}$ $I_2 = 12.5 \times 10^6\,mm^4$

1.2.2 Stiffnesses

Axial stiffness $EA_2 := E_{cm2} \cdot A_2$ $EA_2 = 1.889 \times 10^3\,MN$

First moment of area about the bottom fibre of the precast plank $ES_2 := E_{cm2} \cdot S_2$ $ES_2 = 424.923\,MNm$

Second moment of area about the centroid of the topping $EI_2 := E_{cm2} \cdot I_2$ $EI_2 = 0.393\,MN \cdot m^2$

Composite structure (precast plank + topping)

Axial stiffness $EA := EA_1 + EA_2$ $EA = 10.445 \times 10^3\,MN$

First moment of area about the bottom fibre of the precast plank $ES := ES_1 + ES_2$ $ES = 1.272 \times 10^3\,MNm$

Centroid from the bottom fibre of the precast plank $p := \frac{ES}{EA}$ $p = 121.733\,mm$

Flexural stiffness $EI := EI_1 + EI_2 + EA_1 \cdot (p - p_1)^2 + EA_2 \cdot (p - p_2)^2$ $EI = 53.778\,MN \cdot m^2$

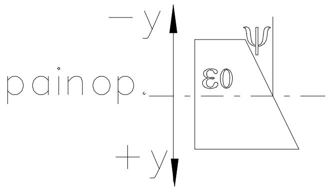
Due to the opping the flexural stiffnes has been increased 87 % $k := \frac{EI}{EI_1}$ $k = 1.867$

2. Stress analysis

2.1. Strains

Axial strain at the centroidal axis $\varepsilon_0 := \frac{N}{EA}$

Curvature $\psi := \frac{M}{EI}$



M and EI are calculated about thye centroidal axis

Strains at different points of the cross-section $\varepsilon(y) := \varepsilon_0 + \psi \cdot y$ y is measured from the centroid;positive to downward

Stresses $\sigma_c(y) := E_{cm} \cdot \varepsilon(y)$

2.2. Prestressing

At the prestressing state the effective cross-section is the precast plank with the prestressing strands.

The losses are supposed to happen totally before casting the topping (generally this is not true, a part of losses happen after casting the topping)

$\sigma_{p\infty} := 1100 \cdot \text{MPa}$ Prestress force $P_\infty := A_p \cdot \sigma_{p\infty}$ $P_\infty = 716.1\,\text{kN}$

Eccentricity of the prestress force about the centroid of the precast plank $e_p := p_1 - c_p$ $e_p = 63.942\,\text{mm}$

The precast plank is loaded by the forces due to the prestressing

$N_p := -P_\infty$ $N_p = -716.1\,\text{kN}$

$M_p := -P_\infty \cdot e_p$ $M_p = -45.789\,\text{kNm}$

Strains due to the prestressing

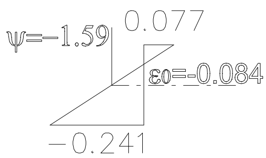
Axail strain at the centroidal axis $\varepsilon_{0P} := \frac{N_p}{EA_1}$ $\varepsilon_{0P} = -0.084\,‰$

Curvature $\psi_P := \frac{M_p}{EI_1}$ $\psi_P = -1.59\,\frac{‰}{m}$

Strains in different point of the cross-section:

Bottom fibre $\varepsilon_{caP} := \varepsilon_{0P} + \psi_P \cdot p_1$ $\varepsilon_{caP} = -0.241\,‰$

Top fibre $\varepsilon_{cyP} := \varepsilon_{0P} + \psi_P \cdot (p_1 - h_1)$ $\varepsilon_{cyP} = 0.077\,‰$



Stress

Bottom fibre $\sigma_{caP} := E_{cm1} \cdot \varepsilon_{caP}$ $\sigma_{caP} = -8.488\,\text{MPa}$

Top fibre $\sigma_{cyP} := E_{cm1} \cdot \varepsilon_{cyP}$ $\sigma_{cyP} = 2.711\,\text{MPa}$

2.3 Stresses due to self-weight of the precast plank

Span of the slab $L := 7 \cdot m$

Self-weight of the precast plank $g_1 := 25 \cdot \frac{kN}{m^3} \cdot h_1 \cdot b$ $\quad g_1 = 6 \frac{kN}{m}$

Bending moment $M_{g1} := \frac{g_1 \cdot L^2}{8}$ $\quad M_{g1} = 36.75\, kNm$

$N_{g1} := 0 \cdot kN$

Axial strain in the centroidal axis $\varepsilon_{0g1} := \frac{N_{g1}}{EA_1}$ $\quad \varepsilon_{0g1} = 0$

Curvature $\psi_{g1} := \frac{M_{g1}}{EI_1}$ $\quad \psi_{g1} = 1.276 \frac{‰}{m}$

Strains:

Bottom fibre $\varepsilon_{cag1} := \varepsilon_{0g1} + \psi_{g1} \cdot p_1$ $\quad \varepsilon_{cag1} = 0.126\, ‰$

Top fibre $\varepsilon_{cyg1} := \varepsilon_{0g1} + \psi_{g1} \cdot (p_1 - h_1)$ $\quad \varepsilon_{cyg1} = -0.129\, m \frac{‰}{m}$

Stresses:

Bottom fibre $\sigma_{cag1} := E_{cm1} \cdot \varepsilon_{cag1}$ $\quad \sigma_{cag1} = 4.447\, MPa$

Top fibre $\sigma_{cyg1} := E_{cm1} \cdot \varepsilon_{cyg1}$ $\quad \sigma_{cyg1} = -4.542\, MPa$

2.4 Stresses due to self-ewight of the topping

Self-weight of the topping $g_2 := 25 \cdot \frac{kN}{m^3} \cdot h_2 \cdot b$ $\quad g_2 = 1.5 \frac{kN}{m}$

Bending moment $M_{g2} := \frac{g_2 \cdot L^2}{8}$ $\quad M_{g2} = 9.188\, kNm$

$N_{g2} := 0 \cdot kN$

Axial strain in the centroidal axis $\varepsilon_{0g2} := \frac{N_{g2}}{EA_1}$ $\quad \varepsilon_{0g2} = 0$

Curvature $\psi_{g2} := \frac{M_{g2}}{EI_1}$ $\quad \psi_{g2} = 0.319 \frac{‰}{m}$

Strains:

Bottom fibre $\varepsilon_{cag2} := \varepsilon_{0g2} + \psi_{g2} \cdot p_1$ $\quad \varepsilon_{cag2} = 0.032\, ‰$

Top fibre $\varepsilon_{cyg2} := \varepsilon_{0g2} + \psi_{g2} \cdot (p_1 - h_1)$ $\quad \varepsilon_{cyg2} = -0.032\, ‰$

Stresses:

Bottom fibre $\sigma_{cag2} := E_{cm1} \cdot \varepsilon_{cag2}$ $\quad \sigma_{cag2} = 1.112\, MPa$

Top fibre $\sigma_{cyg2} := E_{cm1} \cdot \varepsilon_{cyg2}$ $\quad \sigma_{cyg2} = -1.135\, MPa$

2.5 Total stress before casting the topping

Total strain:

Axial strain in the centroidal axis	$\varepsilon_{01} := \varepsilon_{0P} + \varepsilon_{0g1} + \varepsilon_{0g2}$	$\varepsilon_{01} = -0.084\,\mathrm{m}\frac{‰}{\mathrm{m}}$
Curvature	$\psi_1 := \psi_P + \psi_{g1} + \psi_{g2}$	$\psi_1 = 0.0052\frac{‰}{\mathrm{m}}$
Strain in the botton fibre	$\varepsilon_{ca1} := \varepsilon_{caP} + \varepsilon_{cag1} + \varepsilon_{cag2}$	$\varepsilon_{ca1} = -0.083\,‰$
	$\varepsilon_{ca1} := \varepsilon_{01} + \psi_1 \cdot p_1$	$\varepsilon_{ca1} = -0.083\,‰$
Strain in the top fibre	$\varepsilon_{cy1} := \varepsilon_{cyP} + \varepsilon_{cyg1} + \varepsilon_{cyg2}$	$\varepsilon_{cy1} = -0.084\,‰$
	$\varepsilon_{cy1} := \varepsilon_{01} + \psi_1 \cdot (p_1 - h_1)$	$\varepsilon_{cy1} = -0.084\,‰$
Stress in the bottom fibre	$\sigma_{ca1} := \sigma_{caP} + \sigma_{cag1} + \sigma_{cag2}$	$\sigma_{ca1} = -2.929\,\mathrm{MPa}$
	$\sigma_{ca1} := E_{cm1} \cdot \varepsilon_{ca1}$	$\sigma_{ca1} = -2.929\,\mathrm{MPa}$
Stress in the top fibre	$\sigma_{cy1} := \sigma_{cyP} + \sigma_{cyg1} + \sigma_{cyg2}$	$\sigma_{cy1} = -2.966\,\mathrm{MPa}$
	$\sigma_{cy1} := E_{cm1} \cdot \varepsilon_{cy1}$	$\sigma_{cy1} = -2.966\,\mathrm{MPa}$

2.6 Deflection

$$a := \delta_a \cdot \psi \cdot L^2$$

Prestressing causes uniformly distributed bendin moment diagram (straight strands)

Deflection coefficeint for prestressing $\delta_{aP} := \frac{6}{48}$ $\delta_{aP} = 0.125$

Imposed loading is uniformly distributed => the bending moment diagram is parabolic = deflection coefficient for the imposed loading

$\delta_{ag} := \frac{5}{48}$ $\delta_{ag} = 0.104$

Deflection due to prestressing	$a_P := \delta_{aP} \cdot \psi_P \cdot L^2$	$a_P = -9.738\,\mathrm{mm}$	upward
Deflection due to self-weight of the precast plank	$a_{g1} := \delta_{ag} \cdot \psi_{g1} \cdot L^2$	$a_{g1} = 6.513\,\mathrm{mm}$	downward
Deflection due to self-ewight of the topping	$a_{g2} := \delta_{ag} \cdot \psi_{g2} \cdot L^2$	$a_{g2} = 1.628\,\mathrm{mm}$	upward

Total deflection before casting and hardening of the topping $a_1 := a_P + a_{g1} + a_{g2}$

$a_1 = -1.597\,\mathrm{mm}$ upward

Curvature $\psi_1 := \psi_P + \psi_{g1} + \psi_{g2}$ $\psi_1 = 5.163 \times 10^{-3}\frac{‰}{\mathrm{m}}$

3. Allowable imposed live load

Allowable live load if in the bottom surface of the slab does not allowed cracks => the tensile stress in the bottom fibre does not exceed the average tensile strength of concrte fctm1.

Concrete grade in the precast plank slab C40 /50

Average tensile strength of concrete in the precast plank slab $f_{ctm1} = 3.509\,\mathrm{MPa}$

$\Sigma\sigma_{ca} \le f_{ctm1}$

The allowable live load is calculated in two cases:
- A. The structure (precast palank + topping) does no fuction as a composite structure
- B. The structure (precast slab+ topiing) function as a composite structure

3.A. The structure does not function as a composite structure

The topping does not function as a part of a load-bearing structure; the topping is only a load on the precast plank slab. The only load-bearing structure is the precast plank slab.
The precast plank slab is loaded by the imposed live load.
The stiffness values are the one of the precast plank slab.

Axial stiffness $EA_1 = 8.557 \times 10^3\,\mathrm{MN}$

First moment of area $ES_1 = 846.639\,\mathrm{MNm}$

Flexural stiffness $EI_1 = 28.8\,\mathrm{MN \cdot m^2}$

Centroid from the bottom fibre $p_1 = 98.942\,\mathrm{mm}$

Stresses in the bottom fibre:

Prestressing + self-weight of the precast plank slab + self weight of the topping $\sigma_{ca1} = -2.929\,\mathrm{MPa}$

Total stress under the limposed live load $\Sigma\sigma_{ca} := \sigma_{ca1} + \sigma_{caq1} = f_{ctm1}$

Stress due to the imposed live load $\sigma_{caq1} := f_{ctm1} - \sigma_{ca1}$ $\sigma_{caq1} = 6.438\,\mathrm{MPa}$

Strain in the bottom fibre due to the imposed live load $\varepsilon_{caq1} := \dfrac{\sigma_{caq1}}{E_{cm1}}$ $\varepsilon_{caq1} = 0.183\,‰$

$\varepsilon_{caq1} := \varepsilon_{0q1} + \psi_{q1} \cdot p_1$

For bending the axial strain in the centroidal axis $\varepsilon_{0q1} := 0$

$\varepsilon_{caq1} := \psi_{q1} \cdot p_1$

Curvature die to the imposed live load $\psi_{q1} := \dfrac{\varepsilon_{caq1}}{p_1}$ $\psi_{q1} = 1.848\,\dfrac{‰}{\mathrm{m}}$

Curvature $\psi_{q1} := \dfrac{M_{q1}}{EI_1}$

Bending moment due to the imposed live load $M_{q1} := \psi_{q1} \cdot EI_1$ $M_{q1} = 53.209\,\mathrm{kNm}$

$$M_{q1} := \frac{q_1 \cdot L^2}{8}$$

Allowable imposed live load $q_1 := 8 \cdot \frac{M_{q1}}{L^2}$ $q_1 = 8.687 \frac{kN}{m}$

$q_1 := \frac{q_1}{b}$ $q_1 = 7.239 \frac{kN}{m^2}$

If the structure does not function as a composite structure, the allowable imposed live load is $q_1 = 7.239 \frac{kN}{m^2}$

Deflection due to the imposed live load $q_1 = 7.239 \frac{kN}{m^2}$

Curvature $\psi_{q1} = 1.848 \frac{‰}{m}$

Deflection coefficient for a uniformly distributed load $\delta_{aq} := \frac{5}{48}$

Deflection due to the imposed live load $a_{q1} := \delta_{aq} \cdot \psi_{q1} \cdot L^2$ $a_{q1} = 9.43\,mm$

Total deflection $a_{tot1} := a_P + a_{g1} + a_{g2} + a_{q1}$ $a_{tot1} = 7.834\,mm$ $\frac{L}{894}$

3.B. The structure function as a composite structure

The topping function together with the precast plank slab as the part of a composite structure
The composite structure is loaded by the imposed live load.

The stiffness values are the one of the composte structure (precast plan slab + topping):

Axial stiffness	$EA = 1.045 \times 10^4 MN$
First moment of area about the bottom fibre of the precast plank slab	$ES = 1.272 \times 10^3 MNm$
Flexural stiffness	$EI = 53.778 MN \cdot m^2$
Centroid from the bottom fibre of the precast plank slab	$p = 121.733 mm$

Stress in the bottom fibre of the precast plank slab before the composite action due to the permanent loads
(prestressing+ self-weight of the precast plank slab + self-weight of the topping).
These loads affect before hardening the topping to the precast plank slab;
Stresses due to these loads are calculated by using the stiffness values of the precast plank slab.

Stress in the bottom fibre of the precast plank slab before composite action $\sigma_{ca1} = -2.929 MPa$

After the topping has hardened it function together with the precast plank slab as a composite structure.
For the loads affect after hardening of the topping are used the stiffness values of the composte section.

Total allowable stress in the bottom fibre under the imposed live load $\Sigma\sigma_{ca} := \sigma_{ca1} + \sigma_{caq2} = f_{ctm1}$

Stress in the bottyom fibre due to the allowable imposed live load $\sigma_{caq2} := f_{ctm1} - \sigma_{ca1}$ $\sigma_{caq2} = 6.438 MPa$

Strain in the bottom fibre due to imposed loive load $\varepsilon_{caq2} := \frac{\sigma_{caq1}}{E_{cm1}}$ $\varepsilon_{caq2} = 0.183 \text{‰}$

$$\varepsilon_{caq2} := \varepsilon_{0q2} + \psi_{q2} \cdot p$$

Axial strain in the centroidal axis due to bending $\varepsilon_{0q2} := 0$

$\varepsilon_{caq2} := \psi_{q2} \cdot p$ (p=distance of the cenroid of the composite cross-section from the bottom fibre of the precast plank slab)

Curvature due to the imposed love load $\psi_{q2} := \frac{\varepsilon_{caq2}}{p}$ $\psi_{q2} = 1.502 \frac{\text{‰}}{m}$

Curvature $\psi_{q2} := \frac{M_{q2}}{EI}$ (EI = flexural stiffness of the composite cross-section)

Bending moment due to allowable imposed live load $M_{q2} := \psi_{q2} \cdot EI$ $M_{q2} = 80.755 kNm$

$$M_{q2} := \frac{q_2 \cdot L^2}{8}$$

Allowable imposed live load $q_2 := 8 \cdot \frac{M_{q2}}{L^2}$ $q_2 = 13.184 \frac{kN}{m}$ $q_2 := \frac{q_2}{b}$ $q_2 = 10.987 \frac{kN}{m^2}$

If the structure function as a composite structure the allowable imposed live load is $q_2 = 10.987 \frac{kN}{m^2}$

When the structure function as a composite structure the allowable imposed live load increase compared to a non-composite structure

$q_1 = 7.239\frac{kN}{m^2}$ --> $q_2 = 10.987\frac{kN}{m^2}$ so about 52 %

<u>3. B.2 Deflection due to the allowable imposed live load</u> $q_2 = 10.987\frac{kN}{m^2}$

Curvature $\psi_{q2} = 0.15\frac{\%}{m}$

Deflection $a_{q2} := \delta_{aq}\cdot\psi_{q2}\cdot L^2$ $a_{q2} = 7.665\,mm$ = L/913

Total deflection $a_{tot2} := a_P + a_{g1} + a_{g2} + a_{q2}$ $a_{tot2} = 6.068\,mm$ = L/1153

Deflection of the composite structure under the imposed live looad $q_1 = 7.239\frac{kN}{m^2}$

$M_q := \frac{b\cdot q_1\cdot L^2}{8}$ $M_q = 53.209\,kNm$

Curvature $\psi_q := \frac{M_q}{EI}$ $\psi_q = 0.989\frac{\%o}{m}$

Deflection $a_q := \delta_{aq}\cdot\psi_q\cdot L^2$ $a_q = 5.05\,mm$

about 64 % of the deflection without composite action

Total deflection $a_{tot} := a_P + a_{g1} + a_{g2} + a_q$ $a_{tot} = 3.454\,mm$

Thanks to composte action:
- the deflection due to the imposed live load decreased $a_{q1} = 9.43\,mm$ --> $a_q = 5.05\,mm$
- the total deflection decreased $a_{tot1} = 7.834\,mm$ --> $a_{tot} = 3.454\,mm$

about 68 % of the deflection without composite action

3.B.3 Stresses in the composte structure under the imposed live load $q_2 = 10.987\,\frac{kN}{m^2}$

$M_{q2} := \frac{b \cdot q_2 \cdot L^2}{8}$ $M_{q2} = 80.755\,kNm$

Strain and stresses due to the imposed live load

Axial strain in the centroidal axis of the composite cross-section $\varepsilon_{0q2} := 0$

Curvature $\psi_{q2} := \frac{M_{q2}}{EI}$ $\psi_{q2} = 1.502\,\frac{‰}{m}$

Strain at the distance y from the centroid of the compossite cross-section $\varepsilon_{q2} := \varepsilon_{0q2} + \psi_{q2} \cdot y$

Bottom fibre $y := p$ $y = 121.733\,mm$

Strain $\varepsilon_{caq} := \varepsilon_{0q2} + \psi_{q2} \cdot p$ $\varepsilon_{caq} = 0.183\,‰$

Stress $\sigma_{caq} := E_{cm1} \cdot \varepsilon_{caq}$ $\sigma_{caq} = 6.438\,MPa$

Top fibre of the precast plank $y := p - h_1$ $y = -78.267\,mm$

Strain $\varepsilon_{cj1q} := \varepsilon_{0q2} + \psi_{q2} \cdot (p - h_1)$ $\varepsilon_{cj1q} = -0.118\,‰$

Stress $\sigma_{cj1q} := E_{cm1} \cdot \varepsilon_{cj1q}$ $\sigma_{cj1q} = -4.139\,MPa$

Bottom fibre of the topping $y := p - h_1$ $y = -78.267\,mm$

Strain $\varepsilon_{cj2q} := \varepsilon_{0q2} + \psi_{q2} \cdot (p - h_1)$ $\varepsilon_{cj2q} = -0.118\,‰$

Stress $\sigma_{cj2q} := E_{cm2} \cdot \varepsilon_{cj2q}$ $\sigma_{cj2q} = -3.699\,MPa$

Strain is the same as in the top fibre of the precast plank (no slip in the construction joint), but the stress is different because the elastic modulus of the topping concrete is different than the elastic modulus of the concrete of the precast plank.

Top fibre of the topping $y := p - (h_1 + h_2)$ $y = -128.267\,mm$

Strain $\varepsilon_{cy2q} := \varepsilon_{0q2} + \psi_{q2} \cdot [p - (h_1 + h_2)]$ $\varepsilon_{cy2q} = -0.193\,‰$

Stress $\sigma_{cy2q} := E_{cm2} \cdot \varepsilon_{cy2q}$ $\sigma_{cy2q} = -6.063\,MPa$

Total stresses

When the precast plank is concerned the sresses due to the imposed load is added to the stresses due to the permanet loads before the composite action (before the topping has hardened).
Before the composite action there are no stress in the topping, so the topping gets stresses only due to the imposed load (loads acting after the composite action starts)

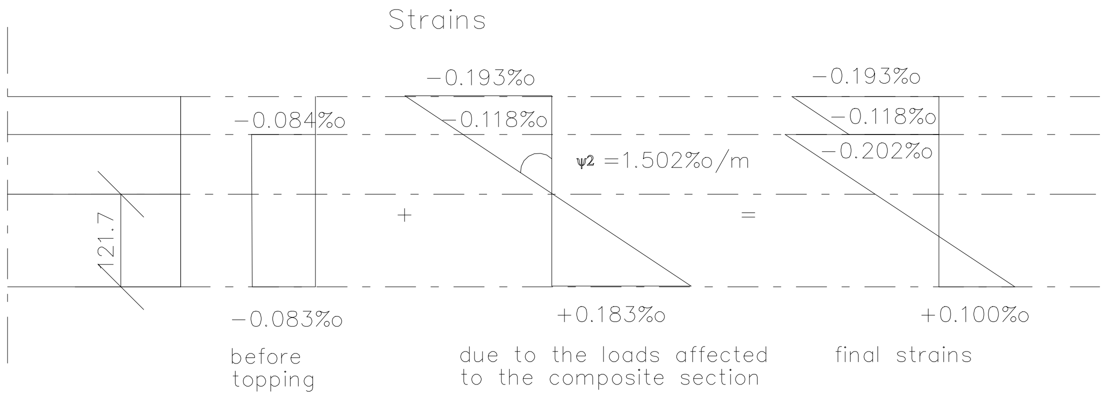
Bottom fibre of the precast plank $\sigma_{ca} := \sigma_{ca1} + \sigma_{caq}$ $\sigma_{ca} = 3.509\,\text{MPa}$ = $f_{ctm1} = 3.509\,\text{MPa}$

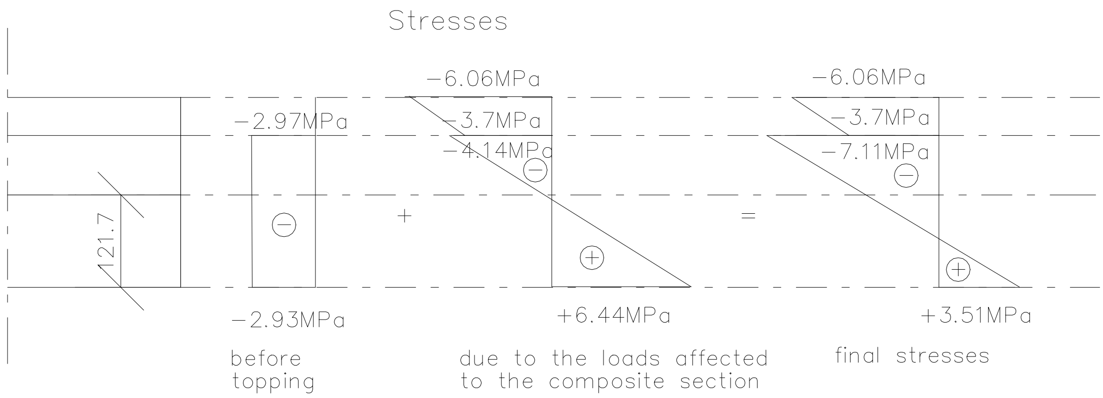
Top fibre of the precast plank $\sigma_{cj1} := \sigma_{cy1} + \sigma_{cj1q}$ $\sigma_{cj1} = -7.105\,\text{MPa}$

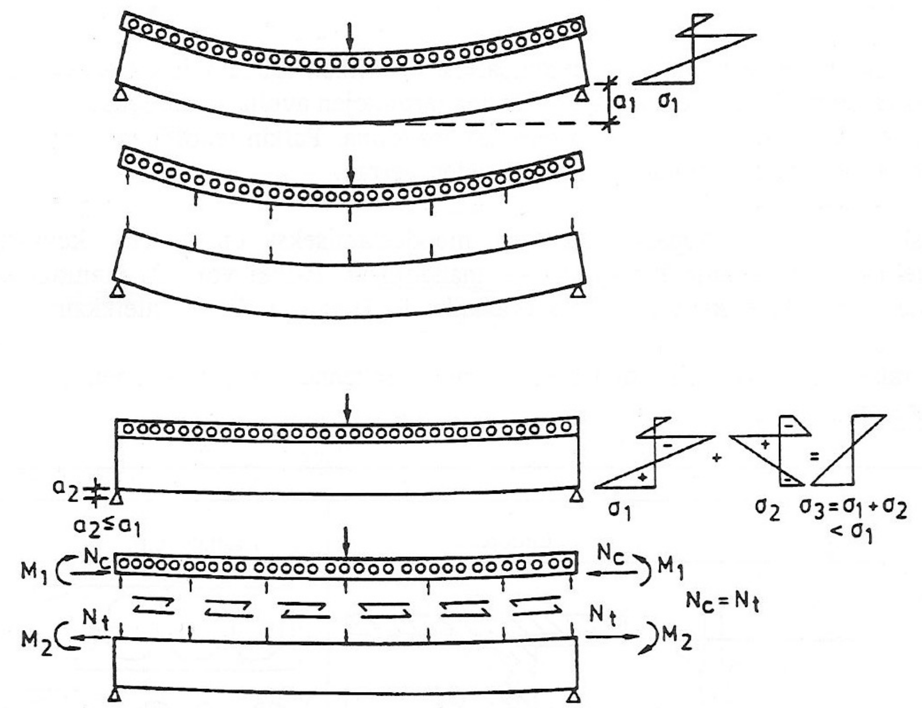
Bottom fibre of the topping $\sigma_{cj2} := \sigma_{cj2q}$ $\sigma_{cj2} = -3.699\,\text{MPa}$

Top fibre of the topping $\sigma_{cy} := \sigma_{cy2q}$ $\sigma_{cy} = -6.063\,\text{MPa}$

Total curvature $\psi := \psi_1 + \psi_{q2}$ $\psi = 1.507\,\frac{‰}{m}$







Non-compost-
e structure

Composite structure

Principle of a composite action

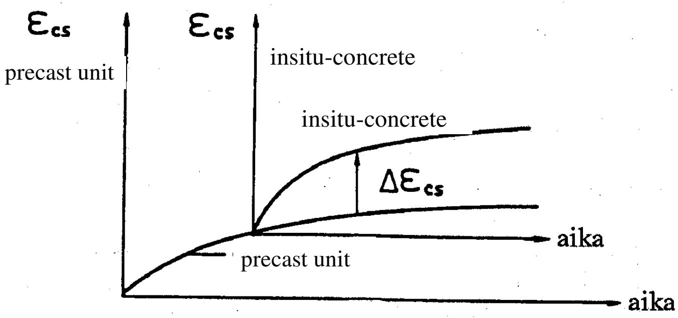
Before the composite action:
- only the precast unit function as a load-bearing structure
- the precast unit is loaded by the self-weights of the precast unit and the insitu-concrete

After the composite action:
- the in-situ concrete has been hardened
- the precast unit and the in-situ concrete function together load-bearing structure as a composite structure
- imposed live load is affected to the composite structure

4. Diffrential shrinkage and creep

Difference of shrinkage creep in the topping and in the precast plank slab

The topping has been casted at the time 28 days from the casting of the precast slab.
At this time a part of the shrinkage of the precast slab has been happened, but the shrinkage of the topping is just starting. So shrinkage which happens after casting of the topping shrinkage of the topping is greater than the shrinkage of the precast slab. This arise strain difference at the joint between the precast slab and the topping which is impossible if shear strength of the joint is adequite. So the strain difference must even out. This cause stresses.



Shrinkage and creep calues:

4. 1.<u>Notational size</u>

Precast plank slab + topping

$$h_o := \frac{A_{c1} + A_2}{b} \qquad h_o = 249.457\,\text{mm}$$

<u>4.2 Shrinkage</u>

<u>4.2.1 Precast slab</u>

Indoor environment, relative humidity $RH := 50\%$

Coefficients which depends on the type of cement $\alpha_{ds1} := 6$ $\alpha_{ds2} := 0.11$

$f_{cmo} := 10\cdot\text{MPa}$ $RH_o := 100\cdot\%$

$$\beta_{RH} := 1.55\cdot\left[1 - \left(\frac{RH}{RH_o}\right)^3\right] \qquad \beta_{RH} = 1.356$$

Drying shrinkage $\varepsilon_{cd0} := -0.85\cdot\left(220 + 110\cdot\alpha_{ds1}\right)\cdot e^{-\alpha_{ds2}\cdot\left(\frac{f_{cm1}}{f_{cmo}}\right)}\cdot\beta_{RH}\cdot 10^{-6}$

$\varepsilon_{cd0} = -0.6\,‰$

Effect of thye notational size kh, EC2. taul 3.3

$h_o = 249\,\text{mm}$ $k_h := 0.8$

Final value of drying shrinkage $\varepsilon_{sd\infty} := k_h\cdot\varepsilon_{cd0}$ $\varepsilon_{sd\infty} = -0.479\,‰$

Develoment of drying shrinkage with time

$$\beta_{ds}(t, t_s) := \frac{t - t_s}{t - t_s + 0.04\sqrt{\left(\frac{h_o}{mm}\right)^3} \cdot vrk}$$

Drying shrinkage at the time interval t1...t2

$$\varepsilon_{sdt1...t2} := \varepsilon_{sd\infty} \cdot \left(\beta_{ds}(t_2, t_s) - \beta_{ds}(t_1, t_s)\right)$$

t_s means the time from which the evaporation of moisture starts so in practic the casting time => $t_s := 0 \cdot days$

Strength at transfer $f_{cmi} := 0.7 \cdot f_{ck1} + 8 \cdot MPa$ $f_{cmi} = 36 MPa$

$$\beta_{cc} := \frac{f_{cmi}}{f_{cm1}}$$ $\beta_{cc} = 0.75$

$s := 0.2$ Cement-class R (rapid)

Theoretical temperature adjusted age of concrete $t_o := 1 \cdot vrk$

$$t_{oT} := \frac{28 \cdot vrk}{\left(1 - \frac{\ln(\beta_{cc})}{s}\right)^2}$$ $t_{oT} = 4.7 vrk$

Final value of autogenous shrinkage

$$\varepsilon_{ca\infty} := -2.5 \cdot \left(\frac{f_{ck1} - 10 \cdot MPa}{MPa}\right) \cdot 10^{-6}$$ $\varepsilon_{ca\infty} = -0.075\ ‰$

Develoment of the autogenous shrinkage with time $\beta_{as}(t) := 1 - e^{-0.2 \cdot \sqrt{\frac{t}{vrk}}}$

Autogenous shrinkage at the time interval t1...t2

$$\varepsilon_{cat1...t2} := \varepsilon_{ca\infty} \cdot \left(\beta_{as}(t_2) - \beta_{as}(t_1)\right)$$

4.2.1.2.Shrinkage at different time intervals; the precast plank slab

It is assumed that after 1 day since the casting of the topping the strength of the topping concrete is enough developed to resist free shrinkage of the topping concrete.

1. Before casting the topping (hardening) $t_1 := 29 \cdot days$ $t_{1T} := t_1 + (t_{oT} - t_o)$ $t_{1T} = 32.7\,vrk$

drying shrinkage $\beta_{ds}(t_{1T}, t_s) = 0.172$ $\varepsilon_{sd11} := \varepsilon_{sd\infty} \cdot \beta_{ds}(t_{1T}, t_s)$ $\varepsilon_{sd11} = -0.082\,‰$

autogenous shrinkage $\beta_{as}(t_{1T}) = 0.681$ $\varepsilon_{as11} := \varepsilon_{ca\infty} \cdot \beta_{as}(t_{1T})$ $\varepsilon_{as11} = -0.051\,‰$

total shrinkage $\varepsilon_{cs11} := \varepsilon_{sd11} + \varepsilon_{as11}$ $\varepsilon_{cs11} = -0.133\,‰$

2. After the hardening of the topping concrete

$t_1 = 29\,vrk$ $t_{1T} = 32.7\,vrk$ $t_\infty := 70 \cdot v$

drying shrinkage $\beta_{ds}(t_\infty, t_s) = 0.994$

$\varepsilon_{sd12} := \varepsilon_{sd\infty} \cdot (\beta_{ds}(t_\infty, t_s) - \beta_{ds}(t_{1T}, t_s))$ $\varepsilon_{sd12} = -0.393\,‰$

autogenous shrinkage $\beta_{as}(t_\infty) = 1$

$\varepsilon_{as12} := \varepsilon_{ca\infty} \cdot (\beta_{as}(t_\infty) - \beta_{as}(t_{1T}))$ $\varepsilon_{as12} = -0.024\,‰$

total shrinkage $\varepsilon_{cs12} := \varepsilon_{sd12} + \varepsilon_{as12}$ $\varepsilon_{cs12} = -0.417\,‰$

4.2.2.Topping

Indoor environment, relative humidity $RH := 50\%$

Coefficients which depends on the type of cement $\alpha_{ds1} := 6$ $\alpha_{ds2} := 0.11$

Cement type R (rapid)

$f_{cmo} := 10 \cdot MPa$ $RH_o := 100 \cdot \%$

$$\beta_{RH} := 1.55 \cdot \left[1 - \left(\frac{RH}{RH_o}\right)^3\right] \qquad \beta_{RH} = 1.356$$

Drying shrinkage $\varepsilon_{cd0} := -0.85 \cdot (220 + 110 \cdot \alpha_{ds1}) \cdot e^{-\alpha_{ds2} \cdot \left(\frac{f_{cm2}}{f_{cmo}}\right)} \cdot \beta_{RH} \cdot 10^{-6}$

$\varepsilon_{cd0} = -0.71\,‰$

Effect of the notational size kh, EC2. taul 3.3

$h_o = 249\,mm$ $k_h := 0.8$

Final value of the drying shrinkage $\varepsilon_{sd\infty} := k_h \cdot \varepsilon_{cd0}$ $\varepsilon_{sd\infty} = -0.565\,‰$

Development of the drying shrinkage with time

$$\beta_{ds}(t, t_s) := \frac{t - t_s}{t - t_s + 0.04\sqrt{\left(\frac{h_o}{mm}\right)^3} \cdot vrk}$$

Drying shrinkage at the time interval t1...t2

$$\varepsilon_{sdt1...t2} := \varepsilon_{sd\infty} \cdot \left(\beta_{ds}(t_2, t_s) - \beta_{ds}(t_1, t_s)\right)$$

t_s means the time from which the evaporation of moisture starts so in practic the casting time of the topping => $t_s := 0 \cdot days$

Final value of autogenous shrinkage

$$\varepsilon_{ca\infty} := -2.5 \cdot \left(\frac{f_{ck2} - 10 \cdot MPa}{MPa}\right) \cdot 10^{-6} \qquad \varepsilon_{ca\infty} = -0.038\,‰$$

Development of the autogenous shrinkage with time $\beta_{as}(t) := 1 - e^{-0.2 \cdot \sqrt{\frac{t}{vrk}}}$

Autogenous shrinkage at the time interval1...t2

$$\varepsilon_{cat1...t2} := \varepsilon_{ca\infty} \cdot \left(\beta_{as}(t_2) - \beta_{as}(t_1)\right)$$

Final stage $t_{\infty} = 70\,v$

drying shrinkage $\beta_{ds}(t_{\infty}, t_s) = 0.994$ $\varepsilon_{sd2} := \varepsilon_{sd\infty} \cdot \beta_{ds}(t_{\infty}, t_s)$ $\varepsilon_{sd2} = -0.561$ %o

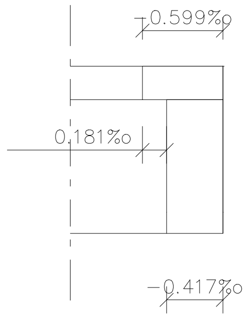
autogenous shrinkage $\beta_{as}(t_{\infty}) = 1$ $\varepsilon_{as2} := \varepsilon_{ca\infty} \cdot \beta_{as}(t_{\infty})$ $\varepsilon_{as2} = -0.037$ %o

total shrinkage $\varepsilon_{cs2} := \varepsilon_{sd2} + \varepsilon_{as2}$ $\varepsilon_{cs2} = -0.599$ %o

4.2.4. Differential shrinkage

Difference between the shrinkages of the precast slab and the topping

$\Delta\varepsilon_{cs} := \varepsilon_{cs2} - \varepsilon_{cs12}$ $\Delta\varepsilon_{cs} = -0.181$ %o



4.3. Creep

Creep of the strains due to the loads which loading times are different develop according to the individual creep function (in accordance with the loading time).
Creep is smaller if the concrete is older at the loading time. This is the reason why creep due to each load which loading time is different must be calculate separately and the creep strain are added together.

4.3.1 Preacast plank slab

Coefficients to consider the influence of the concrete strength on creep, when fcm>35 MPa

$$\alpha_1 := \min\left[\left(\frac{35\cdot MPa}{f_{cm1}}\right)^{0.7}, 1\right] \qquad \alpha_1 = 0.802$$

$$\alpha_2 := \min\left[\left(\frac{35\cdot MPa}{f_{cm1}}\right)^{0.2}, 1\right] \qquad \alpha_2 = 0.939$$

$$\alpha_3 := \min\left[\left(\frac{35\cdot MPa}{f_{cm1}}\right)^{0.5}, 1\right] \qquad \alpha_3 = 0.854$$

Effect of relative humidity on the notational creep

$$\phi_{RH} := \left(1 + \frac{1 - \frac{RH}{100\cdot\%}}{0.1\cdot\sqrt[3]{\frac{h_o}{mm}}}\cdot\alpha_1\right)\cdot\alpha_2 \qquad \phi_{RH} = 1.537$$

Effect of the concrete strength on the notational creep $\beta_{fcm} := \frac{16.8}{\sqrt{\frac{f_{cm1}}{MPa}}}$ $\beta_{fcm} = 2.425$

Effect of the cement type

Cement type R $\alpha := 1$

$$t_o := t_{oT}\cdot\left[\frac{9}{2 + \left(\frac{t_{oT}}{vrk}\right)^{1.2}} + 1\right]^{\alpha}$$

The effect of this equation is included to the calculated time according to βcc

$t_{oT} = 4.709\,vrk$

First the effect of the temperature during hardening of concrete is calculated from the equation SFS-EN 1992-1-1 (B.10); time t_T.
The the effect of the cement type is calclulated from the equation (B.9); $t_o=T_{oT}$ is obtained
The the concrete strength is calculated from the equations (3.1) and (3.2); the result should be $\beta_{cc}=0.75$

Effect of the concrete age (loading time) on the notational creep

$$\beta(t_{oT}) := \frac{1}{0.1 + \left(\frac{t_{oT}}{days}\right)^{0.2}}$$

Notational creep factor $\phi_o(t_{oT}) := \phi_{RH} \cdot \beta_{fcm} \cdot \beta(t_{oT})$ $\phi_{RH} \cdot \beta_{fcm} = 3.726$

Factor of development speed of creep depending on the relative humidity and the notational size

$$\beta_H := \min\left[1.5 \cdot \left[1 + \left(0.012 \cdot \frac{RH}{\%}\right)^{18}\right] \cdot \frac{h_o}{mm} + 250 \cdot \alpha_3, 1500 \cdot \alpha_3\right] \cdot vrk \qquad \beta_H = 587.7\, vrk$$

Development of creep with time (duration of the load t-to)

$$\beta_c(t, t_{oT}) := \frac{t - t_{oT}}{\beta_H + t - t_{oT}}$$

Creep factor at the time t for the loading which loading time is to (the duration of the load is t - to)

$$\phi(t, t_{oT}) := \phi_o \cdot \beta_c(t, t_{oT})$$

$$\phi(t, t_{oT}) := \phi_{RH} \cdot \beta_{fcm} \cdot \beta(t_{oT}) \cdot \beta_c(t, t_{oT})$$

For each load which loding time is different and the stress due to this load is calculated the own creep factor and the own creep strain εcc

4.3.2 Creep factor for loads with different loading times

4.3.2.1 Load affecting to the precast plank slab :

1. Prestress force and self weight of the precast plank slab; loading time is the transfer time $t_o = 1\,\text{days}$

Theoretical temperature adjusted age of to $t_{oT} = 4.7\,\text{days}$

Coeficient for the loading time $\beta_{transfer} := \beta(t_{oT})$ $\beta_{transfer} = 0.683$

Creep factor for the loading time to $\phi_{o.transfer} := \phi_o(t_{oT})$ $\phi_{o.transfer} = 2.546$

Creep factor before the casting (hardening) of the topping:

Time of the casting the topping $t_1 = 29\,\text{days}$

Theoretical temperature adjusted age of precast slab concrete at the time of the casting of the topping $t_{1T} = 32.7\,\text{days}$

Development coefficient for time interval toT...t1T $\beta_c(t_{1T}, t_{oT}) = 0.045$

From the final creep has been been happened before the casting of the topping

$\phi_{transfer.1} := \phi_{o.transfer} \cdot \beta_c(t_{1T}, t_{oT})$ $\phi_{transfer.1} = 0.116$

Creep factor after the casting of the topping for the loads which loading time is to

$\phi_{transfer.2} := \phi_{o.transfer} \cdot (\beta_c(t_\infty, t_{oT}) - \beta_c(t_{1T}, t_{oT}))$ $\phi_{transfer.2} = 2.373$

2. Prestress losses before the casting of the topping

Treatment of the losses is the same as for the pre-tensioned member. For the simplicity the losses is not calculated here.

Prestress losses and the changes of the sresses due to the loasses develop gradually. For this loading case (prestress losses before the casting of the topping) is assumed that the loading time to=1 days is valid for the whole value and the gradually development is taken into account by multiplying the creep factor with the ageing coefficient $\chi := 0.8$

Creep factor before the casting of the topping for this loading case is $\phi_{loss.1} := \chi \cdot \phi_{transfer.1}$

Creep factor after the casting the topping $\phi_{loss.2} := \chi \cdot \phi_{transfer.2}$

3. Self-weight of the topping; loading time $t_{o1} := t_1$ $t_{o1} = 29\,\text{days}$

Theoretical temperature adjusted age $t_{1T} = 32.7\,\text{days}$

Coeficient for the loading time $\beta_{topp} := \beta(t_{1T})$ $\beta_{topp} = 0.474$

$\phi_{o.topp} := \phi_o(t_{1T})$ $\phi_{o.topp} = 1.767$

$\beta_c(t_\infty, t_{1T}) = 0.977$ $\phi_{top} := \phi_{o.topp} \cdot \beta_c(t_\infty, t_{1T})$ $\phi_{top} = 1.727$

4.3.2.2 Loads affecting to the composite structure / Creep factors for the precast slab

4. Differential shrinkage and creep (sh.cr)

Stresses due to differential shrinkage and creep develop gradually.

The loading time of this load case is supposed $t_1 = 29\,\text{days}$

Theoretical temperature adjusted age $t_{1T} = 32.709\,\text{days}$

Coefficient for the loading time $\beta_{sh.cr} := \beta(t_{1T})$ $\beta_{sh.cr} = 0.474$

$\phi_{o.sh.cr} := \phi_o(t_{1T})$ $\phi_{o.sh.cr} = 1.767$

$\beta_c(t_\infty, t_{1T}) = 0.977$

$\phi_{sh.cr} := \phi_{o.sh.cr} \cdot \beta_c(t_\infty, t_{1T})$ $\phi_{sh.cr} = 1.727$

Gradually development of the stresses due to differential shrinkage and creep is taken into account by the ageing coefficient χ.

5. Prestress losses after he casting of the topping

Stresses due to prestress losses develop gradually. The starting time (loading time) of these stresses are supposed time t1.

Prestress losses are calculated in a same way as for ordinary pre-tensioned member but using the cross-section values of the composite structure (the precast plank slab + topping). The calculating of the losses is not presented in this example.

The effect of the prestress losses can be calculated together with differential shrinkage and creep so that the restraint forces are calculated not only from the differential shrinkage and creep between the the precast unit and the topping but the shrinkage and creep happened after casting the topping in the both part.

The effect of shrinkage and creep on the stresses of a memebre is treated in two parts:
- In both part (whole cross-section) is assumed the same shrinkage and creep strain and curvature => prestress loss after casting the topping.

- Because shrinkage and creep in the topping is different than the above assumed values, the calculation is corrected by calculating the stresses to the concrete and the prestressing strands due to this difference of shrinkage and creep (load case number 4).

The same results is got if the restraint forces due to shrinkage and creep of each part are calculated and then the stresses to the concrete and the prestressing strands due to the sum of these restraint forces The change of the stress in the prestressing strands includes both the prestress losses due to shrinkage and creep and also the effect of the diffrential shrinkage and creep between the parts of the composite cross-section.

Creep factor for the losses after the casting of the topping is the same as the one for the diffrential shrinkage and creep.

$\phi_{loss.2} := \phi_{sh.cr}$ $\phi_{loss.2} = 1.727$

6. Long-term (quasi-permanent) imposed live load

The loading time of the quasi-permanent live load is supposed 1 month from the casting of the topping, when the casting of the precast slab has went $t_2 := 60 \cdot days$

Temperature adjusted age of precast slab $t_{2T} := t_2 + (t_{oT} - t_o)$ $t_{2T} = 63.7\, days$

Coefficient for the loading time $\beta_{long} := \beta(t_{2T})$ $\beta_{long} = 0.417$

$\phi_{o.long} := \phi_o(t_{2T})$ $\phi_{o.long} = 1.555$

$\beta_c(t_\infty, t_{2T}) = 0.977$

$\phi_{long} := \phi_{o.long} \cdot \beta_c(t_\infty, t_{2T})$ $\phi_{long} = 1.52$

4.3.2.3 Loads affecting to the composite structure /Creep factors for the topping

Effect of the concrete strength of the topping on creep, when fcm>35 MPa

$$\alpha_1 := \min\left[\left(\frac{35\cdot MPa}{f_{cm2}}\right)^{0.7}, 1\right] \qquad \alpha_1 = 1$$

$$\alpha_2 := \min\left[\left(\frac{35\cdot MPa}{f_{cm2}}\right)^{0.2}, 1\right] \qquad \alpha_2 = 1$$

$$\alpha_3 := \min\left[\left(\frac{35\cdot MPa}{f_{cm2}}\right)^{0.5}, 1\right] \qquad \alpha_3 = 1$$

Nominal creep factor depending on relative humidity

$$\phi_{RH} := \left(1 + \frac{1 - \frac{RH}{100\cdot\%}}{0.1\cdot\sqrt[3]{\frac{h_o}{mm}}}\cdot\alpha_1\right)\cdot\alpha_2 \qquad \phi_{RH} = 1.794$$

Effect of concrete strength on the nominal creep factor $\beta_{fcm} := \frac{16.8}{\sqrt{\frac{f_{cm2}}{MPa}}}$ $\beta_{fcm} = 2.925$

$$\beta(t_{oT}) := \frac{1}{0.1 + \left(\frac{t_{oT}}{days}\right)^{0.2}}$$

Nominal creep factor $\phi_o(t_{oT}) := \phi_{RH}\cdot\beta_{fcm}\cdot\beta(t_{oT})$

Coefficient for creep speed depending on the relative humidity and the nominal size

$$\beta_H := \min\left[1.5\cdot\left[1 + \left(0.012\cdot\frac{RH}{\%}\right)^{18}\right]\cdot\frac{h_o}{mm} + 250\cdot\alpha_3, 1500\cdot\alpha_3\right]\cdot days \qquad \beta_H = 624.2\,days$$

Develoment of creep with time (duration of theloading is t - to)

$$\beta_c(t, t_{oT}) := \frac{t - t_{oT}}{\beta_H + t - t_{oT}}$$

Effect of the loading time on the creep factor

Creep factor at time t for the load which loading time is to (duration of the loading is t - to)

$$\phi(t, t_{oT}) := \phi_o\cdot\beta_c(t, t_{oT})$$

$$\phi(t, t_{oT}) := \phi_{RH}\cdot\beta_{fcm}\cdot\beta(t_{oT})\cdot\beta_c(t, t_{oT})$$

4. Differential shrinkage and creep

Stresses due to the diffrenetial shrinkage and creep develop gradually.
The oading time oif this load case is assumed $t_1 = 29\,days$ from the casting of the precast slab

Age of the topping $t_{12} := t_1 - 28 \cdot days$ $t_{12} = 1\,days$

Coefficient for the loading time $\beta_{sh.cr2} := \beta(t_{12})$ $\beta_{sh.cr2} = 0.909$

$\phi_{o.sh.cr2} := \phi_o(t_{12})$ $\phi_{o.sh.cr2} = 4.77$

$\beta_c(t_\infty, t_{12}) = 0.976$

$\phi_{sh.cr2} := \phi_{o.sh.cr2} \cdot \beta_c(t_\infty, t_{12})$ $\phi_{sh.cr2} = 4.657$

The gradual development of the stresses due to the differential shrinkage and creep are taken into account by the ageing coefficient $\chi 2=0.6$

5. Prestress losses after the casting of the topping

The losses after the casting the topping induce also stresses into the topping and creep affect into the strains due to these stresses.

$\phi_{loss.22} := \phi_{sh.cr2}$ $\phi_{loss.22} = 4.657$

6. Long-term (quasi-permanent) imposed live load

The loading time of the quasi-permanent live load is supposed 1 month from the casting of the topping, when the age of the precast slab is then $t_2 := 60 \cdot days$

Age of the topping at the loading time of the long-term imposed live load $t_{22} := t_2 - 28 \cdot days$ $t_{22} = 32\,days$

Coefficient for the loading time $\beta_{long2} := \beta(t_{22})$ $\beta_{long2} = 0.476$

$\phi_{o.long2} := \phi_o(t_{22})$ $\phi_{o.long2} = 2.499$

$\beta_c(t_\infty, t_{22}) = 0.976$

$\phi_{long2} := \phi_{o.long2} \cdot \beta_c(t_\infty, t_{22})$ $\phi_{long2} = 2.439$

The short-time part of the imposed live load does not cause creep

4.3.3 Influence of creep on the stresses due to restraint forces of gradually developing differential shrinkage and creep

Differential shrinkage and creep develop graudually in time between 29 days... ∞.
Creep is affected on the stresses due to some developed differential shrinkage and creep just after these are arised. In the calculation it is assumed that the differential shrinkage and creep affect as the whole magnitude just after 29 days.

The elastic strains due to differential shrinkage and creep equal the strain difference in the both sides of the construction joint. These strains increase by creep and so the smaller elastic strains are required to eliminate the strain difference because a part of the elastic strains are substituted by creep strains.
This is taken into account by reducing the elastic modulus of concrete by creep factor and the gradual development of differential shrinkage and creep is taken into account by reducing the creep factor with the ageing coefficient of concrete χ = 0.6...0.8.

Alternatively for the calculating the restraint forces due to differential shrinkage and creep the elastic modulus can be reduced by the factor $k_{\varphi} := \frac{1 - e^{-\phi}}{\phi}$.
Both methods give about the same result when the creep factor is ϕ = 2.

The creep factors are reduced by the ageing coefficeient $\chi_1 := 0.8$ (precast slab)

$\chi_2 := 0.6$ (topping)

Defining of the ageing coefficient see Ghali & Favre: Concrete Structures: Stresses and deformations

Precast slab: $\chi_1 \cdot \phi_{sh.cr} = 1.382$

Topping: $\chi_2 \cdot \phi_{sh.cr2} = 2.794$

4.4. Cross-section values for calculating the restraint forces taking into account the influence of creep

Cross-section values of the composite structure used in the calculation of the influence of the differential shrinkage and creep:

Effective elastic modulus of concrete $E_{c¤} := \frac{E_{cm}}{1 + \chi \cdot \phi}$

4.4.1 Precast plank slab:

Creep factor for the differential shrinkage and creep $\chi_1 \cdot \phi_{sh.cr} = 1.382$

Effective elastic modulus of the precast concrete $E_{c1¤} := \frac{E_{cm1}}{1 + \chi_1 \cdot \phi_{sh.cr}}$ $E_{c1¤} = 14788.3\,MPa$

Concrete

$E_{c1¤} \cdot A_{c1} = 3.54 \times 10^3\,MN$

$E_{c1¤} \cdot S_{c1} = 353.619\,MNm$

$E_{c1¤} \cdot I_{c1} = 11.871\,MN \cdot m^2$

Strands:

$E_p \cdot A_p = 126.945\,MN$

$E_p \cdot S_p = 4.443\,MNm$

$E_p \cdot I_p = 0\,MN \cdot m^2$

Precast slab (concrete + strands)

Axial stiffness $EA_{1¤} := E_{c1¤} \cdot A_{c1} + E_p \cdot A_p$ $EA_{1¤} = 3.667 \times 10^3\,MN$

First moment of area about the bottom fibre $ES_{1¤} := E_{c1¤} \cdot S_{c1} + E_p \cdot S_p$ $ES_{1¤} = 358.062\,MNm$

centroid from the bottom fibre of the precast slab $p_{1¤} := \frac{ES_{1¤}}{EA_{1¤}}$ $p_{1¤} = 97.658\,mm$

Flexural stiffness

$EI_{1¤} := E_{c1¤} \cdot I_{c1} + E_{c1¤} \cdot A_{c1} \cdot \left(p_1 - p_{c1}\right)^2 + E_p \cdot A_p \cdot \left(p_{1¤} - c_p\right)^2$ $EI_{1¤} = 12.373\,MN \cdot m^2$

4.4.2 Composite structure

Toppin-
g

Effective elastic modulus if the topping coincrete

$$E_{c2¤} := \frac{E_{cm2}}{1 + \chi_2 \cdot \phi_{sh.cr2}}$$ $E_{c2¤} = 8296.3\,MPa$

Axial stiffness of the topping

$EA_{2¤} := E_{c2¤} \cdot A_2$ $EA_{2¤} = 497.781\,MN$

First moment of area about the bottom fibre of the precast slab

$ES_{2¤} := E_{c2¤} \cdot S_2$ $ES_{2¤} = 112.001\,MNm$

Flexural stiffness

$EI_{2¤} := E_{c2¤} \cdot I_2$ $EI_{2¤} = 0.104\,MN \cdot m^2$

Composite cross-sectoion

Axial stiffness

$EA_{¤} := EA_{1¤} + EA_{2¤}$ $EA_{¤} = 4.164 \times 10^3\,MN$

First moment of are about the boittom fibre of the precast slab

$ES_{¤} := ES_{1¤} + ES_{2¤}$ $ES_{¤} = 470.063\,MNm$

Centroid from the bottom fibre of the precast slab

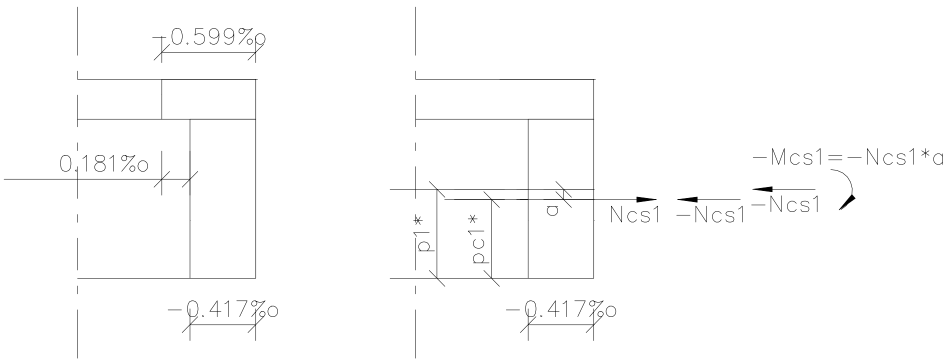
$$p_{¤} := \frac{ES_{¤}}{EA_{¤}}$$ $p_{¤} = 112.88\,mm$

Flexural stiffness

$$EI_{¤} := EI_{1¤} + EA_{1¤} \cdot \left(p_{¤} - p_{1¤}\right)^2 + EI_{2¤} + EA_{2¤} \cdot \left(p_{¤} - p_2\right)^2$$ $EI_{¤} = 19.584\,MN \cdot m^2$

4.5 Shrinkage

In the calculation of the inluence of shrinkage is included both the diffrential shrinkage between the precast slab and the topping also the shrinkage of the precast slab. Then the calcultaion gives also the prestress loss due to shrinkage. Later the calculation (section 4.6) is done only for the differential part of shrinkage.



Shrinkage of the precast slab and the topping   Restraint forces due to the restraint shrinkage of the precast slab

4.5.1 Precast slab $\varepsilon_{cs12} = -0.417\,‰$

The shrinkage strain is assumed to restrained => the tensile restraint force which prevent the free shrinkage strain

$N_{cs1} := -\varepsilon_{cs12} \cdot E_{c1¤} \cdot A_{c1}$ $N_{cs1} = 1.477\,\mathrm{MN}$

at the centroid of the shrinking concrete $p_{c1} = 99.905\,\mathrm{mm}$ from the bottom of the precast slab

(Ac1 is the area of the pure concrete without the strands)

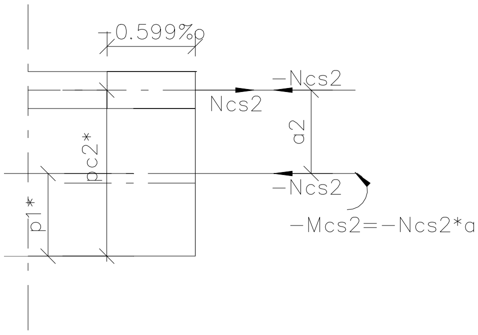
The compressive counterforce which reverse the tensile restraint force at the centroid of the pure concrete $-N_{cs1} = -1.477 \times 10^{3}\,\mathrm{kN}$

The compressive counterforce nis moved to the centroid of the composite section and the movement is substitued with the moment

$M_{cs1} := -N_{cs1} \cdot (p_{¤} - p_{c1}) \quad = \quad N_{cs1} \cdot a_1$ $M_{cs1} = -19.167\,\mathrm{kNm}$

The tensile force Ncs1 affect to the precast slab and it reverse free shrinkage ε_{cs1}
The compressive counterforce -Ncs1 affect to the whole composite section even if it's location is the centroid of the precast slab concrete. -Ncs1 arises to the composite section the centric compressive force -Ncs1 and the moment Mcs1.

4.5.2 Topping $\varepsilon_{cs2} = -0.599\,\%o$



Shrinkage and the restraint forces of the topping

The restraint force which prevent the free shrinkage strain

$N_{cs2} := -\varepsilon_{cs2} \cdot E_{c2¤} \cdot A_2$ $N_{cs2} = 0.298\,MN$

The restraint force affect to the centroid of the topping $p_2 = 225\,mm$ from the bottom of the precast slab

For maintaining the equilibrium to the composite section is loaded by the compressive counterforce $-N_{cs2} = -0.298\,MN$ a affecting to the centroid of the topping

The compressive counterforce -Ncs2 is moved to the centroid of the composite section and the movement is substituted with the moment

$M_{cs2} := -N_{cs2} \cdot (p_{¤} - p_2) = N_{cs2} \cdot a_2$ $M_{cs2} = 33.407\,kNm$

4.5.3 The compsoite section is loaded by the forces -Ncs ja Mcs due to shrinkage

$N_{cs} := N_{cs1} + N_{cs2}$ $-N_{cs} = -1775.2\,kN$

$M_{cs} := M_{cs1} + M_{cs2}$ $M_{cs} = 14.24\,kNm$

4.5.4 Strains and stresses due to the forces -Ncs ja Mcs

Axial strain at the centroid of the composite section $\Delta\varepsilon_{0cs} := \frac{-N_{cs}}{EA_{¤}}$ $\Delta\varepsilon_{0cs} = -0.426\,\%o$

Curvature $\Delta\psi_{cs} := \frac{M_{cs}}{EI_{¤}}$ $\Delta\psi_{cs} = 0.727\,\frac{\%o}{m}$

When strains and streses are calculated at different points of the cross-section, the tensile restraint forces +Ncsi and the correspondent strains εcsi are taken into acoount

Strains due to +Ncs $\varepsilon i := \frac{N_{csi}}{EA_{i¤}}$ $\varepsilon_i := -\varepsilon_{csi}$

4.5.5 Strains and stresses due to shrinkage

$\Delta\varepsilon := \Delta\varepsilon_{0cs} + \Delta\psi_{cs} \cdot y$ — y is calculated from the centroid of the composite section; positive is downward

Precast slab $\varepsilon_{cs12} = -0.417\,‰$

bottom fibre $y := p_{¤}$ $y = 112.88\,mm$

strain $\Delta\varepsilon_{cb} := \Delta\varepsilon_{0cs} + \Delta\psi_{cs} \cdot p_{¤} - \varepsilon_{cs12}$ $\Delta\varepsilon_{cb} = 0.073\,‰$

stress $\Delta\sigma_{cb.sh} := \Delta\varepsilon_{cb} \cdot E_{c1¤}$ $\Delta\sigma_{cb.sh} = 1.082\,MPa$

construction joint; top fibre of the precast slab $y := p_{¤} - h_1$ $y = -87.12\,mm$

strain $\Delta\varepsilon_{cj1} := \Delta\varepsilon_{0cs} + \Delta\psi_{cs} \cdot (p_{¤} - h_1) - \varepsilon_{cs12}$ $\Delta\varepsilon_{cj1} = -0.072\,‰$

stress $\Delta\sigma_{cj1.sh} := \Delta\varepsilon_{cj1} \cdot E_{c1¤}$ $\Delta\sigma_{cj1.sh} = -1.069\,MPa$

Toppin-g $\varepsilon_{cs2} = -0.599\,‰$

construction joint; bottom fibre of the topping $y = -87.12\,mm$

strain $\Delta\varepsilon_{cj2} := \Delta\varepsilon_{0cs} + \Delta\psi_{cs} \cdot (p_{¤} - h_1) - \varepsilon_{cs2}$ $\Delta\varepsilon_{cj2} = 0.109\,‰$

stress $\Delta\sigma_{cj2.sh} := \Delta\varepsilon_{cj2} \cdot E_{c2¤}$ $\Delta\sigma_{cj2.sh} = 0.904\,MPa$

Remark! Δεcj1 ja Δεcj2 are differs from each other only by the shrinkage

$\Delta\varepsilon_{cj2} - \Delta\varepsilon_{cj1} = 0.181\,‰$ $\varepsilon_{cs2} - \varepsilon_{cs12} = -0.181\,‰$ differential shrinkage between the parts

Top fibre of the topping $y := p_{¤} - (h_1 + h_2)$ $y = -137.12\,mm$

strain $\Delta\varepsilon_{cy} := \Delta\varepsilon_{0cs} + \Delta\psi_{cs} \cdot (p_{¤} - h_1 - h_2) - \varepsilon_{cs2}$ $\Delta\varepsilon_{cy} = 0.073\,‰$

stress $\Delta\sigma_{cy.sh} := \Delta\varepsilon_{cy} \cdot E_{c2¤}$ $\Delta\sigma_{cy.sh} = 0.602\,MPa$

Concrete stress at the location of the strands $y := p_{¤} - c_p$ $y = 77.88\,mm$

strain $\Delta\varepsilon_{cp} := \Delta\varepsilon_{0cs} + \Delta\psi_{cs} \cdot (p_{¤} - c_p) - \varepsilon_{cs12}$ $\Delta\varepsilon_{cp} = 0.048\,‰$

stress $\Delta\sigma_{cp.sh} := E_{c1¤} \cdot \Delta\varepsilon_{cp}$ $\Delta\sigma_{cp.sh} = 0.705\,MPa$

4.5.6 Prestress loss due to shrinkage

Prestress loss due to shrinkage $\varepsilon_{cs12} = -0.417\,‰$

and the differential shrinkage between the precast slab and the topping $\Delta\varepsilon_{cs} = -0.181\,‰$

Reinforcement does not shrink, so for ythe strands εcs=0

$y = 77.88\,mm$ location of the strands from the centroid of the composite section

Strain in the strands due to shrinkage (shrinkage of the precast slab and the differential shrinkage

$\Delta\varepsilon_p := \Delta\varepsilon_{0cs} + \Delta\psi_{cs}\cdot(p¤ - c_p)$ $\Delta\varepsilon_p = -0.37\,‰$

$\Delta\sigma_p := E_p\cdot\Delta\varepsilon_p$ $\Delta\sigma_p = -72.083\,MPa$

The prestress loss includes the effect of the shrinkage of the topping and the diffrerential shrinkage and the elastic lengthening of the prestess loss.

The above calculated strains are elastic strains which arise stresses. The total strain of concrete is obtained by adding to the elastic strains the free shrinkage strain εcsi.

4.5.7 Total strains

Precast unit

Bottom fibre	$\varepsilon_{ca.tot} := \Delta\varepsilon_{cb} + \varepsilon_{cs12}$	$\varepsilon_{ca.tot} = -0.344\,‰$	<	$\varepsilon_{cs12} = -0.417\,‰$
Construction joint; top fibre of the precast slab	$\varepsilon_{cj1.tot} := \Delta\varepsilon_{cj1} + \varepsilon_{cs12}$	$\varepsilon_{cj1.tot} = -0.49\,‰$	>	$\varepsilon_{cs12} = -0.417\,‰$
Construction joint; bottom fibre of the topping	$\varepsilon_{cj2.tot} := \Delta\varepsilon_{cj2} + \varepsilon_{cs2}$	$\varepsilon_{cj2.tot} = -0.49\,‰$	<	$\varepsilon_{cs2} = -0.599\,‰$
Top fibre of the topping	$\varepsilon_{cy.tot} := \Delta\varepsilon_{cy} + \varepsilon_{cs2}$	$\varepsilon_{cy.tot} = -0.526\,‰$	<	$\varepsilon_{cs2} = -0.599\,‰$

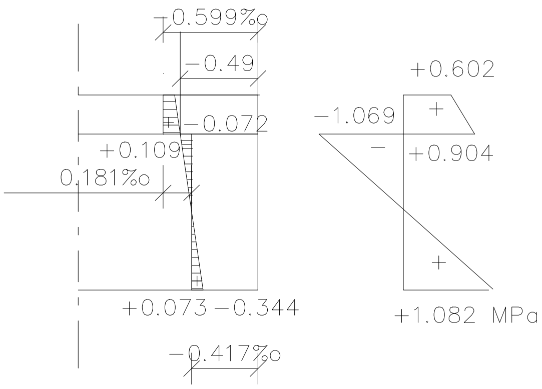
The part of the strain which differs from the free shrinkage strain (εcs12 in the precast slab, εcs2 in the toppping) is restraint shrinkage which induce stresses.

E.g. The strain $\Delta\varepsilon_{cb} = 0.073\,‰$ in the bottom of the precast slab is caused by the differential shrinkage between the precast slab and the topping and the fact that the strands resist free shrinkage of the precast slab concrete.

$\Delta\varepsilon_{cj2} = 0.109\,‰$ in the topping is caused by the fact that the less shrinking precast slab resist the shrinkage of the topping.

The total strains in the both sides of the construction joint are the same $\varepsilon_{cj1.tot} = -0.49\,‰$

$\varepsilon_{cj2.tot} = -0.49\,‰$



Strains and sresses due to shrinkage

4.5.8 Deflection due shrinkage

Curvature due to shrinkage $\Delta\psi_{cs} = 0.727\frac{\%o}{m}$

Shrinkage is equal at the whole span => the curvature (moment Mcs) is equal at the whole span =>

Deflection coefficient $\delta_a := \frac{1}{8}$

Deflection $a_{cs} := \delta_a \cdot \Delta\psi_{cs} \cdot L^2$ $a_{cs} = 4.454\,mm$ downward

4.6 Only the differential shrinkage

4.6.1 Restraint force due to the differential shrinkage

Above have been calculated stresses due to shrinkage. These stresses are included the effect of the differential shrinkage and also the shrinkage of the precast slab. When the shrinkage of the precast slab where the strands are located have been taken into account the prestress loss due to shrinkage of the precast slab is alos obtained.

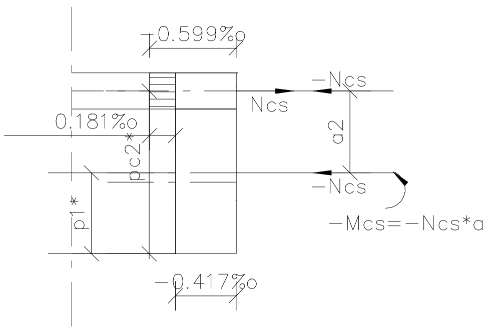
In the next calculation the shrinkage of the precast slab is excluded and the mere differential part of shrinkage is taken into account. The prestress loss due to the shrinkage of the precast slab must be calculated separately.

Shrinkage values

Precast slab $\varepsilon_{cs12} = -0.417\,\%o$

Topping $\varepsilon_{cs2} = -0.599\,\%o$

Differential shrinkage between the precast slab and the topping $\Delta\varepsilon_{cs} := \varepsilon_{cs2} - \varepsilon_{cs12}$ $\Delta\varepsilon_{cs} = -0.181\,\%o$



Restraint forces due to the diffrential shrinkage

The strain difference due to the diffrential shrinkage is reversed by the tensile restraint force affecting to the centroid of the topping

$\Delta N_{cs} := -\Delta\varepsilon_{cs} \cdot E_{c2¤} \cdot A_2$ $\Delta N_{cs} = 90.206\,kN$

For maintaining the equilibrium *the composite section is loaded* by the equal compressive counterforce -ΔNcs is acting to the centroid of the topping.

Because -ΔNcs affects to the composite section, it can be moved to the centroid of the composite section and the movement can be substituted with the moment

$\Delta M_{cs} := -\Delta N_{cs} \cdot (p_{¤} - p_2)$ $\Delta M_{cs} = 10.114\,kNm$

$p_{¤} - p_2 = -112.12\,mm$ is the distance of the centroidal axis of the topping from the centroidal axis of the composite section

Forces acting to the composite section

$-\Delta N_{cs} = -90.206\,kN$

$\Delta M_{cs} = 10.114\,kNm$

In addition the centric restraint force $\Delta N_{cs} = 90.206\,kN$ due to the restraint diffrerential shrinkage is acting to the topping

4.6.2 Strains in the composite section due to the differential shrinkage

Axial strain at the centroid of the composite section $\Delta\varepsilon_{0cs.diff} := \dfrac{-\Delta N_{cs}}{EA_{¤}}$ $\Delta\varepsilon_{0cs.diff} = -0.022\,‰$

Curvature $\Delta\psi_{cs.diff} := \dfrac{\Delta M_{cs}}{EI_{¤}}$ $\Delta\psi_{cs.diff} = 0.516\,\dfrac{‰}{m}$

4.6.3 Sresses of the composite section due to the differential shrinkage

Strains in the different points of the section $\Delta\varepsilon := \Delta\varepsilon_{0cs.diff} + \Delta\psi_{cs.diff} \cdot y$

Precast slab

Bottom fibre of the precast slab $y := p_{¤}$ $y = 112.88\,mm$

$\Delta\varepsilon_{cb} := \Delta\varepsilon_{0cs.diff} + \Delta\psi_{cs.diff} \cdot p_{¤}$ $\Delta\varepsilon_{cb} = 0.037\,‰$

$\Delta\sigma_{cb.diff} := \Delta\varepsilon_{cb} \cdot E_{c1¤}$ $\Delta\sigma_{cb.diff} = 0.542\,MPa$ tension in the bottom fibre of the precast slab

Construction joint; top fibre of the precast slab $y := p_{¤} - h_1$ $y = -87.12\,mm$

$\Delta\varepsilon_{cj1} := \Delta\varepsilon_{0cs.diff} + \Delta\psi_{cs.diff} \cdot (p_{¤} - h_1)$ $\Delta\varepsilon_{cj1} = -0.067\,‰$

$\Delta\sigma_{cj1.diff} := \Delta\varepsilon_{cj1} \cdot E_{c1¤}$ $\Delta\sigma_{cj1.diff} = -0.986\,MPa$ compression in the top fibre of the precast slab

Topping

Centric tensile restraint force accfeting to the te topping $\Delta N_{cs} = 90.206\,\text{kN}$

Strain due to the restraint force $\Delta\varepsilon_{cs2} := \dfrac{\Delta N_{cs}}{E_{c2\text{¤}} \cdot A_2}$ $\Delta\varepsilon_{cs2} = 0.181\,\%\text{o}$

The strain due to the restarint force is taken into account

Construction joint; bottom fibre of the topping $y = -87.12\,\text{mm}$

$\Delta\varepsilon_{cj2} := \Delta\varepsilon_{0cs.diff} + \Delta\psi_{cs.diff} \cdot \left(p_{\text{¤}} - h_1\right) + \Delta\varepsilon_{cs2}$ $\Delta\varepsilon_{cj2} = 0.115\,\%\text{o}$

$\Delta\sigma_{cj2.diff} := \Delta\varepsilon_{cj2} \cdot E_{c2\text{¤}}$ $\Delta\sigma_{cj2.diff} = 0.95\,\text{MPa}$ tension to the bottom fibre of the topping

Remark! The strain difference between the sides of the construction joint

$\Delta\varepsilon_j := \Delta\varepsilon_{cj1} - \Delta\varepsilon_{cj2}$ $\Delta\varepsilon_j = -0.181\,\%\text{o}$ = $\Delta\varepsilon_{cs} = -0.181\,\%\text{o}$ = diffrential shrinkage

Top fibre of the topping $y := p_{\text{¤}} - \left(h_1 + h_2\right)$ $y = -137.12\,\text{mm}$

$\Delta\varepsilon_{cy} := \Delta\varepsilon_{0cs.diff} + \Delta\psi_{cs.diff} \cdot \left(p_{\text{¤}} - h_1 - h_2\right) + \Delta\varepsilon_{cs2}$ $\Delta\varepsilon_{cy} = 0.089\,\%\text{o}$

$\Delta\sigma_{cy.diff} := \Delta\varepsilon_{cy} \cdot E_{c2\text{¤}}$ $\Delta\sigma_{cy.diff} = 0.736\,\text{MPa}$

Total strain in the topping

bottom fibre of the topping $\Delta\varepsilon_{cj2.tot} := \Delta\varepsilon_{cj2} - \Delta\varepsilon_{cs2}$ $\Delta\varepsilon_{cj2.tot} = -0.067\,\%\text{o}$ < $\Delta\varepsilon_{cs} = -0.181\,\%\text{o}$

top fibre of the topping $\Delta\varepsilon_{cy.tot} := \Delta\varepsilon_{cy} - \Delta\varepsilon_{cs2}$ $\Delta\varepsilon_{cy.tot} = -0.092\,\%\text{o}$ < $\Delta\varepsilon_{cs} = -0.181\,\%\text{o}$

The difference of the real happened strain and the free shrinkage cause tensile stress to the topping

$\Delta\sigma_{cj2.diff} := \left(\Delta\varepsilon_{cj2.tot} - \Delta\varepsilon_{cs}\right) \cdot E_{c2\text{¤}}$ $\Delta\sigma_{cj2.diff} = 0.95\,\text{MPa}$

$\Delta\sigma_{cy.diff} := \left(\Delta\varepsilon_{cy.tot} - \Delta\varepsilon_{cs}\right) \cdot E_{c2\text{¤}}$ $\Delta\sigma_{cy.diff} = 0.736\,\text{MPa}$

Concrete strain at the point of the strands $y := p_{\text{¤}} - c_p$ $y = 77.88\,\text{mm}$

$\Delta\varepsilon_{cp} := \Delta\varepsilon_{0cs.diff} + \Delta\psi_{cs.diff} \cdot \left(p_{\text{¤}} - c_p\right)$ $\Delta\varepsilon_{cp} = 0.019\,\%\text{o}$

Change of the stress in the strands

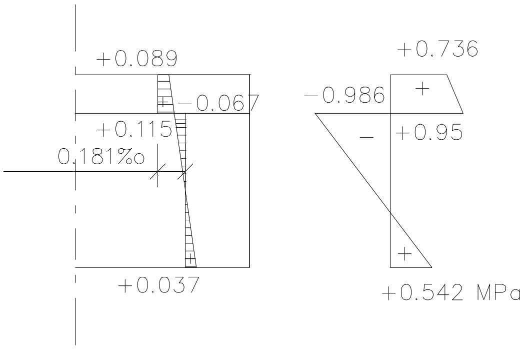
$\Delta\sigma_{p.diff} := \Delta\varepsilon_{cp} \cdot E_p$ $\Delta\sigma_{p.diff} = 3.619\,\text{MPa}$ tension

4.6.4 Deflection due to the differential shrinkage

Curvature $\Delta\psi_{cs.diff} = 0.516\frac{\text{‰}}{m}$

deflection coefficient $\delta_a := \frac{1}{8}$

Deflection $\Delta a_{cs} := \delta_a \cdot \Delta\psi_{cs.diff} \cdot L^2$ $\Delta a_{cs} = 3.163\,mm$ downward



Stresses due to the diffrerential shrinkage

4.6.5 Prestress loss due to shrinkage in the precast slab

When the results due to the mere diffrential shrinkage are compared to to the calculated results where the shrinkage of the precast slab is taken into account.

Precast slab

Bottom fibre of the precast slab $\Delta\sigma_{cb.loss} := \Delta\sigma_{cb.sh} - \Delta\sigma_{cb.diff}$ $\Delta\sigma_{cb.loss} = 0.54\,MPa$

Construction joint; top fibre of the precast slab $\Delta\sigma_{cj1.loss} := \Delta\sigma_{cj1.sh} - \Delta\sigma_{cj1.diff}$ $\Delta\sigma_{cj1.loss} = -0.083\,MPa$

Topping

Bottom fibre of the topping $\Delta\sigma_{cj2.loss} := \Delta\sigma_{cj2.sh} - \Delta\sigma_{cj2.diff}$ $\Delta\sigma_{cj2.diff} = 0.95\,MPa$

Top fibre of the topping $\Delta\sigma_{cy.loss} := \Delta\sigma_{cy.sh} - \Delta\sigma_{cy.diff}$ $\Delta\sigma_{cy.loss} = -0.134\,MPa$

Prestress loss due to shrinkage in the precast slab $\Delta\sigma_{p.loss} := \Delta\sigma_{p} - \Delta\sigma_{p.diff}$ $\Delta\sigma_{p.loss} = -75.702\,MPa$

4.6 Creep

4.6.1 Elastic strains

Strains in the precast slab before the composite action (before the hardening of the topiing concrete)

Prestress

Axail strain at thge centroid of the precast slab $\varepsilon_{0P} = -0.084\,‰$

Curvature $\psi_P = -1.59\,\frac{1}{m}\,‰$

Strain at the bottom fibre of the precast slab $\varepsilon_{caP} = -0.241\,‰$

Strain at the top fibre of the precast slab $\varepsilon_{cyP} = 0.077\,‰$

Self-weight of the precast slab

Axail strain at thge centroid of the precast slab $\varepsilon_{0g1} = 0\,‰$

Curvature $\psi_{g1} = 1.276\,\frac{1}{m}\,‰$

Strain at the bottom fibre of the precast slab $\varepsilon_{cag1} = 0.126\,‰$

Strain at the top fibre of the precast slab $\varepsilon_{cyg1} = -0.129\,‰$

Self-weight of the topping

Axail strain at thge centroid of the precast slab $\varepsilon_{0g2} = 0\,‰$

Curvature $\psi_{g2} = 0.319\,\frac{1}{m}\,‰$

Strain at the bottom fibre of the precast slab $\varepsilon_{cag2} = 0.032\,‰$

Strain at the top fibre of the precast slab $\varepsilon_{cyg2} = -0.032\,‰$

Even if these strains arise before the composite action, a part of the creep strains due to these strains affect to the composite section.

4.6.2 Strains due to creep

Strains due to creep:

Increase of axial strain $\Delta\varepsilon_{cc0} := \phi \cdot \varepsilon_{c0}$

Increase of the curvature $\Delta\psi_{cc} := \phi \cdot \psi$

$\Delta\varepsilon_{cc0}$ is the axial strain due to creep ; equal magnitude at the whole depth of the precast slab; causes the restraint force ΔNcc; can be treated in the same way as shrinkage

$\Delta\psi_{cc}$ is the increase of the curvature due to creep; causes the restraint moment ΔMcc

Prestress

$t_o = 1\,\text{days}$ $\phi_{transfer.2} = 2.373$

$\varepsilon_{0P} = -8.369 \times 10^{-5}$ $\Delta\varepsilon_{cc0P} := \phi_{transfer.2} \cdot \varepsilon_{0P}$ $\Delta\varepsilon_{cc0P} = -0.199\,\text{‰}$

$\psi_P = -1.59\,\frac{\text{‰}}{m}$ $\Delta\psi_{ccP} := \phi_{transfer.2} \cdot \psi_P$ $\Delta\psi_{ccP} = -3.773\,\frac{\text{‰}}{m}$

Self-weight of the precast slab

$t_o = 1\,\text{days}$ $\phi_{transfer.2} = 2.373$

$\varepsilon_{0g1} = 0$ $\Delta\varepsilon_{cc0g1} := \phi_{transfer.2} \cdot \varepsilon_{0g1}$ $\Delta\varepsilon_{cc0g1} = 0$

$\psi_{g1} = 1.276\,\frac{\text{‰}}{m}$ $\Delta\psi_{ccg1} := \phi_{transfer.2} \cdot \psi_{g1}$ $\Delta\psi_{ccg1} = 3.028\,\frac{\text{‰}}{m}$

Self-weight of the topping

$t_{o1} = 29\,\text{days}$ $\phi_{top} = 1.727$

$\varepsilon_{0g2} = 0$ $\Delta\varepsilon_{cc0g2} := \phi_{top} \cdot \varepsilon_{0g2}$ $\Delta\varepsilon_{cc0g2} = 0$

$\psi_{g2} = 0.319\,\frac{\text{‰}}{m}$ $\Delta\psi_{ccg2} := \phi_{top} \cdot \psi_{g2}$ $\Delta\psi_{ccg2} = 0.551\,\frac{\text{‰}}{m}$

Creep strain in the precast slab after the hardening of the topping concrete due to the loads acting before the composite action

Change of the axial strain at the centroid of the precast slab

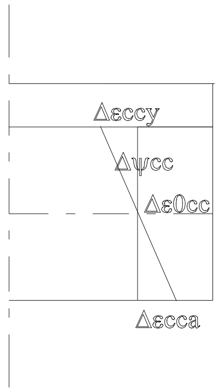
$\Delta\varepsilon_{cc0} := \Delta\varepsilon_{cc0P} + \Delta\varepsilon_{cc0g1} + \Delta\varepsilon_{cc0g2}$ $\Delta\varepsilon_{cc0} = -0.199\,\text{‰}$

Change of the curvature $\Delta\psi_{cc} := \Delta\psi_{ccP} + \Delta\psi_{ccg1} + \Delta\psi_{ccg2}$ $\Delta\psi_{cc} = -0.194\,\frac{\text{‰}}{m}$

Strain in the centroid of the pure concrete (without the strands) $p_{c1} = 99.905\,mm$ from the bottom fibre of the precast slab

$\Delta\varepsilon_{cc0c} := \Delta\varepsilon_{cc0} + \Delta\psi_{cc} \cdot (p_1 - p_{c1})$ $\Delta\varepsilon_{cc0c} = -0.198\,\text{‰}$

Strain at the bottom fibre of the precast slab $\Delta\varepsilon_{cca} := \Delta\varepsilon_{cc0} + \Delta\psi_{cc} \cdot p_1$ $\Delta\varepsilon_{cca} = -0.218\,\text{‰}$



4.6.3 Differential creep at the construction joint

Creep causes to the precast slab after the composite action has been started shortening strain Δεcc0 and curvature Δψcc which do not exist in the topping. So there exist the strain difference Δεcc0 and the curvature diffrerence Δψcc between the precast slab and the topping.

Strain difference at the construction joint $\Delta\varepsilon_{ccj} := \Delta\varepsilon_{cc0} + \Delta\psi_{cc}\cdot(p_1 - h_1)$ $\Delta\varepsilon_{ccj} = -0.179\,‰$

Because at the construction joint cannot exist strain and curvature diffrerences, the state is restored by producing to the precast slab a tensile force ΔNcc which reverse the strain difference Δεcc0 and a moment which restore the curvature difference Δψcc.

4.6.4 Restraint force for restoring the differential creep strain

Tensile restriant force which restore the axial strain Δεcc0 (<0)

$\Delta N_{cc} := -\Delta\varepsilon_{cc0c} E_{c1¤}\cdot A_{c1}$ $\Delta N_{cc} = 702.306\,kN$

ΔNcc affecting at the centroid of the concrete of the precast slab $p_{c1} = 99.905\,mm$

from the bottom fibre of the precast slab

Moment which restore the curvature

$\Delta M_{cc} := -\Delta\psi_{cc}\cdot E_{c1¤}\cdot I_{c1}$ $\Delta M_{cc} = 2.301\,kNm$

For maintaining the equilibrium the composite section is loaded by the compressive counterforce .ΔNcc (location nis at the centroid of the precast slab concrete) and the countermoment -ΔMcc.

$-\Delta N_{cc} = -702.306\,kN = \Delta\varepsilon_{cc}\cdot E_{c1¤}\cdot A_{c1}$

$-\Delta M_{cc} = -2.301\,kNm = \Delta\psi_{cc}\cdot E_{c1¤}\cdot I_{c1}$

Because -ΔNcc affecting to the composite section it can me moved to the centroidal axis of the composite section and the movement is subsituted with the moment

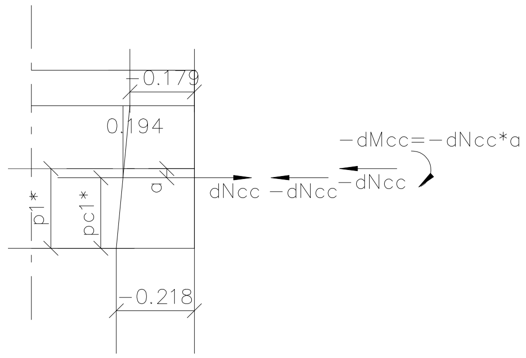
$M_e := -\Delta N_{cc}\cdot(p_{¤} - p_{c1})$ $M_e = -9.112\,kNm$

Total moment due to creep difference $\Delta M_{cc.tot} := -(-\Delta M_{cc} + M_e)$ $-\Delta M_{cc.tot} = -11.414\,kNm$

Forces acting to the centroid of the composite section:

$-\Delta N_{cc} = -702.306\,kN$

$-\Delta M_{cc.tot} = -11.414\,kNm$



4.6.5 Strains and stresses in the composite section due to the creep difference

Axial strain at the centroid of the composite section $\Delta\varepsilon_{cc00} := \frac{-\Delta N_{cc}}{EA_{¤}}$ $\Delta\varepsilon_{cc00} = -0.169\,‰$

Curvature $\Delta\psi_{cc0} := \frac{-\Delta M_{cc.tot}}{EI_{¤}}$ $\Delta\psi_{cc0} = -0.583\,\frac{‰}{m}$

Strains and stresses at the different points of the cross-section

Total strain $\varepsilon_{ctot} := \Delta\varepsilon_{cc00} + \Delta\psi_{cc0}\cdot y$

Elastic strain which arise stress

$$\varepsilon_{ce} := \varepsilon_{ctot} + \left[-\left(\Delta\varepsilon_{cc0} + \Delta\psi_{cc}\cdot y_{c1}\right)\right]$$

$$\varepsilon_{ce} := \Delta\varepsilon_{cc00} + \Delta\psi_{cc0}\cdot y + \left[-\left(\Delta\varepsilon_{cc0} + \Delta\psi_{cc}\cdot y_{c1}\right)\right]$$

acting to the composite section -ΔNcc,-ΔM.cc.tot

acting to the precast slab ΔNcc, ΔMcc only in the region of the precast slab

yc1 from the centroid of the precast slab concrete the reference point of Δεcc0

Stress $\sigma_c := E_{c¤}\cdot\varepsilon_{ce}$

Precast slab

Bottom fibre $y := p_{¤}$ $y = 112.88\,\text{mm}$ $y_{c1} := p_{c1}$ $y_{c1} = 99.905\,\text{mm}$

$\varepsilon_{ca1.tot} := \Delta\varepsilon_{cc00} + \Delta\psi_{cc0} \cdot p_{¤}$ $\varepsilon_{ca1.tot} = -0.234\,\text{‰}$

$\varepsilon_{ca1e} := \varepsilon_{ca1.tot} + \left[-\left(\Delta\varepsilon_{cc0c} + \Delta\psi_{cc} \cdot p_{c1}\right)\right]$ $\varepsilon_{ca1e} = -0.017\,\text{‰}$

$\sigma_{cb1.cre} := E_{c1¤} \cdot \varepsilon_{ca1e}$ $\sigma_{cb1.cre} = -0.246\,\text{MPa}$

Construction joint;
top fibre of the precast slab $y := p_{¤} - h_1$ $y = -87.12\,\text{mm}$ $y_{c1} := p_{c1} - h_1$ $y_{c1} = -100.1\,\text{mm}$

$\varepsilon_{cj1.tot} := \Delta\varepsilon_{cc00} + \Delta\psi_{cc0} \cdot \left(p_{¤} - h_1\right)$ $\varepsilon_{cj1.tot} = -0.118\,\text{‰}$

$\varepsilon_{cj1e} := \varepsilon_{cj1.tot} + \left[-\left[\Delta\varepsilon_{cc0c} + \Delta\psi_{cc} \cdot \left(p_{c1} - h_1\right)\right]\right]$ $\varepsilon_{cj1e} = 0.061\,\text{‰}$

$\sigma_{cj1.cre} := E_{c1¤} \cdot \varepsilon_{cj1e}$ $\sigma_{cj1.cre} = 0.904\,\text{MPa}$

Topping

In the topping there is no stresses which cuse creep befor the composite action, so

In the topping there is no restraint force due to creep => $\Delta\varepsilon_{cc02} := 0$ $\Delta\psi_{cc2} := 0$

=> total strain = elastic strain

Construction joint
; bottom fibre of the topping $y := p_{¤} - h_1$ $y = -87.12\,\text{mm}$

$\varepsilon_{cj2.tot} := \Delta\varepsilon_{cc00} + \Delta\psi_{cc0} \cdot \left(p_{¤} - h_1\right)$ $\varepsilon_{cj2.tot} = -0.118\,\text{‰}$

$\varepsilon_{cj2e} := \varepsilon_{cj2.tot}$

$\sigma_{cj2.cre} := E_{c2¤} \cdot \varepsilon_{cj2e}$ $\sigma_{cj2.cre} = -0.978\,\text{MPa}$

top fibre $y := p_{¤} - h_1 - h_2$ $y = -137.12\,\text{mm}$

$\varepsilon_{cy2.tot} := \Delta\varepsilon_{cc00} + \Delta\psi_{cc0} \cdot \left(p_{¤} - h_1 - h_2\right)$ $\varepsilon_{cy2.tot} = -0.089\,\text{‰}$

$\varepsilon_{cy2e} := \varepsilon_{cy2.tot}$

$\sigma_{cy2.cre} := E_{c2¤} \cdot \varepsilon_{cy2e}$ $\sigma_{cy2.cre} = -0.736\,\text{MPa}$

<u>Concrete stress at the level of the strands</u>

$y := p_{¤} - c_p$ $\quad y = 77.88\,mm$ $\quad y_{c1} := p_{c1} - c_p$ $\quad y_{c1} = 64.905\,mm$

Total strain

$\varepsilon_{cp.tot} := \Delta\varepsilon_{cc00} + \Delta\psi_{cc0} \cdot (p_{¤} - c_p)$ $\quad \varepsilon_{cp.tot} = -0.214\,‰$

Elastic strain

$\varepsilon_{cpe} := \varepsilon_{cp.tot} + [-[\Delta\varepsilon_{cc0c} + \Delta\psi_{cc} \cdot (p_{c1} - c_p)]]$ $\quad \varepsilon_{cpe} = -3.04 \times 10^{-3}\,‰$

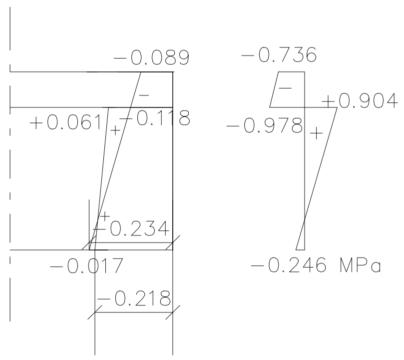
$\sigma_{cp} := E_{c1¤} \cdot \varepsilon_{cpe}$ $\quad \sigma_{cp} = -0.045\,MPa$

Change of the stress in the strands due to the creep difference

$\Delta\varepsilon_p := \varepsilon_{cp.tot}$ $\quad \Delta\varepsilon_p = -0.214\,‰$

$\Delta\sigma_{pcc} := E_p \cdot \varepsilon_{cp.tot}$ $\quad \Delta\sigma_{pcc} = -41.738\,MPa$

$\Delta\sigma pcc$ is the prestress loss due to creep



<u>4.6.6 Deflection due to creep</u>

Creep causes due to

- prestress force, constant curvature $\delta_{aP} := \frac{1}{8}$

- self-weights of the precast slab and the topping; parabolic curvature $\delta_{ag} := \frac{5}{48}$

The curvature of the composite section due to creep is divided into parts:

-<u>prestress</u>

$\Delta\varepsilon_{cc0P} = -0.199\,\%o$ $\Delta\psi_{ccP} = -3.773\,\frac{\%o}{m}$

$\Delta N_{ccP} := -\Delta\varepsilon_{cc0P}\cdot E_{c1¤}\cdot A_{c1}$ $\Delta N_{ccP} = 702.967\,kN$

$\Delta M_{ccP} := -\Delta\psi_{ccP}\cdot E_{c1¤}\cdot I_{c1}$ $\Delta M_{ccP} = 44.791\,kNm$

ΔNccP is moved to the centroid of the composite section

$\Delta M_{ccP1} := \Delta M_{ccP} + \Delta N_{ccP}\cdot\left(p_{¤} - p_{c1}\right)$ $-\Delta M_{ccP1} = -53.912\,kNm$

Strains of the composite section due to the forces -ΔNccP ja -ΔMccP1:

$\Delta\varepsilon_{cc00P} := \frac{-\Delta N_{ccP}}{EA_{¤}}$ $\Delta\varepsilon_{cc00P} = -0.169\,\%o$

$\Delta\psi_{cc0P} := \frac{-\Delta M_{ccP1}}{EI_{¤}}$ $\Delta\psi_{cc0P} = -2.753\,\frac{\%o}{m}$

Deflection coefficient $\delta_{aP} := \frac{1}{8}$

Deflection $a_{ccP} := \delta_{aP}\cdot\Delta\psi_{cc0P}\cdot L^2$ $a_{ccP} = -16.862\,mm$ upward

<u>- Self-weights of the precast slab + the topping</u>

$\Delta\varepsilon_{cc0g1} + \Delta\varepsilon_{cc0g2} = 0$ $\Delta\psi_{ccg1} + \Delta\psi_{ccg2} = 3.579\,\frac{\%o}{m}$

$\Delta N_{ccg} := 0$

$\Delta M_{ccg} := -\left(\Delta\psi_{ccg1} + \Delta\psi_{ccg2}\right)\cdot E_{c1¤}\cdot I_{c1}$ $-\Delta M_{ccg} = 42.49\,kNm$

Strains of the composite section due to the forces -ΔNccg ja -ΔMccg:

$\Delta\varepsilon_{cc00g} := 0$

$\Delta\psi_{cc0g} := \frac{-\Delta M_{ccg}}{EI_{¤}}$ $\Delta\psi_{cc0g} = 2.17\,\frac{\%o}{m}$

$\delta_{ag} := \frac{5}{48}$

Deflection $a_{ccg} := \delta_{ag}\cdot\Delta\psi_{cc0g}\cdot L^2$ $a_{ccg} = 11.074\,mm$ downward

Total deflection due to creep $a_{cc} := a_{ccP} + a_{ccg}$ $a_{cc} = -5.787\,\text{mm}$ upward

Total curvature due to creep $\Delta\psi_{cc0} := \Delta\psi_{cc0P} + \Delta\psi_{cc0g}$ $\Delta\psi_{cc0} = -0.583\,\frac{\text{‰}}{\text{m}}$

4.8 Total stresses dueto shrinkage and creep

Precast slab; bottom fibre $\Delta\sigma_{cb} := \Delta\sigma_{cb.sh} + \sigma_{cb1.cre}$ $\Delta\sigma_{cb} = 0.835\,\text{MPa}$

Precast unit; top fibre; construction joint $\Delta\sigma_{cj1} := \Delta\sigma_{cj1.sh} + \sigma_{cj1.cre}$ $\Delta\sigma_{cj1} = -0.165\,\text{MPa}$

Topping; biottom fibre; constructuion joint $\Delta\sigma_{cj2} := \Delta\sigma_{cj2.sh} + \sigma_{cj2.cre}$ $\Delta\sigma_{cj2} = -0.074\,\text{MPa}$

Topping; top fibre $\Delta\sigma_{cy} := \Delta\sigma_{cy.sh} + \sigma_{cy2.cre}$ $\Delta\sigma_{cy} = -0.134\,\text{MPa}$

Because the shrinkage and creep values are not very exact, it is recommended:
- For studying tensile stresses and cracking at the bottom fibre, it is recommended to take into account only a part, maybe 70 % of the compression stress due to ceep

- For studying tensile stresses at to top fibre of the topping it is recommended to take into account only a part maybe 70 % of the compression stress due to creep

Stress at the bottom fibre due to shrinkage and creep

$\Delta\sigma_{cb} := \Delta\sigma_{cb.sh} + 0.7\cdot\sigma_{cb1.cre}$ $\Delta\sigma_{cb} = 0.909\,\text{MPa}$

Stress at the top fibre due to shrinkage and creep

$\Delta\sigma_{cy} := \Delta\sigma_{cy.sh} + 0.7\cdot\sigma_{cy2.cre}$ $\Delta\sigma_{cy} = 0.087\,\text{MPa}$

a small tension at the top fibre of the topping

5. CONTINUITY

The structure is made continuous with the topping. The reinforcement required the support moment is in the topping. The joint between the precast slabs is grouted at the same time as the casting the topping. So the joint can take the compression resultant due to the support moment.

5. 1 General

Because of diffrerential shrinkage the structure has curvature which causes rotation at the support. Rotation causes rotation difference at the both sides of the support, if the structure does not act as a continuous structure.

There cannot be rotation difference at the middle support of the continuous structure and so there exist a restraint moment which reverse the rotation diffrerence.

Due to the diffrerential shrinkage between the precast unit and the topping (shrinkage is greater in the topping) the structure will curve downward arising a negative moment at the middle support.

The creep of the precast unit causes a negative moment at the middle support if the precast unit has a downward curvature (reinforced structure) due to the loads before the structure has been made continuous and a positive moment if the precast unit has a upward curvature (prestressed structure).

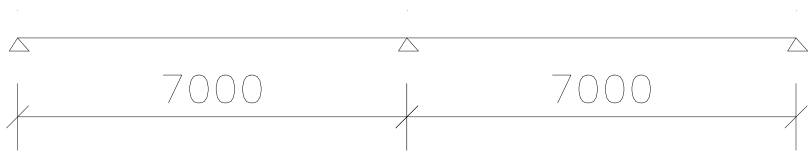
Development of a negative moment means that part of the loads (self weights of the precast unit and the topping) which affect a simply supported precast slab before the continuity transmits to carrying of the continuous compooste structure structure.

For loads which affect after the hardening of the topping and the joint concrete the structure acts like a normal continuous structure.

The precast slab is prestressed so the flexural stiffness at the field corresponds the stiffness of a uncracked cross-section.

The topping is not prestressed, so the flexural stiffness at the support corresponds a stiffness of a cracked reinforced cross-section.

The smaller flexural stiffness at the support reduces a support moment.

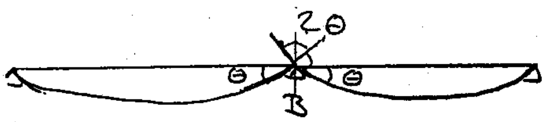


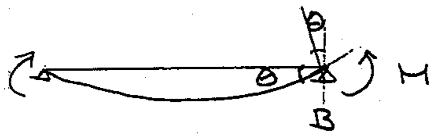
5.2 Support moment due to differential shrinkage

Curvature due to the diffrerential shrinkage $\Delta\psi_{cs} = 0.727 \frac{‰}{m}$ downward

The curvature is equal at the whole span => coreresponds a equal moment at the whole span

$$\theta_{Bcs} := \frac{1}{2} \cdot \frac{M}{EI} \cdot L = \frac{1}{2} \cdot \Delta\psi_{cs} \cdot L$$

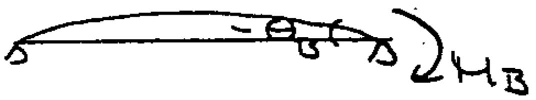




Curvature at a free support due to equal curvature $\theta_{Bcs} := \frac{1}{2} \cdot \Delta\psi_{cs} \cdot L$ $\theta_{Bcs} = 2.545 \times 10^{-3}\,\text{rad}$

The required restraint moment at the middle support for reversing the roatation difference

$$M_{Bcs} := 3 \cdot \frac{EI_{\text{¤}}}{L} \cdot \left(-\theta_{Bcs}\right) = \frac{-3}{2} \cdot EI_{\text{¤}} \cdot \Delta\psi_{cs} \qquad M_{Bcs} = -21.36\,\text{kNm}$$



$EI_{\text{¤}} = 19.584\,\text{MN}\cdot\text{m}^2$

Because M_B and $EI_{\text{¤}}$ is calculated by using the elastic modulus $E_{c\text{¤}} := \frac{E_{cm}}{1 + \chi \cdot \phi}$
a reducing influence of creep is taken into account

In the field the bottom side of the structure is prestressed but the top side of the structure corresponds a non-prestressed reinforced cross-section and so the flexural stiffness of the support (negative moment) region is smaller than the one in the field. Because of the smaller stiffness of the support region the support moment is reduced by the factor 0.7.

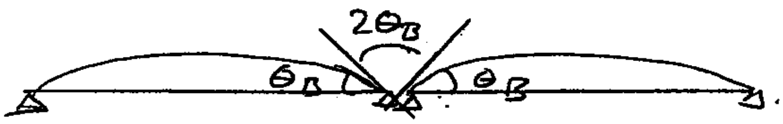
$M_{Bcs} := 0.7 \cdot M_{Bcs}$ $M_{Bcs} = -14.952\,\text{kNm}$

5.3 Support moment due to a differential creep

Support moment is calclulated as above.

The curvature due to differential creep $\Delta\psi_{cc0} = -0.583 \frac{\%o}{m}$

Beause a part of the creep is due the prestressing which causes the equal curvature to the whole span; a part of the creep is due to the sefl-weights of the precast slab and the topping which cause a parabolic curvature distribution.



The curaveture of the creep is divided into two parts:

Due to prestressing $\Delta\psi_{cc0P} = -2.753 \frac{\%o}{m}$

Rotation at the middle support due to the equal curvature $\theta_{BccP} := \frac{1}{2} \cdot \Delta\psi_{cc0P} \cdot L$ $\theta_{BccP} = -9.635 \times 10^{-3}\,\text{rad}$

Restraint moment which reverse the curvature difference $M_{BccP} := 3 \cdot \frac{EI_{\text{¤}}}{L} \cdot \left(-\theta_{BccP}\right)$ $M_{BccP} = 80.868\,\text{kNm}$

Because of the stiffness ratio of the support region and the field region the support moment is reduced by the factor 0.7

$M_{BccP} := 0.7 \cdot M_{BccP}$ $M_{BccP} = 56.608\,\text{kNm}$

Due to equal distributed load;
self-weights of the precast slab and the toping $\Delta\psi_{cc0g} = 2.17 \frac{\%o}{m}$

Rotation at the middle support due to the parabolic curvature distribution $\theta_{Bccg} := \frac{M \cdot L}{3 \cdot EI_{\text{¤}}} = \frac{1}{3} \cdot \Delta\psi_{cc0g} \cdot L$

$\theta_{Bccg} := \frac{1}{3} \cdot \Delta\psi_{cc0g} \cdot L$ $\theta_{Bccg} = 5.063 \times 10^{-3}\,\text{rad}$

Restraint moment which reverses the rotation difference due to the parabolic curvature distribution $M_{Bccg} := 3 \cdot \frac{EI_{\text{¤}}}{L} \cdot \left(-\theta_{Bccg}\right) = EI_{\text{¤}} \cdot \Delta\psi_{cc0g}$

$M_{Bccg} = -42.49\,\text{kNm}$

Because of the stiffness ratio of the support region and the field region the support moment is reduced by the factor 0.7

$M_{Bccg} := 0.7 \cdot M_{Bccg}$ $M_{Bccg} = -29.743\,\text{kNm}$

Total support monent due to the differential creep $M_{Bcc} := M_{BccP} + M_{Bccg}$ $M_{Bcc} = 26.865\,\text{kNm}$

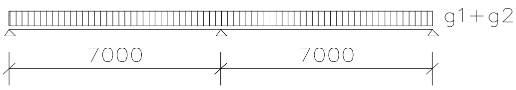
Total support moment due to the differential shrinkage and creep $M_B := M_{Bcs} + M_{Bcc}$ $M_B = 11.913\,\text{kNm}$

The total support moment is positive in this case

Deflection due to the support moment $a_{MB} := \dfrac{M_B \cdot L^2}{15.6 \cdot EI_{¤}}$ $a_{MB} = 1.911\,\text{mm}$ downward

The support moment due to the differential creep is compared to the situation where the self-weights of the precast slab and the topping is loaded to the continuous structure

self-weight of the precast slab $g_1 = 6\dfrac{\text{kN}}{\text{m}}$



self -weight of the topping $g_2 = 1.5\dfrac{\text{kN}}{\text{m}}$

$$M_g := \frac{-(g_1 + g_2) \cdot L^2}{8} \qquad M_g = -45.938\,\text{kNm}$$

The shrinkage diffrenence and the creep difference due to self-weights of the precast slab and the topping are caused the support moment $M_{Bcs} + M_{Bccg} = -44.695\,\text{kNm}$ which is about

97 % of bending moment due to the self-weights of the precast slab and the topping in the simply supported structure.

Beacuse of the shrinkage and creep differencies about 97 % of the loads acting on the simply supported precast slab transmits gradually to a load of the continuous composite structure (a corresponding situation in a reinforced structure which has a downward curvature before a composite and a continuous action.

A prestressing change the situation so that a upward curvature before composite action of a precast unit due to a prestressing causes positive support moment

Because shrinkage- and creep-values are not very excat and they depend on many factors like time, humidity and so on, one may be careful about an utilization of the support moment for dimensioning the field. A little different values of shrinkage and creep the support moment may be positive which increase a field moment.

Recommendation:
For calculating the positive field moment at the cracking limit state the negative support moment due to the differential shrinkage is utilized only e.g. about 70 % and the positive support moment totally (100 %).

For dimensioning a reinforcement against cracking at the support region the negative suppoort moment due to the differential shrinkage is utilized toatally (100 %) but the positive support moment due to creep only e.g. about 70 %.

Influence to the max. field moment:

$$\Delta M_{field} := 0.5 \cdot (0.7 \cdot M_{Bcs} + M_{Bcc}) \qquad \Delta M_{field} = 8.199\,\text{kNm}$$

This is added to the field moment due to the imposed loads and it increase a tensile stress at the bottom fibre of the slab.

For the studying the tensile stress at the bottom fibre of the slab the support moment is

$$M_{B1} := 0.7 \cdot M_{Bcs} + M_{Bcc} \qquad M_{B1} = 16.398\,\text{kNm}$$

If the absolute value of the support moment due to the imposed loads is smaller than this value M_{B1} the continuity does not give a significant benefit for the stresses in the field.

The support moment due to the imposed live load $M_{Bq} := -0.7 \cdot 0.125 \cdot q \cdot L^2$

Factor 0.7 takes into account the stffness difference between the support region and the field region.

If the imposed live load q < $\dfrac{M_{B1}}{0.7 \cdot 0.125 \cdot L^2} = 3.825 \dfrac{kN}{m}$ the coninuity does not give any benefit for the cracking limit state in field.

The top reinforcement at the support is dimensioned for in addition to the support moment due to the imposed live load the support moment due to differential shrinkage and creep

$$M_{B.neg.dim} := M_{Bcs} + 0.7 \cdot M_{Bcc} \qquad M_{B.neg.dim} = 3.853\,kNm$$

The top reinforcement at the support is dimensioned against in addition to the support moment due to the imposed live load (frequent / quasi-permanent load combination) this support moment due to diffrerential shrinkage and creep so that the allowable crack width does not exceed.

In addition to the above support moment the possible positive support moment due to differential creep should be considered. This positive support moment is decresed by the negative support moment due to the possible imposed permanent load:

$$M_{B.pos.dim} := 0.7 \cdot M_{Bcs} + M_{Bcc} \qquad M_{B.pos.dim} = 16.398\,kNm$$

The ties (joint reinforcement) 2 T12 in the longitudinal joint between the precast plan slab units are adequite for this positive support moment.

If the support moment caused by the differential shrinkage and creep the deflection due to this positive moment should be calculated with using the moment $M_{B1} = 16.398\,kNm$

$$a_{MB} := \frac{M_{B1} \cdot L^2}{15.6 \cdot EI_{¤}} \qquad a_{MB} = 2.63\,mm$$

The joint between the precast units is designed so that it can resist this positive support moment:
- reinforcement bars come out from the precast units into the joint which
  - are welded together (this is concerned usually only for beams with great positive moments)
  - outcoming reinforcement bars are hooks or loops
- joint reinforcement in longitudinal joints between the slabs

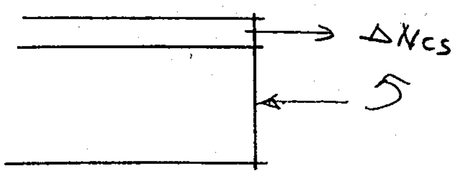
At the service limiyt state the stress in this renforcement can be near the yield point if the restricting of cracking does not require smaller stress.

The joint can be also done so that there does not exist positive moment:
- in the joint there is some soft flexible material which allows rotation at the middle support; when a rotation and a support reaction is big there is a risk for breaking of the corner of the support due to a movement caused by the rotation
=> using bearing pads between a precast unit and the support and strengthening of the support with reinforcement or other ways.
- great amount of reinforcement at the support which allow great compressive strain in concete due to a rotation without breaking
=> an effective cross-section at the support is only the topping which flexural stiffness is small compared to the stiffness of the total composite cross-section in the field.

These solutions are usually used in bridges where forces and spans are big.
In customary building structures a positive moment at the support does not require any special actions especially for slabs.

At the ultimate limit state these restraint moments do not need to take into account. At the ultimate limit state the yield moment according to the support reinforcement can be utilized for studying the flexural resistance of the field.

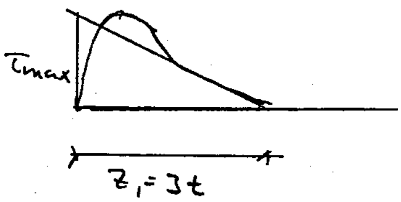
5.4 Shear stress at the construction joint due to differential shrinkage



Restraint force in the topping due to diffrerential shrinkage $\Delta N_{cs} = 90.206\,kN$

Shear stress at the end of the slab is distributed at the distance $z_1 := 3 \cdot \min(h_1, h_2)$ $z_1 = 150\,mm$

The stear stress is supposed to distribute triangular over this distance so that at the end of the member is the greates shear stress and at the distance z1 from the end of the slab shear stress = 0



The maximum shear stress at the end of the slab $\tau_{max} := 2 \cdot \frac{\Delta N_{cs}}{z_1 \cdot b}$ $\tau_{max} = 1.002\,MPa$

Suppose the partial safety factor for the shear stress due to differential shrinkage the partial safety factor for live load beacuse shrinkage values are not very acuurate) $\gamma_q := 1.5$

Design value of the shear stress in the joint $v_{Ed} := \gamma_q \cdot \tau_{max}$ $v_{Ed} = 1.503\,MPa$ $\beta := 1$

(the total restraint force is above the construction joint => β = 1)

The shear strength of the construction joint

Partial safety factor for concrete $\gamma_c := 1.5$

Partial safety factor for the shear reinforcement $\gamma_s := 1.15$

Concrete grade of the topping concrete C25/30

Design value of the tensile strength of the topping concrete

$$f_{ctd2} := \frac{0.7 \cdot 0.3 \cdot \left(\frac{f_{ck2}}{MPa}\right)^{0.667} \cdot MPa}{\gamma_c}$$

$f_{ctd2} = 1.198\,MPa$

Concrete grade of the precast slab concrete C40/50

Design value of the tensile strength of the precast slab concrete

$$f_{ctd1} := \frac{0.7 \cdot 0.3 \cdot \left(\frac{f_{ck1}}{MPa}\right)^{0.667} \cdot MPa}{\gamma_c}$$

$f_{ctd1} = 1.639\,MPa$

The surface of the precast plank slab is an extruded surface => smooth SFS-EN-1992-1-1 6.2.5(2)

Cohesion coefficient $c := 0.2$ friction coefficient $\mu := 0.6$

In the Eurocode the shear resistance of a construction joint is based on the friction-cohesion theory (bond between the concretes coreresponds cohesion and the force in the shear reinforcement causes friction onto the surface)

The design value of the tensile strength of concrete is the smaller value of the concretes at the both sides of the joint

$f_{ctd} := \min(f_{ctd1}, f_{ctd2})$ $\quad f_{ctd} = 1.198\,MPa$

Design value of the strength of the joint reinforcement (A500HW) $\quad f_{yd} := \frac{500 \cdot MPa}{\gamma_s}$ $\quad f_{yd} = 434.783\,MPa$

Vertical stirrups, the inclination angle of the shear reinforcement $\quad \alpha := 90 \quad \sin\alpha := 1 \quad \cos\alpha := 0$

SFS-EN-1992-1-1 equation (6.25)

$$v_{Rdi} := c \cdot f_{ctd} + \mu \cdot \sigma_n + \rho \cdot f_{yd} \cdot (\mu \cdot \sin\alpha + \cos\alpha)$$

Compression stress perpendicular to the joint $\quad \sigma_n := 0 \cdot MPa$

The required amount of the shear reinforcement os solved, when the shear resistance of the joint $v_{Rdi} := v_{Ed}$

$$\rho := \frac{v_{Ed} - c \cdot f_{ctd}}{f_{yd}} \qquad \rho = 2.907 \times 10^{-3}$$

Amount of the joint reinforcementat the end of the slab due to differerential shrinkage

$$A_{sv.sh} := \frac{\rho \cdot b \cdot z_1}{2} \qquad A_{sv.sh} = 261.601\,mm^2$$

=> 3 T 8 2-leggs links c/c 50.

6. Cracking moment

For calculating the cracking moment it must be taken into account that a part of the load is affected to the precast unit (precast plank slab) before the composte action and a part to the composite section.

Stress state of the precast slab before the composite action:

$\sigma_{cb1} := -2.929 \cdot MPa$ due to the bending moment $M_1 := M_{g1} + M_{g2}$ $M_1 = 45.938\,kNm$

(prestress, losses before composite action, self-weights of the precast slab and the topping)

Stresses due to the loads acting to the composite cross-section:
- prestress losses after the composite action
- differential shrinkage and creep
- bending moment M2 due to imposed dead and live loads after composite action

The prestress loss and the differential shrinkage cause usually tensile stresses to the bottom fibre of the slab, so the effect of these actions should be taken into account when calculating the cracking moment. The differential creep due to creep of the precast unit cause often compression stress to the bottom fibre of the slab, so the effect of this is recommended to ignore totally or partly because of a inaccurancy of creep calculations.

Prestress loss after the beginning of the composite action

due to shrinkage $\Delta\sigma_{cb.loss} = 0.54\,MPa$ (page 61)

due to creep $\sigma_{cb1.cre} = -0.246\,MPa$ ignored or taking into accout not more than 70 %

due to relaxation $\Delta\sigma_{c.rel} := 0.23 \cdot MPa$ (not calculated in this example)

due to differential shrikage and creep $\Delta\sigma_{cb.diff} = 0.542\,MPa$

The change of concrete stress at the bottom fibre of the slab due to deformation loads (the value which is in a safe side, the effect of the creep diffrerence is not taken into account)

$\Delta\sigma_{cb} := \Delta\sigma_{cb.loss} + \Delta\sigma_{c.rel} + \Delta\sigma_{cb.diff}$ $\Delta\sigma_{cb} = 1.312\,MPa$

Solving such moment M2 for which the total sress at the bottom fibre is $\Sigma\sigma_{ca} = f_{ctm1}$

Stress before the composite action $\sigma_{cb1} = -2.929\,MPa$ moment M1

Stress at the bottom fibre due to bending moment M2 to the composite section M2 σ_{ca2}

Total stress at the bottom fibre $\Sigma\sigma_{ca} := \sigma_{ca1} + \Delta\sigma_{ca} + \sigma_{ca2} = f_{ctm1} = 3.509\,MPa$

$$\sigma_{cb2} := E_{cm1} \cdot \frac{M_2 \cdot p}{EI} = \frac{M_2}{W_{b.comp}}$$

Wb.comp is the section modulus of the composite cross-section about the bottom fibre of the precast slab

$$W_{b.comp} := \frac{EI}{E_{cm1} \cdot p} \qquad W_{b.comp} = 0.0125\,m^3$$

$p = 121.733\,mm$ is the distance of the cenroid from the bottom fibre of the precast slab

$E_{cm1} = 3.522 \times 10^4 MPa$ is the elastic modulus of the precast slab concrete

$f_{ctm1} = 3.509\,MPa$ is the average tensile strength of the precast slab concrete

The allowable additional stress Δσcb2 due to the bending moment M2 before cracking

$\sigma_{cb2} := f_{ctm1} - \sigma_{cb1} - \Delta\sigma_{cb}$ $\sigma_{cb2} = 5.126\,MPa$

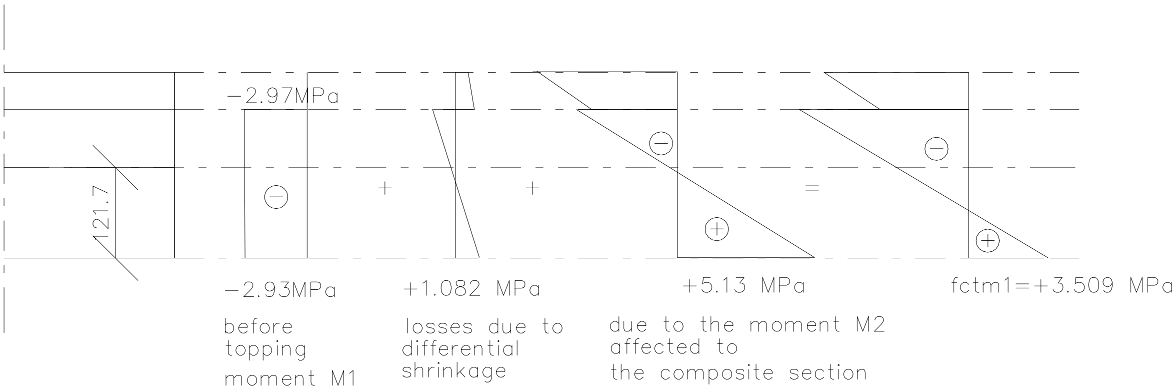
$M_2 := \sigma_{cb2} \cdot \dfrac{EI}{E_{cm1} \cdot p}$ $M_2 = 64.298\,kNm$

Before the composite action the structure has loaded by the moment M1 due to the self-weights of the precast slab and the topping, and fue to the moment M1 the stress at the bottom fibre is

$\sigma_{cb1} = -2.929\,MPa$

$M_1 := M_{g1} + M_{g2}$ $M_1 = 45.938\,kNm$

Total bending moment (the cracking moment) which causes cracking at the bottom fibre $M_{cr} := M_1 + M_2$ $M_{cr} = 110.235\,kNm$



According to the above study the equation for the cracking moment is obtained

$$\sigma_{cb1} := \frac{-P_o + \Delta P_1}{A_1} + \frac{(-P_o + \Delta P_1) \cdot e_{p1}}{W_{b1}} + \frac{M_1}{W_{b1}}$$

$$\Delta\sigma_{cb} := \frac{\Delta P_2}{A_{comp}} + \frac{\Delta P_2 \cdot e_{p2}}{W_{b.comp}}$$

$$\sigma_{cb2} := \frac{M_2}{W_{b.comp}}$$

Total stress at the bottom fibre of a precast unit

$$\Sigma\sigma_{cb} := \sigma_{cbq1} + \Delta\sigma_{cb} + \sigma_{cb2}$$

$$\Sigma\sigma_{cb} := \frac{-P_o + \Delta P_1}{A_1} + \frac{(-P_o + \Delta P_1)\cdot e_{p1}}{W_{b1}} + \frac{M_1}{W_{b1}} + \left(\frac{\Delta P_2}{A_{comp}} + \frac{\Delta P_2 \cdot e_{p2}}{W_{b.como}}\right) + \frac{M_2}{W_{b.comp}}$$

$$= f_{ctm1}$$

$$\Rightarrow \quad M_2 := f_{ctm1}\cdot W_{a.liitto} + \left[\frac{P_o - \Delta P_1}{A_1} + \frac{(P_o - \Delta P)\cdot e_{p1}}{W_{a1}}\right]\cdot W_{a.liitto} - \left(\frac{\Delta P_2}{A_{liitto}} + \frac{\Delta P_2 \cdot e_{p2}}{W_{a.liitto}}\right)\cdot W_{a.aliitto}$$

Cracking moment

$$M_r := M_1 + M_2$$

$$M_r := f_{ctm1}\cdot W_{b.comp} + \left[(P - \Delta P_1)\cdot\left(\frac{1}{A_{.1}} + \frac{e_{p1}}{W_{b1}}\right) - \Delta P_2\cdot\left(\frac{1}{A_{comp}} + \frac{e_{p2}}{W_{b.comp}}\right)\right]\cdot W_{b.comp} + M_1\cdot\left(1 - \frac{W_{b.comp}}{W_{b.1}}\right)$$

A1 is the area of the cross-section of the precast unit included the reinforcement
Acomp is the area of the cross-section of the composite structure
W.b1 is the section modulus of the cross-section of the precast unit about the bottom fibre
Wb.comp is the section modulus of the composite cross-sectio about the bottom fibre of the precast unit
ep1 is the eccentricity of the prestress force about the centroid of the precast unit
ep2 is the eccentricity of the presress force about the centroid of the composite cross-section

7. Effect of temporary supports (propping)

<u>7.1 Principle</u>

The aim of the using of temporary supports (propping) is to transmit the stresses due to the self-weights of the precast unit and the in-situ concrete from the precast unit to the composite structure which has a greater stiffness.

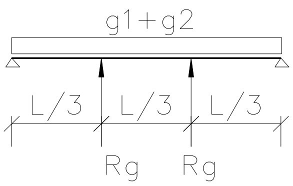
The effect of the propping is concern only to the sresses at the service limit state, not to the resistancies (flexural and shear resistancies) at the ultimate limit state.

Using the propping decreases deflection of the composite structure, tensile stresses at the bottom fibre and increases the resistance against cracking.

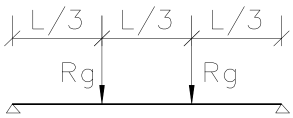
A precast unit is supported by temporary supports (propped) in the field. When the cast-insitu concrete has been hardened and the structure acts as a composite structure, the temporary supports are removed. Then the coposite structure is loaded by the point loads which corresponds the support reactions of the temporary supports.

Because the section modulus and the flexural stiffness of the composite cross-section are significantly greater than the one of the precast unit, the deflection and stresses due to these point loads are smaller than the same point loads acts to the pure precast unit (= no temporary supports, unpropped).

Propped:
Precast unit



Removing the temporary supports
Composite structure



Section modulus W1

$$M_1 := M_g - R_g \cdot \frac{L}{3}$$

$$R_g := 1.1 \cdot g \cdot \frac{L}{3}$$

$$M_g := \frac{g \cdot L^2}{8}$$

$$\sigma_1 := \frac{M_1}{W_1} = \frac{M_g}{W_1} - \frac{R_g \cdot \frac{L}{3}}{W_1}$$

Section modulus W2 > W1

$$M_2 := R_g \cdot \frac{L}{3}$$

$$\sigma_2 := \frac{M_2}{W_2} = \frac{R_g \cdot \frac{L}{3}}{W_2}$$

Total stress
$$\sigma_1 + \sigma_2 = \frac{M_g}{W_{.1}} - \frac{R_g \cdot \frac{L}{3}}{W_1} + \frac{R_g \cdot \frac{L}{3}}{W_2} = \frac{M_g}{W_1} - R_g \cdot \frac{L}{3} \cdot \left(\frac{1}{W_1} - \frac{1}{W_2} \right)$$

$$= \frac{M_g}{W_1} - R_g \cdot \frac{L}{3} \cdot \frac{\left(W_2 - W_1\right)}{W_1 \cdot W2}$$

$$= \frac{M_g - R_g \cdot \frac{L}{3} \cdot \left(1 - \frac{W_1}{W_2} \right)}{W_1}$$

$$= \frac{\frac{g \cdot L^2}{8} - \frac{1.1 \cdot g \cdot L^2}{9} \cdot \left(1 - \frac{W_1}{W_2} \right)}{W_1}$$

$$= \frac{\frac{g \cdot L^2}{8}}{W_1} \cdot \left[1 - \frac{8.8}{9} \cdot \left(1 - \frac{W_1}{W_2} \right) \right] \qquad W_2 > W_1$$

Un-propped
$$M_1 := M_g = \frac{g \cdot L^2}{8} \qquad M_2 := 0$$

$$\sigma_1 := \frac{M_g}{W_1} \qquad \sigma_2 := 0$$

$$\sigma_1 + \sigma_2 = \frac{M_g}{W_1} = \frac{\frac{g \cdot L^2}{8}}{W_1}$$

During the casting of the in.situ concrete must also be checked the flexural resistance at the point of the temporary support and possible the cracking resistance of the top fibre of the precast unit againts the support moment due to self-weights of the precast unit and the cast-in situ concrete together with the live load during the construction.

$$M_{Ed.supp} := \gamma_g \cdot M_{g1} + \gamma_g \cdot M_{g2} + \gamma_q \cdot M_{q.const} - \left(\gamma_g \cdot R_{g1} + \gamma_g \cdot R_{g2} + \gamma_q \cdot R_{q.const}\right) \cdot \frac{L}{3} \le M_{Rd.supp}$$

Mg1 is the support moment at the temporary support due to self-weight of the precast unit
Mg2 is the support moment at the temporary support due to self-weight of the in-situ concrete
Mq.const is the support moment due to live load during construction
Rg1 is the support reaction of the temporary support due to the self-weight of the precast unit
Rg2 is the support reaction of the temporary support due to the self-weight of the in-situ concrete
Rq.const is the support reaction of the temporary support due to the live load under the construction
$\gamma g=1.2$ is the partial safety factor for the dead loads
$\gamma q=1.5$ is the partial safety factor for the live loads
L is the span; the temporary supports at the 1/3-points

The stresses and the deflection are calculated at the casting time of the in-situ concrete only for the self-weights of the precast unit and the in-situ concrete.
In addition to this the stress (cracking) at the top fibre of the precast unit is checked for live load under construction ogether with the dead loads.

When the temporary supports are removed the composite structure is loaded by the point loads Rg1 and Rg2 corresponding only the support reaction of the sefl-weights of the precast unit and the cast in-situ concrete (without live load during construction) Rg1 ja Rg2

The camber of the precast unit can be controlled by the temporary supports: The precast unit can be given a desired camber by the the temporary supports (the support reactions of the temporary supports are greater than the support reactions of a 3-span continuous structure) or allow the precast unit to deflect a desired magnitude when only a part of the self-weights of the precast unit and the cast in.situ concrete causes support reactions to the temporary supports.

7.2 Example

Stresses and deformations of the precast plank slab at the casting stage of the in-situ concrete without temporary supports (un-propped):

Self-weight of the precast unit $g_1 = 6\frac{kN}{m}$ $M_{g1} = 36.75\,kNm$

Self-weight of the topping $g_2 = 1.5\frac{kN}{m}$ $M_{g2} = 9.188\,kNm$

Total dead load before the composite action

$g := g_1 + g_2$ $g = 7.5\frac{kN}{m}$

$M_1 := M_{g1} + M_{g2}$ $M_1 = 45.938\,kNm$

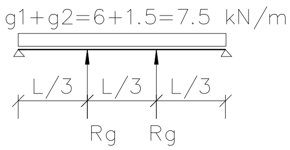
$\sigma_{cb1} = -2.929\,MPa$

Deflection $a_1 = -1.597\,mm$

Propped: Temporary supports at the 1/3-points of the span.

The desired camber of the precast unit at the casting stage of the in-situ concrete

$a_{camber} := \frac{-L}{1000}$ $a_{camber} = -7\,mm$



At the casting stage of the in-situ concrete the precast slab acts as a 3-span continuous slab which span is L/3.

The support reaction due to the self-weights of the precast slab and the topping (uniformly distributed load in all spans of the 3-span slab)

$R_g := 1.1 \cdot g \cdot \frac{L}{3}$ $R_g = 19.25\,kN$ /1.2 m

Support moment due to the support reactions $M_{Rg} := -R_g \cdot \frac{L}{3}$ $M_{Rg} = -44.917\,kNm$

Stress due to the support reactions at the point $x := \frac{L}{2}$

$M_{Rg} = -44.917\,kNm$

Axial strain at the centroid $\varepsilon_{0Rg} := 0$

Curvature $\psi_{Rg} := \frac{M_{Rg}}{EI_1}$ $\psi_{Rg} = -1.56\frac{‰}{m}$

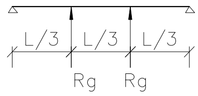
Strain at the bottom fibre of the precast slab $\varepsilon_{cbRg} := \varepsilon_{0Rg} + \psi_{Rg} \cdot p_1$ $\varepsilon_{cbRg} = -0.154\,‰$ compression

Stress at the bottom fibre of the precast slab $\sigma_{cbRg} := \varepsilon_{cbRg} \cdot E_{cm1}$ $\sigma_{cbRg} = -5.435\,MPa$ compression

Total stress at the bottom fibre at the middle point of the span before the composite action $\sigma_{cb11} := \sigma_{cb1} + \sigma_{cbRg}$ $\sigma_{cb11} = -8.364\,MPa$

Deflection at the middle point of the span

Point load at 1/3-poinst; deflection coefficient $\delta_{aRg} := \frac{1}{9.39}$ $\delta_{aRg} = 0.1065$



$a_{Rg} := \delta_{aRg} \cdot \psi_{Rg} \cdot L^2$ $a_{Rg} = -8.139\,\text{mm}$ upward

corresponds the deflection due to self-weights of the precast slab and the topping $a_{g1} + a_{g2} = 8.141\,\text{mm}$

(a small difference comes from the fact that the self-weights of the precast unit and the topping are uniformly distributed but the support reactions of the temporary supports are point loads)

Deflection before the composite action $a_{11} := a_1 + a_{Rg}$ $a_{11} = -9.735\,\text{mm}$

$> \; a_{camber} = -7\,\text{mm}$

The upward deflection is greater than the desired camber

If the certain camber is desired the support reactions (the level of the temporary supports) are chosen so that the desired camber is reached.

If the deflection $a_{11} = -9.735\,\text{mm}$ before the composite action < the desired camber => the support reaction are increased (stress at the top fibre of the precast slab at the point of the temporary support should be checked for sake of cracking)

If the deflection $a_{11} = -9.735\,\text{mm}$ before the composite action > the desired camber => the support reactions are decreased (the level of the temporary supports falls from the value a.11)

$a_{11} = -9.735\,\text{mm} > \; a_{camber} = -7\,\text{mm}$

=> the support reaction is decreased $R_{g1} := \frac{a_{Rg} - (a_{11} - a_{camber})}{a_{Rg}} \cdot R_g$ $R_{g1} = 12.78\,\text{kN}$

Deflection due to the support reaction $a_{Rg1} := a_{Rg} \cdot \frac{R_{g1}}{R_g}$ $a_{Rg1} = -5.403\,\text{mm}$

Deflection befor the composite action $a_{12} := a_1 + a_{Rg1}$ $a_{12} = -7\,\text{mm}$

$= \; a_{camber} = -7\,\text{mm}$

Moment due to the support reaction $M_{Rg} := -R_{g1} \cdot \frac{L}{3}$ $M_{Rg} = -29.821\,\text{kNm}$

Curvature $\psi_{Rg} := \frac{M_{Rg}}{EI_1}$ $\psi_{Rg} = -1.035\,\frac{‰}{\text{m}}$

Compressive strain at the bottom fibre $\varepsilon_{cbRg} := \psi_{Rg} \cdot p_1$ $\varepsilon_{cbRg} = -0.102\,‰$

Stress due to the temporary support reaction at the bottom fibre $\sigma_{cbRg} := \varepsilon_{cbRg} \cdot E_{cm1}$ $\sigma_{cbRg} = -3.608\,\text{MPa}$

Total stress at the bottom fibre $\sigma_{cb12} := \sigma_{cb1} + \sigma_{cbRg}$ $\sigma_{cb12} = -6.537\,\text{MPa}$

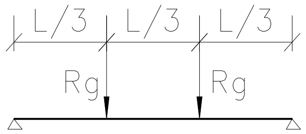
7.3 Removing the temporary supports

The temporray supports afre removed when the is-situ concrete has nbeen hardened and the structure acts as a composite structure.

Flexural stiffness of the composite section $EI = 53.778\,MN\cdot m^2$

Centroid of the composite section from the bottom fibre of the precast slab $p = 121.733\,mm$

The composte structure are loaded the downward point loads $R_{g1} = 12.78\,kN$ corresponding the support reactions of the temporary supports at the 1/3-points of the span



Moment due to the poiint loads $M_{Rg2} := R_{g1}\cdot\frac{L}{3}$ $M_{Rg2} = 29.821\,m\,kN$

Stresses and deformations due to the removing the temporary supports

Curvature $\psi_{Rg2} := \frac{M_{Rg2}}{EI}$ $\psi_{Rg2} = 0.555\,\frac{‰}{m}$

Strain at the bottom fibre $\varepsilon_{cbRg2} := \psi_{Rg2}\cdot p$ $\varepsilon_{cbRg2} = 0.068\,‰$

Stress at the bottom fibre $\sigma_{cbRg2} := \varepsilon_{cbRg2}\cdot E_{cm1}$ $\sigma_{cbRg2} = 2.378\,MPa$ tension

Stress at the bottom fibre after the removing the temporary supports before the imposed live load and the losses $\sigma_{cb21} := \sigma_{cb12} + \sigma_{cbRg2}$ $\sigma_{cb21} = -4.16\,MPa$ compression

Without th temporray supports (un-propped) the stress at the bottom fibre of the slab $\sigma_{cb1} = -2.929\,MPa$

The benefit of the temporary suppoorts $\sigma_{cb21} - \sigma_{cb1} = -1.231\,MPa$

Deflection due to the removing the temporary supports

deflection coefficient $\delta_{aRg} = 0.1065$

$a_{Rg2} := \delta_{aRg}\cdot\psi_{Rg2}\cdot L^2$ $a_{Rg2} = 2.894\,mm$ alaspäin

Deflection before the imposed live load $a_2 := a_{12} + a_{Rg2}$ $a_2 = -4.106\,mm$ upward

Deflection wihout the temporary supports before the imposed live load $a_1 = -1.597\,mm$

Effect of the temporary supports on the deflection $a_2 - a_1 = -2.51\,mm$

7.4 Effect of the temporary supports on teh cracking resistance

The effect of the temporray supports are taken into account the stresses.

Stress at the bottom fibre before the removing the temporary supports $\sigma_{cb12} = -6.537\,\mathrm{MPa}$

Stress due to removing the temporary supports $\sigma_{cbRg2} = 2.378\,\mathrm{MPa}$

Stress due to the losses and the differential shrinkage and creep $\Delta\sigma_{cb} = 1.312\,\mathrm{MPa}$

Stress due to the moment M22 affecting to the composite section σ_{cb22}

Total stress when cracking occurs $\Sigma\sigma_{cb} := \sigma_{cb12} + \sigma_{cbRg2} + \Delta\sigma_{cb} + \sigma_{cb22} = f_{ctm1} = 3.509\,\mathrm{MPa}$

$\sigma_{cb22} := f_{ctm1} - \sigma_{cb12} - \sigma_{cbRg2} - \Delta\sigma_{cb}$ $\sigma_{cb22} = 6.357\,\mathrm{MPa}$

(without the temporary supports $\sigma_{cb2} = 5.126\,\mathrm{MPa}$)

Strain corresponding the stress σcb22 at the bottom fibre $\varepsilon_{cb} := \dfrac{\sigma_{cb22}}{E_{cm1}}$ $\varepsilon_{cb} = 0.18\,‰$

Curvature due to the moment M22 $\psi_{22} := \dfrac{\varepsilon_{cb}}{p}$ $\psi_{22} = 1.483\,\dfrac{‰}{m}$

$M_{22} := \sigma_{cb22}\cdot\dfrac{EI}{E_{cm1}\cdot p}$ $M_{22} = 79.736\,\mathrm{kNm}$

or $M_{22} := \psi_{22}\cdot EI$

(without the temporary supports $M_2 := 64.3\cdot\mathrm{kNm}$)

Moment M11 before the composite action, when the stress at the bootom fibre is $\sigma_{cb12} = -6.537\,\mathrm{MPa}$

$M_{11} := M_{g1} + M_{g2} + M_{Rg}$ $M_{11} = 16.117\,\mathrm{kNm}$

Bending moment acting on the composite section $M_{Rg2} + M_{22} = 109.557\,\mathrm{kNm}$

Cracking moment $M_{cr2} := M_{11} + M_{Rg2} + M_{22}$ $M_{cr2} = 125.673\,\mathrm{kNm}$

$= M_{g1} + M_{g2} + M_{22} = 125.673\,\mathrm{kNm}$

Without the temporary supports $M_{cr} = 110.235\,\mathrm{kNm}$

Temporary supports increase the cracking resistance about 14 %

7.5 Allowable imposed live load if cracking is not allowed at the bottom fibre of the slab and the precast slab is propped

How much the allowable imposed live load can be increased by using temporary suppots ?

The influence of the shrinkage and creep difference are ignored, the the situation is comparable to the result in the pages 16...17 => the benefit of the using of the tenporary supports

Stress at the bottom fibre before the composite action when the slab is propped $\sigma_{cb12} = -6.537\,MPa$

Stress at the bottom fibre due to the removing the tenmporray supports $\sigma_{cbRg2} = 2.378\,MPa$

Stress at the bottom fibre due to the imposed live load (bending moment Mq) σ_{caq}

Total sress at the bottom fibre (effect of the diffrerential shrinkage and creep is ignored) when cracking occurs

$$\Sigma\sigma_{cb} := \sigma_{cb12} + \sigma_{cbRg2} + \sigma_{cbq} \quad = \quad f_{ctm1} = 3.509\,MPa$$

The allowable stress due to the imposed live load $\sigma_{cbq} := f_{ctm1} - \sigma_{cb12} - \sigma_{cbRg2}$ $\quad \sigma_{cbq} = 7.669\,MPa$

(without temporary supports $\sigma_{cb2} = 5.126\,MPa$)

Strain at the bottom fibre $\varepsilon_{cbq} := \dfrac{\sigma_{cbq}}{E_{cm1}}$ $\quad \varepsilon_{cbq} = 0.218\,\%o$

Curvature $\psi_q := \dfrac{\varepsilon_{cbq}}{p}$ $\quad \psi_q = 1.789\,\dfrac{\%o}{m}$

Allowable bending moment due to the imposed live load $M_q := \psi_q \cdot EI$ $\quad M_q = 96.187\,kNm$

or $M_q := \dfrac{\sigma_{caq}}{E_{cm1}} \cdot \dfrac{EI}{p}$

$M_q := \dfrac{q \cdot L^2}{8}$ => $q := \dfrac{8 \cdot M_q}{L^2}$ $\quad q = 13.184\,\dfrac{kN}{m}$

=> the allowable imposed live load $q_{adm} := \dfrac{q}{b}$ $\quad q_{adm} = 10.987\,\dfrac{kN}{m^2}$

Without the temporary supports $q_2 = 10.987\,\dfrac{kN}{m^2}$

increase due to the temporary supports 19 %

If the effect of the diffrerential shrinkage (and creep) is taken into account (calculating for a safe side if the effect of the creep difference is ignored)

$\Sigma\sigma_{cb} := \sigma_{cb12} + \sigma_{cbRg2} + \Delta\sigma_{cb} + \sigma_{cbq} \quad = \quad f_{ctm1} = 3.509\,\text{MPa}$

$\sigma_{cbq} := f_{ctm1} - \sigma_{cb12} - \sigma_{cbRg2} - \Delta\sigma_{cb} \qquad \sigma_{cbq} = 6.357\,\text{MPa}$

Strauin at the bottom fibre $\varepsilon_{cbq} := \dfrac{\sigma_{cbq}}{E_{cm1}} \qquad \varepsilon_{cbq} = 0.18\,\text{‰}$

Curvature $\psi_q := \dfrac{\varepsilon_{cbq}}{p} \qquad \psi_q = 1.483\,\dfrac{\text{‰}}{\text{m}}$

Bending moment due to the imposed live load $M_q := \psi_q \cdot EI \qquad M_q = 79.736\,\text{kNm}$

$M_q := \dfrac{q_3 \cdot L^2}{8}$ => $q_3 := \dfrac{8 \cdot M_q}{L^2} \qquad q_3 = 13.018\,\dfrac{\text{kN}}{\text{m}}$

=> the allowable imposed live load $q_{adm} := \dfrac{q_3}{b} \qquad q_{adm} = 10.848\,\dfrac{\text{kN}}{\text{m}^2}$

Without the temporary supports $q_{adm1} := \dfrac{8 \cdot M_2}{L^2 \cdot b} \qquad q_{adm1} = 8.748\,\dfrac{\text{kN}}{\text{m}^2}$

increase due to the temporary supports 24 %

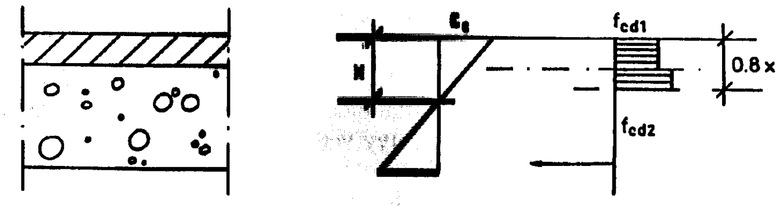
8. Flexural resistance

8.1 Principle

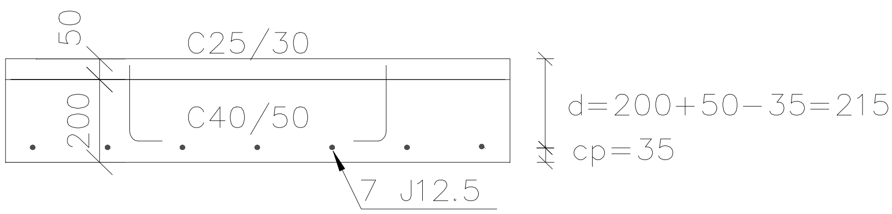
At the ultimate limit state a composite section can be treated as a monolitic structure (like a same casting) for the whole load regardless of which a part of the load is affected to the precast unit and a part of the load is affected to the composite section. A construction and loading hostory has no meaning at the ultimate limit state. **The essential prequisite for this is that the construction joint is dimensioned to resist the whole shear force due to the whole design load, so in addition to the imposed live load affecting to the composite section also the dead loads affecting to the precast unit due to the self-weights of the precast unit and the in-situ concrete.**
The flexural resistance of the composite structure is the plastic resistance of the composite section.

The effective depth of the composite section is the distance from the reinforcement in the precast unit to the top fibre of the in-situ concrete so. d=h1+ h2 - cp (for field moment).

The effectevie depth of the composite section depends on the in-situ concrete and it's tolerancies, so the partial safety factor for concrete is γc=1.5 and for the prestressing steel γs=1.15.



8.2 Cross-section and the strength values



Effective depth os obtained according to the whole depth $d := h_1 + h_2 - c_p$ $d = 215\,mm$

Widthd of the compression zone $b = 1200\,mm$

Concrete $\gamma_c := 1.5$

In-situ concfrete $f_{cd2} := 0.85 \cdot \frac{f_{ck2}}{\gamma_c}$ $f_{cd2} = 14.167\,MPa$

Precast unit $f_{cd1} := 0.85 \cdot \frac{f_{ck1}}{\gamma_c}$ $f_{cd1} = 22.667\,MPa$

(is the compression zone is reached to the precast unit)

Prestressing steel St11630 /1860 $\gamma_s := 1.15$

0.1-proof stress limit $f_{po.1k} := 1630 \cdot MPa$

Design strength $f_{pd} := \frac{f_{po.1k}}{\gamma_s}$ $f_{pd} = 1417.4\,MPa$

Ultimate strength $f_{pk} := 1860 \cdot MPa$

Yielding strain $\varepsilon_{yd} := \frac{f_{pd}}{E_p}$ $\varepsilon_{yd} = 7.269\,‰$

Ultimate strain $\varepsilon_{uk} := 35 \cdot \%o$

Max. allowable strain $\varepsilon_{ud} := 20 \cdot \%o$

Stress for the max. allowable strain $f_{pdmax} := f_{pd} + \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) \cdot \frac{(\varepsilon_{ud} - \varepsilon_{yd})}{(\varepsilon_{uk} - \varepsilon_{yd})}$

Elastic modulus $E_p = 195000\,MPa$

Amount of the strands 7 ϕ_{p7} 12.5 $A_p = 651\,mm^2$

8.3 Design loads:

Self-weight of the precast slab $g_1 = 6\,\frac{kN}{m}$

Self.weight of the topping $g_2 = 1.5\,\frac{kN}{m}$

Imposed live load $q := 12 \cdot \frac{kN}{m}$

Partial safety factors for loads: dead load $\gamma_g := 1.2$

live load $\gamma_q := 1.5$

Design load $p_d := \gamma_g \cdot (g_1 + g_2) + \gamma_q \cdot q$ $p_d = 27\,\frac{kN}{m}$

Bending moment due to the design load $M_{Ed} := \frac{p_d \cdot L^2}{8}$ $M_{Ed} = 165.375\,kNm$

8.4. Horizontal branch of the stress-strain figure of the prestressing steel

Fist the flexural resistance is calculated by using the horizontal branch of the stress-strain figure of the prestressing steel after yielding so. the stress if the prestressing steell is resitricted to the value $f_{pd} = 1.417 \times 10^3 \, MPa$ and with no limit to max. strain.

The reinforcement is assumed to yield and then the force of the reinforcement is

$N_s := f_{pd} \cdot A_p$ $\quad N_s = 922.722 \, kN$

Concrete compression resultant $\quad N_c := N_s \quad N_c = 922.722 \, kN$

Required effective height of the compression zone $\quad y := \dfrac{N_c}{b \cdot f_{cd2}} \quad y = 54.278 \, mm \; > \; h_2 = 50 \, mm$

=> height of the required compression zone if greater than the height of the topping => the compression zone reaches to the upper part of the precast slab.

Concrete compression resultant in the topping $\quad N_{c2} := b \cdot h_2 \cdot f_{cd2} \quad N_{c2} = 850 \, kN$

Concrete compression resultant in the precast slab $\quad N_{c1} := N_c - N_{c2} \quad N_{c1} = 72.722 \, kN$

Required height of the compression zone in the precast slab $\quad y_1 := \dfrac{N_{c1}}{b \cdot f_{cd1}} \quad y_1 = 2.674 \, mm$

Effective height of the compression zone $\quad y := y_1 + h_2 \quad y = 52.674 \, mm$

Height of the comporession zone $\quad \lambda := 0.8 \quad x := \dfrac{y}{0.8} \quad x = 65.842 \, mm$

Location of the concrete compression resultant from the top fibre $\quad \eta x := \dfrac{N_{c2} \cdot \dfrac{h_2}{2} + N_{c1} \cdot \left(h_2 + \dfrac{y_1}{2} \right)}{N_c} \quad \eta x = 27.076 \, mm$

Lever arm $\quad z := d - \eta x \quad z = 187.924 \, mm$

Flexural resistance $\quad M_{Rd} := N_s \cdot z \quad M_{Rd} = 173.402 \, kNm \; > \; M_{Ed} = 165.375 \, kNm$

Strains

Concrete strain $\quad \varepsilon_c := -3.5 \cdot \text{‰} = \varepsilon_{cu2}$

Additional strain of the prestressing steel $\quad \Delta\varepsilon_s := \varepsilon_c \cdot \dfrac{(x - d)}{x} \quad \Delta\varepsilon_s = 7.929 \, \text{‰}$

Strain due to prestressing $\quad \varepsilon_{p\infty} := \dfrac{\sigma_{p\infty}}{E_p} \quad \varepsilon_{p\infty} = 5.641 \, \text{‰}$

Prestress after the losses $\quad \sigma_{p\infty} = 1100 \, MPa$

Total strain of the prestressing steel $\quad \varepsilon_{p.tot} := \varepsilon_{p\infty} + \Delta\varepsilon_s \quad \varepsilon_{p.tot} = 13.57 \, \text{‰}$

$< \; \varepsilon_{ud} = 20 \, \text{‰}$

<u>8.5. Inclined branch of the stress-strain figure of the prestresssing steel</u>

The inclined branch of the stress-strain figure of the prestressing steel is taken ino account

Stress of the prestressing steel for the strain $\varepsilon_{p.tot} = 13.57\,‰$

$$f_{pd1} := f_{pd} + \left(f_{pdmax} - f_{pd}\right)\cdot\frac{\left(\varepsilon_{p.tot} - \varepsilon_{yd}\right)}{\varepsilon_{ud} - \varepsilon_{yd}} \qquad f_{pd1} = 1462.8\,\mathrm{MPa}$$

about. 3.2 % greater than fpd

Tensile force of the prestressing steel $N_s := f_{pd1}\cdot A_p$ $N_s = 952.306\,\mathrm{kN}$

Concrete compression resultant $N_c := N_s$ $N_c = 952.306\,\mathrm{kN}$

Concrete compression resultant in the topping is the same as above $N_{c2} = 850\,\mathrm{kN}$

Concrete compression resultant in the precast slab $N_{c1} := N_c - N_{c2}$ $N_{c1} = 102.306\,\mathrm{kN}$

Effective height of the compression zone in the precast slab $y_1 := \dfrac{N_{c1}}{b\cdot f_{cd1}}$ $y_1 = 2.674\,\mathrm{mm}$

Effective height of the compression zone $y := y_1 + h_2$ $y = 53.761\,\mathrm{mm}$

Height of the compression zone $\lambda := 0.8$ $x := \dfrac{y}{0.8}$ $x = 67.202\,\mathrm{mm}$

Location of the concret ecompression resultant from the top fibre

$$\eta x := \frac{N_{c2}\cdot\frac{h_2}{2} + N_{c1}\cdot\left(h_2 + \frac{y_1}{2}\right)}{N_c} \qquad \eta x = 27.888\,\mathrm{mm}$$

Lever arm $z := d - \eta x$ $z = 187.112\,\mathrm{mm}$

Flexural resistanbce $M_{Rd} := N_s\cdot z$ $M_{Rd} = 178.188\,\mathrm{kNm}$ $>$ $M_{Ed} = 165.375\,\mathrm{kNm}$

Additional strain in the prestressing steel $\Delta\varepsilon_s := \varepsilon_c\cdot\dfrac{(x - d)}{x}$ $\Delta\varepsilon_s = 7.698\,‰$

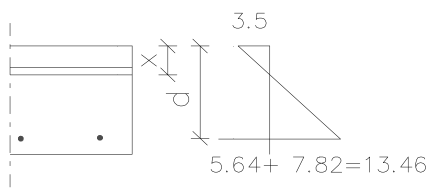
Total strain of the prestressing steel $\varepsilon_{p.tot} := \varepsilon_{p\infty} + \Delta\varepsilon_s$ $\varepsilon_{p.tot} = 13.339\,‰$

$< \ \varepsilon_{ud} = 20\,‰$

Stress of the prestressing steel for strain $\varepsilon_{p.tot} = 13.339\,‰$

$$f_{pd1} := f_{pd} + \left(f_{pdmax} - f_{pd}\right)\cdot\frac{\left(\varepsilon_{p.tot} - \varepsilon_{yd}\right)}{\varepsilon_{ud} - \varepsilon_{yd}} \qquad f_{pd1} = 1461.2\,\mathrm{MPa}$$

error 0,03 %; close to the used value



9. Shear resistance

9.1 Common

The shear resistance is calculated for the total design load by using **the effective depth (lever arm) of the whole composite section.**

Member wiht shear reinforcement:
Stirrups extend from under the tension reinforcement upto the top side of the topping.
Effective depth $d = 215\,mm$
Design strength of the shear reinforcement; partial safety factor $\gamma_S = 1.15$

Member without shear reinforcement:

a) Region, where does not exist flexural cracks; SFS-EN-1992-1-1 equation (6.4)
(hollow-core slabs (HC-slabs) SFS-EN 1168 clause 4.3.3.2.2.1 and composite slabs (HC-slabs with the topping) appendix F clause F2.2 and SFS-7016 clause F2.2)

The equation (6.4) is based on the elastic theory on uncracked cross-section; so with using the equation (6.4) should be take into account the loading history of the structure; ie. how much of total load is affected to the precast unit and how much to the composite section.

b) Region, where exist flexural cracks at the ultimate limit state; SFS-EN-1992-1-1 equation (6.2)
Effective depth $d = 215\,mm$ according to the total depth of the composite section.

Effect of the prestressing σcp is calculated for the precast unit, because the prestressing force affected before the composite action and does not cause copmpression into the in-situ concfrete.

In the continuous composite structure where the top side of the structure is tensioned due to the negative moment, the compression nstress due to prestressing is σcp=0.

The relative amount ρl of the tensile reinforcement is calculated for the whole composite section.

Concrete characteristic cylinder strength is the value of the weakest concrete in the composite section. Partial safety factor for conrete is γc=1.5.

When calculating the factor k $d = 215\,mm$

Slabs are usually without shear reinforcement.

9.2 Point from the support where exist the first crack at the ultimate limit state

Stresses at the bottom fibre (at the elastic stage):

due to the prestress after the losses $\sigma_{caP} = -8.488\,MPa$ (includes the losses due to the shrinkage an creep of the precast slab)

due to the differential shrinkage $\Delta\sigma_{cb.diff} = 0.542\,MPa$

due to the dead loads (self-weight of the precast slab and the topping $\sigma_{cag1} + \sigma_{cag2} = 5.558\,MPa$

The ultimate limit state is concerned, so the above stresses should be multiplied by the partial safety factors;

stress due to the prestress force $\gamma_p := 0.9$

stress due to the differential shrinkage $\gamma_q := 1.5$

stress due to the dead loads $\gamma_g = 1.2$

Design value of the stress at the bottom fibre before the impoised live load:

$$\sigma_{cbd} := \gamma_p \cdot \sigma_{caP} + \gamma_q \cdot \Delta\sigma_{cb.diff} \qquad \sigma_{cbd} = -6.826\,MPa$$

Stress due to the imposed live load σ_{caqd}

Total stress $\Sigma\sigma_{cb} := \sigma_{cbd} + \sigma_{cbpd} \;=\; f_{ctd1} := \dfrac{f_{ctk1}}{\gamma_c}$

Characteristic value of conrete tensile strength $f_{ctk1} := 0.7 \cdot f_{ctm1}$ $f_{ctk1} = 2.456\,MPa$

Partila safety factor for concrete $\gamma_c := 1.5$

Design value of concrete tensile strength $f_{ctd1} := \dfrac{f_{ctk1}}{\gamma_c}$ $f_{ctd1} = 1.637\,MPa$

Stress due to the design load when the cracking occurs

$$\sigma_{cbpd} := f_{ctk1} - \sigma_{cbd} \qquad \sigma_{cbpd} = 9.283\,MPa$$

A part of the stress is caused by the dead load which affect to the precast slab and rest of the stress is caused by the imposed live load which affect to the compoiste section. The stresses are divided in a same ratio than the loads.

The part of the stress to the precast slab $\sigma_{cbgd} := \sigma_{cbpd} \cdot \dfrac{\gamma_g \cdot (g_1 + g_2)}{p_d}$ $\sigma_{cbgd} = 3.094\,MPa$

The part of the stress to the composite section $\sigma_{cbqd} := \sigma_{cbpd} \cdot \dfrac{\gamma_q \cdot q}{p_d}$ $\sigma_{cbqd} = 6.188\,MPa$

For calculating the moment due to the dead load the stiffness values EI1 and p1 of the precast slab are used

Design moment due to the dead load $M_{gdx} := \sigma_{cbgd} \cdot \dfrac{EI_1}{E_{cm1} \cdot p_1}$ $M_{gdx} = 25.572\,kNm$

For calculating the moment due to the imposed live load the stiffness values EI and p of the composite section are used

Design moment due to the imposed live load $M_{qdx} := \sigma_{cbqd} \cdot \frac{EI}{E_{cm1} \cdot p}$ $M_{qdx} = 77.621\,kNm$

Total design moment when the first crack appears $M_{crdx} := M_{gdx} + M_{qdx}$ $M_{crdx} = 103.193\,kNm$

Point where the first crack appears $M_{Edx} := p_d \left(\frac{L}{2} \cdot x_{cr} - \frac{x_{cr}^2}{2} \right)$ = $M_{crdx} = 103.193\,kNm$

Solving $x_{cr} := \frac{L}{2} \cdot \left(1 - \sqrt{1 - \frac{8 \cdot M_{crdx}}{p_d \cdot L^2}} \right)$

$x_{cr} = 1.354\,m$ from the support

> the transmission length of the strand

If xcr > lpt2 so xcr must be sove by iteration because teh stresses due to the prestressing force and the bending chance with x.

9.3 Shear resistance in the region where does not excist flexural cracks

a) The region where does not exist flexural cracks at the ultimate limit state $x < x_{cr}$

Section where the shear resistance is checked is $x_v := p$ $p = 0.122\,m$ from the inner edge of the support

Support length $l_{supp} := 60 \cdot mm$

Desig value of the shear force $V_{Ed} := p_d \left(\frac{L}{2} - \frac{l_{supp}}{2} - x_v \right)$ $V_{Ed} = 90.403\,kN$

The equatioin (6.4) is based on the elastic theory; so it is taken into account how much of the load is affected to the precast unit and how much to the compoiste section.
The shearforce is divided into two parts:
- shear force Vd1 due to the laods affecting to the precast slab
- shear force Vd2 due to the imposed live load affecting to the comoosite section

$$V_{Ed1} := \gamma_g \cdot (g_1 + g_2) \cdot \left(\frac{L}{2} - x_v \right) \qquad V_{Ed1} = 30.404\,kN$$

$$V_{Ed2} := \gamma_q \cdot q \cdot \left(\frac{L}{2} - x_v \right) \qquad V_{Ed2} = 60.809\,kN$$

When calculating shear stresses due to Vd1 the section values of the prerecast slab are used and calulating shaer stresses due Vd2 the section values of the composite section are used.

The shear force Vd1 causes the greates shear stress at the centroid of the precast slab and the shear stress distribution is parabolic over the precast slab.
The shear force Vd causes the greatest shear stress at the centroid of the composiet cross-section and the shear stress distribution over the depth of the composite section is parabolic.
The locations of the greatest shear stresses do not coincide. The greatres value of the total shear stress is somewhere between the centroids of the precast slab and the coposite section. The value on the safe side can be obtained by adding the both greatest values together.

Precast slab

The first moment of area below the point concerned about the centroid p1 of the precast slab

$$ES_1(y) := E_{cm1} \cdot b \cdot y \cdot \left(p_1 - \frac{y}{2} \right) + E_p \cdot A_p \cdot (p_1 - c_p)$$

The first moment of area below the centroid $ES_{1p} := ES_1(p)$ $ES_{1p} = 204.015\,MN \cdot m$

$V_{Ed1} = 30.404\,kN$

The greatest shear stress $\tau_{1max} := \frac{V_{Ed1} \cdot ES_{1p}}{b \cdot EI_1}$ $\tau_{1max} = 0.179\,MPa$

$$\tau_1(y) := \frac{V_{Ed1} \cdot ES_1(y)}{b \cdot EI_1}$$

Composite section

The first moment of area below the point concerned about the centroid p of the composite section

$$ES_2(y) := E_{cm1} \cdot b \cdot y \cdot \left(p - \frac{y}{2}\right) + E_p \cdot A_p \cdot \left(p - c_p\right)$$

The first moment of area below the centroid $ES_{2p} := ES_2(p)$ $ES_{2p} = 324.169\,MN \cdot m$

$V_{Ed2} = 60.809\,kN$

The greatest shear stress $\tau_{2max} := \dfrac{V_{Ed2} \cdot ES_{2p}}{b \cdot EI}$ $\tau_{2max} = 0.305\,MPa$

$$\tau_2(y) := \frac{V_{Ed2} \cdot ES_2(y)}{b \cdot EI}$$

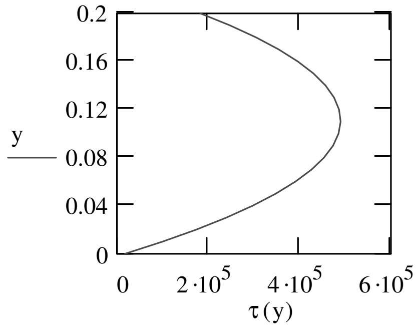
Total shear stress $\tau_{max} := \tau_{1max} + \tau_{2max}$ $\tau_{max} = 0.485\,MPa$

$$\tau(y) := \tau_1(y) + \tau_2(y)$$

<u>9.3.1 Total shear stress at different points due to the shear forces Vd1 and Vd2</u>

$y := 0\cdot mm, 10\cdot mm .. 200\cdot mm$

y = (mm)	$\tau_1(y)$ = (MPa)	$\tau_2(y)$ = (MPa)	$\tau(y)$ = (MPa)
0	$7.141\cdot 10^{-3}$	0.01	0.018
10	0.042	0.057	0.099
20	0.073	0.099	0.173
30	0.101	0.138	0.239
40	0.125	0.172	0.297
50	0.145	0.203	0.348
60	0.161	0.23	0.391
70	0.174	0.252	0.426
80	0.182	0.271	0.453
90	0.188	0.285	0.473
100	0.189	0.296	0.485
110	0.187	0.303	0.49
120	0.181	0.305	0.486
130	0.171	0.304	0.475
140	0.158	0.299	0.457
150	0.141	0.29	0.43



The point of the greatest shear stress can be obtained by derivatibg τ(y)

$$\frac{d}{dy}\tau(y) = \frac{V_{Ed1}\cdot E_{cm1}\cdot b\cdot(p_1 - y)}{b\cdot EI_1} + \frac{V_{Ed2}\cdot E_{cm1}\cdot b\cdot(p - y)}{b\cdot EI}$$

Solving $y := \dfrac{V_{Ed1}\cdot p_1\cdot EI + V_{Ed2}\cdot p\cdot EI_1}{V_{Ed1}\cdot EI + V_{Ed2}\cdot EI_1}$ $y = 110.729\,mm$

$\tau_1(y) = 0.187\,MPa$ $\tau_2(y) = 0.303\,MPa$ $\tau(y) = 0.49\,MPa$

9.3.2 Prestress at the point $x_v = 0.122\,m$ from the support

Transfer (release) of the prestress force

Average concrete compressive strength at the time of the transfer $f_{cmi} := 0.75\cdot(f_{ck1} + 8\cdot MPa)$ $f_{cmi} = 36\,MPa$

$\beta_{cci} := \dfrac{f_{cmi}}{f_{ck1} + 8\cdot MPa}$ $\beta_{cci} = 0.75$

Concrete tensile strength at the time of the transfer $f_{ctdi} := \dfrac{\beta_{cc}\cdot f_{ctk1}}{1.5}$ $f_{ctdi} = 1.228\,MPa$

Prestess just before the transfer is assumed $\sigma_{pm0} := 1250\cdot MPa$

Bond coefficient $\eta_{p1} := 3.2$

Bottom strands; "good" bond condition $\eta_1 := 1.0$

Bond strength $f_{bpt} := \eta_{p1}\cdot\eta_1\cdot f_{ctdi}$ $f_{bpt} = 3.93\,MPa$

The strands are cutted by sawing => sudden release => $\alpha_1 := 1.25$

7-wire strand ϕp7 $\phi_p := 12.5\cdot mm$

Factor $\alpha_2 := \dfrac{A_{p1}}{\pi\cdot\phi_p^{\,2}}$ $\alpha_2 = 0.19$

The basic value of the transmission length $l_{pt} := \alpha_1\cdot\alpha_2\cdot\phi_p\cdot\dfrac{\sigma_{pm0}}{f_{bpt}}$ $l_{pt} = 942\,mm$

The design value of the transmission length:

Lower design value $l_{pt1} := 0.8\cdot l_{pt}$ $l_{pt1} = 753\,mm$

Upper design value $l_{pt2} := 1.2\cdot l_{pt}$ $l_{pt2} = 1130\,mm$

Prestress at the point $x_v = 0.122\,m$ $\sigma_{px} := \sigma_{p\infty}\cdot\dfrac{x_v + l_{supp}}{l_{pt2}}$ $\sigma_{px} = 176.922\,MPa$

Prestress force $P_x := \sigma_{px}\cdot A_p$ $P_x = 115.176\,kN$

The average compressive stress due to the prestress force (here compression stress is positive)

$\sigma_{cp} := \dfrac{P_x}{EA}\cdot E_{cm1}$ $\sigma_{cp} = 0.388\,MPa$

<u>9.3.3 Principal tensile stress</u> $\sigma_1 := \frac{-\sigma_{cp}}{2} + \sqrt{\frac{\sigma_{cp}^2}{4} + \tau(y)^2}$ $\sigma_1 = 0.333\,\text{MPa}$

$< \; f_{ctd1} = 1.637\,\text{MPa}$

The shear resistance is adequite

<u>9.3.4 Shear resistance when the principal tensile stress</u> $\sigma_1 := f_{ctd1}$

Solving the shear strength $\tau_{Rd} := \sqrt{f_{ctd1} \cdot (f_{ctd1} + \sigma_{cp})}$ $\tau_{Rd} = 1.821\,\text{MPa}$

The greatest shear stress at the distance $y = 110.729\,\text{mm}$ from the bottom fibre

Shear stress due to the dead load before the composite action $\tau_1(y) = 0.187\,\text{MPa}$

Shear strength for the loads after the composite action $\tau_{Rd2} := \tau_{Rd} - \tau_1(y)$ $\tau_{Rd2} = 1.635\,\text{MPa}$

Shear resistance due to the live load $V_{Rd2} := \tau_{Rd2} \cdot \frac{EI \cdot b}{ES_2(y)}$ $V_{Rd2} = 328.023\,\text{kN}$

Shear resistance $V_{Rd} := V_{Ed1} + V_{Rd2}$ $V_{Rd} = 358.427\,\text{kN}$

$> \; V_{Ed} = 90.403\,\text{kN}$

We want to know what is the greatest imposed live load which the structure can resist when the structure bears alsothe dead loads. The dead load does not change but the imposed live load is variable. So the ratio of the shear forces VRd2/VEd1 is variable and they affect to the different corss-sections.
In the first step the location of the greates shear stress is calculated according to the shear forces due to the design loads. When the shear resistance is greater than the shaer force due to the design loads the structure canresist greater imposed load than the design value when the dead load does not change. So the location of the greates shear stress changes a little. In the next step the location of the greatest shear stress is calcultated by using VRd2=VRd-VEd1.

Because the ratio VRd2/VEd1 is different than the ratio VEd2/VEd1, so the greatest shear stress is nearer to the centroid of the composite section than the value used above.

$$y := \frac{V_{Ed1} \cdot p_1 \cdot EI + V_{Rd2} \cdot p \cdot EI_1}{V_{Ed1} \cdot EI + V_{Rd2} \cdot EI_1} \qquad y = 118.371\,\text{mm}$$

Shear stress due to the dead loads before the composite action $\tau_1(y) = 0.182\,\text{MPa}$

Shear strength for the loads after the composite action $\tau_{Rd2} := \tau_{Rd} - \tau_1(y)$ $\tau_{Rd2} = 1.639\,\text{MPa}$

Shear resistance due to the live load $V_{Rd2} := \tau_{Rd2} \cdot \frac{EI \cdot b}{ES_2(y)}$ $V_{Rd2} = 326.557\,\text{kN}$

Shear resistance $V_{Rd} := V_{Ed1} + V_{Rd2}$ $V_{Rd} = 356.961\,\text{kN}$

$> \; V_{Ed} = 90.403\,\text{kN}$

$$y := \frac{V_{Ed1} \cdot p_1 \cdot EI + V_{Rd2} \cdot p \cdot EI_1}{V_{Ed1} \cdot EI + V_{Rd2} \cdot EI_1} \qquad y = 118.358\,\text{mm}$$

enough close to the value calculated above

9.3.5 Shear resistance of the precast slab

$\tau_1(y) = \tau_{Rd}$ $\quad V_{Rd1} := \tau_{Rd} \cdot \frac{EI_1 \cdot b}{ES_{1p}}$ $\quad V_{Rd1} = 308.527\,kN$

The shear resistance of the composite section is about 16 % greater than the one of the precast slab.

9.4 Shear resistance in the region where exist flexural cracks

b) Region where exist flexural cracks at the ultimate limit state x> $x_{cr} = 1.354\,m$

Design shear force $V_{Ed} := p_d \cdot \left(\frac{L}{2} - x_{cr} \right)$ $\quad V_{Ed} = 57.947\,kN$

Because the section concerned is cracked at the ultimate limit state and the calculation equations (6.2) do not based on the elastic theory, the construction and the loading history does not affect on the shear resistance; so how much load affect on the precast slab and how much on the composite section is meaningless.

The characteristic value of concrete strength is the one of the weakest concrete of the parts

Characteristic value of the concrete strength $f_{ck} := \min(f_{ck1}, f_{ck2})$ $\quad f_{ck} = 25\,MPa$

artia safety factor for concrete $\gamma_c = 1.5$

9.4.1 Anchorage length of the strand

7-wire strand; bond coefficient for achoring against the load at the ultimate limit state $\eta_{p2} := 1.2$

Bottom strands; good bond condition => $\eta_1 = 1$

Design tensile strength of the precast slab concrete (where the strands are located) $f_{ctd1} = 1.637\,MPa$

Bond strength for anchoring the load at the ultimate limit state

$f_{bpd} := \eta_{p1} \cdot \eta_1 \cdot f_{ctd}$ $\quad f_{bpd} = 3.83\,MPa$

Anchorage length corresponding the full design strength of the strand $\sigma_{sd} := f_{pd}$ $\quad f_{pd} = 1417.4\,MPa$

Prestress after the losses $\sigma_{p\infty} = 1100\,MPa$

Anchorage length corresponding the design strength of the strands

$$l_{bpd} := l_{pt2} + \alpha_2 \cdot \phi_p \cdot \frac{\left(f_{pd} - \gamma_p \cdot \sigma_{p\infty}\right)}{f_{bpd}} \qquad l_{bpd} = 1394\,mm$$

Anchorage length of the strand form the point cr to the end of the slab

$$l_{bpd.real} := \frac{l_{supp}}{2} + x_{cr}$$

$$l_{bpd.real} = 1384\,mm \;\sim\; l_{bpd}$$

The strands are anchored from the considerd point xr almost for the full design strength

Inclination angle of the compressive strut $\cot\theta := 2.5$

Tensile stress in the reinforcement at the point considered

$$N_s := \frac{M_{crdx}}{z} + V_{Ed} \cdot \cot\theta \qquad N_s = 696.371\,kN$$

Stress in the strands at the ultimate limit stat $\quad \sigma_{sd} := \frac{N_s}{A_p} \qquad \sigma_{sd} = 1.07 \times 10^3\,MPa \;<\; f_{pd}$

Required anhorage length $\quad l_{bpd} := l_{pt2} + \alpha_2 \cdot \phi_p \cdot \frac{\left(\sigma_{sd} - \gamma_p \cdot \sigma_{p\infty}\right)}{f_{bpd}} \qquad l_{bpd} = 1179\,mm$

$$< \; l_{bpd.real} = 1384\,mm$$

9.4.2. Shear resistance

The whole amount of the reinforcement can be taken into the calculation of the longitudinal reinforcement ratio ρl

Reinforcement ratio $\rho_1 := \frac{A_p}{b \cdot d}$ $\rho_1 = 0.252\,\%$

Shear strength $C_{Rdc} := \frac{0.18 \cdot MPa}{\gamma_c}$ $C_{Rdc} = 0.12\,MPa$

$k := 1 + \sqrt{\frac{200 \cdot mm}{d}}$ $k = 1.964$ < 2.0

Web width $b_w := b$ $b_w = 1200\,mm$

Effective depth $d = 215\,mm$

Prestress at the point xcr: $\sigma_{p\infty} = 1100\,MPa$

$P_\infty := A_p \cdot \sigma_{p\infty}$ $P_\infty = 716.1\,kN$

Normal force $N_{Ed} := \gamma_p \cdot P_\infty$ $N_{Ed} = 644.49\,kN$

Average compression stress due to prestress $\sigma_{cp1} := \frac{N_{Ed}}{EA_1} \cdot E_{cm1}$ $\sigma_{cp1} = 2.653\,MPa$

here compression is positive

Factor $k_1 := 0.15$

The compression stress due to the prestress force is only in the precast slab, not in nthe topping. According to BY 210 in a composite structure a compressive stress due to prestress should not be taken into account. Then the calculation of the shear rsistance is in the safe side.

Minimum bvalue of the shear strength $v_{min} := 0.035 \cdot k^{\frac{3}{2}} \cdot \sqrt{\frac{f_{ck}}{MPa}} \cdot MPa$ $v_{min} = 0.482\,MPa$

Shear resistance of the structure without shear reinforcement

$$V_{Rdc1} := \left[C_{Rdc} \cdot k \cdot \left(100 \cdot \rho_1 \cdot \frac{f_{ck}}{MPa} \right)^{\frac{1}{3}} + k_1 \cdot \sigma_{cp1} \right] \cdot b_w \cdot d \qquad V_{Rdc1} = 215.039\,kN$$

The minimum valaue of the shear resistance

$$V_{Rdc2} := \left(v_{min} + k_1 \cdot \sigma_{cp1} \right) \cdot b_w \cdot d \qquad V_{Rdc2} = 226.978\,kN$$

Shear resistance $V_{Rdc} := \max\left(V_{Rdc1}, V_{Rdc2}\right)$ $V_{Rdc} = 226.978\,kN$

$> V_{Ed} = 57.947\,kN$

9.4.3 Shear resistance of the precast slab in the cracked region

Effective depth $d_1 := h_1 - c_p$ $d_1 = 165\,mm$

Reinforecement ratio $\rho_1 := \frac{A_p}{b \cdot d_1}$ $\rho_1 = 0.329\,\%$ < 2 %

Factor $k := \min\left(1 + \sqrt{\frac{200 \cdot mm}{d_1}}, 2\right)$ $k = 2$

Average compressive stress due to prestress $\sigma_{cp1} = 2.653\,MPa$

Characteristic compressive strength $f_{ck1} = 40\,MPa$

Shear resistance of the precast slab without shear reinforcement

$$V_{Rdc11} := \left[C_{Rdc} \cdot k \cdot \left(100 \cdot \rho_1 \cdot \frac{f_{ck1}}{MPa}\right)^{\frac{1}{3}} + k_1 \cdot \sigma_{cp1}\right] \cdot b_w \cdot d_1 \qquad V_{Rdc11} = 190.954\,kN$$

Minimum value of the shear resistance

$$V_{Rdc21} := \left(v_{min} + k_1 \cdot \sigma_{cp1}\right) \cdot b_w \cdot d_1 \qquad V_{Rdc21} = 174.192\,kN$$

Shear resistance $V_{Rdc} := \max\left(V_{Rdc11}, V_{Rdc21}\right)$ $V_{Rdc} = 190.954\,kN$

$> V_{Ed} = 57.947\,kN$

Due to the composite action shear resistance is about 19 % greater than without composite action

The minimum shear reinforcement may be omitted in the slabs (SFS-EN-1992-1-1 clause 6.2.1(4))

10. Shear resistance of the construction joint (horizontal shear)

The compression resultant locates in the topping above the construction joint;
the tensile counterfoce is in the reinforcement of the precast slab under the construction joint.
The compression resultant transmits across the construction joint to the precast unit and there to the support by the inclined strut at the distance of the max. bending moment and the zero point of the bending moment.
So in the construction joint there is a horizontal shear flow (a horizontal component of the inclined compression strut), which resultant at the distance between the point of the max. bending moment and the point of the zero moment corresponds the compression resultant above the construction joint due to the maximum bending moment.
At the distance of the length unit Δx the horizontal shear force is the same as the change of the compression resultant at this distance.

The construction joint is dimensioned to resist the shear force due to the total load (in addition to the imposed live load after composite action also the dead load before the composiet action.

Concrete compression resultant above the construction joint at the point x $N_{c2x} := \frac{\beta \cdot M_x}{z}$ $\beta \le 1$

Concrete compression resultant above the construction joint at the point x-Δx $N_{c2\Delta x} := \frac{\beta \cdot M_{\Delta x}}{z}$

The change of the compression resultant above the construction joint at the distance Δx

$$\Delta N_{c2} := N_{c2x} - N_{c2\Delta x} = \frac{\beta \cdot M_x}{z} - \frac{\beta \cdot M_{\Delta x}}{z} = \beta \cdot \frac{(M_x - M_{\Delta x})}{z} = \beta \cdot \frac{\Delta M}{z}$$

$$v \cdot b_i := \frac{\Delta Nc2}{\Delta x} = \beta \cdot \frac{\Delta M}{z \cdot \Delta x} = \frac{\beta V}{z} \qquad v := \frac{\beta V}{bi \cdot z}$$

bi is the width of the joint

According to the Eurocode the shear stress distribution is assumed according to the elastic theory to be the same as the distribution of the shear force.

The critical point is at the distance of effective height d from the inner edge of the support

Design value of the shear force $V_{Ed} := p_d \cdot \left(\frac{L}{2} - \frac{l_{supp}}{2} - d \right)$ $V_{Ed} = 87.885\,kN$

Lever arm $z = 187.112\,mm$

The compression resultant corresponding the flexural resistanc $N_c = 952.306\,kN$

The compression resultant above the construction joint $N_{c2} = 850\,kN$

Factor $\beta := \frac{N_{c2}}{N_c}$ $\beta = 0.893$

Width of the joint $b_i := b$ $b_i = 1200\,mm$

Design value of the shear stress in the contruction joint (SFS-EN-1992-1-1 equation (6.24))

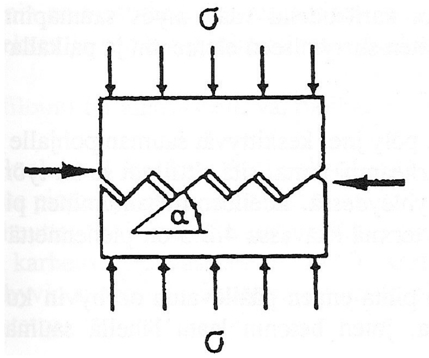
$$v_{Ed} := \beta \cdot \frac{V_{Ed}}{b_i \cdot z} \qquad v_{Ed} = 0.349\,MPa$$

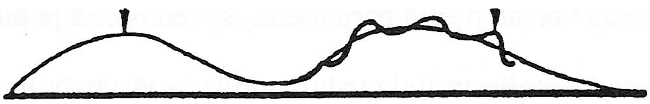
In the Eurocode the shear resistance of the construction joint is based on the shear-friction theory with a cohesion. The joint surfaces is assumed to be rough (even a smooth surface is not perfectllhy smoot but there is always somekind small roughness). When the joint is loaded by a horizontal shear flow the parts gets a small displacement in the direction of the shear force. So the knots of the both parts due to the roughness slides to one upon another overlapps) and the parts will tend to separate from each others. The reinforcement which goes across the joint and is achored for the full yield force to the both parts will resist this separation with it's axial stiffness getting tension and pushimg the parts close to each other. In the joint surface arise the compression stress which resultant is the counterforce of the tensile froce of the reinforcement. Beacuse the joint surface is rough there is friction at the surfaces of the joint due to this compression stress and so the joint is capable to transmit shear force parallel to the joint surface. When the shear deformation is enough large the reinforcement yields and deflects. Shear reinforcement is dimensioned for a tensile force due to the shear friction not a shear force. If the bending stiffness of the reinforcment bar is greater then it acts as a dowel which transmit shear force by the shear resistance of the dowel bar. In this case is assumed that the bending stiffness of the shear reinforcement bar is so small that it deflects adue to shear deformation geting tension stress instead of shear stress. An essential prerequisite is that the shear reinforcement is anchored for the full yield force into the both parts.

In the joint there is greater overall roughness and alsop smalle microroughness.

In the Eurorcode shear resistance equation the friction force represents the term $\mu \rho f_{yd}$ where μ is the friction coefficient of the joint surface, ρ is the ratio of the shear reinforcement which goes across the joint and fyd is the design strength of the reinforcement. If in the joint surface is additional compression stress perpendicular the surface it causes also friction.

In addition to the friction in the joint there is also bond (cohesion) between the concrete surfaces. This is taken into account with the term c fctd, where c is the cohesion coefficient and fctd is the design tensile strength of the weakest concrete of the parts. The cohesion term represents a microroughness.





Shear-friction mechanism

Overall roughness microroughness

The top surface of the prestressed plank slab (and a hollow.core slab) is extruded surface => smooth SFS-EN-1992-1-1.6.2.5(2).

Cohesio-coefficient $c := 0.2$ friction coefficient $\mu := 0.6$

Design value of the concrete tensile strength is the lower value of the tensile strengths of the concretes in the both sides of the joint

$f_{ctd} := \min\left(f_{ctd1}, f_{ctd2}\right)$ $f_{ctd} = 1.198\,\text{MPa}$

Design strength of the joint reinforcement (A500HW) $f_{yd} := \dfrac{500 \cdot \text{MPa}}{\gamma_s}$ $f_{yd} = 434.783\,\text{MPa}$

Vertical links, the inclination of the reinforcement $\alpha := 90$ $\sin\alpha := 1$ $\cos\alpha := 0$

SFS-EN-1992-1-1 equation (6.25)

$$v_{Rdi} := c \cdot f_{ctd} + \mu \cdot \sigma_n + \rho \cdot f_{yd} \cdot (\mu \cdot \sin\alpha + \cos\alpha)$$

Compression stress perpendicular the joint $\sigma_n := 0 \cdot MPa$

(Remark! $\sigma n < 0.6 \cdot \min(fcd1, fcd2)$ If σn is tension, so the cohesion term c fctd=0)

Joingt reinforcement A500HW characteristic strength $f_{yk} := 500 \cdot MPa$

partial safety factor $\gamma_s := 1.15$

design strength $f_{yd} = 434.783\,MPa$

The required amount of the joint reinfrocement is solved when the resistance of the joint is $v_{Rdi} := v_{Ed}$

$$\rho := \frac{v_{Ed} - c \cdot f_{ctd}}{f_{yd}} \qquad \rho = 2.523 \times 10^{-4}$$

Amount of the reinforcement $A_{sv} := \dfrac{\rho \cdot b_i \cdot 1000 \cdot \frac{mm}{m}}{2}$ $\qquad A_{sv} = 151.395 \dfrac{mm^2}{m}$

The point from which the cohesion alone is enough without reinforcement; so vEdx=c*fctd

$$v_{Rdc} := c \cdot f_{ctd} \qquad v_{Rdc} = 0.24\,MPa$$

$$x_1 := \frac{v_{Rdc}}{v_{Ed}} \cdot \left(\frac{L}{2} - \frac{l_{supp}}{2} - d\right) \qquad x_1 = 2.233\,m$$ form the support

The joint reinforcement can be stepped according to SFS-EN-1992-1-1 figure 6.10 but Eurocode does not give the distance where the stepping can be made. The measure of the stepping can be used the distance z cotθ = z ... 2.5 z (SFS-EN-1992-1-1 6.2.3(5) because it is same thing as the dimensioning of the normal shear reinfoercement in a web.

Stepping distance $z = 187\,mm$ ... $2.5 \cdot z = 468\,mm$

Eurocode does not give a minimun amount of the joint freinforcement, but beacuse it is a same thing as the dimensioning of the web reinforcement, the same minimum amount is used also for the joint reinforcement,.

Concrete compressive strength is the lower value of the one of the concretes on the both side of the joint.

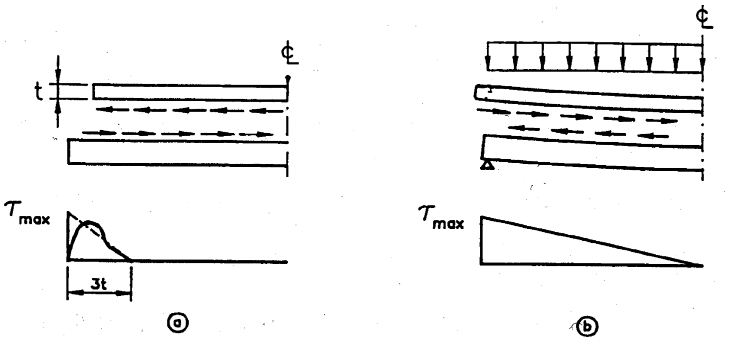
$$f_{ck} := \min(f_{ck1}, f_{ck2}) \qquad f_{ck} = 25\,MPa$$

$$\rho_{w.min} := \frac{0.08 \cdot \sqrt{\frac{f_{ck}}{MPa}} \cdot MPa}{f_{yk}} \qquad \rho_{w.min} = 0.08\,\%$$

$$A_{sv.min} := \rho_{w.min} \cdot b_i \cdot 1000 \cdot \frac{mm}{m} \qquad A_{sv.min} = 960 \frac{mm^2}{m} \quad > \quad A_{sv} = 151.395 \frac{mm^2}{m}$$

The shear stress in the joint due to the differential shrinkage is in the opposite direction than the shear stresses du to the imposed loads, so the required joint reinforcement is the greater amount of the required values for the differential shrinkage and the impoised loads.

The differential shrinkage: the bottom fibre of the topping will tend to shorten
Due to imposed loads: the bottom fibre of the topping will tend to lengthen



Shear stress in the construction joint due to the differential shrinkage — due to the imposed loads

The required joint reinforcement due to the differential shrinkage at the end of the member at the distance z1

$z_1 = 150\,\text{mm}$ $\quad A_{sv.sh} = 261.601\,\text{mm}^2$

The required joint reinforcement at the distance 150 mm from the end of the slab

$A_{sv.150} := \max(A_{sv} \cdot z_1, A_{sv.sh}, A_{sv.min} \cdot z_1)$ $\quad A_{sv.150} = 261.601\,\text{mm}^2$

3 T 8 2-legg links c/c 50 $\quad A_{sv.150} := 3 \cdot 2 \cdot 50.3 \cdot \text{mm}^2$ $\quad A_{sv.150} = 301.8\,\text{mm}^2$

At the distance $z_1 = 150\,\text{mm}$ .... $x_1 = 2.233\,\text{m}$ the required joint reinforcement

$A_{sv2} := \max(A_{sv}, A_{sv.min})$ $\quad A_{sv2} = 960 \dfrac{\text{mm}^2}{\text{m}}$

2 pcs T8 2-legg links c/c 200 $\quad A_{sv2} := 2 \cdot 2 \cdot \dfrac{50.3 \cdot \text{mm}^2}{200 \cdot \text{mm}}$ $\quad A_{sv2} = 1006 \dfrac{\text{mm}^2}{\text{m}}$

At the distance $x_1 = 2.233\,\text{m}$ .... $\dfrac{L}{2} = 3.5\,\text{m}$ no joint reinforcement is needed, the cohesion alone is adequite

The links must be anchored for the full yield force into the both sides of the joint.