

Lecture 10

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I. SINGLE QUBIT OPERATIONS

Initialization, gates, and readout:

- To execute quantum algorithms, one needs three main operations: **initialization, gates, readout**.
- Usually one wants to **isolate** the quantum circuits as much as possible, however, the above operations need a certain **interaction**.
- Due to noise and other parasitic couplings, the operations we perform are **never ideal** and we will always make errors.

The basic elements of quantum algorithms:

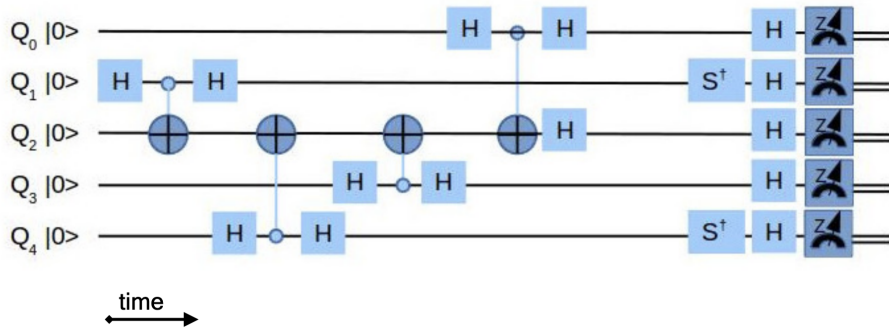


FIG. 1: An example quantum circuit.

As seen in the above figure, it is essential to reliably reset the states, i.e. initialize all qubits to state $|0\rangle$ (2nd DiVincenzo criteria). This is followed by Hadamard, cNOT, and S^\dagger gates, all before readout.

Initialization:

- To **start a computation**, it is necessary that all registers are in a well-known state, typically $|0\rangle$.
- For experimental implementation, **thermal noise** always drives us out of equilibrium when approaching $|0\rangle$.

- We can compensate this by **actively cooling** or using more advanced feedback or pulse schemes.

Short recap: the qubit state

We can effectively describe our non-linear superconducting circuit as a quantum 2-level system following the Hamiltonian

$$\mathcal{H}_q = \frac{\omega_q}{2} \hat{\sigma}_z . \quad (1)$$

In the absence of decoherence, we describe the qubit dynamics by a time evolution of the Bloch vector

$$|\Psi\rangle = \cos \frac{\theta}{2} |g\rangle + e^{i\varphi} \sin \frac{\theta}{2} |e\rangle = \begin{pmatrix} \cos \frac{\theta}{2} \\ e^{i\varphi} \sin \frac{\theta}{2} \end{pmatrix} . \quad (2)$$

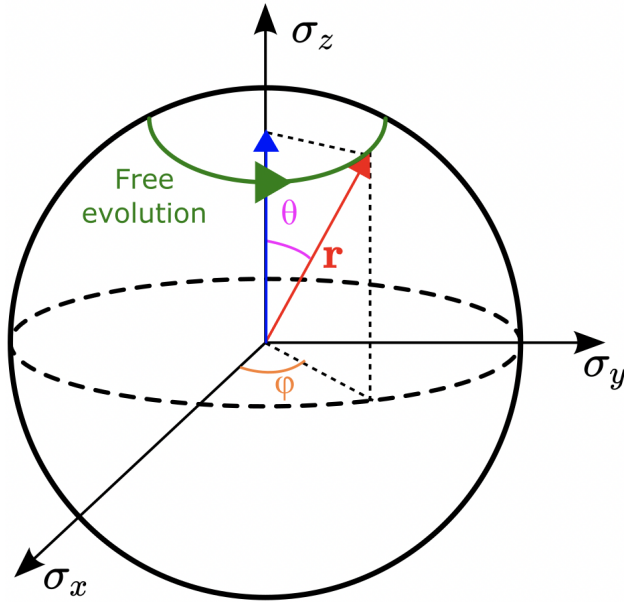


FIG. 2: A qubit in superposition as represented on the Bloch sphere.

This allows us to define qubit operations as rotations with certain angles carried out on the qubit state in the basis

$$|g\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} , \quad |e\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} . \quad (3)$$

Note: Phases, such as $e^{i\varphi}$ matter so long as they are not global, in which case they can be neglected. That's why it's important to initialize states such that global phases can be ignored. Initialization means bringing the controlled qubit into the ground state.

In an experimental setup, circuits are never isolated but coupled to an environment, e.g., the silicon substrate. Hence, they will end up in a thermal state.

The probability to find the circuit in a thermally excited state is given by the Maxwell-Boltzmann distribution

$$P_{|i\rangle} = \frac{1}{Z} g_i \exp(-E_i/k_B T) . \quad (4)$$

Here, $Z = \sum_j g_j \exp(-E_j/k_B T)$ is the partition function, g_i is the degeneracy of each energy level E_i . For convenience, we set $E_0 = 0$.

For a “hot” 2-level system without degeneracy ($g_i = 1$), $Z = 2$, we find $P_{|i\rangle}(T_{\text{hot}}) = 1/2$.

In experiments, we want $P_{|1\rangle}(T_{\text{cold}}) \ll 1\%$. Hence, we need $k_B T \ll \hbar\omega_q$.

- Superconducting circuits need to be **cooled to mK temperatures** to avoid thermal excitation (50 mK \longleftrightarrow 1 GHz).
- Initialization can be done by waiting several qubit lifetimes ($5T_1 \sim 0.5$ ms).

Feedback-based reset

Instead of physical cooling, the effective temperature of a qubit can be reduced by feedback control.

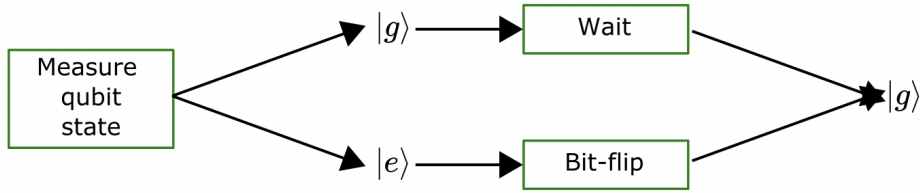


FIG. 3: Feedback-based reset.

The disadvantage of this approach is the relatively long feedback cycle $\sim 1.5 \mu\text{s}$.

Alternatively, instead of waiting, one can **actively cool the system** further (on-chip or feedback), or one can apply pulse-based reset schemes. See for example, arXiv:1801.07689.

On the chip, active cooling can be achieved through a quantum circuit refrigerator (QCR). See for example, Nat. Commun. 8, 15189 and Phys. Rev. B 96, 094524.

Qubit readout

- To **determine the qubit state**, we must project it onto an eigenstate and measure the expectation value.
- Hence, measurements must be **averaged many times** to distill the relevant probability distribution.
- We don't read out the qubit directly but use the interaction with a superconducting resonator. This is a precise and fast way to perform **single-shot qubit readout**.

Dispersive readout of qubits

We consider a qubit strongly coupled to a resonator in the dispersive limit of the Jaynes-Cummings model:

$$H_{\text{disp}} = \left(\omega_r + \chi \sigma_z \right) \left(a^\dagger a + \frac{1}{2} \right) + \frac{\tilde{\omega}_q}{2} \sigma_z . \quad (5)$$

Where the coupling term $\chi = g^2/\Delta$ is the qubit-state dependent frequency shift, a so-called *dispersive shift*.

The phase shift can be detected either by recording a complete spectrum or in the time trace of a single shot.

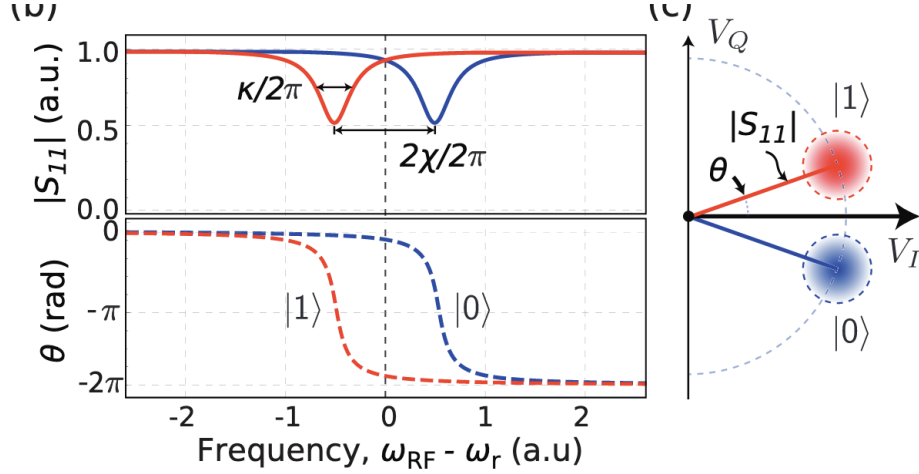


FIG. 4: From arXiv:1904.06560.

In the above figure, we see the reflected magnitude $|S_{11}|$ and phase response θ of the resonator when the qubit is in the ground state (blue) $|0\rangle$ and excited state (red) $|1\rangle$. They are separated by a frequency χ/π . To the right, we have the complex plane representation, where each point has an in-plane $\text{Re}[S_{11}]$ and quadrature $\text{Im}[S_{11}]$ component.

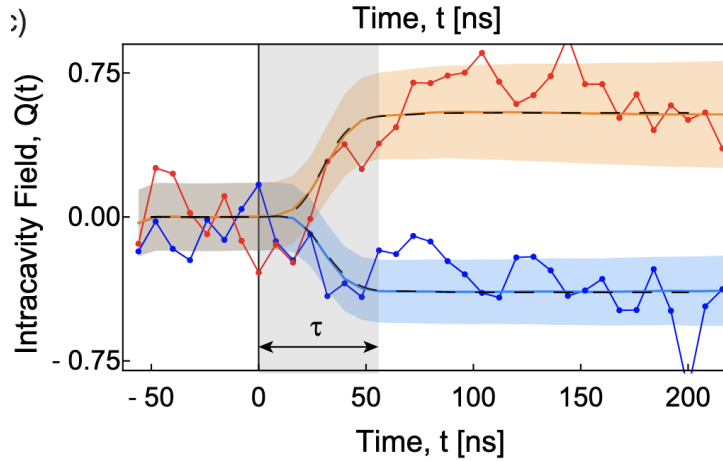


FIG. 5: From arXiv:1701.06933.

Fig. 5 shows the characteristic ground (blue) and excited (red) state single-shot tra-

jectories plotted along with the mean of each trajectory (solid lines), and with the their respective standard deviations (shaded area).

Qubit readout is done **through a resonator** and can be achieved in less than 100 ns for a **single shot** approach.

Qubit Control

- To **control the qubit state**, we must be able to control the direction of the Bloch vector.
- We do this by applying short microwave pulses with well defined amplitude and phase to the qubits.
- By applying certain pulse sequences, we are able to extract important qubit parameters like its decay and decoherence rate.

Controlling the qubit state through microwave pulses

To control the qubit state, we use an on-chip control line and apply coherent microwave signals. They add an energy term to the Hamiltonian:

$$\mathcal{H}_{d,q} = \frac{\hbar}{2} \left[\omega_q \hat{\sigma}_z + 2\Omega_d \cos(\omega_d t) \hat{\sigma}_x \right]. \quad (6)$$

Using a unitary transformation, we can bring the Hamiltonian into the following form:

$$\mathcal{H}'_d = \hbar\Omega_R \hat{\sigma}_x. \quad (7)$$

Hence, we can rotate the state vector about the x-axis in this way. We can apply rotations about the y-axis by introducing a finite phase to the drive.

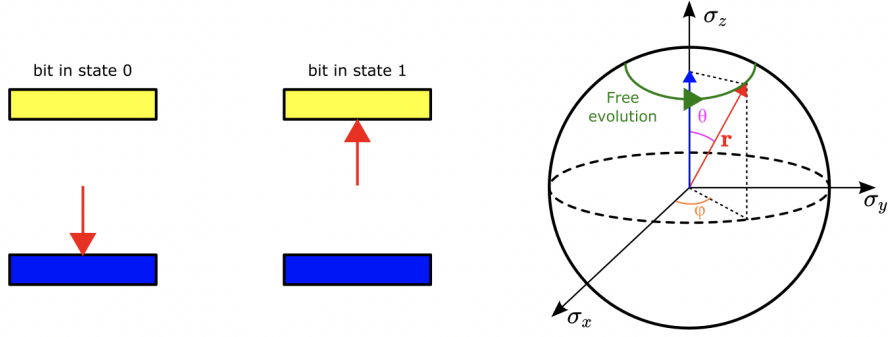


FIG. 6

Non-resonant driving

For a non-resonant drive with a detuning $\delta_d = \omega_q - \omega_d$, we transform into a frame rotating with the drive frequency ω_d . That way, the time-independent interaction Hamiltonian in a rotating wave approximation can be expressed as $\mathcal{H}'_d = \hbar[\delta_d \hat{\sigma}_z + \Omega_d \hat{\sigma}_x]/2$. This Hamiltonian shows that the rotation changes to $\Omega_R^* = \sqrt{\Omega_R^2 + \delta_d^2}$.

Single qubit gates

There are several single qubit gates of importance in quantum computation. However, we will only introduce the Pauli gates (X,Y,Z) and the S, T, and Hadamard gates (H).

- As the name suggests, the Pauli gates have matrix representations which are just the three Pauli matrices. The effect of these gates is to rotate the qubit state by π radians about their respective axes. For instance, the X gate rotates the qubit state π radians about the x-axis. In matrix form, the Pauli gates are

$$X = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Each of these gates have the property $XX = YY = ZZ = I$, where I is the identity gate, i.e., $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

- The S gate (sometimes referred to as \sqrt{Z} -gate) rotates the qubit state by $\pi/2$ radians about the z-axis. In matrix form this is

$$S = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{pmatrix}.$$

- The T gate (sometimes referred to as $\sqrt[4]{Z}$ gate) rotates the qubit state by $\pi/4$ radians about the z-axis. In matrix form this is

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}.$$

- The Hadamard gate rotates the qubit by π radians about an axis diagonal in the x-z plane. This is equivalent to an X-gate followed by a $\pi/2$ rotation about the y-axis. In matrix form this is $H = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$.

Two consecutive Hadamard gates yield the identity gate. With the input state $|0\rangle$, we obtain the equatorial state

$$H|0\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}},$$

and with the input state $|1\rangle$, we obtain the equatorial state

$$H|1\rangle = \frac{|0\rangle-|1\rangle}{\sqrt{2}}.$$

Coherence Measurements

The dynamics of quantum two-level systems or qubits can conveniently be described within the Bloch-Redfield theory introducing the longitudinal relaxation (depolarization) rate

$$\gamma_1/2\pi = T_1^{-1}, \tag{8}$$

and the transverse relaxation (dephasing) rate

$$\gamma_2/2\pi = T_2^{-1}. \quad (9)$$

As indicated in Fig. 7, these rates are determined by relaxation and Ramsey measurements (first and second pulse sequences from the top). The dephasing process itself is a combination of energy decay and pure dephasing (homogeneous broadening), which obey

$$\gamma_2 = \frac{\gamma_1}{2} + \gamma_\varphi, \quad (10)$$

characterized by the pure dephasing rate γ_φ . This can be ascertained by performing a so-called spin-echo measurement.

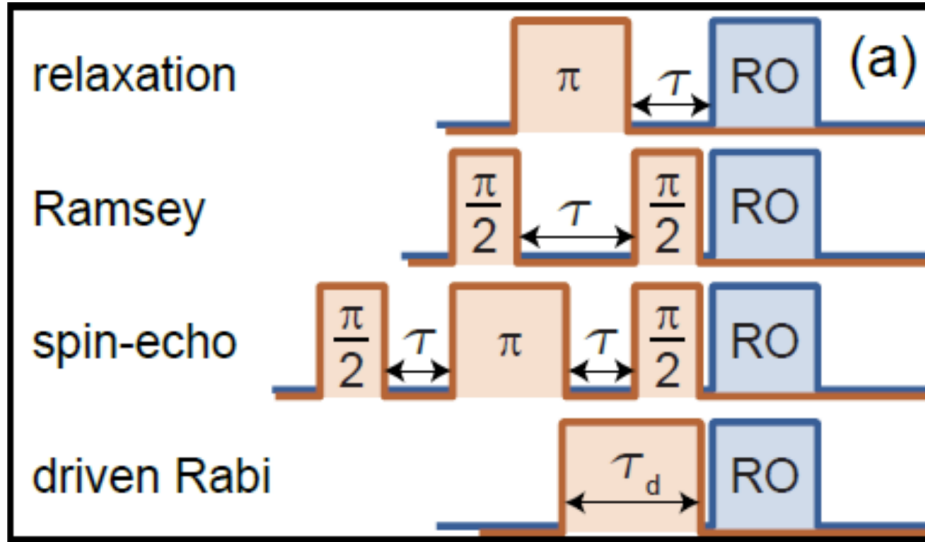


FIG. 7

Randomized benchmarking:

In randomized benchmarking a sequence of Clifford operations of length n is chosen at random ($U\{X, Y, \dots\}$), followed by the operation which inverts the effect of the sequence (U_{corr}).

- To benchmark the fidelity of a certain gate U_x (for example a π -pulse), this gate is inserted into the gate sequence and the same analysis is performed.
- By comparing the results with and with U_x , one can extract the error rate that this particular gate introduces.

References

- <http://mediatum.ub.de/1326240>