Problem set II – Model solutions

Question 1

1.

Based on Figure 6 of the reading we observe the following:

- The higher the education level, the more wages have increased between 1964 and 2010. This is true for both males and females.
- Among males, the wages of those having a higher than bachelor's degree have almost doubled. Wages of those with a bachelor's degree have increased by 40 %. Among "some college" and "high school graduate" groups wages have increased by 10-20 %. The wages of high school drop outs have decreased compared to year 1964.
- The trends in wage growth between different wage levels departed in the 1980s among males. After that, the wages of the highest-educated have increased stably and strongly, whereas the wages of the lowest-educated have even decreased.
- Among females, the wages have also increased the more the higher is the education level, but the differences are considerably smaller than among males. The highest-educated females have experienced a bit over 80 % wage growth and in other groups the changes are 20-60 %.
- Wage growth trends among females also departed in the 1980s when education started to matter, but wages have still grown since that in all except the lowest-educated group.

We can make the following conclusion considering the development of the Gini-coefficient:

- The Gini coefficient has probably increased among both males and females because the earnings have increased relatively more the higher is the education level. Those with the highest education level had probably already initially higher earnings (on average). When the wages of the highest-earnings individuals grow relatively more than those of the lowest-earning individuals.
- The Gini coefficient has probably increased more among males than among females. This is because the wage growth among males differs considerably between education groups, but among females the differences are considerably smaller.
- In is unclear whether the total Gini coefficient has increased, because we are just comparing individuals that are working. For instance, increased labor market participation of women may have decreased the Gini coefficient by reducing the earnings gap between males and females. We also do not account for the earnings of those that are not working, the unemployed in slide 16. However, it is true that the Gini coefficient in the whole of the U.S.A. has increased since the 1960s.

2.

If intergenerational mobility is low, parents' earnings predict their children's earnings well. That is, the children of the poor will be poor and the children of rich will be rich (in general; not true for everyone, if the intergenerational income elasticity is less than one). High income inequality is associated with low intergenerational mobility because, for instance, the social and economic opportunities between individuals differ a lot based on their parents' income. Wealthy parents are more able to invest in their children's economic and social resources and , for instance, pay for their college fees. It is hard for a child of a poor family to climb up the income ladders. A child of a rich family may be able to retain her position even if she was not particularly high-skilled. Then, if high income inequality decreased intergenerational mobility, decreased mobility could in turn increase income inequality as more resource accumulate to the higher end of the income distribution. Therefore, we could expect that the Gini coefficient in a country with a high inequality and low mobility would increase over time. This is drawn in the hypothetical graph below with the Lorentz curve of two generations. In the latter generation (gen2), the Lorentz curve is below the former generation (gen1). This mean that the Gini coefficient increases.

3.

- In the Figure, the effect of automation is to increase unemployment and earnings inequality. Automation reduces the demand for low-skilled manual work that can be replaced with robots. In turn, automation increases demand for high-skilled work that completements robots. The total demand for human work decreases, since part of jobs are totally replace with robots and new jobs are not created. The lowest-paying wages become unacceptably low. Consequently, unemployment increased (from 5% to 10 in the figure).

  Moreover, the decreased bargaining power of the low-skilled workers (65 % of the population) means that their income share decreases and the Lorentz-curve between 10% and 65 of the income distribution becomes less steep. The slope of the Lorentz-curve between 65% and 95 of the income distribution becomes steeper; the income share of these workers increases since their productivity increases considerably thanks to robots. The income share of the highest-earning 5% (employers and robot owners) is expected to remain unchanged, since the reduced bargaining power of low-skilled workers is balanced with increased bargaining power of the high-skilled workers and total labor share of income remains unchanged (this is not maybe a reasonable assumption, since decreased demand for human labor is likely to increase the income share of the capital owners).

  In total, the Gini-coefficient increases in the Figure, since the income share of the low-income workers decreases and that of the high-income workers increases.

- Automation is expected to increased income inequality, i.e., increase the Gini coefficient. The income share of the low-skilled workers decreases and that of the high-skilled workers increases. This occurs because robots are substitutes for the low-skilled workers, that is, robots and low-skilled workers compete for jobs in a way. Robots either replace the low-skilled workers that become unemployed or force them to accept lower wages by reducing their bargaining power. The high-skilled workers in turn are complements to robots; their work input makes robots more productive and the other way round. By this way, the bargaining power of the high-skilled workers increases, and they can demand higher wages. The ”missing middle” refers to the fact that automation may destroy good-paying middle-class jobs and divide the labor force to low-skilled, low-earning individuals and high-skilled, high-earnings individuals.

Question 2

1.

Let us first start thinking, whether it is reasonable to form a pair instead of working individually.

Consider first pair LS, which can produce 15 in total. If L and S worked alone, they could both produce 6. Cooperation then produces 3 additional units. Clearly cooperation pays off, since both can receive at least the same amount or more as alone.

Consider pair SP. This pair produces 16. S and P would produce 6 alone, so this again is a profitable coalition.

Consider pair LP, which produces 18. L and P would alone produce 6, so this is also a profitable coalition.

Now, think about the pairs. Is it reasonable to accept a third party? Consider first LS. If they accept P to join, the total output will increase by 9 units, from 15 to 24. P joins if he is offered at least 6, and this ca be offered without reducing the share of LS, who can offer 9 at most without reducing their share. So, the coalition of three is formed.

Now, consider pair SP. If L joins, total output increases from 16 to 24. L will join is she is offered at least 6, and SP can offer 8 at most without reducing their share. So, L can join without reducing anyone's share and this increases total output, so the coalition of three is formed.

Finally, if LP accepts S, total output increases from 18 to 24. LP can offer S 6, which is also the least S can accept (can produce this also alone). Both LP and S are in this case indifferent between forming and not forming the coalition of three, so coalition is formed (it is not evident, since they are indifferent, but let us suppose that a coalition of three is formed if possible).

2.

- First consider everybody working alone. Each can get 6. If they form a coalition of three, the total output is 24. Everybody can be given at least 6, and at least one of them more. Since at least one of them can be made better-off while making nobody worse-off, this is a Pareto improvement. It is better to work all the three together than alone.

  So, now we concluded that everybody working individually is not Pareto efficient. What about the pairs? Consider LS with total output 12 and P then gets 9 while working alone. If P joins LS, total output increases by 3 units. Therefore, at least one of the members can be made better-off without making anyone worse-off, so this is a Pareto improvement.

  Consider pair SP that produces 13 while L makes 9 alone. If SP accept L, the total output increases by 2 units. Therefore, at least one of the members can be made better-off without making anyone worse-off, so this is a Pareto improvement.

  Lastly, pair LP can produce 14 and S produces 9 alone. If S is accepted to the team, total output increases by one unit. This one unit can be given to somebody (or divided somehow) without taking anything away from anyone, so this is a Pareto improvement.

  So, we have concluded that forming a coalition of three is a Pareto improvement compared to any other working mode; forming a coalition of three can make at least someone better-off without making anybody worse-off. Conversely, if the coalition of three broke down, the total output would decrease and at least someone would be worse-off. Therefore, the coalition of three is a Pareto efficient outcome, and the only Pareto efficient outcome.

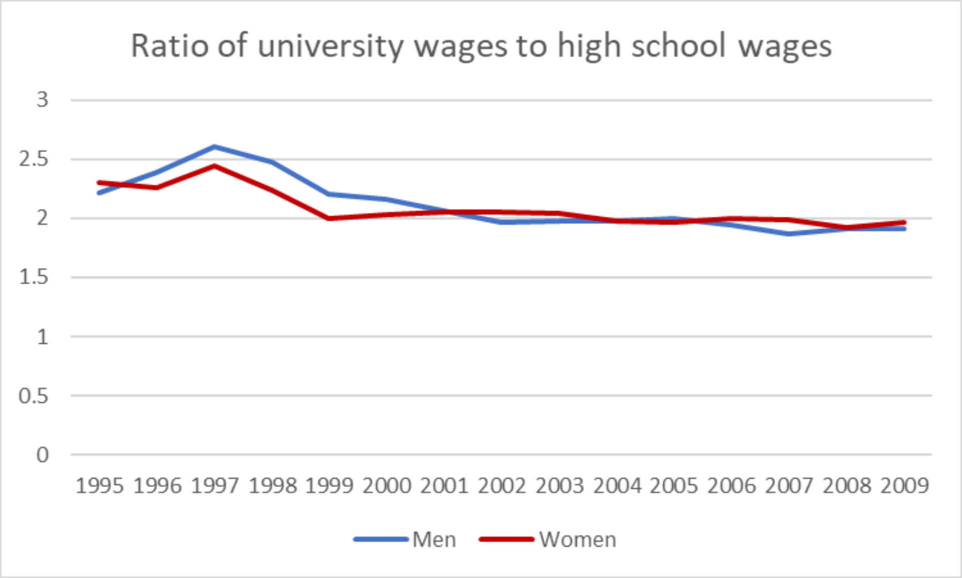
- Forming this coalition of three is now more difficult due to externalities. In Question 1, everybody could produce 6 alone. In the coalition of three, everybody could be guaranteed at least 6 and at least somebody more since total output was 24. Therefore, nobody had incentives to leave the coalition. In Question 2, total output of the coalition of three is still 24. However, if somebody leaves the team, she can alone produce 9 but in the team, not everybody can receive 9. Therefore, the one (or all of them if output is divided equally) that receive less than 9 has incentives to leave the team. So, even though the coalition of three is Pareto efficient, it is not sustainable!

Question 3

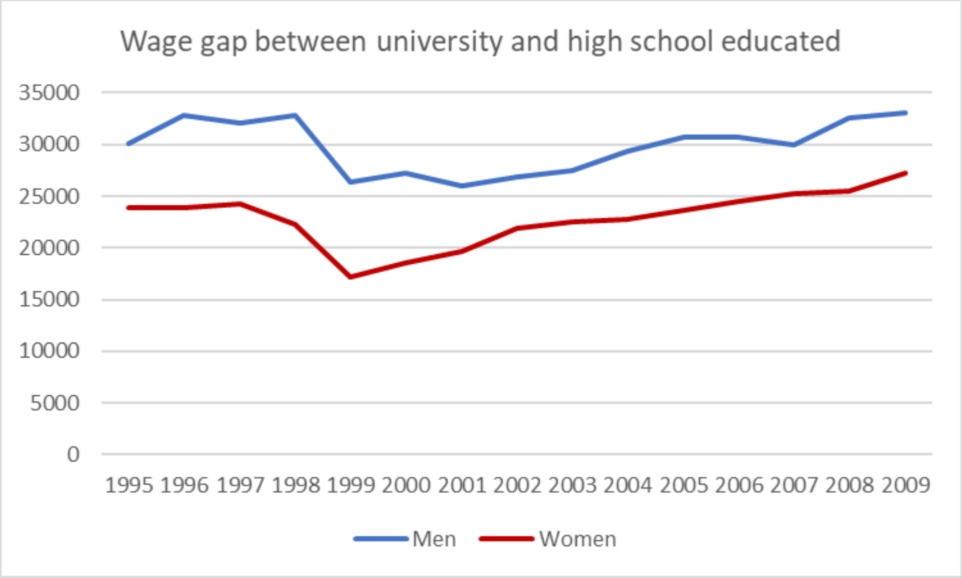
1.

(a)

This is the ratio of earnings of university educated to the earnings of high school educated. The ratio is currently about the same, around 2, for men and women and has developed similarly. The ratio declined in the late 1990s and has remained stable in the 2000s. The ratio was at its highest value in 1997, when it was about 2.45 for women and 2.61 for men. The ratio has thus declined a bit more among the men than among women, since the ratio if now about the same for both.



If you reproduced Figure 1 of the reading, which graphs the development of the absolute earnings gap instead of the ratio, the figure looks like this:



The absolute wage gap has obviously increased since the beginning of the 2000s, since the incomes of both groups has increased and the ratio between the incomes has remained about the same, i.e., the growth has relatively been about the same, but the university educated had larger income to begin with, so their absolute wage gains have been larger.

(b) The formula of the present value sum is: $PV = \sum_{t=0}^{T} \frac{y_t}{(1+r)^t}$. Now we set year 1995 to be year 0, so T = 14. Using this formula, the present value of the wage gap is about 345,141 euros for men and 262,112 euros for women.

2.

The code is attached separately.

Question 4

1.

The firm maximizes its profits by choosing an optimal price. Quantity is a function of price: $Q = AP^{\epsilon}$. Profits are the difference between sales revenue PQ and production costs CQ. Firm maximizes its profit by setting an optimal price:

$$max_P \ \Pi = PQ - CQ = P * AP^{\epsilon} - 400AP^{\epsilon} = AP^{\epsilon+1} - CAP^{\epsilon}$$

The optimal P is solved by derivating the profit function with respect to P and setting the derivative to zero:

$$\frac{\partial \Pi}{\partial P} = (\epsilon + 1)AP^{\epsilon} - C\epsilon AP^{\epsilon-1} = 0$$

$$(\epsilon + 1)AP^{\epsilon} = C\epsilon AP^{\epsilon-1}$$

$$P = \frac{C\epsilon}{\epsilon + 1}$$

Plugging in C = 400 and $\epsilon = -2$ we get:

$$P = \frac{-2 * 400}{-2 + 1} = 800$$

You could have found this also simply by remembering the Lernex index from the lecture slides. Lerner index is L = $\frac{1}{-\epsilon} = \frac{P-MC}{P}$. Given MC = 400 and $\epsilon = -2$ we get:

$\frac{1}{2} = \frac{P-400}{P}$ <=> $0.5P = P - 400$ <=> $P = 800$.

So, the optimal price is 800.

2.

Given Q = 10, and profit function is PQ – CQ, we get profits (innovation rent):

$$\Pi = 800 * 10 - 400 * 10 = 4000$$

So, the innovation rent is 4000.

3.

This can be again calculated with Excel, for instance. The formula of the present value sum is:

$$PV = \sum_{t=0}^{T} \frac{y_t}{(1+r)^t}$$

Let us assume that investment is made in period 0 and innovation rents start flowing in beginning from period 0 and lasting until period T = 19. (It is ok, if you began calculating from period 1 and continued until period 20; in this case the present value sum is a bit smaller). The annual innovation rent is 4000 and discount rate is 4%, so we calculate:

$$PV = \sum_{t=1}^{20} \frac{4000}{(1+0.04)^t} \approx 56{,}536$$

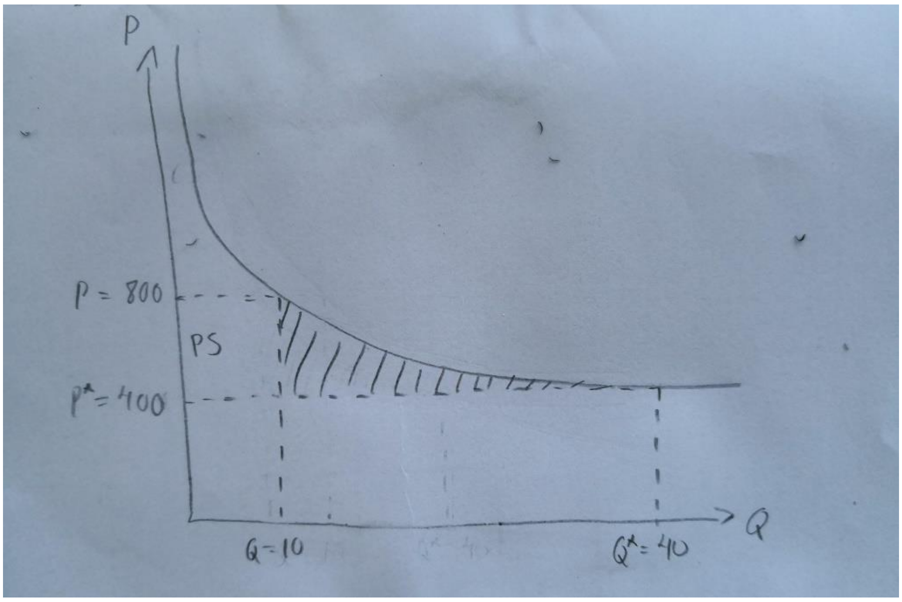
The present value sum of innovation rents is about 56,536. This is the most the firm can invest without making losses.

(If you calculated the present value sum for innovation rents flowing during periods 1-20, the present value sum would be about 54,361).

4.

When the patent expires, profits of the patent owner decrease substantially, since competitors that supply an (almost) identical substitute enter the market. If competition was perfect, profits would go to zero (competition is rarely perfect in any market, so the earlier patent owner and its competitors probably still make some profits).

5.



Profit maximizing firm sells quantity Q = 10 at price P = 800. Consumer surplus is area CS and producer surplus area PS. If the firm sold at a price P* = 400, i.e., at marginal cost, the quantity sold would be Q* = 40. You can find this quantity by using the fact that Q is 10 when P is 800, so that $A * 800^{-2} = 10 <=> A = 6{,}400{,}000$ so that $6{,}400{,}000 * 400^{-2} = 40$. With price 400, the area PS would turn into consumer surplus and the shaded area would also become consumer surplus. With price 800, the shaded area is no one's surplus: it is the deadweight loss caused by patent protection.

In this subquestion, it is enough if you answered with a graphical illustration of the situation as above. However, the deadweight loss can be calculated by integration or approximated it by treating the shaded area as a triangle with sides 400 and 30.

The deadweight loss is the area under the inverted demand curve (P = $\sqrt{\frac{6,400,000}{Q}}$) in range Q = [10,40] less the area of the rectangle between price 400 and quantities 10 and 40

$$\left(\int_{10}^{40} \sqrt{\frac{6,400,000}{Q}}\, dQ\right) - (40 - 10) * 400 = 16000 - 12000 = 4000$$

If approximated with a triangle, the estimated loss would be $(40 - 10) * (800 - 400)/2 = 6000$, so the approximation is not very precise in this exercise.