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Conservation laws: linear momentum conserved in the absence of forces

$$\dot{\bar{p}} = \bar{F} = 0 \Rightarrow \bar{p} = \text{constant}$$

Angular momentum: $\bar{L} = \bar{r} \times \bar{p}$ if $\bar{p} = m\bar{v}$

$$\Rightarrow \dot{\bar{L}} = m\dot{\bar{r}} \times \bar{v} + m\bar{r} \times \dot{\bar{v}}, \quad \dot{\bar{r}} \times \bar{v} = \bar{v} \times \bar{v} = 0$$

$$\Rightarrow \frac{d\bar{L}}{dt} = m\bar{r} \times \dot{\bar{v}} = \bar{r} \times \bar{F} \equiv \Gamma \quad (\text{torque})$$

Component of $\bar{L}$ conserved if corresponding comp. of torque vanishes.

Note: $\bar{L}$ depends on frame. shift origin by $\bar{r}_0 \rightarrow \bar{r} \Rightarrow \bar{r} + \bar{r}_0$ and $\bar{L} \Rightarrow \bar{r} \times \bar{p} + \bar{r}_0 \times \bar{p}$

Energy and work: Force field $\bar{F}(\bar{r})$. Test particle moves from $\bar{r}$ to $\bar{r} + d\bar{s}$. Work done is

$$dW = \bar{F}(\bar{r}) \cdot d\bar{s}$$

and from $1 \rightarrow 2$: $W_{1\rightarrow 2} = \int_1^2 d\bar{s} \cdot \bar{F}(\bar{r})$

Starts at $\bar{r}_1$ and passes through $\bar{r}_2$: $d\bar{s} = \bar{v}\, dt$

also $\bar{F} = m \frac{d\bar{v}}{dt}$

$$\Rightarrow W_{1\rightarrow 2} = \int_1^2 m\, dt \frac{d}{dt} \frac{1}{2} v^2 = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \quad \text{independent of path}$$

$T = \frac{1}{2} m v^2 =$ kinetic energy

Work done = increase in kinetic energy

If force is conservative there is a potential $U(\bar{r})$ such that

$$\bar{F}(\bar{r}) = -\nabla U(\bar{r})$$

In this case

$\int_1^2 d\bar{s} \cdot \bar{F} = -\int_1^2 d\bar{s} \cdot \nabla U(\bar{r})$ $\Rightarrow$ integrand is differential change in U in moving from $\bar{r}$ to $\bar{r} + d\bar{s}$

$$\Downarrow$$

$$-\int_1^2 dU = -U_2 + U_1$$

$T_2 - T_1 = -U_2 + U_1 \iff T_1 + U_1 = T_2 + U_2$ total energy $E = T + U$

Note: Force is conservative also if

$$\nabla \times \bar{F}(\bar{r}) = 0 \quad \forall \bar{r}$$

$\oint d\bar{s} \cdot \bar{F}(\bar{r}) = 0$ for all closed paths

Musings on radiation pressure vs. ablation pressure as rocket engines

a) radiation pressure: E = pulse energy, $\hbar\omega$ = photon energy

$\#\text{photons} \sim E/\hbar\omega$, $p_{photon} = \hbar k = \hbar\omega/c$

impulse from reflecting the pulse

$$\Delta p_{rad} = 2(\#\text{photons}) \cdot \frac{\hbar\omega}{c} = \frac{2E}{c}$$

1 MW laser for 1 s $\Rightarrow E = 10^6$ J $\Rightarrow \Delta p_{rad} \sim 7 \cdot 10^{-3}$ kgm/s

b) boil water with 1 MW laser pulse (for 1 s)

$\Rightarrow$ boil about 0.4 litres or $N_w = 10^{25}$ molecules

molecule leaves at energy $k_B T \sim 5 \cdot 10^{-21}$ J $= \Delta p_w^2/2m$

impulse $\sim \sqrt{2mk_BT} \cdot N_w \sim 120$ kgm/s

$\Rightarrow$ even if only small fraction of pulse energy ends up in molecules flying in right direction, ablation can beat radiation pressure

Newton II: muuttuva massa esimerkki

suhteellisuusteoriassa kappaleen massa nousee nopeuden kasvaessa niin, että liikemäärä

$$p = \frac{mv}{\sqrt{1-v^2/c^2}}, \quad m = \text{massa levossa}, \quad c = \text{valon nopeus.}$$

Kiihdytetään hiukkasta vakiovoimalla F_0 levosta. Mikä on hiukkasen nopeus ajan funktiona?

$$\frac{dp}{dt} = F_0 \Rightarrow \int_0^{v(t)} d\left[\frac{mv}{\sqrt{1-v^2/c^2}}\right] = \int_0^t dt\, F_0$$

olisi helppo yleistää
$F_0 \to F(t)$
$\int_0^t dt\, F(t) = I(t)$
eli impulssi

$$\Rightarrow \frac{mv}{\sqrt{1-v^2/c^2}} = F_0 t$$

$$\Rightarrow m^2v^2 = (F_0 t)^2 (1 - v^2/c^2)$$

$$v^2\left(m^2 + \left(\frac{F_0 t}{c}\right)^2\right) = (F_0 t)^2$$

$$\Rightarrow v(t) = \frac{F_0 t}{\sqrt{m^2 + \left(\frac{F_0 t}{c}\right)^2}}$$

Kun $t \gg \frac{mc}{F_0}$, jälkimmäinen dominoi neliöjuuren sisällä

$\Rightarrow \lim_{t\to\infty} v(t) = c \Rightarrow$ ei koskaan saavuta valonnopeutta.