MEC-E1005

MODELLING IN APPLIED MECHANICS 2023

Week 17 Mon 10:15-12:00

MODELLING ASSIGNMENTS

LEARNING OUTCOMES

MEC-E1005 Modelling in Applied Mechanics is a project course, which combines modelling, numerical methods, experiments, and scientific writing in applied mechanics. Most projects are related with displacement, vibration, and stability analyses on toy structures but projects on rigid body dynamics and Arctic technology may be also offered. After the course a student

(1) understands the interplay of modelling and experiments in engineering work

(2) is able to use a numerical software in structural analyses

(3) knows how to write a technical report

Assessment: Participation in industry presentations (2p) and 3 reports (6p each)

EXPERIMENT VS. MODELLING

In design of a simple pendulum of a tall-case clock, the required information is the dependency of period T on mass m, initial angle ϕ_0 from the stable equilibrium position, acceleration by gravity g, and length L. The main options are

Straightforward experiment: Measurement of T on various physical structures (characterized by m and L), with various initial angles ϕ_0, and on various places on earth (characterized by g).

Dimension analysis: Application of generic principles of physics to get $T = \sqrt{L / g}\, f(\phi_0)$ and measurement of $T\sqrt{g / L}$ as the function of ϕ_0.

Mathematical modelling: Application of simplifying assumptions, the basic laws of mechanics, and rules of mathematics to get $T = 2\pi\sqrt{L / g}$.

MODELLING IN MECHANICS

- **Crop**: Decide the boundary of structure. Interaction with surroundings need to be described in terms of known forces, moments, displacements, and rotations.
- **Idealize**: Simplify the geometry. Ignoring the details, not likely to affect the outcome, may simplify analysis a lot.
- **Parameterize**: Assign symbols to geometric and material parameters of the idealized structure. Measure or find the values needed in calculations.
- **Model**: Write/decide the mathematical description consisting of equilibrium equations, constitutive equations, and boundary conditions.
- **Solve**: Use an analytical or numerical method and hand calculations or software to find the solution.

1 PERIODIC STRUCTURE DISPLACEMENT

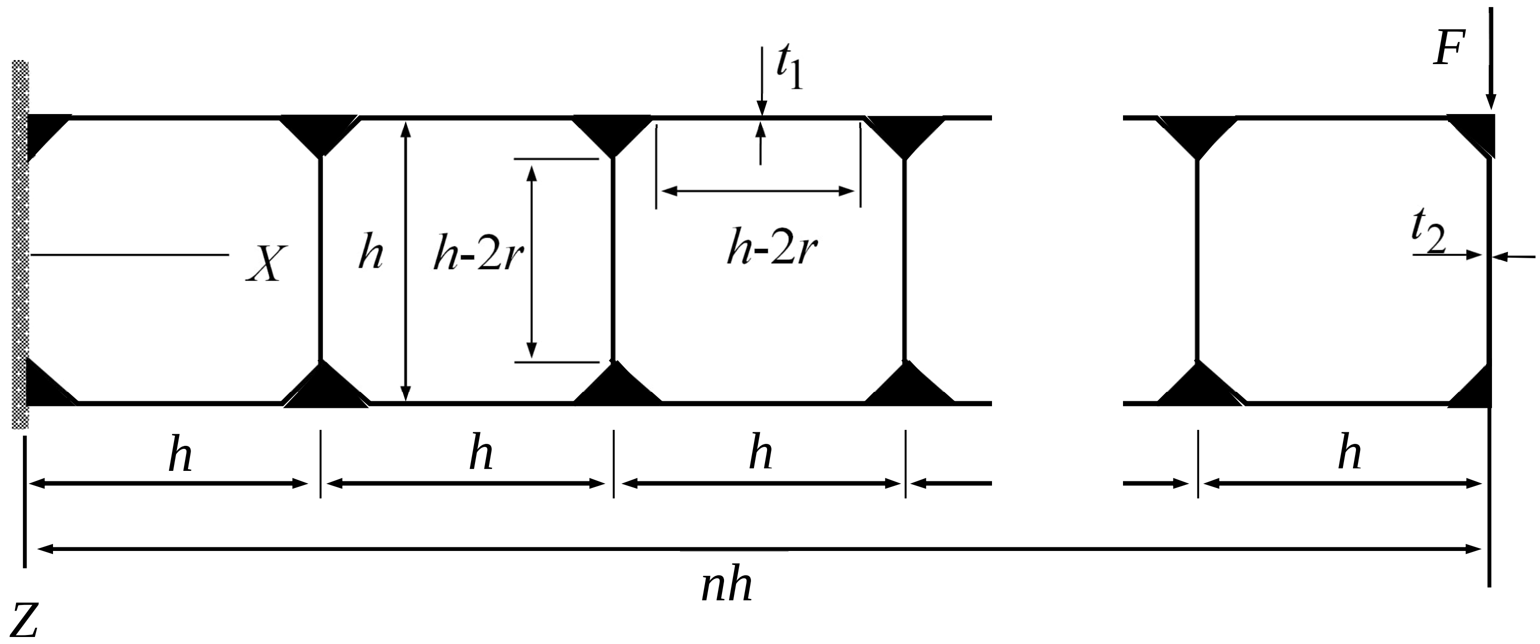
ASSIGNMENT

A structure is considered as periodic if one may identify a part which repeats itself in one or several directions. Regularity in that sense can be used to simplify, e.g., displacement analysis with variable number of parts.

The aim is to find the effect of geometrical and material parameters to the displacement-load relationship of a periodic steel structure of the type shown in the figure. The left end is attached to a wall. The right end is loaded by a point force. The starting point is a generic expression predicted by dimension analysis. First, a simplified model is used for a more specific relationship. After that, analysis by FEM is used to study the effects of details. Finally, the displacement-force relationship is compared with an experimental one.

IDEALIZATION AND PARAMETERIZATION

In the idealization, *n* identical parts are connected by welds of triangular shape. The left end is rigidly connected to a wall and load is acting at the right end.



DIMENSION ANALYSIS

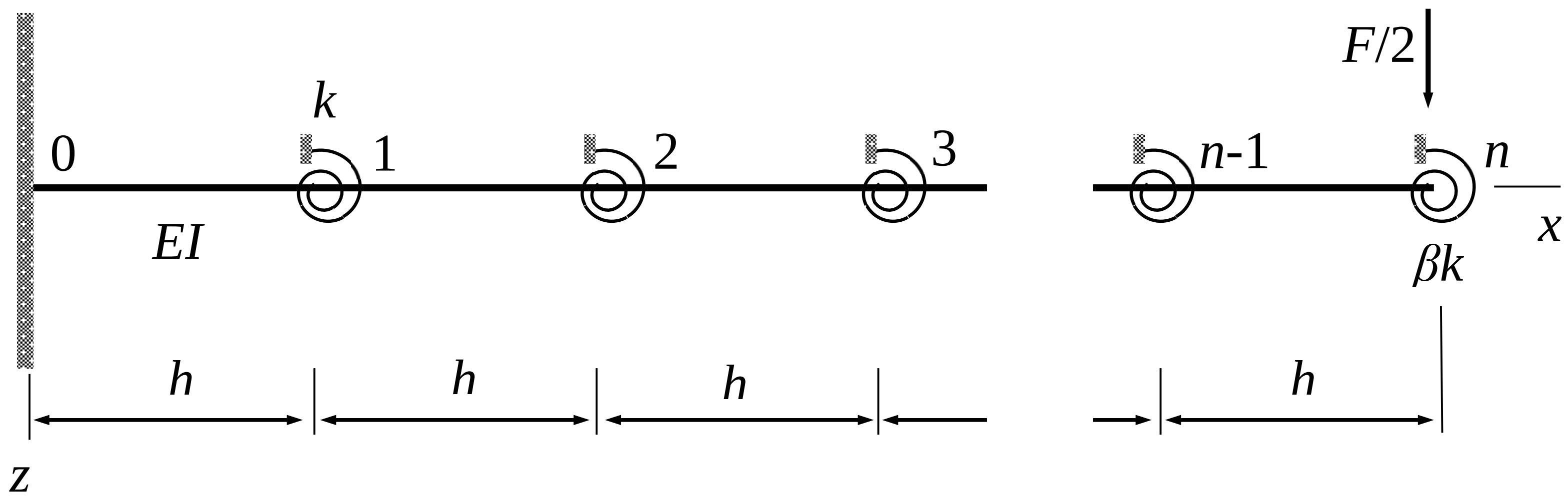
Assuming that the effect of the axial deformation of the material is negligible compared to that of bending, dimension analysis, based on quantities E, I_1, I_2, h, r, F, and w, gives the relationship

$$\frac{w}{h} = \frac{Fh^2}{EI_1}\alpha(\frac{r}{h}, \frac{I_2}{I_1}, n), \tag{1}$$

in which the form of the right-hand side requires a more detailed analysis. The indices in the second moments refer to the two different thicknesses of the material. Equation (1) assumes linearity between displacement and load, vanishing displacement with zero loading, and that the straight parts behave essentially as Bernoulli beams in bending.

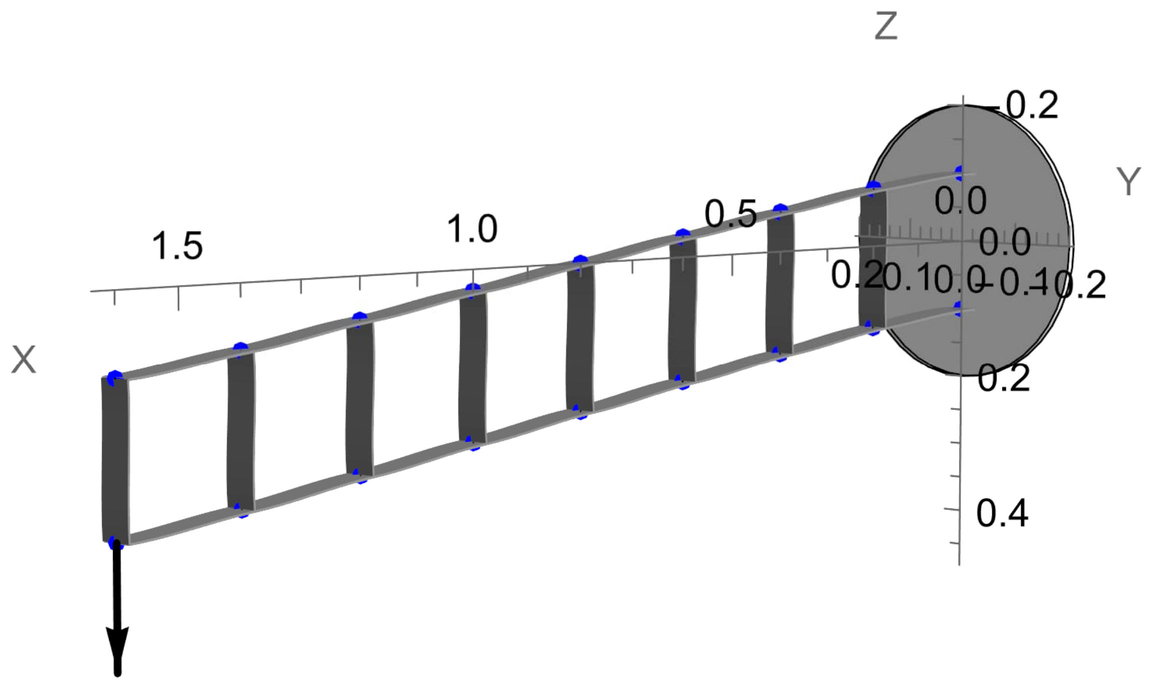
SIMPLIFIED ANALYSIS

Simplification assumes that $r/h = 0$ and $I_2/I_1 = 1$. As the end points of the vertical parts translate downwards but not sideways, the vertical parts just increase the bending stiffness of the upper and lower horizontal parts which deform in the same manner. Therefore, one may consider an inextensible Bernoulli beam shown.



FINITE ELEMENT ANALYSIS

Finite element analysis with solid, plate or beam elements give the force-displacement relationship without (too many) simplifying assumptions. However, analysis requires numerical values for all the problem parameters.



EXPERIMENT

The set-up is located in Puumiehenkuja 5L (Konemiehentie side of the building). The hall is open for measurements during the office hours (09:00-16:00) from Thu of week 17 to Thu of week 18.

Place a mass on the loading tray and record the displacement shown on the laptop display. Gather enough mass-displacement data to find the displacement-force relationship reliably. For example, you may repeat a measurement with certain loading several times to reduce the effect of random error by averaging etc. Do not overload the structure.

2 SINGING BOWL FREQUENCY

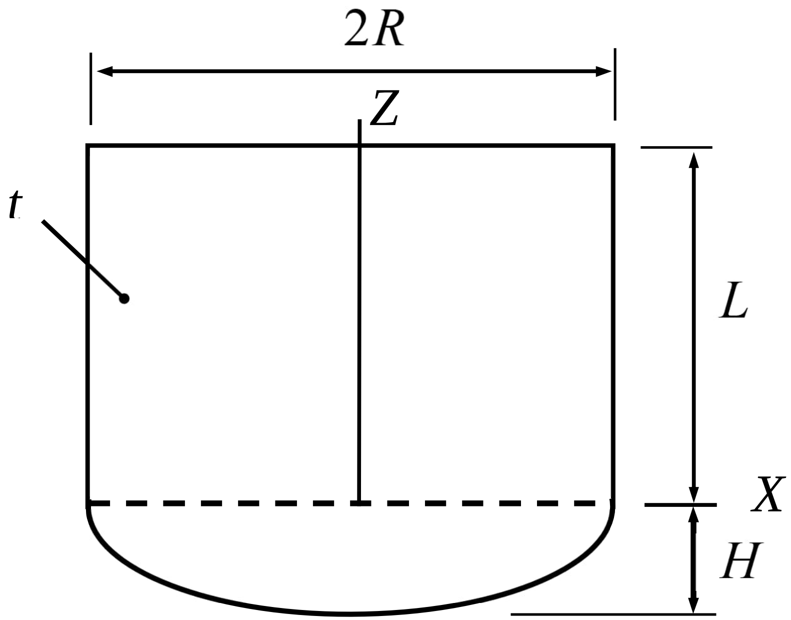
ASSIGNMENT

Vibration of a structure can be thought of superposition of modes harmonic variation in time with certain frequencies. In engineering work and design of, the lowest frequencies (or even the lowest one) are important as they determine, e.g., the response for external excitation. Singing bowls are designed to vibrate with a certain frequency under excitation.

In the modelling assignment, the aim is to find the effect of geometrical and material parameters to the lowest frequency of a singing bowl. The starting point is a generic expression predicted by dimension analysis. First, a simplified model is used for qualitative understanding. After that, analysis by FEM is used to study the effects of the geometrical parameters defining the shape of the bowl. The predicted lowest frequency is compared with the experimental one.

IDEALIZATION AND PARAMETERIZATION

The bowl geometry is simplified by considering cylindrical and ellipsoidal parts of constant thickness. Material is linearly elastic and isotropic homogeneous quartz crystal. The effect of the supporting rubber ring is omitted as negligible (no external forces nor displacement constraints).



DIMENSION ANALYSIS

Assuming that the relevant quantities determining the lowest (non-zero) frequency f are Young's modulus E, Poisson's ratio ν, density ρ, and geometrical parameters t, H, L, R, dimension analysis implies the relationship

$$f R^2 \sqrt{\frac{\rho}{E}} = \alpha(\frac{t}{R}, \frac{H}{R}, \frac{L}{R}, \nu), \qquad (2)$$

where the expression on the right-hand side is to be found by a more detailed analysis or/and additional assumptions. For example, considering shell model and bending dominated behavior, the proper mass and stiffness measures inside the square root are the mass per unit area $\mu = t\rho$ and $D = t^3 / 12E / (1-\nu^2)$. Therefore, to have the correct power of t on the left hand side

$$\alpha(\frac{t}{R}, \frac{H}{R}, \frac{L}{R}, \nu) = \frac{t}{R}\alpha_0(\frac{H}{R}, \frac{L}{R}, \nu).$$

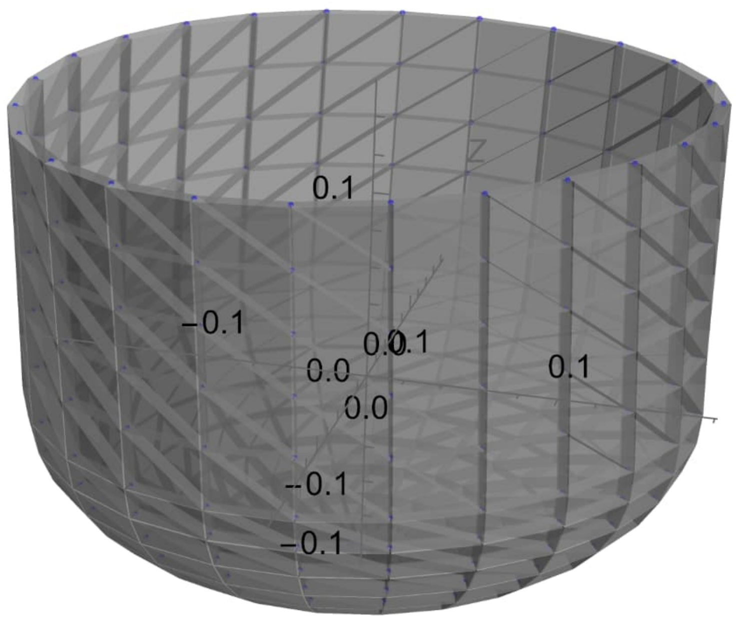
SIMPLIFIED ANALYSIS

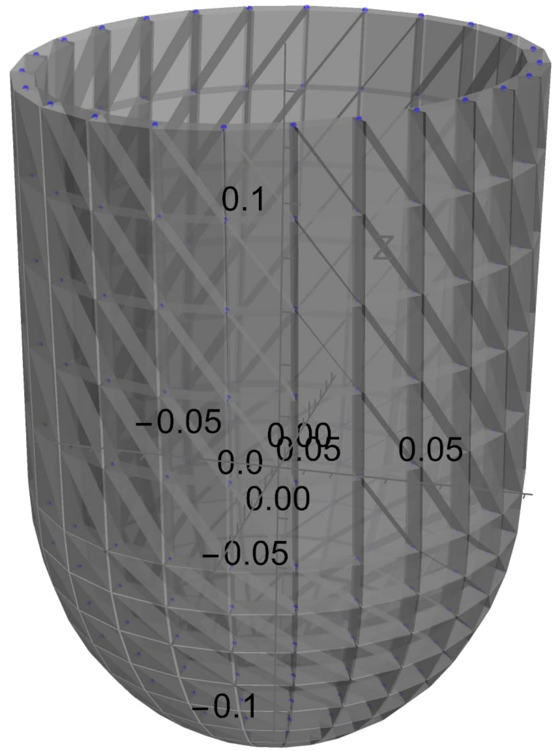
Use simplified analysis with the cylindrical part and curved beam or cylindrical shell equations to find expression (2) when the cylindrical part is long so that the effect of the closed end to the smallest frequency becomes negligible (MEC-E8003):

$$\begin{Bmatrix} \dfrac{\partial N}{\partial s}-\dfrac{1}{R}Q-\rho A\dfrac{\partial^2 u}{\partial s^2} \\ \dfrac{\partial Q}{\partial s}+\dfrac{1}{R}N-\rho A\dfrac{\partial^2 v}{\partial s^2} \\ \dfrac{\partial M}{\partial s}+Q \end{Bmatrix}=0,\quad \begin{Bmatrix} 0 \\ 0 \\ M \end{Bmatrix}=\begin{Bmatrix} \dfrac{\partial u}{\partial s}-\dfrac{1}{R}v \\ \dfrac{\partial v}{\partial s}+\dfrac{1}{R}u-\psi \\ EI\dfrac{\partial \psi}{\partial s} \end{Bmatrix}.$$

FINITE ELEMENT ANALYSIS

Vibration analysis by the finite element method and solid or plate elements gives the frequency without (too many) simplifying assumptions. However, analysis requires numerical values for all the problem parameters.



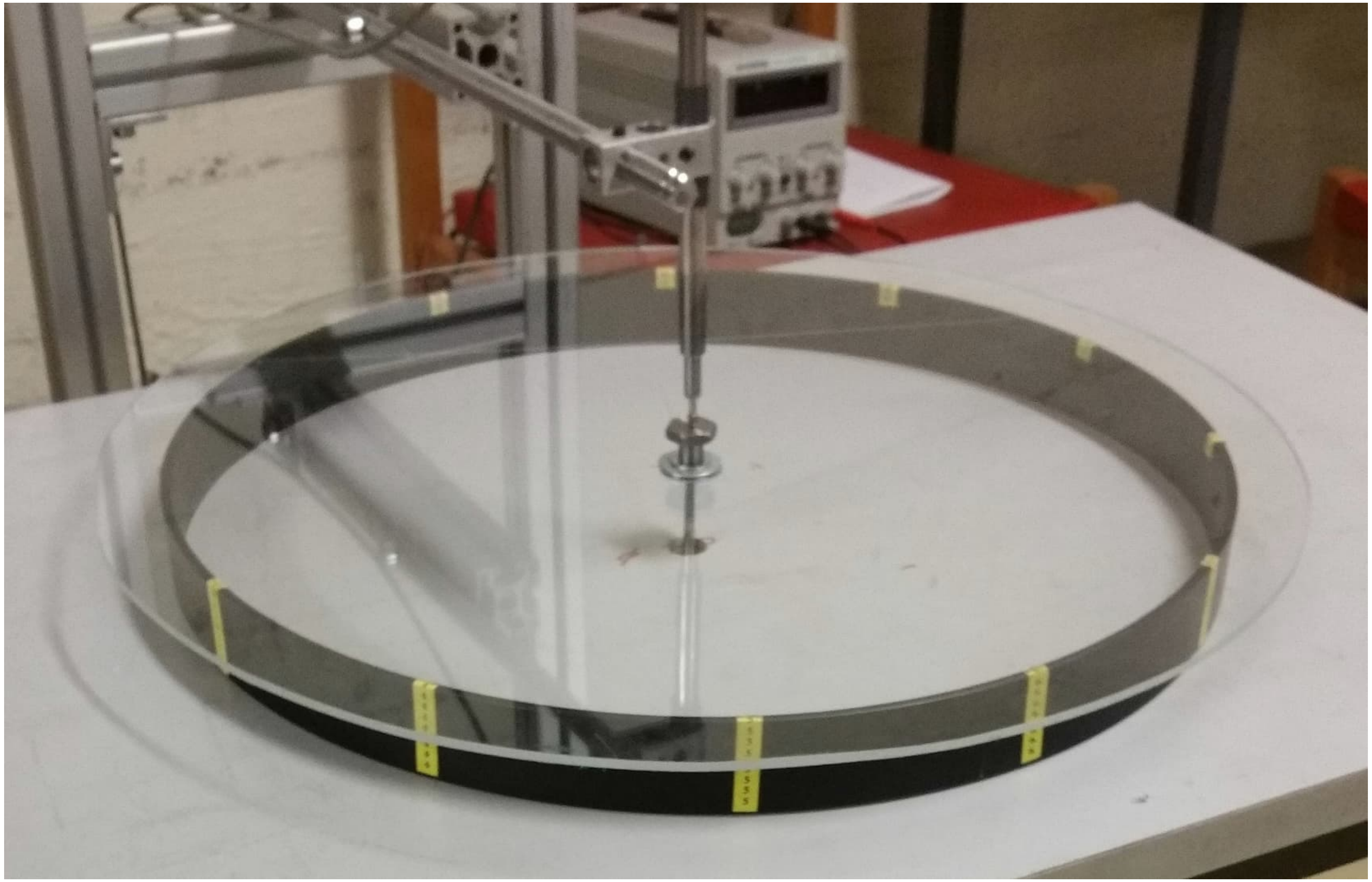


EXPERIMENT

"MEINL Sonic Energy Crystal Singing Bowl 14" is available in Puumiehenkuja 5L (Konemiehentie side of the building). The hall is open for the experiment during the office hours (09:00-16:00) from Thu of week 19 to Thu of week 20.

Record the sound of the bowl and transform the time series to the frequency domain. The lowest frequency of free vibrations is indicated by a peek in the frequency domain representation. You may hit the bowl (gently) to start vibration or try to cause a resonance at the lowest frequency. You may use, for example, Mathematica to record and analyse the sound.

3 RECTANGULAR PLATE RIGIDITY

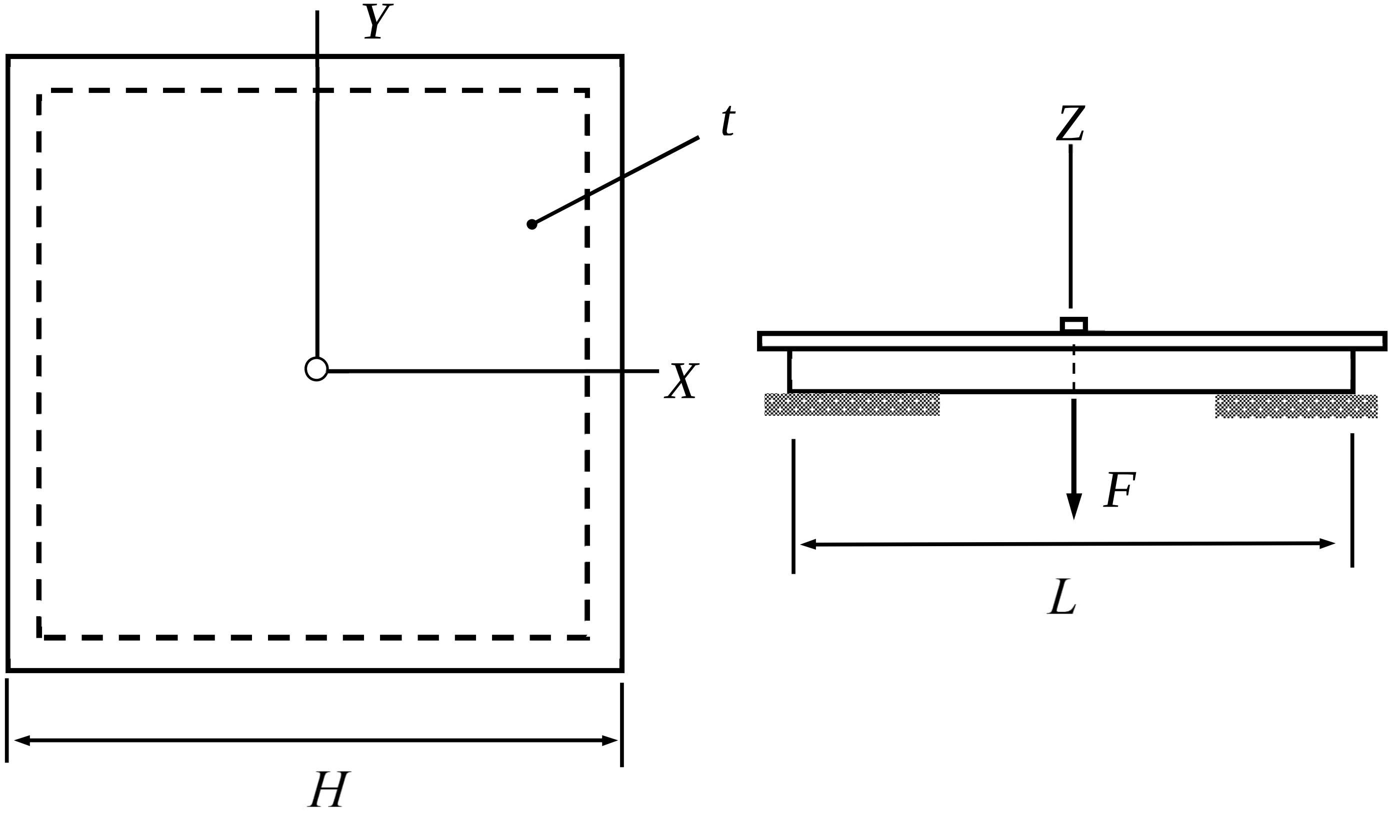


ASSIGNMENT

According to linear theory, rigidity of a simply supported plate is constant whose value depends on the plate thickness, size of the plate, and the plate material. Experiments indicate, however, that rigidity increases rapidly in the transverse displacement. The actual boundary conditions at the support may also affect the setting. For example, in rectangle geometry, the contact between the plate and support may be lost at the corner regions (if the displacement at the support is constrained only downwards like in the figure).

In the modelling assignment, you will study the effects of geometrical and material parameters, and displacement on rigidity of a rectangular plate on a rectangular support (circular example in the figure). The starting point is a generic expression predicted by dimension analysis. First, a simplified linear model is used for a more specific relationship. After that, analysis by FEM is used to study the effects of large displacements. Finally, prediction by the model is compared with an experimental one.

IDEALIZATION AND PARAMETERIZATION



DIMENSION ANALYSIS

Assuming that the quantities related with the setting are E, ν, H, L, t, w, and F, dimension analysis implies the relationship

$$\frac{FL^2}{Et^4} = \alpha(\frac{w}{t}, \frac{H}{L}, \nu) \tag{3}$$

The dimensionless groups are based on plate theory. The expression on the right hand requires a more detailed analysis or/and additional assumptions. For example, truncated Taylor expansion with respect to the first argument gives

$$\alpha(\frac{w}{t}, \frac{H}{L}, \nu) = \alpha_1(\frac{H}{L}, \nu)\frac{w}{t} + \alpha_3(\frac{H}{L}, \nu)(\frac{w}{t})^3$$

if displacement vanishes without loading, and force-displacement response is similar for negative and positive force values.

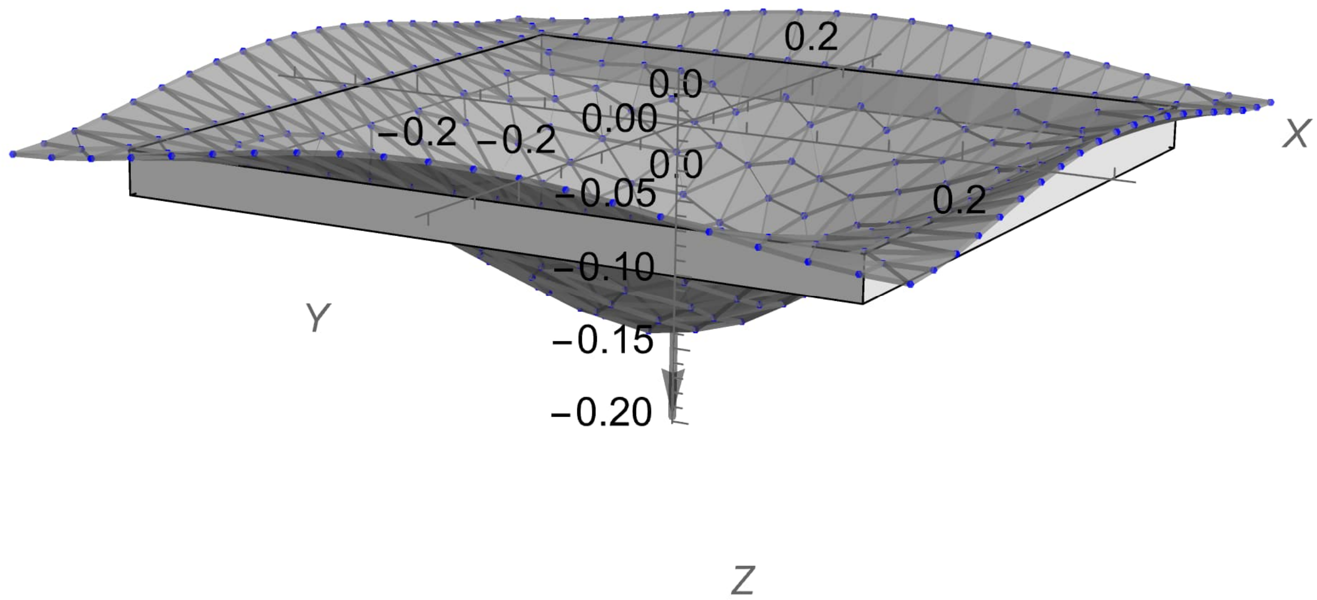
SIMPLIFIED ANALYSIS

Simplified small displacement analysis can be based on the double-sine series solution to a rectangular simply supported plate. One may also use the virtual work density of Kirchhoff plate (either small or large displacement) and a few-parameter displacement approximation (MEC-E1050, MEC-E8001, MEC-E8003)

$$\delta W = -\int_\Omega \begin{Bmatrix} \dfrac{\partial^2 \delta w}{\partial x^2} \\ \dfrac{\partial^2 \delta w}{\partial y^2} \\ 2\dfrac{\partial^2 \delta w}{\partial x \partial y} \end{Bmatrix}^{\mathrm{T}} D \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix} \begin{Bmatrix} \dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial^2 w}{\partial y^2} \\ 2\dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} dA - \delta w(x_F, y_F) F$$

FINITE ELEMENT ANALYSIS

Analysis by the finite element method and solid or plate/shell elements gives the displacement without (too many) simplifying assumptions. Numerical method requires numerical values for all the problem parameters.



EXPERIMENT

The set-up is located in Puumiehenkuja 5L (Konemiehentie side of the building). The hall is open for the experiment during the office hours (09:00-16:00) from Thu of week 21 to Thu of week 22.

Place a mass on the loading tray and record the displacement shown on the laptop display. Disk material is not purely elastic so wait for the displacement reading to settle (almost). Gather enough mass-displacement data to find the displacement-force relationship reliably. For example, you may repeat a measurement with certain loading several times to reduce the effect of random error by averaging etc. You may also consider different loading sequences (like increasing and decreasing the mass) to minimize the effect of the viscous part of material response.