BUCKLING FORCE OF A BEAM STRUCTURE

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SUMMARY

[1]This report evaluates two different methods for determining the buckling force of a beam structure. [2]The first method is based on a simplified engineering model and hand calculation. [3]The second method uses a non-linear beam theory and a numerical model. [4]The results of both methods are validated against experimental data. [5]Comparison of the results indicate that both approaches predict the buckling force within engineering accuracy.

1. INTRODUCTION

[6]An **important** criterion for designing structures composed of thin or slender structural parts is the stability of the structure. [7]The aim of stability analyses is to ensure that structural parts, such as slender beam- or plate-like structures, do not buckle and thereby threaten the integrity of the structure. [8]To avoid this problem, it is necessary to have a reliable model for predicting buckling load. [9]For this purpose, simplified engineering models are typically used to predict critical loading under rather severe assumptions, such as loading aligned with the beam axis. [10]A well-known example of this is the simple Euler formulas [1] for buckling of a beam. [11]**However,** although imperfections or post-buckling behaviour can be better predicted using more precise models, this would require more complex analysis to account for large displacements.

[12]**In order to** evaluate the applicability of using a simplified engineering model instead of a more precise method based on a non-linear beam theory [2] to account for large displacements**, this report compares** the results yielded by these two models with experimental results. [13]The structure used for this comparison is a beam of high-strength steel supported by two hinges acting as cylindrical joints.

[14]**The rest of this report** is divided into **five sections**. [15]**Section 2 describes** the structure of the beam studied in this report. [16]**Section 3 discusses** the Bernoulli beam equation used in the simplified methods for determining the buckling force. [17]**Section 4 reviews** the theory on non-linear buckling analysis, and **Section 5 describes** the buckling experiment**.** [18]**Section 6 presents and compares** the experimental results to those of the two approaches for solving the buckling problem.

2. BEAM STRUCTURE

[19]The beam structure and its parts **are shown** schematically **in Figure 1**. [20]The structure is loaded at Hinge B with a horizontal force F. [21]Hinge B allows free rotation and horizontal displacement u_B, while Hinge A at the other end allows only rotation. [22]The hinges located at the ends are much stiffer than the flexible parts of the beam structure. [23]Rotation centers of the cylindrical joints have small offsets from the centerline of the flexible beam part.

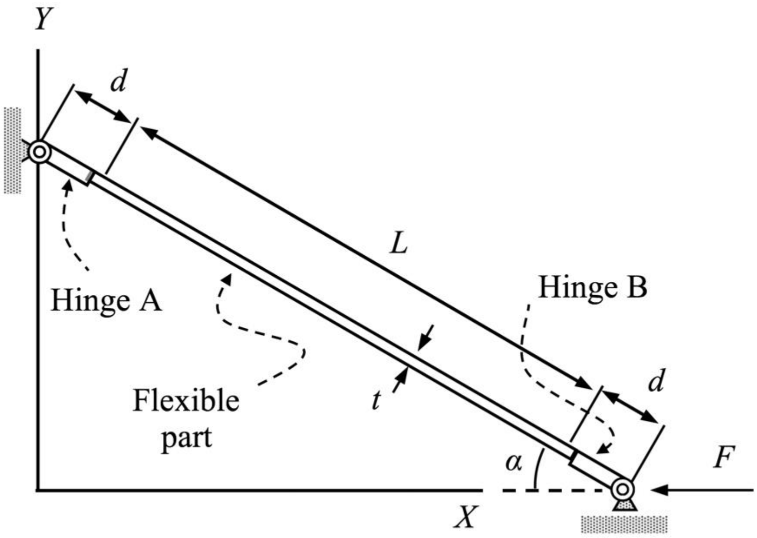

Figure 1. Illustration of the beam structure and its four main geometrical parameters: length L, thickness t, width b of the flexible part, and hinge length d.

[24]The values of the geometrical parameters defined in Figure 1 **are given in Table 1**. [25]The beam is composed of high strength steel and **has <u>a</u>** Young's modulus **<u>of</u>** $E = 210\,\text{GPa}$ and **<u>a</u>** Poisson's ratio **<u>of</u>** $\nu = 0.3$.

Table 1. Geometrical parameters of the structure

d	L	b	t	α
$0.06\,\text{m}$	$0.61\,\text{m}$	$0.04\,\text{m}$	$0.003\,\text{m}$	$\pi / 3$

3. BUCKLING ANALYSIS

[26]Engineering models for an axially loaded beam are based on the Bernoulli beam equation modified by the bending effect of the axial load. [27]The critical axial loading N_{cr} of the beam is identified by the non-uniqueness of the bending solution. [28]The outcome is a simple analytical expression attributed to Euler [2].

[29]For the beam structure in Figure 1, the axial force N acting on the beam **can be deduced from** the equilibrium of the moving joint of Hinge B **as**

$$N \cos \alpha = F. \tag{1}$$

[30]The buckling force yielded by the engineering model **is given by**

$$N_{\text{cr}} = \pi^2 \frac{EI}{L^2}. \tag{2}$$

[31]As the equilibrium Equation (1) holds also at the critical loading, equations can be solved for the critical loading F_{cr} acting on Joint B (Figure 1). [32]The axial buckling force expression of a simply supported beam in Equation (2) assumes constant bending rigidity between the two joints. [33]Therefore, EI is chosen as the bending stiffness for the flexible part of the beam, and L the distance between the joints.

4. NON-LINEAR ANALYSIS

[34]Post-buckling analysis and finding the full force-displacement relationship requires a model that would also be valid for large displacements. [35]In variational form, the planar beam problem **can** be stated **as follows** [3]: Find the corresponding displacement components $u(x)$ and $v(x)$ in the directions of $X-$ and $Y-$axis (Figure 1), such that

$$\delta W = -\int_{x_A}^{x_B} (\delta\varepsilon EA\varepsilon + \delta\kappa EI\kappa) dx - \delta u_B F = 0 \tag{3}$$

for all δu and δv. [36]With the Lagrange notation for a derivative with respect to the material coordinate x along the axis of the beam, the Green-Lagrange strain ε and curvature κ in the virtual work expression **are defined by**

$$\varepsilon = u' + \frac{1}{2} u'^2 + \frac{1}{2} v'^2 \tag{4}$$

and

$$\kappa = \frac{v'u'' - (1+u')v''}{[(1+u')^2 + v'^2]^{3/2}}. \tag{5}$$

[37]Equations (3), (4) , and (5) assume that ε and κ vanish at the initial geometry when $F = 0$.

[38]The finite element method and a cubic element approximation for the displacement components are used to find a numerical solution. [39]In the displacement-controlled algorithm, displacement u_B is decreased (a negative quantity) step-by-step. [40]Starting from a known equilibrium solution, u_B is given a decrement with $\delta u_B = 0$, Newton's method is used to find a new equilibrium solution, and force F is calculated by considering $\delta u_B \neq 0$.

[41]**Figure 2 shows** the force F acting on node B as function of the displacement $u_F = -u_B$ in the direction of the force when the joints have an offset of 1 mm. [42]The range for the displacement is from the initial position to the position where the nodes of Hinges A and B are on the same vertical line**.** [43]**From the figure, it can be observed that** buckling occurs with a small displacement at the point $dF/du_F = 0$. [44]Consequently, a buckling experiment based on control of F is not feasible.

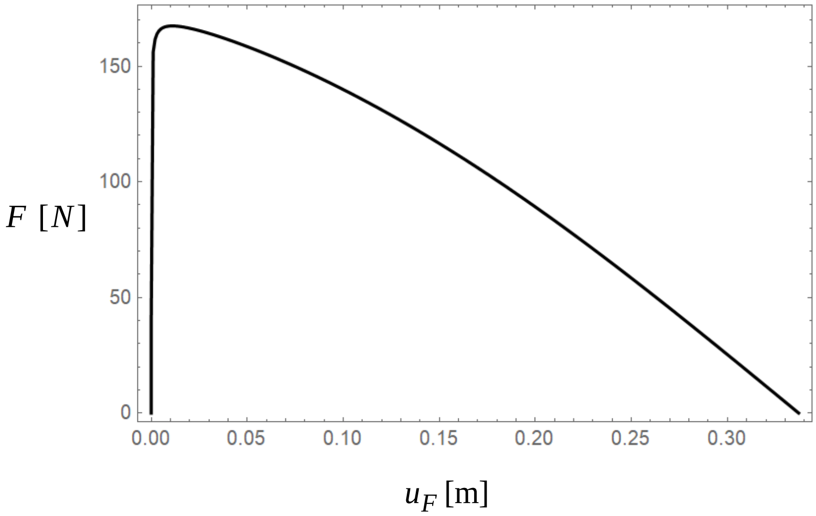

Figure 2. Displacement-force relationship given by the large displacement beam model.

5. BUCKLING EXPERIMENT

[45]The set-up of the buckling experiment **is shown** schematically **in Figure 3**. [46]A slider-threaded bar-wrench system is used to adjust the horizontal position X_B of Joint B. [47]The force F acting on the joint is given by a force-transducer connected to a computer through an amplifier.

[48]During the experiment, Hinge B is moved to 25 positions starting from the initial position ($F = 0$) to the position where the line connecting the joints is vertical ($F = 0$). [49]Thereafter, the 25 positions are measured in the reverse order. [50]The two measurements allow elimination of the friction force acting on Hinge B from the slider. [51]The outcome of the experiment **is given in Appendix A**.

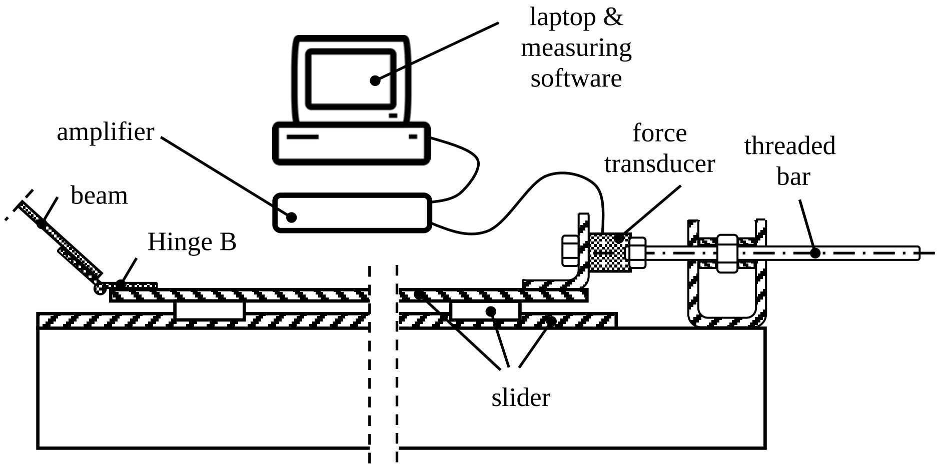

Figure 3. Set-up of the buckling experiment.

6. RESULTS AND CONCLUSION

[52]**Table 2 shows** the critical force values given by the simplified engineering model, the method based on non-linear beam theory and a numerical model, and the experiments. [53]**As can be seen from the table,** the predictions by the two models yielded results that are in fair agreement and well within the precision needed for design: the results yielded by the approaches differ by less than 5%.

Table 2. Critical loading of the structure

Method	F_{cr} [N]
Simplified	175
Non-linear	177
Experiment	168

REFERENCES

[1] Parnes, R., 2001, *Solid Mechanics in Engineering*, John Wiley & Sons Ltd., Chisester, U.K.

[2] Reddy J.N., 2003, *Mechanics of laminated composite plates and shells*. CRC Press, Boca Raton, FL, USA.

APPENDIX A. Measured force-displacement relationship.

N:o	X_B [mm]	F_+ [N]	F_- [N]	$(F_- + F_+)/2$ [N]
1	82	3	2	3
2	90	181	153	167
3	100	177	159	168
4	110	176	157	167
5	120	172	153	163
6	130	168	149	159
7	140	165	146	156
8	150	164	145	155
9	160	158	139	149
10	170	154	136	145
11	180	150	136	143
12	190	145	129	137
13	200	141	127	134
14	220	133	114	124
15	240	121	106	114
16	260	110	97	104
17	280	106	87	97
18	300	98	76	87
19	320	84	67	76
20	340	75	55	65
21	360	59	42	51
22	380	46	33	40
23	400	33	24	29
24	420	18	5	12
25	440	6	6	6