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MS-EV0011: Codes over nonstandard alphabets

Problem Set I

Problem 1:

- **(a)** Find 8 distinct points in $\mathbb{F}_2^6$ that pairwise differ in at least 3 coordinates.
- **(b)** Show that there are no 9 distinct points in $\mathbb{F}_2^6$, that have this property. Do this by grouping such a set of points according to their first entry and then applying the pigeonhole principle.

Problem 2: We generalise the situation in problem 1 in the following way: Denote by $A_q(n,d)$ the maximal number of points in $\mathbb{F}_q^n$ that pairwise differ in at least d positions. Prove that

$$A_q(n,d) \;\leq\; q\,A_q(n-1,d).$$

Problem 3: Prove the Singleton bound, which says: If $C \subseteq F^n$ is a q-ary block code of Hamming minimum distance d, then $|C| \leq q^{n+1-d}$.

Problem 4: Prove the Gilbert-Varshamov bound, which says, that

$$A_q(n,d) \;\geq\; \frac{q^n}{\sum_{j=0}^{d-1} \binom{n}{j}(q-1)^j}.$$

Do this by assuming a code C that has $M = A_q(n,d)$ words, and concluding that the (Hamming) disks with radius $d-1$ centered in the codewords then must cover the ambient space.

Problem 5:

- **(a)** Factorize the polynomial x^9+1 over $\mathbb{F}_2$.
- **(b)** Determine all *distinct* cyclic binary linear codes of length 9, and find their parameters.
- **(c)** Calling two codes equivalent, if they result from each other by a co-ordinate permutation, determine all *inequivalent* cyclic binary linear codes of length 9.