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MS-EV0011: Codes over nonstandard alphabets

Problem Set III

Problem 1: Let R be a finite ring, and let $R^{\times}x$ denote the set of all generators of the ideal $Rx \leq R$, and let μ be the Moebius function on the poset of all principal left ideals of R.

(a) Show that for all $x \in R$ there holds:

$$\dot{\bigcup_{Ry \leq Rx}} R^{\times}y \;=\; Rx.$$

(b) Use Moebius inversion in order to derive that for all $x \in R$ there holds:

$$|R^{\times}x| \;=\; \sum_{Ry \leq Rx} |Ry|\, \mu(Ry, Rx).$$

Problem 2: Let R be a finite ring, and let μ again denote the Moebius function on the poset of its left principal ideals. For arbitrary $\gamma > 0$, verify that the function $w : R \longrightarrow \mathbb{R}$ with

$$w(x) \;:=\; \gamma \left[1 - \frac{\mu(0, Rx)}{|R^{\times}x|}\right],$$

is indeed a (left) homogeneous weight, as it satisfies all the 3 of its defining criteria.

Problem 3:

(a) Compute the homogeneous weight of average 1 on $\mathbb{Z}_6$ and verify that this weight does not satisfy the triangle inequality.

(b) Compute the homogeneous weight w on $R := \mathbb{Z}_2 \times \mathbb{Z}_2$ and verify that $w(1,1) = 0$, so that w is not strictly positive.

Problem 4: Draw the Hasse diagram for the poset of all left ideals of $M_2(\mathbb{F}_q)$ and compute the homogeneous weight of average 1 for this ring.

Please feel free to collaborate, but submit individual solutions.