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MS-EV0011: Codes over nonstandard alphabets

Problem Set V

Problem 1: Find the Hamming weight enumerator of the binary $[7,4,3]$ Hamming code and compute the weight enumerator of its dual code. Verify that this code is a $[7,3,4]$ code.

Problem 2: Extend the binary $[7,4,3]$ Hamming code by a parity check bit, and write down its Hamming weight enumerator. Compute the MacWilliams' transform of this enumerator. What do you observe? Verify your conjecture about self-duality of the extended Hamming code.

Problem 3: The Gaussian binomial coefficients $\left[{n \atop k}\right]_q$ describe the number of k-dimensional subspaces in an n-dimensional vector space over the q-element field.

(a) Show that $\left[{n \atop k}\right]_q = \frac{(1-q^n)(1-q^{n-1})\cdots(1-q^{n+1-k})}{(1-q^k)(1-q^{k-1})\cdots(1-q)}$.

(b) Verify that $\lim_{q\to 1}\left[{n \atop k}\right]_q = \binom{n}{k}$.

Problem 4: Feel encouraged to use the book by Blyth and Janowitz, which you find in the assignments folder.

(a) Show that if K and L are complete lattices, then a mapping $f : K \longrightarrow L$ is residuated if and only if $f(\sum X) = \sum f(X)$ for all $X \subseteq K$.

(b) Show that in this case the residual g of f is given by $g(y) = \sum\{x \in L \mid f(x) \leq y\}$.

As usual, please feel free to collaborate, but submit individual write-ups.