

Summary of Part 2

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Midterm Exam

Topics covered: Lectures 12–22

- ▶ problem sets 5–8
- ▶ practice questions published during the second part (the first is related to functions)

Material in the textbook

- ▶ Multivariate calculus; 12-15
- ▶ Optimization: chapters 16-19, 21-22, 30.1, 30.3
- ▶ note: in exam there will be no questions on topics not covered in the lecture and exercise material, e.g. no chapters 18.6, 19.1, 19.2, 19.5, 21.4, 21.5

Review - Calculus of several variables

- ▶ Multivariate functions
- ▶ Continuous functions
- ▶ Open, closed, and compact sets
- ▶ Partial derivatives, gradient (and Jacobian)
- ▶ Chain rule
- ▶ Hessian matrix
- ▶ Implicit function theorem

Review - Unconstrained optimization

- ▶ Local and global extrema (minimizers or maximizers)
- ▶ Weierstrass's Theorem
- ▶ First order conditions (for interior extrema)
- ▶ Second order conditions (for interior extrema)
- ▶ Quadratic forms
- ▶ Definite and semi-definite matrices
- ▶ Convex sets, concave and convex functions
- ▶ Monotone transformations

Review - Constrained Optimization

Optimization problems with equality constraints:

- ▶ NDCQ
- ▶ first order necessary conditions
- ▶ second order sufficient conditions for local extrema

Optimization problems with inequality constraints

- ▶ NDCQ, first order necessary conditions
- ▶ concave problems
- ▶ second order sufficient conditions for local extrema

Review - Some fundamentals

We have seen several propositions of the form “If P , then Q ”

- ▶ We say that Q is a **necessary condition** for P
- ▶ In more compact form, this is often indicated as $P \implies Q$, i.e. P implies Q
- ▶ E.g., If the matrix A is positive definite, then its diagonal entries are strictly positive

We've also seen propositions of the form “ P if and only if Q ”

- ▶ In this case, we say that P is both a **necessary and sufficient condition** for Q , and vice versa
- ▶ In more compact form, this is often indicated as $P \iff Q$, i.e. P is equivalent to Q
- ▶ E.g., A function f is concave if and only if its Hessian matrix is negative semi-definite

Tips

You need to be able to differentiate the usual functions in economic models!

- ▶ linear and quadratic functions
- ▶ Cobb-Douglas (its gradient and Hessian)
- ▶ log-transformed Cobb-Douglas
- ▶ chain rule

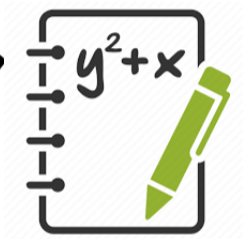
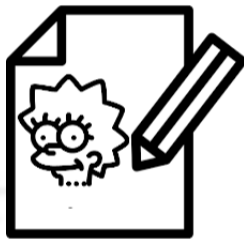
Understanding key concepts and theories

- ▶ implicit function theorem
- ▶ Weierstrass theorem
- ▶ role of concavity in maximization
- ▶ verifying the concavity of a function and the convexity of a set

First order conditions

- ▶ finding solutions
- ▶ be smart: utilize symmetry and monotonicity whenever possible

Final Advice



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Mathematical Methods for Economics Research: Optimization

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Feedback

The ideas for the development of this course come from students!

- ▶ examples: recap of calculus, in-class exercises, format of TA sessions, ...
- ▶ please give feedback