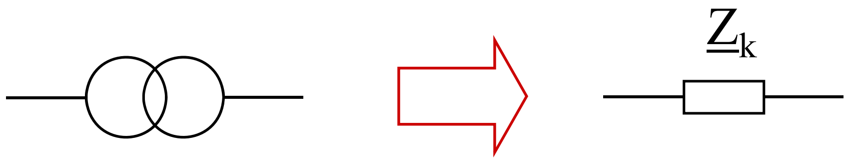
Transformer

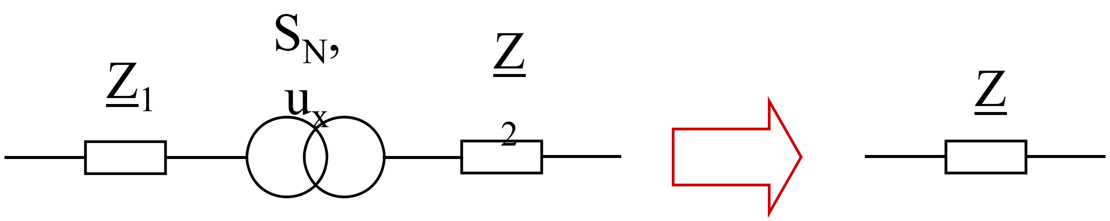


U_N = rated voltage (kV)
S_N = rated power (MVA)
u_x = p.u. short circuit reactance

$$\underline{Z}_k = R_k + jX_k$$

$$R_k = u_r \frac{U_N^2}{S_N}$$

$$X_k = u_x \frac{U_N^2}{S_N}$$

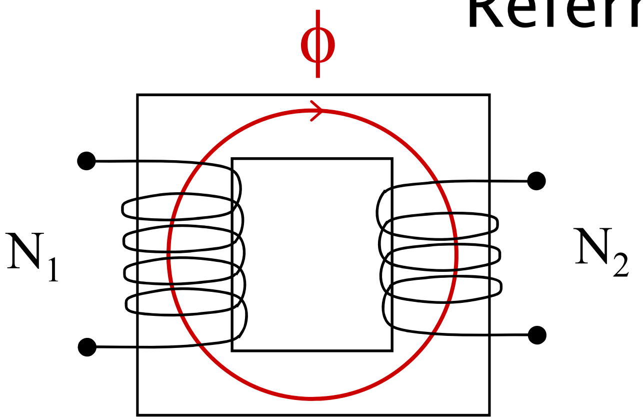


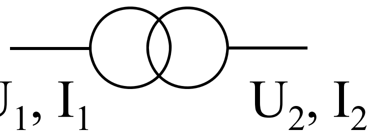
μ = transforming ratio

$$\mu = \frac{U_{1n}}{U_{2n}}$$

$$\underline{Z} = \underline{Z}_1 + ju_x \frac{U_{1n}^2}{S_N} + \mu^2 \cdot \underline{Z}_2$$

Referring over a transformer





Voltage over a turn is constant

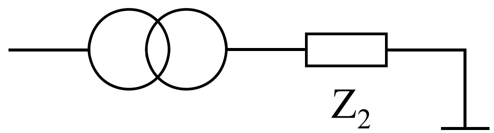
$$\frac{U_1}{U_2} = \frac{N_1}{N_2}$$

Power is constant

$$S = \sqrt{3}\, U_2 I_2 = \sqrt{3}\, U_1 I_1$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{U_2}{U_1} = \frac{N_2}{N_1}$$

Secondary impedance Z_2 referred to primary ?



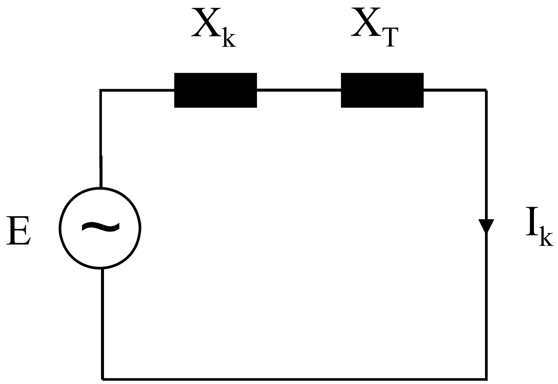
in secondary $Z_2 = \frac{U_2}{I_2}$

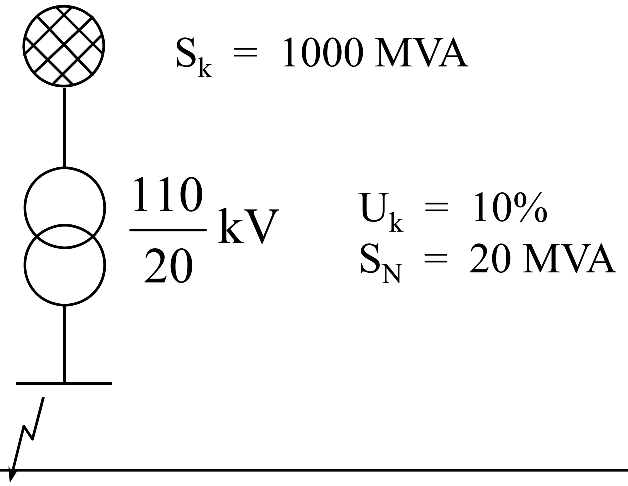
in primary $Z_2' = \frac{U_1}{I_1} = \frac{U_2 \cdot \frac{N_1}{N_2}}{I_2 \cdot \frac{N_2}{N_1}} = \left(\frac{N_1}{N_2}\right)^2 \cdot Z_2$

usually we write $\frac{N_1}{N_2} = \frac{U_{1n}}{U_{2n}}$

Referring over a transformer

An example





$$X_k = \frac{U_1^{\,2}}{S_k} = \frac{110^2}{1000} = 12{,}1\,\Omega \qquad (110\,\mathrm{kV})$$

$$X_T = u_k \frac{U_2^{\,2}}{S_N} = 0{,}1 \cdot \frac{20^2}{20} = 2\,\Omega \qquad (20\,\mathrm{kV})$$

20 kV level $\quad E = \frac{U_2}{\sqrt{3}}$

$$I_{k_2} = \frac{U_2}{\sqrt{3}\left(X_k{}' + X_T\right)} = \frac{20\,\mathrm{kV}}{\sqrt{3}\left(\left(\frac{20}{110}\right)^2 \cdot 12{,}1 + 2\right)\Omega} = \underline{\underline{4{,}81\,\mathrm{kA}}}$$

110 kV level $\quad E = \frac{U_1}{\sqrt{3}}$

$$I_{k_1} = \frac{U_1}{\sqrt{3}\left(X_k + X_T{}'\right)} = \frac{110\,\mathrm{kV}}{\sqrt{3}\left(12{,}1 + \left(\frac{110}{20}\right)^2 \cdot 2\right)\Omega} = \underline{\underline{0{,}875\,\mathrm{kA}}}$$

check :

$$I_{k_1} = \frac{U_{2n}}{U_{1n}} I_{k_2} = \frac{20}{110} \cdot 4{,}81\,\mathrm{kA} = 0{,}875\,\mathrm{kA}$$

Referring and p.u. values

1. Referring over a transformer (from U_1 to U_2)

$$U_1' = (U_{2n}/U_{1n}) \cdot U_1$$
$$I_1' = (U_{1n}/U_{2n}) \cdot I_1$$
$$Z_1' = (U_{2n}/U_{1n})^2 \cdot Z_1$$

Referring is easy when only two voltage levels

2. In case of several voltage levels p.u. computation

Select for the base values some base power S_b and voltage of one selected voltage level U_b

⇨ Other base voltages by transforming ratios:

$$U_{2b} = (U_{2n}/U_{1n}) \cdot U_{1b}$$

⇨ Current base value by base power and base voltage:

$$I_b = S_b / \sqrt{3}\, U_b$$

⇨ Impedance base value by base power and base voltage:

$$Z_b = U_b^{\,2} / S_b$$

3-ph power as a p.u. value using the equation $\underline{s} = \underline{u}\underline{i}^*$
Other computation as with single line diagram

Calculations using p.u. values

The actual U(kV), I(A), Z(Ω), S (MVA), P(MW), Q(Mvar) values are first scaled by corresponding base values of the voltage level in question

$$u = U/U_b \qquad i = I / I_b \qquad z = Z/Z_b \qquad s = S / S_b$$

As the result we have per unit (p.u.) values. Now all the different voltage Levels have been transformed to the same general domain and no any Referring over transformers is needed any more.

The calculations are done with single line diagram. For 3ph powers $\underline{s} = \underline{u}\underline{i}^*$

Once the calculations are done, we have the results in the form of Dimensionless p.u. values.

Finally the results are transformed back to actual physical values by Multiplying the p.u. values by the base values of the concerned voltage level.

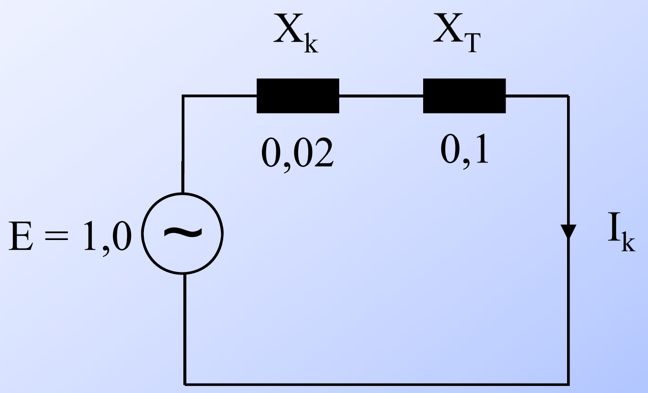
$$U = uU_b \qquad I = iI_b \qquad Z = zZ_b \qquad S = sS_b$$

Referring over a transformer

The previous example using p.u. computation

Let´s select

$$S_b = 20\,\text{MVA}$$
$$U_{b1} = 110\,\text{kV}$$
$$U_{b2} = \frac{U_{2n}}{U_{1n}} \cdot U_{b1} = 20\,\text{kV}$$
$$Z_{b1} = \frac{U_{b1}^{\;2}}{S_b} = \frac{110^2}{20} = 605\,\Omega$$
$$Z_{b2} = \frac{U_{b2}^{\;2}}{S_b} = \frac{20^2}{20} = 20\,\Omega$$
$$I_{b1} = \frac{S_b}{\sqrt{3}\,U_{b1}} = 105\,\text{A}$$
$$I_{b2} = \frac{S_b}{\sqrt{3}\,U_{b2}} = 577\,\text{A}$$



$$X_T = \frac{2}{20} = 0{,}1\,\text{p.u.}$$
$$X_k = \frac{12{,}1}{605} = 0{,}02\,\text{p.u.}$$

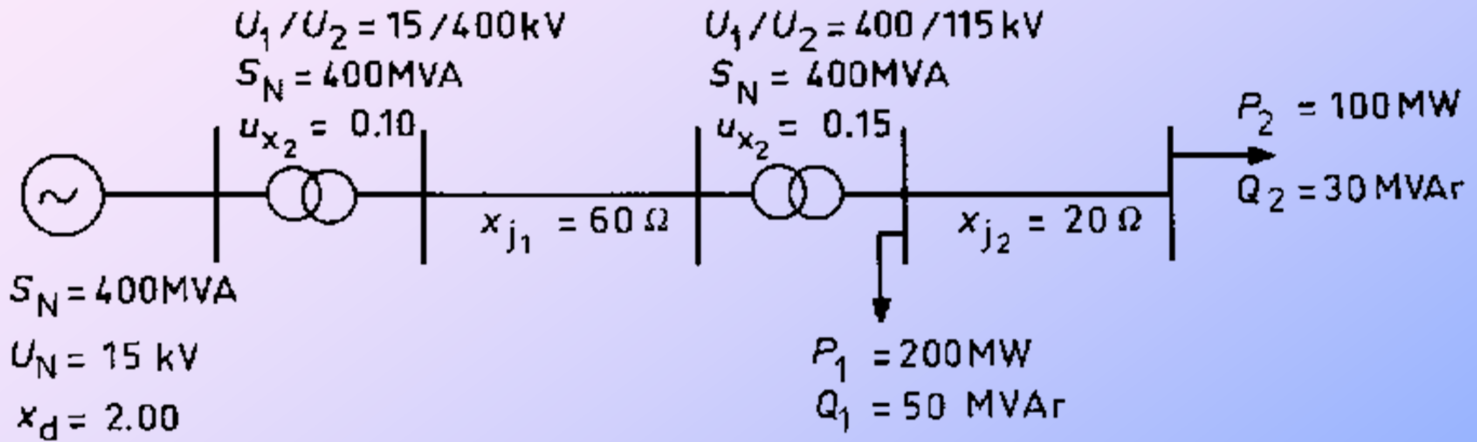
$$\Rightarrow I_k = \frac{E}{X_k + X_T} = \frac{1{,}0}{0{,}02 + 0{,}1} = \frac{1}{0{,}12} = 8{,}333\,\text{p.u.}$$

$$20\,\text{kV}: \quad I_k = 8{,}333 \cdot I_{b2} = 8{,}333 \cdot 577\,\text{A} = \underline{\underline{4{,}81\,\text{kA}}}$$

$$110\,\text{kV}: \quad I_k = 8{,}333 \cdot I_{b1} = 8{,}333 \cdot 105\,\text{A} = \underline{\underline{0{,}875\,\text{kA}}}$$

Computation using p.u. values (1/4)

Example network



$S_b = 400\,\text{MVA} \quad \& \quad U_b = 400\,\text{kV}$

Base voltages:

400 kV selected

$$100\,\text{kV level}: \frac{115}{400} \cdot 400\,\text{kV} = 115\,\text{kV}$$

$$15\,\text{kV level}: \frac{15}{400} \cdot 400\,\text{kV} = 15\,\text{kV}$$

Base currents: $I_b = S_b / \sqrt{3}\, U_b$

$$400\,\text{kV level}: 400\,\text{MVA} / \sqrt{3} \cdot 400\,\text{kV} = 577{,}4\,\text{A}$$

$$110\,\text{kV level}: 400\,\text{MVA} / \sqrt{3} \cdot 115\,\text{kV} = 2008{,}2\,\text{A}$$

$$15\,\text{kV level}: 400\,\text{MVA} / \sqrt{3} \cdot 15\,\text{kV} = 15{,}4\,\text{kA}$$

Base impedances: $Z_b = U_b^2 / S_b$

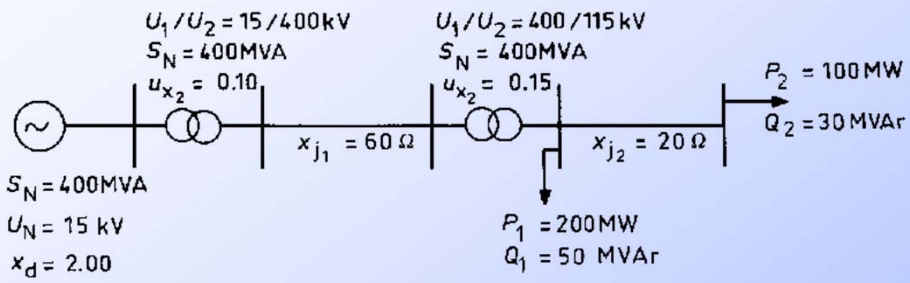
$$400\,\text{kV level}: \frac{400^2}{400} \Big/ \frac{\text{kV}^2}{\text{MVA}} = 400\,\Omega$$

$$110\,\text{kV level}: \frac{115^2}{400} \Big/ \frac{\text{kV}^2}{\text{MVA}} = 33{,}06\,\Omega$$

$$15\,\text{kV level}: \frac{15^2}{400} \Big/ \frac{\text{kV}^2}{\text{MVA}} = 0{,}5625\,\Omega$$

Computation using p.u. values (2/4)

Example network



Base impedances:

400 kV	110 kV	15 kV
400 Ω	33,06 Ω	0,5625 Ω

line reactances:

$$x_{j1} = \frac{60\,\Omega}{400\,\Omega} = 0{,}15\ \text{p.u.}$$

$$x_{j2} = \frac{20\,\Omega}{33{,}06\,\Omega} = 0{,}6\ \text{p.u.}$$

generator reactance:

$$X_d = x_d \cdot \frac{U_n^2}{S_n} = 2{,}00 \cdot \frac{15^2\ \text{kV}}{400\ \text{MVA}} = 1{,}125\,\Omega$$

$$x_d = \frac{1{,}125}{0{,}5625} = 2{,}0\ \text{p.u.}$$

transformer reactances:

$$X_{m1} = x_{m1} \cdot \frac{U_n^2}{S_n} = 0{,}1 \cdot \frac{400^2\ \text{kV}}{400\ \text{MVA}} = 40\,\Omega$$

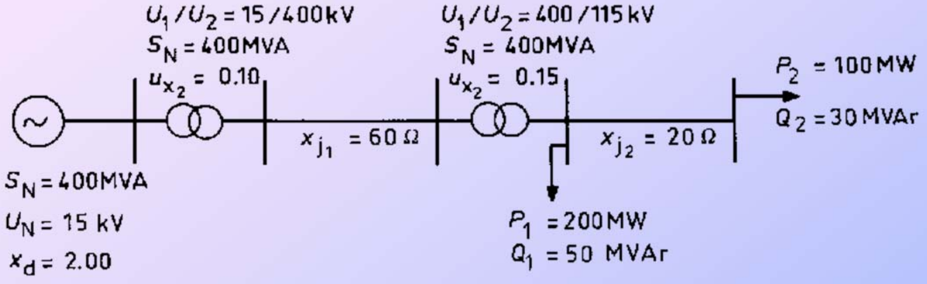
$$x_{m1} = \frac{X_{m1}}{X_b} = \frac{40}{400}\ \text{p.u.} = 0{,}1\ \text{p.u.}$$

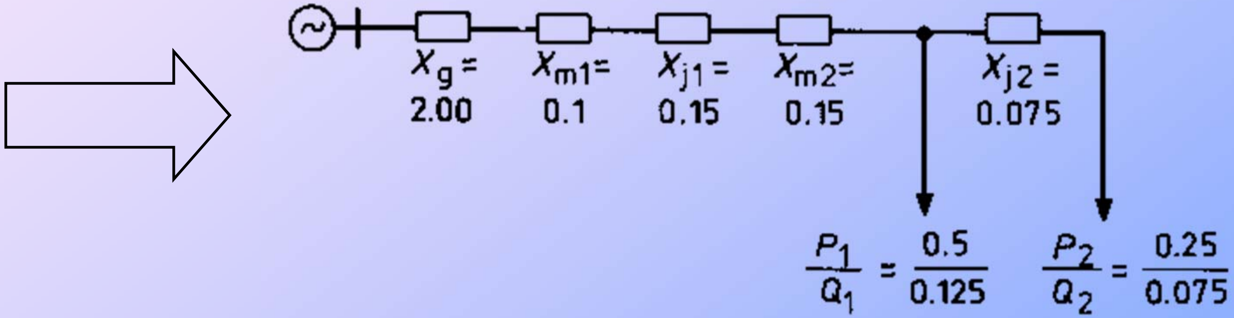
$$X_{m2} = x_{m2} \cdot \frac{U_n^2}{S_n} = 0{,}15 \cdot \frac{400^2\ \text{kVA}}{400\ \text{MVA}} = 60\,\Omega$$

$$x_{m2} = 60/400\ \text{p.u.} = 0{,}15\ \text{p.u.}$$

Computation using p.u. values (3/4)

Example network



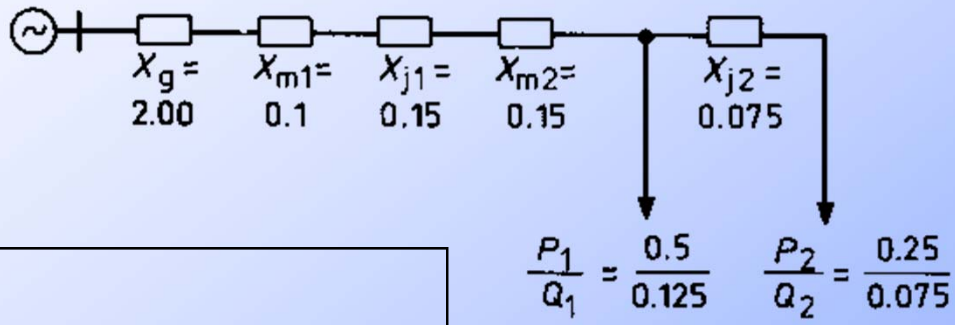


Loads as p.u. values

$$P_1 = 200/400 = 0{,}5$$
$$P_2 = 100/400 = 0{,}25$$
$$Q_1 = 50/400 = 0{,}125$$
$$Q_2 = 30/400 = 0{,}075$$

Computation using p.u. values (4/4)

Example network



Computation by p.u. values

a) the load current of transformer 2 ?

$$S = (P_1 + P_2) + j(Q_1 + Q_2) = 0{,}75 + j\,0{,}2 = 0{,}776 \angle 14{,}4°$$

$$\underline{s} = \underline{u}\underline{i}^* \quad ; \quad \text{let us take } \; u = 1 \quad \Rightarrow \quad i = 0{,}776$$

$$\text{current in } 115\,\text{kV level:} \quad I = 0{,}776 \cdot 2008\,\text{A} = 1558\,\text{A}$$

$$\text{current in } 400\,\text{kV level:} \quad I = 0{,}776 \cdot 577{,}4\,\text{A} = 448\,\text{A}$$

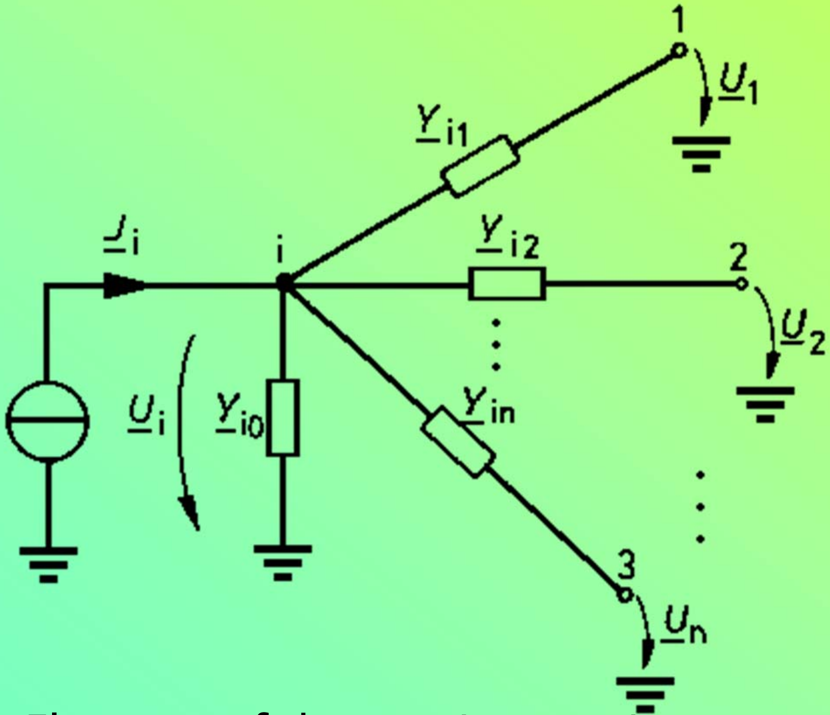
b) short circuit current in tranf. 2 secondary ?

$$x = x_d + x_{m1} + x_{j1} + x_{m2} = 2{,}4$$

$$i_k = \frac{u}{x} = \frac{1{,}0}{2{,}4} = 0{,}417 \quad (\text{assumption: } u = 1{,}0)$$

$$\text{Absolute value:} \quad I_k = 0{,}417 \cdot 2008\,\text{A} \approx 837\,\text{A}$$

Load flow computation in a meshed network



Kirchhoff's current law:

$$\underline{J}_i = \underline{Y}_{i0}\,\underline{U}_i + \sum_{j=1}^{n} \underline{Y}_{ij} \left(\underline{U}_i - \underline{U}_j\right)$$

Rearranging the terms

$$\underline{J}_i = \left(\underline{Y}_{i0} + \sum_{\substack{j=1 \\ j \neq i}}^{n} \underline{Y}_{ij} \right) \underline{U}_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} \underline{Y}_{ij}\,\underline{U}_j$$

Which is in matrix form:

$$\mathbf{J}_r = \mathbf{Y}_{r0}\mathbf{U}_0$$

Elements of the matrix equation:

$$\mathbf{I} = \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \\ . \\ . \\ . \\ \underline{I}_n \end{bmatrix} = \begin{bmatrix} \underline{y}_{11} \cdots \underline{y}_{1n} \\ \underline{y}_{21} \cdots \underline{y}_{2n} \\ \cdots\cdots \\ \cdots\cdots \\ \cdots\cdots \\ \underline{y}_{n1} \cdots \underline{y}_{nn} \end{bmatrix} \begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \\ . \\ . \\ . \\ \underline{U}_n \end{bmatrix} = \mathbf{YU}$$

$\mathbf{Y}$ = nodal admittance matrix

$\underline{Y}_{ii}$ = diagonal element, self-admittance
= sum of admittances in node i

$\underline{Y}_{ij}$ = non-diagonal elements
= admittance between nodes ij multiplied by -1

Load flow computation

Node types in a meshed network

Load node

- P, Q are known
- U, δ are computed

Generator node

- P, U are known
- Q, δ are computed
- $Q_{min} < Q < Q_{max}$

Reference node

- U, δ are known
- P, Q are computed

Polar presentation of network equations

Matrix equation in sum form:

$$\underline{J}_k = \sum_{i=1}^{n} \underline{y}_{ki} \underline{U}_i \qquad k = 1 \ldots n$$

For power it holds:

$$P_k - jQ_k = \underline{U}_k^* \underline{J}_k = \underline{U}_k^* \sum_{i=1}^{n} \underline{y}_{ki} \underline{U}_i$$

In polar form:

$$\underline{U}_k = U_k e^{j\delta_k}, \quad \underline{y}_{ki} = y_{ki} e^{-j\theta ki}, \quad \underline{U}_i = U_i e^{j\delta_i}$$

The equation for power comes:

$$P_k - jQ_k = \sum_{i=1}^{n} U_k y_{ki} U_i e^{-j(\theta_{ki} - \delta_i + \delta_k)}$$

Writing the exponents by sin and cos:

$$e^{-j(\theta_{ki} - \delta_i + \delta_k)} = \cos(\theta_{ki} - \delta_i + \delta_k) - j \sin(\theta_{ki} - \delta_i + \delta_k)$$

The equations for power flow become:

$$P_k = \sum_{i=1}^{n} U_k y_{ki} U_i \cos(\theta_{ki} - \delta_i + \delta_k)$$

$$Q_k = \sum_{i=1}^{n} U_k y_{ki} U_i \sin(\theta_{ki} - \delta_i + \delta_k)$$

$$k = 1 \ldots n - 1$$

Load flow computation

The solution for network equations

1. The equations are arranged so that first are the load nodes and then are the generator nodes. In the number of nodes is **n**, and the number of generator nodes is **m**, the number of load nodes is **n-m-1**

2. The network equations are written:

$$\begin{bmatrix} \Delta P_1 \\ . \\ . \\ \Delta P_{n-1} \\ \Delta Q_1 \\ . \\ . \\ \Delta Q_{n-m-1} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_1 & \mathbf{J}_2 \\ \\ \mathbf{J}_3 & \mathbf{J}_4 \end{bmatrix} \cdot \begin{bmatrix} \Delta \delta_1 \\ . \\ . \\ \Delta \delta_{n-1} \\ \Delta U_1 \\ . \\ . \\ \Delta U_{n-m-1} \end{bmatrix}$$

$J_1 \ldots J_4$ are Jacobian matrices of network equations :

$$J_{1ij} = \frac{\partial P_i}{\partial \delta_j} \qquad J_{2ij} = \frac{\partial P_i}{\partial U_j}$$

$$J_{3ij} = \frac{\partial Q_i}{\partial \delta_j} \qquad J_{4ij} = \frac{\partial Q_i}{\partial U_j}$$

Load flow computation

1. Give initial values for voltage amplitudes and phase angles

2. Compute P and Q for load, generator and reference nodes

$$P_k = \sum_{i=1}^{n} U_k y_{ki} U_i \cos(\theta_{ki} - \delta_i + \delta_k)$$

$$Q_k = \sum_{i=1}^{n} U_k y_{ki} U_i \sin(\theta_{ki} - \delta_i + \delta_k)$$

3. Compute ΔP for load and generator and ΔQ
   for load nodes (difference of calculated and target values)

4. Calculate the Jacobian matrix

$$J_{1ij} = \frac{\partial P_i}{\partial \delta_j} \qquad J_{2ij} = \frac{\partial P_i}{\partial U_j}$$

$$J_{3ij} = \frac{\partial Q_i}{\partial \delta_j} \qquad J_{4ij} = \frac{\partial Q_i}{\partial U_j}$$

5. Invert the matrix and compute the corrections for node voltages ΔU, $\Delta\delta$

$$\begin{bmatrix} \Delta\delta \\ \Delta U \end{bmatrix} = \boldsymbol{J}^{-1} \begin{bmatrix} \Delta \boldsymbol{P} \\ \Delta \boldsymbol{Q} \end{bmatrix}$$

6. Back to 2 and calculate new P and Q, until convergence …

Load flow computation

The flow of computation

1. Give initial values for voltage amplitudes and phase angles

2. Compute P and Q for load, generator and reference nodes

3. Compute ΔP for load and generator and ΔQ for load nodes

4. Make Jacobian, invert the matrix and compute the corrections for node voltages ΔU, $\Delta \delta$

5. Back to 2 until convergence

Defining the load flow

When the voltages are known, the currents are obtained:

$$I_{ki} = Y_{ki}(U_k - U_i)$$

The power flowing into a line:

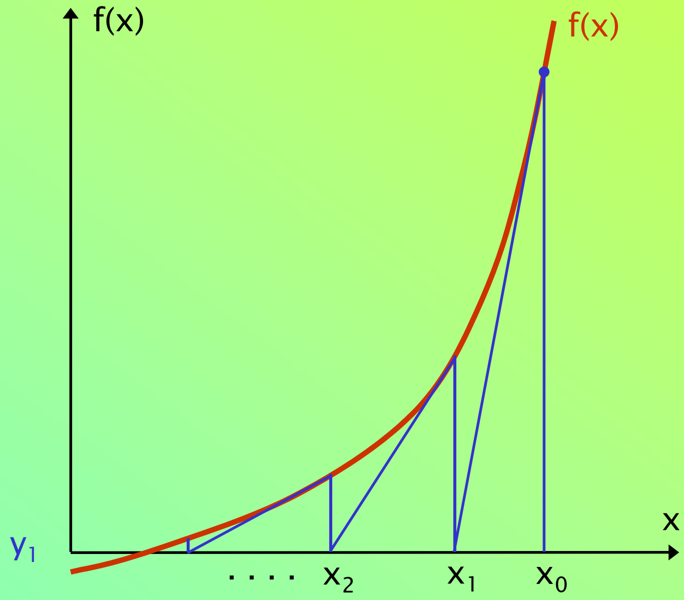
$$\underline{S}_{ki} = \underline{U}_k \underline{I}_{ki}^* = \underline{U}_k \underline{Y}_{ki}^* (\underline{U}_k^* - \underline{U}_i^*)$$

The power loss of a line:

$$\underline{S}_{hki} = \underline{Y}_{ki}^* [\underline{U}_k - \underline{U}_i]^2$$

Load flow computation

Newton-Raphson method for iteration



$$f(x_0 + \Delta x_1) = y_1$$

$$= f(x_0) + \frac{\partial f}{\partial x}(x_0)\ \Delta x_1$$

$$\Rightarrow \Delta x_1 = \frac{y_1 - f(x_0)}{\frac{\partial f}{\partial x}(x_0)}$$

$$\Rightarrow x_1 = x_0 + \Delta x_1 = x_0 + \frac{y_1 - f(x_0)}{\frac{\partial f}{\partial x}(x_0)}$$

Load flow computation

The sets of equations

$$\begin{cases} y_1 - f_1(x_1, x_2) = \dfrac{\partial f_1}{\partial x_1}\Delta x_1 + \dfrac{\partial f_1}{\partial x_2}\Delta x_2 \\ y_2 - f_2(x_1, x_2) = \dfrac{\partial f_2}{\partial x_1}\Delta x_1 + \dfrac{\partial f_2}{\partial x_2}\Delta x_2 \end{cases}$$

In matrix form:

$$\mathbf{y} - \mathbf{f}(k) = \mathbf{J}(k)\,\Delta x$$

$$\Leftrightarrow$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} f_1(k) \\ f_2(k) \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

$\mathbf{J}(k)$ is the Jacobian matrix

$$\Rightarrow \mathbf{x}(k+1) = \mathbf{x}(k) + \mathbf{J}(k)^{-1}(\mathbf{y} - \mathbf{f}(k))$$

Admittance vs impedance matrix

$$\mathbf{I} = \mathbf{YU}$$

$$\Rightarrow \mathbf{U} = \mathbf{Y}^{-1}\mathbf{I} = \mathbf{Z}\,\mathbf{I}$$

$\mathbf{Z}$ is the nodal impedance matrix

z_{ii} node self impedance (corresponds to short-circuit impedance)

z_{ij} transfer impedance of nodes i and j