3.1 (Construction of a zero-coupon bond and forward rates) Consider two 4-year and 5-year bonds as presented in table below.
a) Find the prices of 4- and 5-year zero-coupon bond.
b) Find the short rate at year 4.

Bond	Coupon rate	Maturity (years)	Price
A	8%	4	98.30
B	7%	4	95.00
C	9%	5	101.00
D	7%	5	93.20

3.2 (L4.13) (Stream immunization) A company faces a stream of obligations over the next 8 years as shown in the table below: where the numbers denote thousands of dollars. The spot rate curve is also presented in this table. The company has decided to invest in two bonds. Bond 1 has a maturity of 12 years, 6% coupon and price $P_1 = 65.95$ and bond 2 has maturity of 5 years, 10% coupon and price $P_2 = 101.66$. Find a portfolio, consisting of the two bonds, that has the same present value as the obligation stream and is immunized against an additive shift in the spot rate curve.

Year	1	2	3	4	5	6	7	8	9	10	11	12
Payment	500	900	600	500	100	100	100	50	0	0	0	0
Spot rate	7.67	8.27	8.81	9.31	9.75	10.16	10.52	10.85	11.15	11.42	11.67	11.89

3.3 (L4.5) (Instantaneous rates) Let $s(t)$, $0 \le t \le \infty$ denote a spot rate curve; that is, the present value of a dollar to be received at time t is $e^{-s(t)t}$. For $t_1 < t_2$, let $f(t_1, t_2)$ be the forward rate between t_1 and t_2 implied by the given spot rate curve.
a) Find an expression for $f(t_1, t_2)$ (using $s(t)$).
b) Let $r(t) = \lim_{t_2 \to t} f(t, t_2)$. $r(t)$ is the instantaneous interest rate at time t. Show that $r(t) = s(t) + s'(t)t$.
c) If an amount x_0 is invested at $t = 0$ in a bank which pays the instantaneous rate of interest $r(t)$ at all t (compounded), the bank balance $x(t)$ will satisfy $\mathrm{d}x(t)/\,\mathrm{d}t = r(t)x(t)$. Find an expression for $x(t)$.

3.4 (L4.7) (Bond taxes) An investor is considering the purchase of 10-year US Treasury bonds and plans to hold them to maturity. Federal taxes on coupons must be paid during the year they are received, and tax must also he paid on the capital gain realized at maturity (defined as the difference between face value and original price). Federal bonds are exempt from state taxes. This investor's federal tax bracket rate is $t = 30\%$, as it is for most individuals. There are two bonds that meet the investor's requirements. Bond 1 is a 10-year, 10% bond with a price (in decimal form) of $P_1 = 92.21$. Bond 2 is a 10-year, 7% bond with a price of $P_2 = 75.84$. Based on the price information contained in those two bonds, the investor would like to compute the theoretical price of a hypothetical 10-year zero-coupon bond that has no coupon payments and requires tax payment only at maturity equal in amount to 30% of the realized capital gain (the face value minus the original price). This theoretical price should be such that the price of this bond and those of bonds 1 and 2 are mutually consistent on an after-tax basis. Find this theoretical price, and show that it does not depend on the tax rate t. (Assume all cash flows occur at the end of each year.)

3.5 (L4.15) (Short rate sensitivity) The quasi-modified measures the sensitivity of a price of an asset to a parallel shift in the spot rate curve. A measure for the sensitivity of an asset's price to a parallel shift in the short rates (that is, $r_k \to r_k + \lambda$.) can also be useful. This can be solved using the running present value method. Specifically, letting P_k be the present value as seen at time k and $S_k = \mathrm{d}P_k / \mathrm{d}\lambda|_{\lambda=0}$, the S_k's can be found recursively by an equation of the form $S_{k-1} = -a_k P_k(\lambda = 0) + b_k S_k$, while P_k's are found by the running method. Find a_k and b_k.