6.1 (L7.1) (Capital market line) Assume that the expected rate of return on the market portfolio is 23% and the rate of return on T-bills (the risk-free rate) is 7%. The standard deviation of the market is 32%. Assume that the market portfolio is efficient.
a) What is the equation of the capital market line?
b) (i) If an expected return of 39% is desired, what is the standard deviation of this position? (ii) If you have 1 000 € to invest, how should you allocate it to achieve the above position?
c) If you invest 300 € in the risk-free asset and 700 € in the market portfolio, how much money should you expect to have at the end of the year?

6.2 (L7.5) (Uncorrelated assets) Suppose there are n mutually uncorrelated assets. The return on asset i has variance σ_i^2. The expected rates of return are unspecified at this point. The total amount of asset i in the market is X_i. We let $T = \sum_{i=1}^{n} X_i$ and then set $x_i = X_i/T$, for $i = 1, 2, \ldots, n$. Hence the market portfolio in normalized form is $\mathbf{x} = (x_1, x_2, x_3, \ldots, x_n)$. Assume there is a risk-free asset with rate of return r_f. Find an expression for $\beta_i = \mathrm{Cov}[r_i, r_M]/\mathrm{Var}[r_M]$ in terms of the x_i's and σ_i's.

6.3 (L7.7) (Zero-beta assets) Let $\mathbf{w_0}$ be the portfolio (weights) of risky assets corresponding the minimum-variance point in the feasible region. Let $\mathbf{w_1}$ be any other portfolio on the efficient frontier. Define r_0, r_1, σ_0^2 and σ_1^2 to be the corresponding returns and variances of the returns.
a) There is a formula of the form $\sigma_{01} = A\sigma_0^2$. Find A. (*Hint*: Consider portfolios $\mathbf{p} = (1-\alpha)\mathbf{w_0} + \alpha\mathbf{w_1}$, and consider small variations of the variance of such portofolios near $\alpha = 0$. Note that $\mathrm{dVar}[r_\mathbf{p}]/\,\mathrm{d}\alpha\;|_{\alpha=0} = 0$, because $\mathbf{w_0}$ is the minimum variance point.)
b) Corresponding to the portfolio $\mathbf{w_1}$ there is a portfolio $\mathbf{w_z}$ on the minimum-variance set that has zero beta with respect to $\mathbf{w_1}$; that is, $\sigma_{1z} = 0$. This portfolio can be expressed as $\mathbf{w}_z = (1-\alpha)\mathbf{w_0} + \alpha\mathbf{w_1}$. Find the proper value of α.
c) Show the relation of the three portfolios on a diagram that includes the feasible region.
d) If there is no risk-free asset, it can be shown that other assets can be priced according to the formula

$$\bar{r}_i - \bar{r}_z = \beta_{iM}(\bar{r}_M - \bar{r}_z),$$

where the subscript M denotes the market portfolio and $\bar{r}_z$ is the expected rate of return of the portfolio that has zero beta with the market portfolio. Suppose that the expected returns on the market and the zero-beta portfolio are 15% and 9%, respectively. Suppose that stock i has a correlation with the market of 0.5. Assume also that the standard deviation of the returns of the market and stock i are 15% and 5%, respectively. Find the expected return of stock i.

6.4 (L7.9) Show that for a fund with return $r = (1-\alpha)r_f + \alpha r_M$, both CAPM pricing formulas (pricing form of the CAPM and certainty equivalent pricing formula) give the price of 100 € worth of fund assets as 100 €.