2

Physics of Sound

The word *sound* in English has two confusingly different meanings. It may refer to the physical phenomenon or to the subjective percept. The old philosophical dilemma ponders, ‘If a tree in the forest falls down and there is no observer, does it make a sound?’ Based on experimental evidence and by making a clear distinction between these two meanings, we may say that a falling tree causes a physical sound event that can be recorded and analysed every time, but it does not make any sound in the sense of an auditory perception if there is no subject to hear the event.

In this chapter, we look at the physical side of the concept of sound. Without a physical basis there can be no sound event; the emergence of sound requires a physical substrate, and its perception a physiological one. Although complicated in practice, the physics of sound has a basis that is well formulated mathematically and is thus a widely studied topic in science and engineering. We do not go deeply into physical acoustics here, but rather we present an overview of the most basic concepts necessary or helpful to understand communication by sound and voice. Finding more specialized textbooks and publications is easy, and this chapter includes references to many such sources for more information. This overview serves to refresh the memory of those who have already studied physical acoustics and as a starting point for those who have not.

2.1 Vibration and Wave Behaviour of Sound

Sound, from a physical point of view, is a wave physically propagating in a medium, usually air, and in most cases is caused by a vibrating mechanical object. Exceptions can also be found, such as the electrical discharge of lightning. Some sources of sound waves, interesting within the scope of this book, are the speech organs of a speaker, the vibrating plates and the air column of a guitar body, and the diaphragm of a loudspeaker.

Sound radiated from machines is often considered undesirable sound, *noise*, that can degrade the performance of human hearing if it is exposed to too loud a sound and for a long time. Even not-so-loud sound may be annoying and disturbing, for example while sleeping or when concentrating on a specific task. On the other hand, noise-like sounds also carry information

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Ville Pulkki and Matti Karjalainen.

that helps us to orientate and manage our complex environments. It is good to realize that the energy level of sound is almost always so low that its physical effects can be neglected, but its effects on humans and animals are the reason we are interested in sound.

2.1.1 From Vibration to Waves

The event that causes a sound or vibration is called *excitation* or the *source*. A sound wave or vibration can propagate in a physical *medium*, it may be boosted by *resonance* effects and attenuate due to *losses* that transform it into other energy forms (mostly to heat). Sound waves and vibrations can be explained as an alternation between two forms of energy, potential energy and kinetic energy, starting from a simple case and approaching a general case of sound propagation.

2.1.2 A Simple Vibrating System

There are three basic variables that are used to describe the state of a physical particle:

- *Position* or *displacement* from a reference position. Let us denote this by y.
- *Velocity*, the time derivative of position or the ratio of the difference in position and interval in time t in which the change in position occurs, $v = \mathrm{d}y/\mathrm{d}t = \dot{y}$.
- *Acceleration*, the time derivative of velocity or the second derivative of position, $a = \mathrm{d}v/\mathrm{d}t = \mathrm{d}^2y/\mathrm{d}t^2 = \ddot{y}$.

Note that for a particle of non-zero mass and size, there are three rotational variables that may be important in some other contexts.

A mass (Figure 2.1a) and a spring (Figure 2.1b) can be combined to make the simplest possible vibrating system, as shown in Figure 2.1d. The force acting on the mass due to acceleration is

$$F = ma = m\ddot{y}, \tag{2.1}$$

where F is force [kg m/s^2] = [N, Newton], m is *mass* [kg] and a is acceleration, showing the linear dependence of force and acceleration. For an ideal spring we can write

$$F = -\mathrm{K}y, \tag{2.2}$$

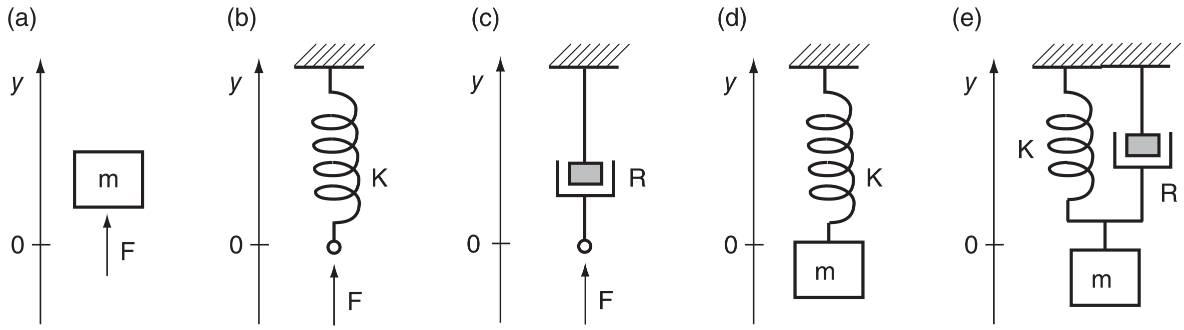

Figure 2.1 (a) A mass, (b) a spring, (c) a dashpot, (d) a mass–spring system and (e) a damped mass–spring system.

where K is the *spring constant* [N/m] and y is the displacement from the equilibrium position when the force is zero. Figure 2.1c shows a *dashpot* that represents energy losses in a vibrating system, typically converting energy to heat, which, in an ideal case, can be expressed as

$$F = -Rv = -R\dot{y}. \tag{2.3}$$

Here, R is a coefficient that relates force F and velocity v linearly. This linear relation does not hold, for example, in the case of mechanical friction.

Figure 2.1e represents a simple vibrating system with a damping element. Assuming that there are no external forces acting on the system, we can write

$$m\ddot{y} + R\dot{y} + \mathrm{K}y = 0 \tag{2.4}$$

since the sum of the forces must be zero. If there are no losses ($R = 0$) and the mass is initially displaced by $y = A$, the system starts to vibrate according to the expression

$$y(t) = A\cos\omega_0 t = A\cos 2\pi f_{\mathrm{res}} t, \tag{2.5}$$

where the maximum displacement A is called the *amplitude*, f_{res} is the *frequency* (*characteristic frequency, resonance frequency, eigenfrequency*), and $\omega_0 = 2\pi f_{\mathrm{res}}$ is the corresponding *angular frequency* of vibration. Curve a) in Figure 2.2 characterizes the oscillation of Equation (2.5). If the initial state is zero displacement but non-zero velocity, the equation has function $\sin\omega_0 t$ instead of $\cos\omega_0 t$. In either case, such a vibration is called a sinusoidal oscillation.

The eigenfrequency of a mass–spring system can be computed from its parameters:

$$f_{\mathrm{res}} = \frac{1}{2\pi}\sqrt{\frac{\mathrm{K}}{m}}. \tag{2.6}$$

For any periodic oscillation the relation between frequency f [Hz, hertz] and period T [s, seconds], for the duration of one oscillation, is

$$f = 1/T. \tag{2.7}$$

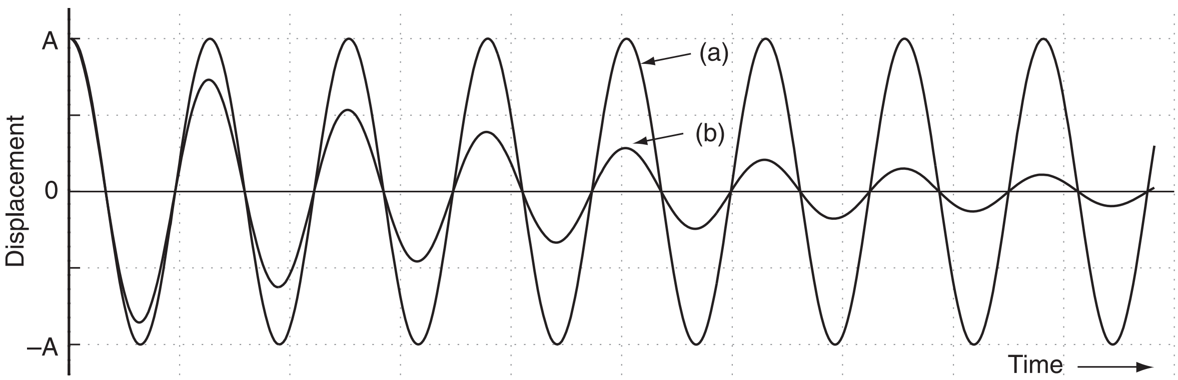

Figure 2.2 The displacement of a simple mass–spring system as a function of time for (a) the lossless case and (b) the lossy (damped) case.

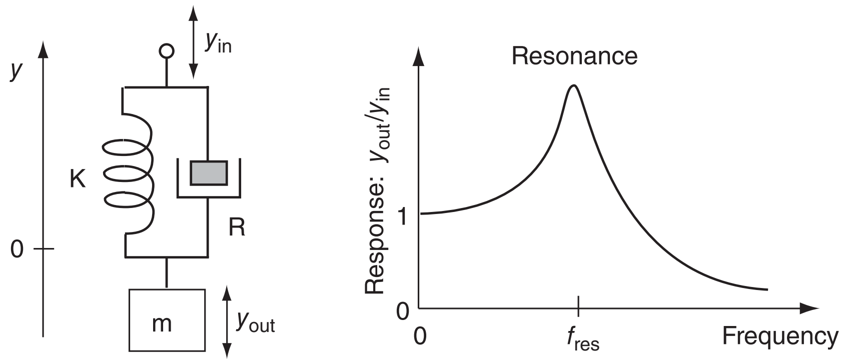

Figure 2.3 The resonance phenomenon in a mass–spring system. If the upper end of the string is moved sinusoidally (excitation), the movement of the mass (response) is increased around the resonance frequency (eigenfrequency).

In practice, losses damp the oscillation of real mass–spring systems, as a result of which the amplitude of the oscillation decreases exponentially in time as

$$y(t) = Ae^{-\alpha t} \cos(\omega_d t + \phi) = A(t) \cos(\psi(t)), \tag{2.8}$$

where α is a damping coefficient and ω_d is the angular frequency of the damped oscillation. $A(t)$ is the *amplitude envelope* and $\psi(t)$ the *instantaneous phase*. The damped oscillation is characterized by curve b) in Figure 2.2. Losses often mean energy conversion to heat, but the energy may also be transferred to another kind of oscillation, such as electrical vibration (see Section 4.1).

Mechanical vibration is typically the source of acoustic waves, and the radiation of sound also drains some of the energy from the mechanical vibration system.

2.1.3 Resonance

The simple mass–spring combination oscillates most easily at its eigenfrequency given in Equation (2.6) or in its vicinity. If the mass–spring damping system of Figure 2.3 is excited by a constant-amplitude sinusoidal movement at different frequencies at the top end of the spring, the response – the amplitude of the mass oscillation – follows the resonator response curve also shown in the figure. When the vibration frequency is near f_{res}, we say that the system is in *resonance* or that it resonates. Such a system is called a *resonator*.

Resonance is a phenomenon found frequently in physical systems. It may be desired, undesirable, or even harmful, depending on the case. If the radiation of sound of a musical instrument is too weak, then building a body or sound board with stronger resonances may help to increase the loudness of the sound source. At the same time, the resonance colours the sound which, if properly designed, can make it more appealing. On the other hand, resonances may amplify noise or even cause a machine to malfunction due to strong resonant behaviour. In such a case, damping helps to reduce undesired or dangerous vibration or harmful noise.

A simple but important type of resonator in acoustics is the *Helmholtz resonator*, which is shown in Figure 2.4. The air inside a closed volume V, due to its compressibility, acts as a mechanical spring, and the moving air in the tube or opening above behaves like a mass. Together they make a mass–spring system that works as an acoustic resonator. For instance, by properly blowing into the opening of an empty bottle, this effect can be easily demonstrated.

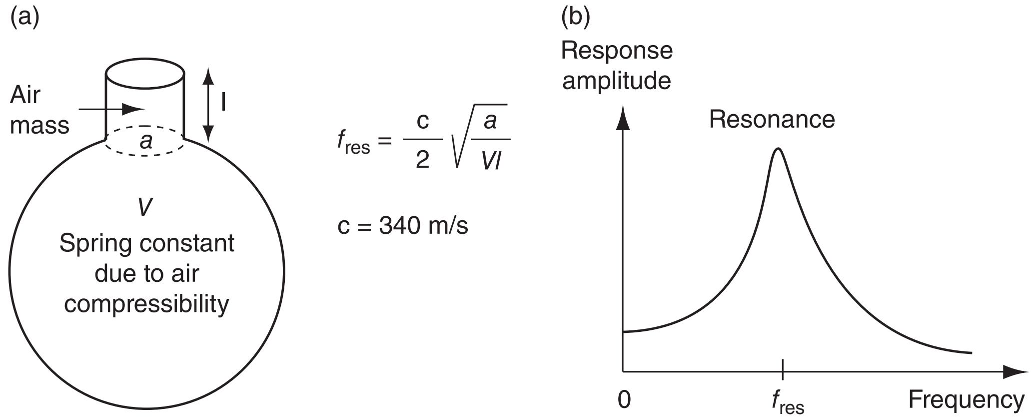

Figure 2.4 Helmholtz resonator: (a) principle of structure and (b) response (e.g., pressure variation in the bottle) as a function of frequency due to (external pressure) excitation.

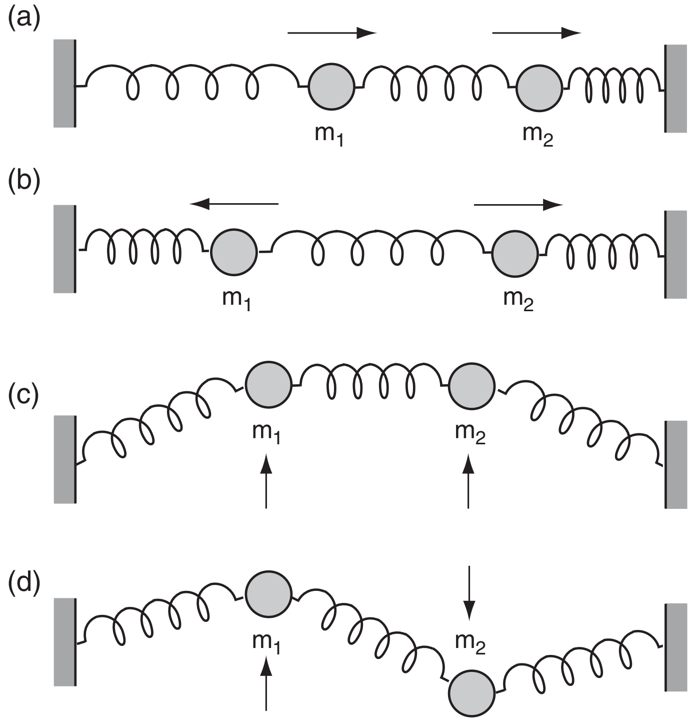

Figure 2.5 (a and b) Longitudinal and (c and d) transversal (vertical) vibration modes in a spring-coupled, two-mass system.

The principle of Helmholtz resonance takes different shapes. The body of the violin or the guitar is a good example where the air hole(s) and the body result in the lowest resonance frequency of the instrument. A bass-reflex type of loudspeaker enclosure adds a desired boost to the low-frequency response of the loudspeaker (see Section 4.1.1). The interior of a car cabin, when a window is slightly opened while driving fast, may strongly amplify turbulent noise at low or very low (infrasound) frequencies.

2.1.4 Complex Mass–Spring Systems

When several masses are coupled through springs, the resulting system shows more complicated oscillatory behaviour. Figures 2.5a and 2.5b illustrate a case where two masses, coupled together by springs and attached to a fixed (non-movable) support, vibrate in the horizontal direction. Such movement is called *longitudinal* vibration. The case of the same two spring-coupled masses now moving in the vertical direction is depicted in Figures 2.5c and 2.5d. The movement occurs perpendicular to the coupling spring direction and is called *transversal* vibration.

In both cases each mass shows one degree of freedom to vibrate. If not limited to move in some direction, each mass can move in three dimensions, thus exhibiting three degrees of freedom (independent components of vibration).

If we expand the system to include three masses, we will have longitudinal vibrations as shown in Figure 2.6a–c, or the vertical vibrations of Figure 2.6d–f.

2.1.5 Modal Behaviour

Since masses coupled through springs cannot vibrate independently, the resonance behaviour of such systems is more complex than in the case of the simple mass–spring system of Figure 2.3.

Figures 2.5 and 2.6 characterize vibration patterns that are called *normal modes* or *eigen-modes*, or simply *modes*. At and near a mode frequency (= resonance frequency), the vibration

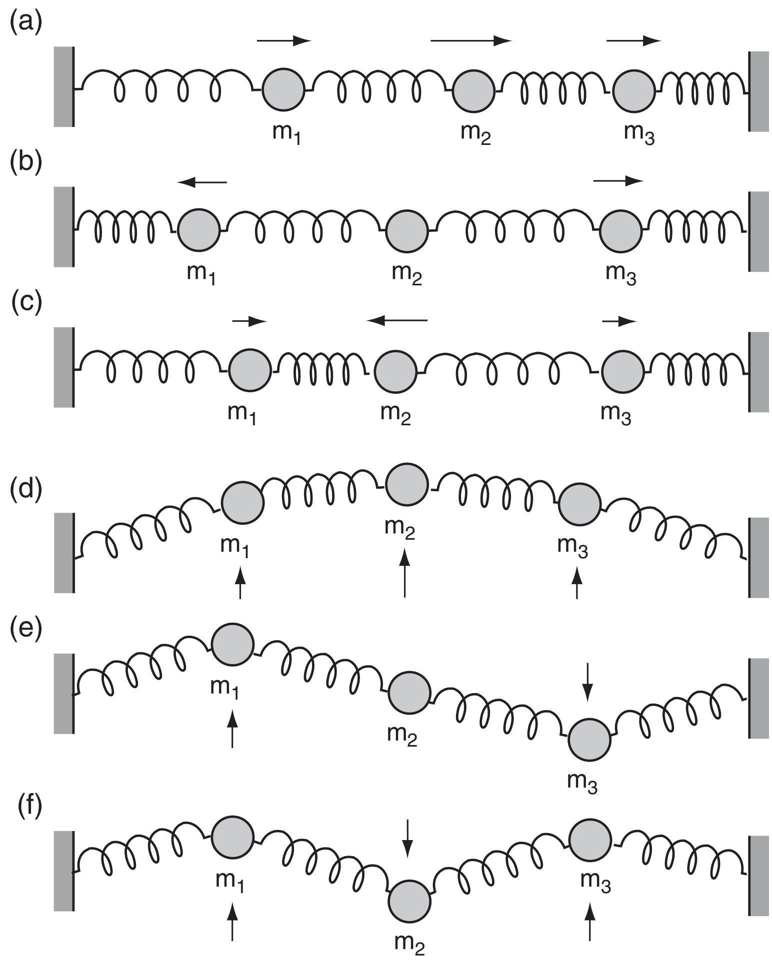

Figure 2.6 (a–c) Longitudinal and (d–f) transversal vibration modes in a three-mass system.

response grows in amplitude when the system is excited, say, by moving one of the terminal supports.

Such modal behaviour is characteristic of simple and complex resonators. As will be explained below, for example the string, the air column, the membrane of a musical instrument, or the human vocal tract show a multitude of mode frequencies. Three-dimensional systems, such as rooms and concert halls, show the same kind of modal behaviour.

Modal frequency vibrations, such as those characterized in Figures 2.5 and 2.6, are only a small subset of possible vibrations in these systems. The special role of modes is that all other vibrations can be expressed as linear combinations of the modes, or, in other words, as movements where the modal oscillations at each point are each summed together with proper amplitude scaling.

The infinitely growing complexity of mass–spring systems is understood when analysing the behaviour of a vibrating string, such as the guitar string, supported at both ends. Figure 2.7 illustrates the eight lowest modes of such a string. We can consider the string as a continuum of infinitely small masses and springs. This means that, in theory, there are infinitely many modal frequencies (see also Sections 2.3 and 2.4.4).

2.1.6 Waves

In a spatially distributed homogeneous mass–spring medium, a vibratory movement (excitation, source) causes a wave to propagate from the source. Figure 2.8a characterizes a transversal one-dimensional wave starting to propagate from a moving source. The medium may be, for example, a rope that is moved rapidly up and down by hand. Transversal means that the movement of the particles in the medium is perpendicular to the direction of the wave propagation.

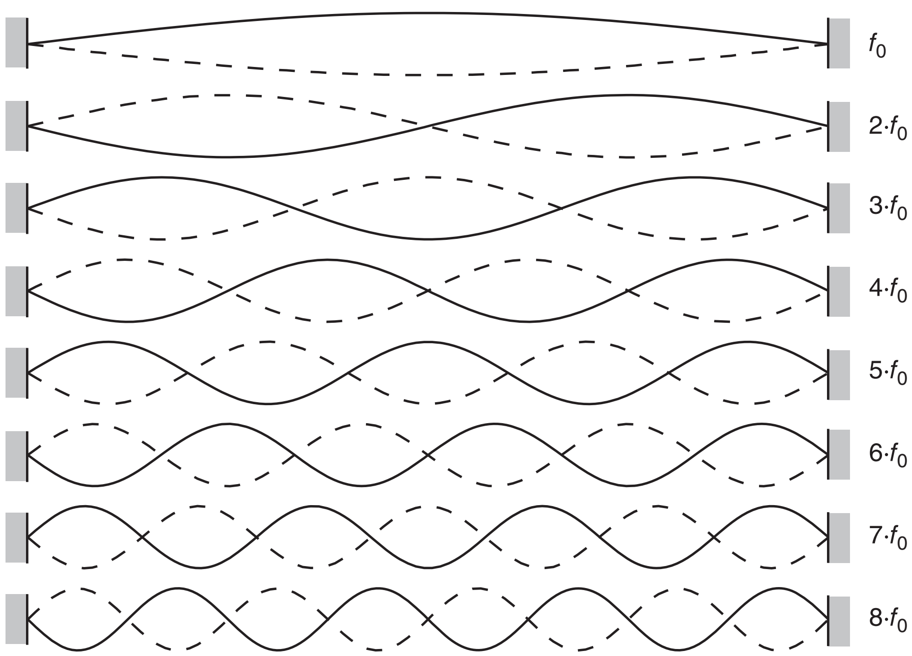

Figure 2.7 The eight lowest resonance modes of a vibrating string. In each mode, the string vibrates between the ends shown by solid and dashed lines. Nodes are points where the vibration amplitude is zero.

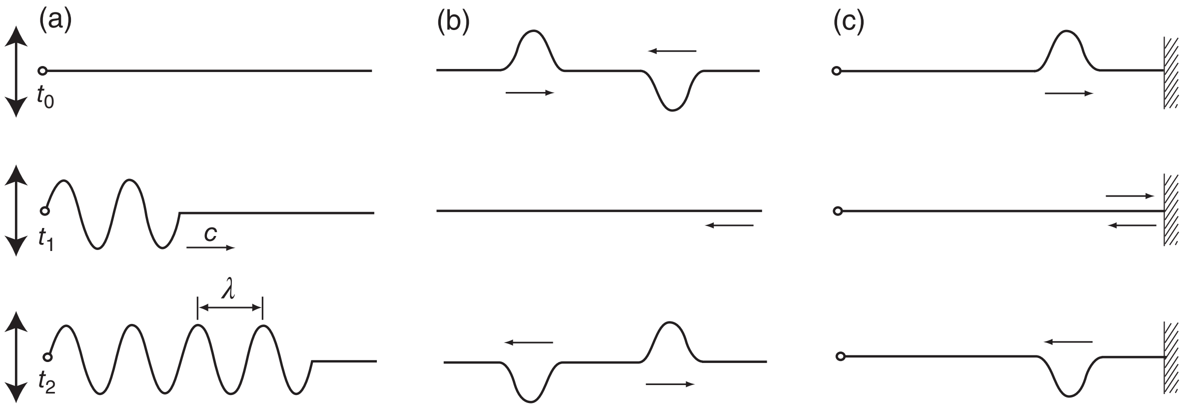

Figure 2.8 (a) Wave propagation in a continuously distributed mass–spring chain, such as a rope, (b) the passing of two wave components travelling in opposite directions, and (c) the wave reflection at a fixed termination.

In Figure 2.8c, the mass–spring medium (such as a rope) is terminated rigidly. The discontinuity of the medium causes a reflection of the wave at the termination. As will be shown below, any discontinuity of the medium can result in full or partial reflection of a wavefront.

In mathematical terms, a wave in a homogeneous and lossless medium follows a simple rule called the *wave equation*:

$$\ddot{y} = c^2 y''. \tag{2.9}$$

Here y is displacement, c is the velocity of propagation of the wavefront, $\ddot{y}$ is the second time derivative of displacement equalling $\partial^2 y/\partial t^2$, and y'' is the second position derivative of displacement equalling $\partial^2 y/\partial x^2$.

One-dimensional wave propagation, as characterized in Figure 2.8, obeys the general solution of the wave equation for a wave propagating in a homogeneous medium,

$$y(t, x) = g_1(ct - x) + g_2(ct + x), \tag{2.10}$$

where y is a physical wave variable (such as displacement), c is the propagation velocity of the wave, t is time, and x is a position coordinate. This equation states that any wave, here $y(t, x)$, can be expressed as a sum of two waveforms, g_1 travelling in the negative x-axis direction and g_2 in the positive x-axis direction. This travelling-wave solution was published by d'Alembert in 1747.

Figure 2.8b illustrates an interesting case where two waves of the same form but opposite polarity pass each other in opposite directions. It may first appear illogical that the waves do not cancel each other totally. Remember, however, that the energy of the waves cannot disappear; it is just in kinetic form in the middle situation and reappears as potential energy after passing.

For a periodic waveform, the distance between equal phase points in the wave (for example, peaks) is called the *wavelength*. The wavelength λ is related to the frequency f and the wave propagation velocity c by

$$\lambda = c/f \tag{2.11}$$

The frequency f in hertz [Hz] is the number of oscillations per second. A more general case, such as sound propagating in air, is a three-dimensional phenomenon. Concepts related to wave behaviour are discussed further in Section 2.3.

When a sound wave encounters a discontinuity, such as a new medium, the simple wave equation, Equation (2.10), for undisturbed propagation of a waveform is not valid anymore. Often, the case is similar to the one in Figure 2.8c where the termination of a medium causes reflection of a wave. In this specific case, the displacement is bound to be zero at the termination, so that the reflecting wave component must be equal to the arriving wave but opposite in sign.

The vibration modes of the string with a distributed mass–spring in Figure 2.7 can also be explained through wave propagation and reflection. It can be shown that two sinusoidal wave components having the same mode frequency but travelling in opposite directions on the string result in a *standing wave* where no net energy transfer takes place. Each subwave reflects back from the terminations and the vibratory energy remains 'in place'. As Figure 2.7 shows, there are points of maximum vibration as well as points of zero (or minimal) vibration called *nodes*.

2.2 Acoustic Measures and Quantities

Acoustics is the branch of physics studying mechanical vibrations. Acoustics uses a set of concepts, variables, quantities, and measures to characterize waves, fields, and signals. Some of the physical measures and quantities most important in the field of communication acoustics are mentioned here. Hearing-related concepts are discussed in later chapters.

2.2.1 Sound and Voice as Signals

When the value of a physical variable is registered at a specified spatial point as a function of time, a signal is obtained. Signals and signal processing discussed in more detail in Chapter 3.

We first need to define some concepts that are needed to characterize sounds as physical phenomena:

- A *pure tone* is a sinusoidally varying sound signal, such as the cosine-form vibration of Equation (2.5) and Figure 2.2a. Thus, it consists only of a single frequency component. Although generating an ideal pure tone is not possible in practice, it can be approximated and is a useful abstraction.
- A *combination tone* consists of a set of pure tones called *partials*, each having its own frequency, amplitude, and phase.
- In *periodic sound signals*, the waveform repeats itself, and the signal consists of partials that have a harmonic relationship. The lowest frequency is called the *fundamental frequency*, often denoted f_0. Partials are called *harmonics*, and their frequencies are integer multiples of the fundamental, $f_n = nf_0$. Most musical instruments in Western music generate harmonic or almost harmonic signals.
- *Non-periodic sounds* may consist of discrete frequencies that are not in harmonic relationships or of a continuous distribution of partial frequencies. In the latter case the signal sounds noise-like.

The 'strength' or 'intensity' of a signal can be characterized by many measures. For a periodic signal, such as a sine wave, the maximum value of the waveform, called the *amplitude*, is often used. If the signal is not symmetric about the abscissa, the negative peak value may have a larger absolute value than the positive and is sometimes used as the amplitude value. Also, if the average value of a signal is not zero – this is often referred to as the DC value (direct current value, due to the analogy from electrical engineering) – the amplitude may be expressed as the maximum deviation from the average (DC) level.

Another common measure is the *root mean square* or *RMS value*, also called the *effective value*. It is defined, for example, for pressure $p(t)$ as

$$p_{\text{rms}} = \frac{1}{t_2 - t_1}\sqrt{\int_{t_1}^{t_2} p(t)^2 \,\mathrm{d}t}, \tag{2.12}$$

where the time range of integration can be over one period for a periodic signal or a long enough – ideally infinite – time span for non-periodic signals. For a pure tone (sinusoidal signal), the *peak value* $\hat{p} = \sqrt{2}\,p_{\text{rms}}$.

2.2.2 Sound Pressure

The most important physical measure in acoustics is *sound pressure*. Pressure, in general, is the force applied per unit area in a direction perpendicular to the surface of an object. Its unit is the Pascal [Pa] = [N/m^2]. Sound pressure, on the other hand, is the deviation of pressure from the static pressure in a medium, most often air, due to a sound wave at a specific point in space. Sound pressure values in air are typically much smaller than the static pressure. Sounds that human hearing can deal with are within the range of $20 \cdot 10^{-6}$ to 50 Pa.

Sound pressure is also an important measure due to the fact that it can be measured easily. A good condenser microphone (see Section 4.1.2) can transform sound pressure into an electrical signal (voltage) with high accuracy.

2.2.3 Sound Pressure Level

Because sound pressure varies over a large range in Pascal units, using a logarithmic unit, the *decibel* [dB], is more convenient. The ratio of two amplitudes, A_1 and A_2, in decibels is computed from

$$L = 20\log_{10}(A_2/A_1), \tag{2.13}$$

and it is used widely in acoustics, electrical engineering, and telecommunications. Some decibel values that are worth remembering are given in Table 2.1.

Table 2.1 Some values in decibels worth remembering.

Ratio	Decibels	Ratio	Decibels
1/1	0		
$\sqrt{2} \approx 1.41$	$\approx 3.01 \approx 3$	$\sqrt{1/2} \approx 0.71$	$\approx -3.01 \approx -3$
2/1	$\approx 6.02 \approx 6$	1/2	$\approx -6.02 \approx -6$
$\sqrt{10} \approx 3.16$	10	$\sqrt{1/10} \approx 0.316$	−10
10/1	20	1/10	−20
100/1	40	1/100	−40
1000/1	60	1/1000	−60

The concept of the decibel in acoustics is used in a special way. If the denominator A_1 in Equation (2.13) is a fixed reference, decibels are then absolute *level* units. The reference sound pressure $p_0 = 20 \cdot 10^{-6}$ Pa is used so that the *sound pressure level* (SPL) L_p [dB] is

$$L_p = 20 \log_{10}(p/p_0) \tag{2.14}$$

This value of p_0 is selected so that it roughly corresponds to the threshold of hearing, the weakest sound that is just audible, at 1 kHz. Human hearing is able to deal with sounds in the range 0–130 dB, from the threshold of hearing to the threshold of pain (for more details, see Chapters 7 and 9). SPL values in dB are more convenient to remember than sound pressure values in Pascals.

SPL values can be converted to sound pressure from the inverse of Equation (2.14):

$$p = p_0\, 10^{L_p/20}. \tag{2.15}$$

2.2.4 Sound Power

Sound power P, like physical power in general, is defined in watts [W] as the physical work done in one second. In acoustics, sound power is considered to be the property of a sound source radiating energy along with the sound wave it creates.

Only a fraction of the primary power of a sound source is transformed into acoustic power. The *efficiency* η of a sound source is

$$\eta = P_\mathrm{a}/P_\mathrm{m}, \tag{2.16}$$

where P_a is the radiated sound power and P_m is the primary power, such as mechanical or electrical power.

Sound power can also be expressed using a logarithmic measure. The *sound power level* L_W in decibels is defined as

$$L_W = 10 \log_{10}(P/P_0), \tag{2.17}$$

where P is sound power [W] and P_0 is the reference power of $1 \cdot 10^{-12}$ W. Note the coefficient 10 for power instead of 20 for pressure. This follows simply by expressing power P in terms of pressure p, $10 \log_{10}(P_1/P_2) = 10 \log_{10}(p_1^2/p_2^2) = 20 \log(p_1/p_2)$, in Equation (2.14).

2.2.5 Sound Intensity

Sound intensity I [W/m^2], as a physical measure, is defined as the sound power through a unit area, describing the flow of sound energy. Mathematically speaking, it is a vector – it has magnitude and direction, as will be further discussed in Section 14.4.7. *Sound intensity level* L_I is defined as

$$L_I = 10 \log_{10}(I/I_0), \tag{2.18}$$

where the reference $I_0 = 1 \cdot 10^{-12}$ [W/m^2].

2.2.6 Computation with Amplitude and Level Quantities

When two or more sounds contribute to a sound field, they can affect the total field in different ways. At any single moment in time, the pressure values add up. This linear superposition of waves is valid in air at normal sound levels. Non-linearities may appear elsewhere, for example in sound reproduction systems.

The resulting amplitude or RMS values and level quantities can be categorized as follows:

1. Sound sources are *coherent* if they or their partials have the same frequencies. Depending on their phase difference they can:
   - add constructively if they have the same phase;
   - add destructively if they are in opposite phase; or
   - in other cases the result depends on the amplitudes and phases of the components.
2. Sound sources are *incoherent* if their frequencies do not coincide, in which case the powers of the signals are summed.

When two coherent signals with the same amplitude A are added constructively, the resulting signal has the amplitude $2A$, which means an increase in level of $20\log_{10}(2A/A) \approx 6$ dB. If two incoherent signals with the same amplitude are added, the resulting level will increase by $10\log_{10}(2A^2/A^2) \approx 3$ dB.

In the general case of incoherent sounds with sound levels L_1 and L_2, the resulting level, based on the addition of powers, will be

$$L_{\text{tot}} = 10\log_{10}\left(\frac{P_1 + P_2}{P_0}\right) = 10\log_{10}\left(10^{L_1/10} + 10^{L_2/10}\right). \tag{2.19}$$

2.3 Wave Phenomena

Sound waves behave similarly in gases and liquids. Such a medium is called a *fluid*. In an ideal fluid, only longitudinal wave propagation is possible, there will be no transversal propagation.

Each medium where sound waves propagate has a characteristic *sound velocity*. For air, the most important medium in acoustics, the sound velocity c_{air} depends on temperature:

$$c_{\text{air}}(T) = 331.3 + 0.6T, \tag{2.20}$$

where T is temperature in °C and velocity is in m/s. This approximation of c_{air} is valid for typical room temperatures.

When a vibrating body generates a sound field in the surrounding fluid, the geometry of the field depends on the form of the sound source and its size compared to the wavelength. In practical cases, the sound field is so complex that it cannot be solved analytically and can only be approximated numerically. With some simplifying assumptions, the characteristics of a radiated sound field can, in many cases, be understood quite easily. The two simplest idealizations are spherical and planar wave fields.

2.3.1 Spherical Waves

A pulsating sphere as a sound source emits a *spherical wave field* that propagates from the source at the velocity of sound c, as illustrated in Figure 2.9. The sound pressure in the wave is inversely proportional to the distance r from the mid-point (symmetry point) of the source,

$$p(r) \propto q/r. \tag{2.21}$$

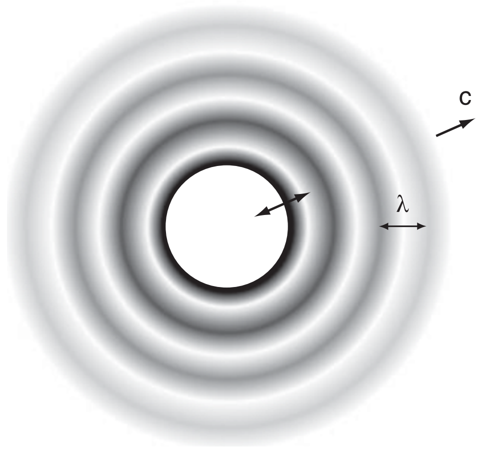

Figure 2.9 Wave propagation from a spherical, sinusoidally vibrating source. The greyscale outside the spherical source denotes the pressure of the sound field; a darker colour means a higher value. The disturbances propagate at the velocity of sound c, and the distance between pressure maxima is the wave length λ.

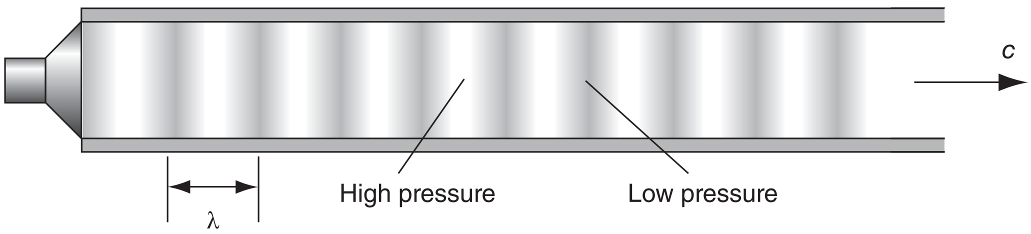

Figure 2.10 Plane wave propagation in a homogeneous tube.

In this formula, q is the volume velocity, a concept defined later. Note that the size of the sphere does not have an effect on the wave field, although a larger sphere is a more efficient radiator. As a mathematical idealization, a *point source* is a useful abstraction.

Any form of sound source that is small in dimension compared to the wavelength and vibrates quite homogeneously can be approximated as a spherical or point source. Thus, at low frequencies such sources, like typical loudspeakers or human speakers, are practically spherical wave sources.

2.3.2 Plane Waves and the Wave Field in a Tube

Another important special case of wave fields is a *plane wave*. A large and homogeneously vibrating planar surface emits a plane wave. In a lossless medium, the planar wavefront preserves its waveform (see Figure 2.8) and propagates without attenuation.

In a homogeneous tube, as shown in Figure 2.10, at frequencies where the cross-sectional dimensions are smaller than the wavelength, only a plane wave can propagate.

Two physical variables are used to fully describe a wave in a tube: sound pressure p and *volume velocity* q in m^3/s. The volume velocity characterizes the flow of the medium through a cross-sectional area in unit time. The relation between pressure and volume velocity is

$$p = Z_a q, \tag{2.22}$$

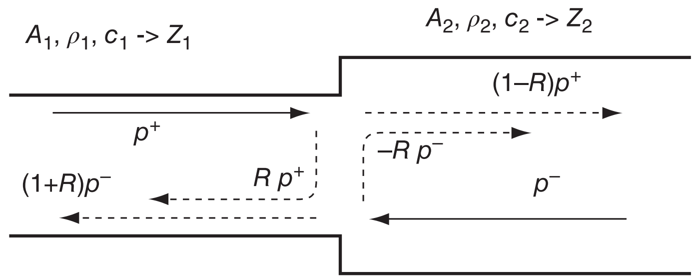

Figure 2.11 Reflection and transmission of a plane wave at a discontinuity in a tube.

where Z_a is the *acoustic impedance*. It can be obtained from the physical properties of the medium as

$$Z_a = \rho c/A, \tag{2.23}$$

where ρ is the density of the medium [kg/m^3], c is sound velocity, and A is the cross-sectional area [m^2].

Another related concept is the *characteristic impedance* Z_0, which is an inherent property of the medium in which the wave is travelling. It is defined as the ratio of pressure and particle velocity in a plane wave,

$$Z_0 = \rho c. \tag{2.24}$$

If the cross-sectional area or any acoustic parameter of the medium in a tube changes at any position so that the acoustic impedance Z_a changes, the simple propagation of a wavefront is disturbed. At such a discontinuity, an arriving wave is split so that part of it reflects back and part of it propagates through the discontinuity. In Figure 2.8c, the wave in a rope reflects back completely due to the fixed termination. In the tube of Figure 2.11, when a plane wave p^+ meets a change in acoustic impedance from Z_1 to Z_2, the wave component p_r^+ that reflects back is

$$p_r^+ = Rp^+, \tag{2.25}$$

and the component p_f^+ that propagates through the junction is

$$p_f^+ = Tp^+ = (1 - R)\,p^+. \tag{2.26}$$

Here, R is the *reflection coefficient*

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{2.27}$$

and $T = 1 - R$ is the *transmission coefficient*. Figure 2.11 also shows a wave p^- propagating to the left and being reflected by the factor $-R$ and transmitted by the factor $T = 1 + R$. From Equations (2.25)–(2.27), it can be concluded that if $Z_1 = Z_2$, which means that there is a perfect impedance match, the reflection coefficient $R = 0$ and no reflection occurs. If impedances Z_1 and Z_2 are very different (impedance mismatch), a strong reflection occurs.

For a tube terminated in a rigid wall, the impedance of the termination $Z_2 >> Z_1$, the impedance of the tube, which implies from Equation (2.27) that $R \to 1$. When the termination is an open end, the impedance Z_2 is very small (at low frequencies) and $R \to -1$.

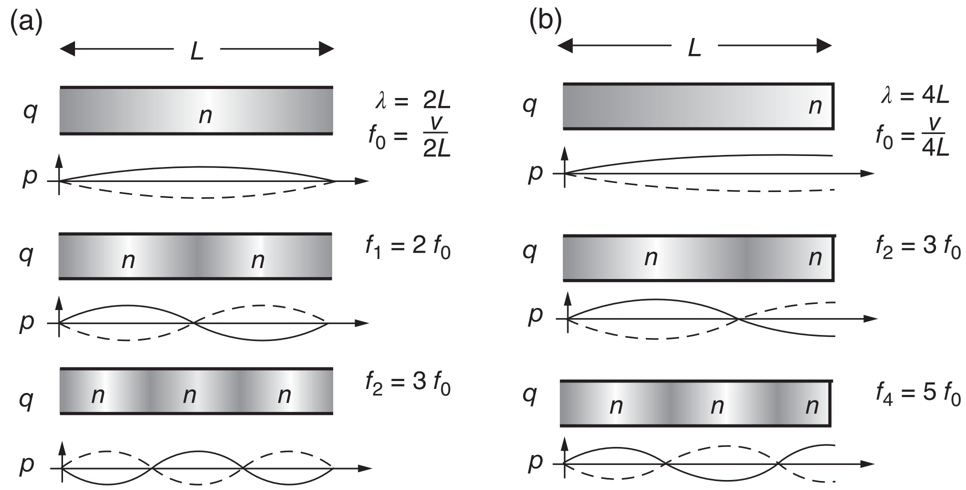

Figure 2.12 The behaviour of the lowest modes in a tube with (a) open ends and (b) one end closed. The pressure p of each mode is shown as a curve below the tube, and the volume velocity q is shown as a colouring of the tube, with darker colour shades denoting higher velocity. n means a nodal point where the volume velocity is minimal. Adapted from Rossing *et al.* (2001).

Table 2.2 The velocity of longitudinal waves in some media.

Medium	Velocity [m/s]
Air (20 °C)	343
Helium	970
Water	1410
Steel	5100
Glass	12000–16000

In a tube of finite length, the terminations reflect waves back so that standing waves appear, resulting in modes and resonance frequencies. Figure 2.12 depicts the lowest modes in a tube that is left open at both ends and in another tube that is closed at one end. The first case is approximated, for example, in the flute, generating all harmonics, and the second case in the clarinet, where (low order) even harmonics are weak due to lack of resonance.

2.3.3 Wave Propagation in Solid Materials

The behaviour of solid matter deviates from that of fluids since transversal (shearing) forces are possible in solids, which results in the possibility of transversal waves forming in addition to longitudinal waves. The velocity of longitudinal waves in several media is listed in Table 2.2.

Transversal waves appear, for example, in a string under tension (Figures 2.7 and 2.13) or in a bar (Figure 2.14). In a non-stiff string (a wire where negligible force is needed to bend it), the velocity c_t of the transversal wave depends on the string tension T in N and mass density μ in kg/m as

$$c_t = \sqrt{(T/\mu)}. \tag{2.28}$$

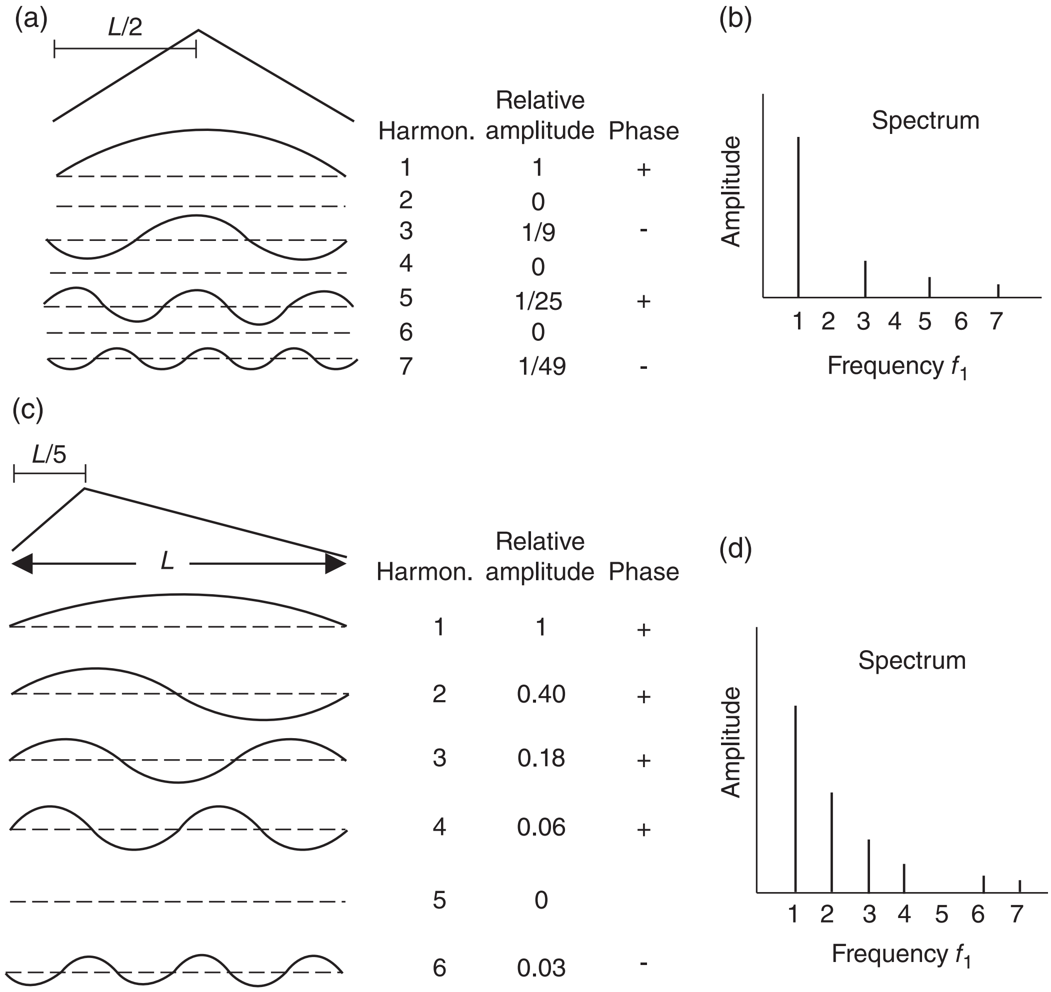

Figure 2.13 (a) The initial displacement of a string when plucked in the middle and (b) the harmonic content of the vibration. When the string is plucked asymmetrically (one fifth of L from the end), the initial displacement is shown in (c) and the harmonic components in (d). Source: Rossing *et al.* 2001.

Figure 2.13 depicts the behaviour of a string when plucked (a) in the middle and (c) one fifth of the length of the string from the end. The spectra consist of harmonic components, but with every Nth harmonic missing for $N = L/L_{pp}$, where L is the length of the string and L_{pp} is the plucking point distance from the end.

An example of transversal waves that resemble string behaviour but which are not purely one-dimensional and harmonic in spectral content is wave propagation in a bar. Due to bending stiffness, the transversal waves are *dispersive*, implying that different frequencies propagate at different velocities (higher frequencies propagate faster than lower ones). This results in modal frequencies that do not have a harmonic relationship. Figure 2.14a illustrates the lowest modes in a free bar and Figure 2.14b the case where the bar is clamped at one end. The resulting sounds are strongly inharmonic. Some degree of inharmonicity can also be found in string instruments, especially in the piano at low frequencies, due to the stiffness of the strings.

Membranes and plates are also often used as sound sources. For example, the membrane of a drum under tension is a two-dimensional equivalent of the vibrating string. Some of the lowest modal patterns of a circular membrane are shown in Figure 2.15. The patterns show

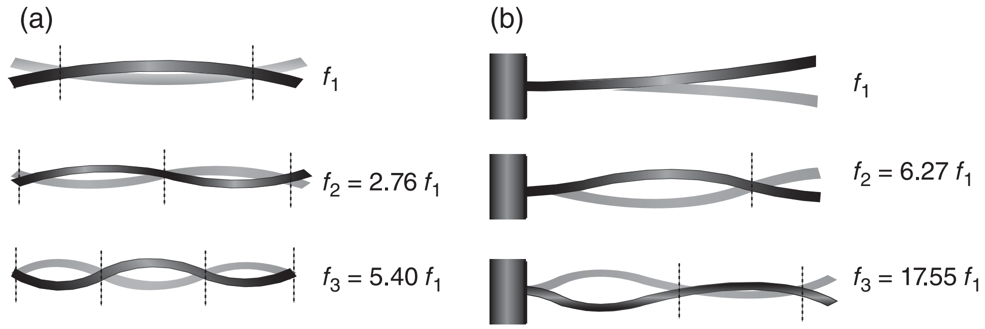

Figure 2.14 Modal patterns and frequencies of a bar: (a) freely vibrating (unsupported) and (b) rigidly clamped at one end. The nodes of vibration are shown with vertical dashed lines. Adapted from Rossing *et al.* (2001).

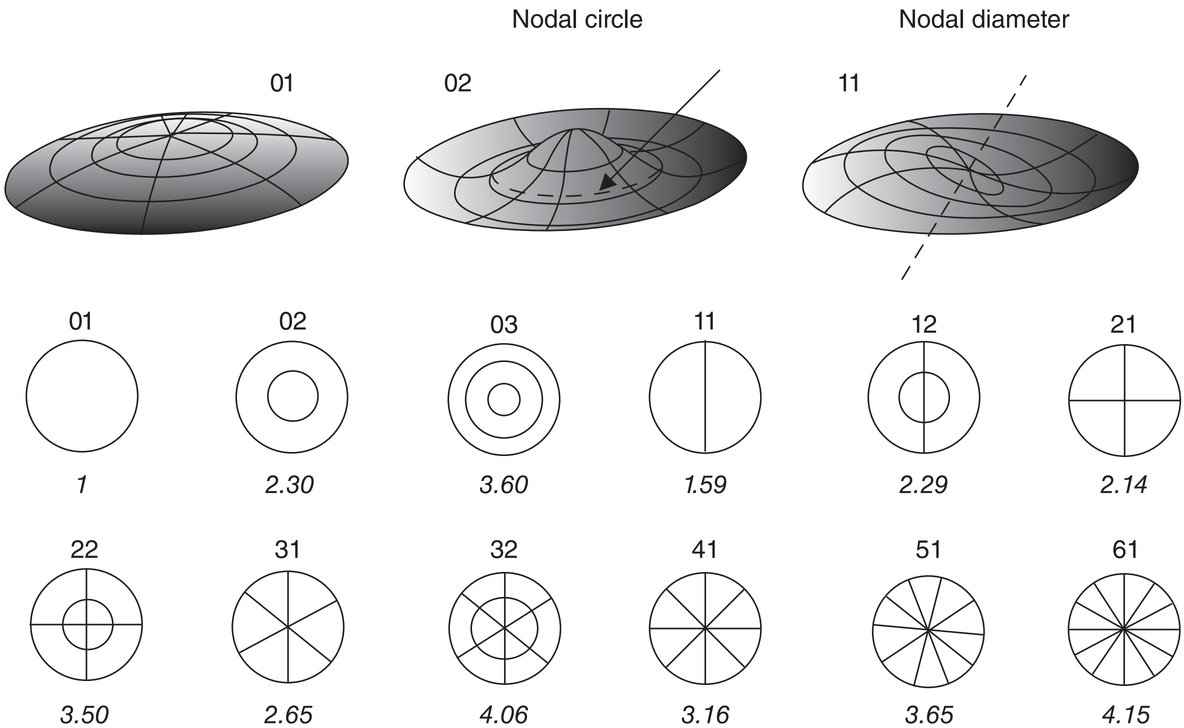

Figure 2.15 Modal vibration of a circular membrane supported at the edge: membrane shapes at moments of maximal displacement (top row) and nodal patterns and corresponding relative resonance frequencies (two lowest rows). Adapted from Rossing *et al.* (2001).

different symmetries, both circular and diametral. Once again, the modal frequencies are not in harmonic relationships, and it is characteristic that the density of mode frequencies increases towards high frequencies.

2.3.4 Reflection, Absorption, and Refraction

When a wavefront encounters the surface of another medium, such as a hard wall, a fraction of the wave energy reflects back and the rest propagates into the other medium or transforms into thermal energy. Figure 2.16 illustrates these phenomena when a plane wave hits a wall with

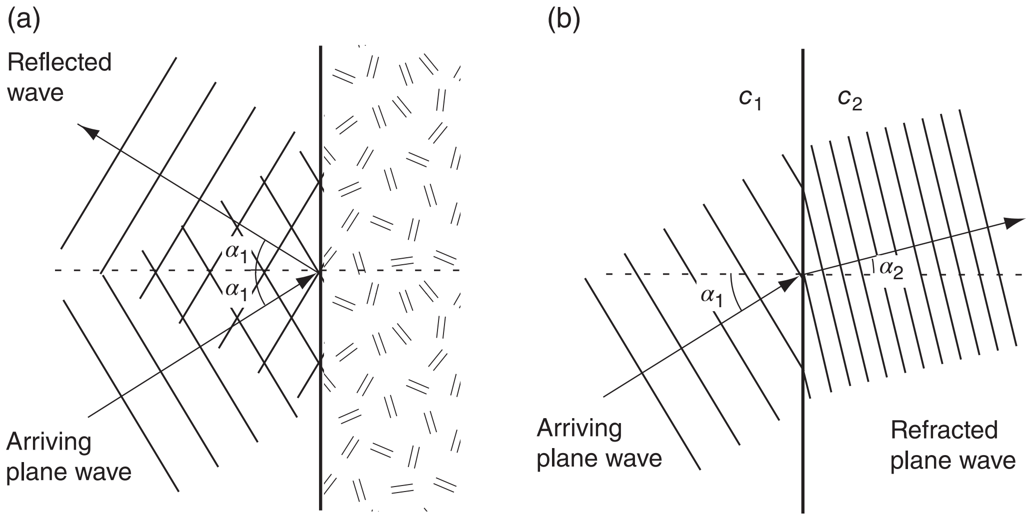

Figure 2.16 (a) Reflection of a plane wave at a hard surface, and (b) refraction (bending) of the wavefront when entering another medium with a different sound velocity.

an incidence angle α_1. In an ideal case, the angle of reflection is equal to α_1. If the reflecting surface is not perfectly flat, spreading of the wave in directions around this mirror image angle will occur (diffuse reflection).

The fraction of sound energy that is not reflected is absorbed from the sound field of the first medium. The ratio of absorbed energy to the incident energy is called the *absorption coefficient*. When the reflection of the wave is described by the coefficient R, as defined in Equation (2.25), the absorption coefficient a is obtained from the equation

$$a = 1 - |R|^2. \tag{2.29}$$

Refraction is the phenomenon of the wavefront bending when entering a medium with a different sound velocity. Figure 2.16b characterizes this when the sound velocity is lower in the second medium. According to Snell's law, $c_1 \sin\alpha_2 = c_2 \sin\alpha_1$, where c_1 and c_2 are sound velocities as defined in Figure 2.16b.

2.3.5 Scattering and Diffraction

When a wave interacts with a geometrical discontinuity like a boundary, *scattering* occurs. Reflection can be interpreted as the simple case of *backscattering* from a boundary layer. More general scattering in acoustics is called *diffraction*. It is a complex phenomenon that is difficult to analyse and model. A solid object or its edge acts as a kind of secondary source. Huygens' principle interprets a wavefront as a set of directed secondary sources that maintains a regular wave propagation. A geometrical discontinuity disturbs this propagation and makes diffraction the secondary observable source.

Figure 2.17 shows a typical case where a noise barrier is used to mask noise sources, such as vehicles on a highway, to decrease disturbance in the environment. At high frequencies, where the wavelength is small compared to the barrier, shadowing due to the barrier is efficient. At low frequencies, however, the sound waves propagate around the barrier so that the desired masking remains relatively small.

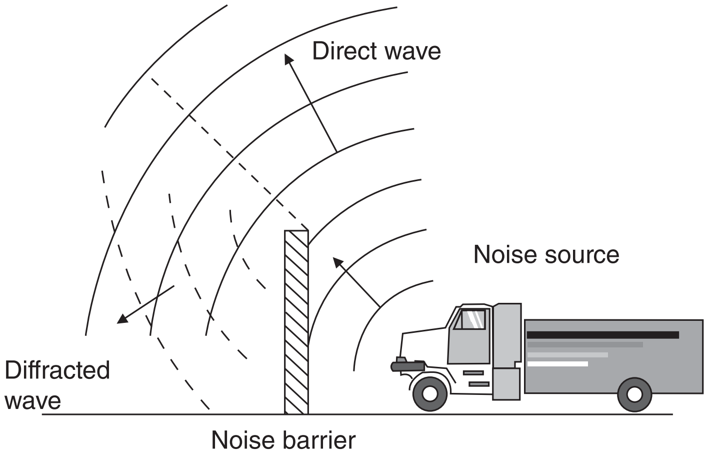

Figure 2.17 Diffraction of sound at the edge of a sound barrier.

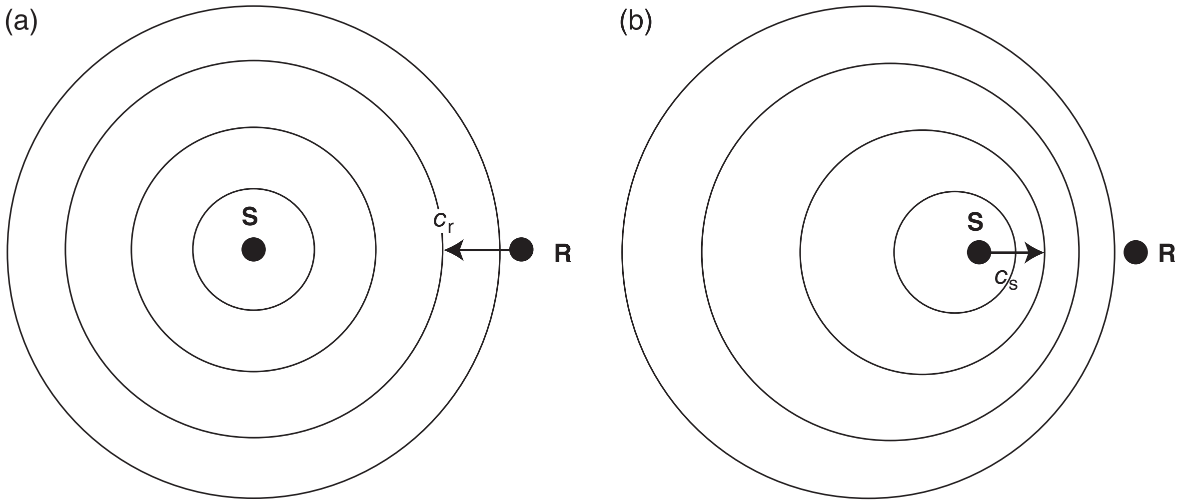

Figure 2.18 Doppler effect (a) when the receiver moves and the source is immobile, (b) when the receiver is immobile and the source moves.

2.3.6 *Doppler Effect*

Imagine a free-field condition with a single receiver and a single sound source emitting one sinusoid with frequency f. If the distance between the source and the receiver diminishes with time, meaning that the source moves towards the receiver or vice versa, as shown in Figure 2.18, each successive wave is emitted from a position closer to the receiver than the previous wave. Therefore, the time between the arrival of successive waves at the observer decreases, causing an increase in the frequency compared to the static situation.

Conversely, if the wave source moves away from the observer, each wave is emitted from a position farther from the observer than the previous wave, thus increasing the arrival time between successive waves and reducing the frequency. The distance between successive wavefronts is increased, so the frequency of the sinusoid is lower. This can be observed when a vehicle producing a harmonic tone (such as an ambulance) passes by at high speed, the perceived pitch of the tone lowers rapidly when the vehicle goes past the observer. This phenomenon is called the *Doppler effect*.

The frequency of the sinusoid observed by the receiver f can be computed as

$$f = \left(\frac{c + c_\mathrm{r}}{c + c_\mathrm{s}}\right) f_0, \tag{2.30}$$

where the velocity of the receiver is c_r, which is positive if the receiver moves towards the source and negative in the other direction. The velocity of the source is c_s, which is positive if the source moves away from the receiver and negative in the other direction.

2.4 Sound in Closed Spaces: Acoustics of Rooms and Halls

A major part of communication by sound and voice takes place in spaces constrained by walls or surfaces, such as living rooms, auditoria, offices, concert halls, etc. The surfaces reflect impinging sound waves, and in a fraction of a second there are thousands of reflections which result in reverberation that gradually decays. Reflections and reverberation make sound and voice louder in the more distant parts of the room. They may also render the sound more ‘colourful’ and pleasant if the room has proper acoustics but, on the other hand, may make it difficult and annoying to communicate if the acoustic properties are not well matched to the form of communication. Background noise is another common reason for problems in communication.

In this section, we study the basic physical properties of closed space acoustics – rooms, auditoria, and concert halls. Subsequent sections will discuss some related aspects, and finally in Chapter 17 we study the subjective and objective quality factors, especially those related to performing spaces.

2.4.1 Sound Field in a Room

In a room with simple geometry, sound propagation from a source to a receiver (listener) can be characterized as shown in Figure 2.19. The first wavefront arrives along the direct path (if the source is visible), and soon after this the first reflections from walls, ceiling, and floor arrive.

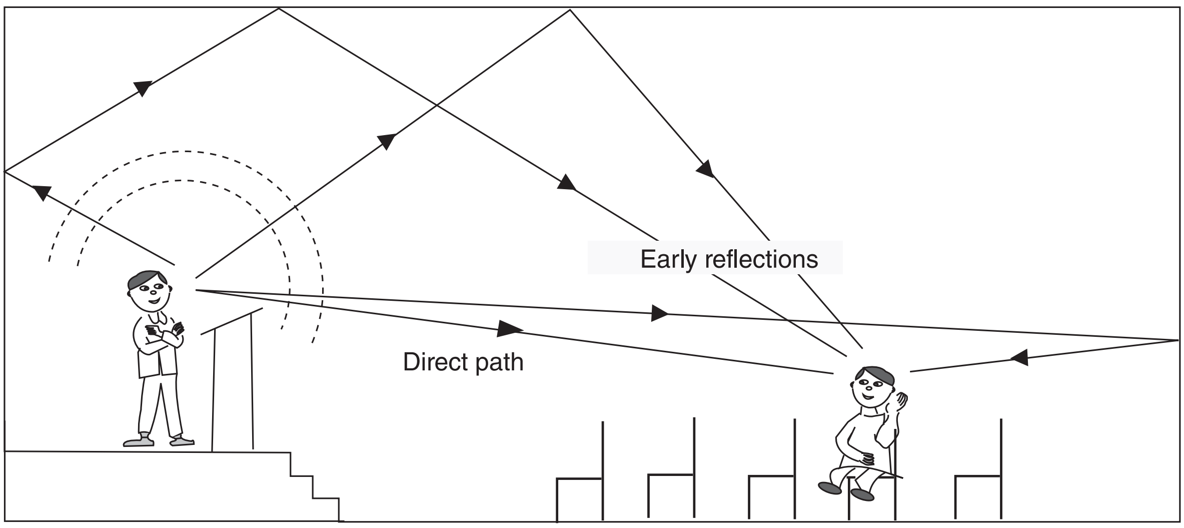

Figure 2.19 Paths of direct sound and the few first reflections in a rectangular room from a speaker to a listener, assuming specular reflections from walls.

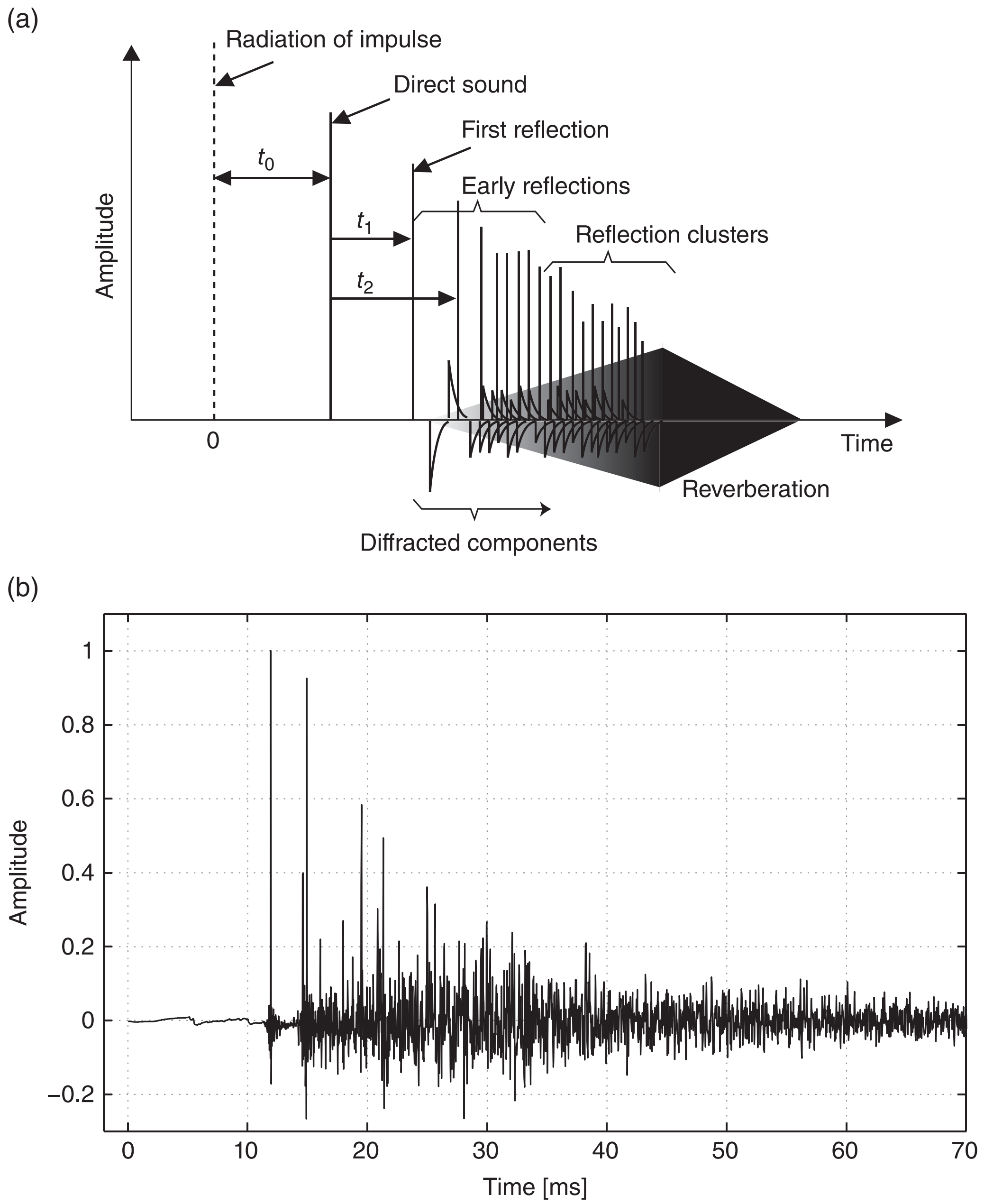

Figure 2.20 (a) The different components of an impulse response measured from the source to the receiver in the case of Figure 2.19, consisting of direct sound, early reflections, diffracted components, reflection clusters, and reverberation. (b) The impulse response measured with a laser-induced pressure pulse in a listening room.

If the source sends an impulse-like sound, the response at the point of the receiver is as depicted in the *reflectogram* of Figure 2.20. Direct sound is followed by early reflections, diffracted components, and then by reverberation, where individual reflections may not be separately visible. Early reflections, for speech up to about 60 ms and for music up to about 100 ms, increase the loudness of sound. Early reflections also strongly contribute to spatial perception, such as in the estimation of room size and source distance. Late reverberation, if too loud, decreases intelligibility of speech or fast passages of music.

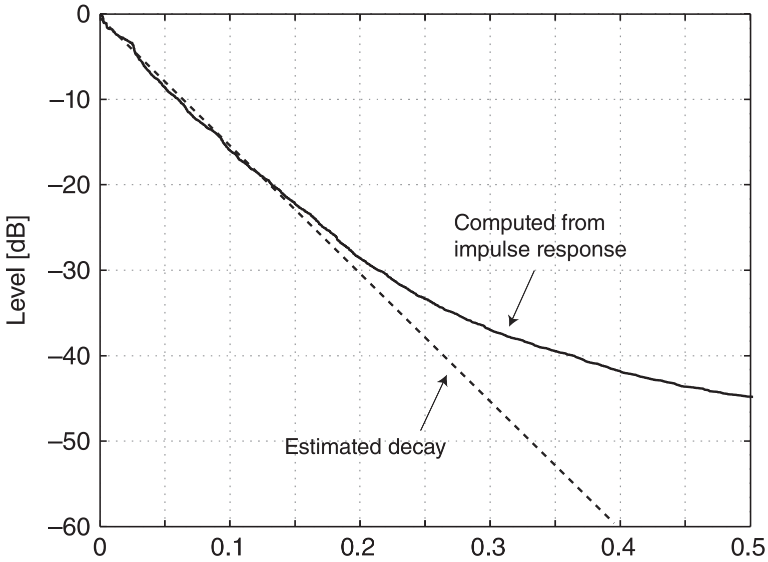

Figure 2.21 Decay of reverberation computed from an impulse response. Due to noise in the measurement, the computed curve does not follow exponential decay. The extrapolated reverberation time T_{60}, the time taken for the response to decay 60 dB, is approximately 0.4 s.

For comparison with the principle in Figure 2.20a, Figure 2.20b plots a measured impulse response of a relatively damped listening room. Note that the reflections have the same polarity as the direct sound, but the diffracted components may have either positive or negative polarity, depending on the geometry (Svensson *et al.*, 1999). Also, the reverberation has both positive and negative polarities. This is also shown in the measured impulse response. Measured room impulse responses can be downloaded, for example, from OpenAIR (2014).

2.4.2 Reverberation

Figure 2.21 plots the decay of the sound pressure level in a room, assuming that a steady-state sound field of white noise is first injected into the room and then interrupted at moment t_0. The curve is plotted on a logarithmic dB scale against a linear time scale. The decay of the sound energy is exponential, which is characteristic of many physical resonator systems (see Equation (2.8)), and corresponds to linear decay on a logarithmic decibel scale.

The single most important parameter describing the acoustics of a room is the *reverberation time*. It is defined as the time period T_{60} during which the sound pressure level decays 60 dB, and it can be estimated from Sabine's formula (Sabine, 1922):

$$T_{60} = 0.161\frac{V}{S}, \tag{2.31}$$

where V is the volume of the room [m^3] and S is the total *absorption area* of the room surfaces. There are other formulations, such as Eyring's formula, that can be more accurate in specific conditions (Kuttruff, 2000). Notice that in many textbooks symbols S and A are used in roles opposite to those here. Here, area is A for consistency with other formulas. The absorption area of room surfaces can be computed from

$$S = \sum a_i A_i \tag{2.32}$$

Table 2.3 Absorption coefficients of materials in different frequency bands.

Frequency	125	250	500	1000	2000	4000
Glass window	0.35	0.25	0.18	0.12	0.07	0.04
Painted concrete	0.10	0.05	0.06	0.07	0.09	0.08
Wooden floor	0.15	0.11	0.10	0.07	0.06	0.07

by summing the product of the absorption coefficient a (Equation (2.29)) and the surface area A in m^2 over each surface i. There exist tables giving the absorption coefficient for different materials as a function of frequency, for example, in octave bands, as shown in Table 2.3. Note that each person inside a room adds to the absorption area S by approximately 0.5 m^2.

Recommendable T_{60} values are about 2 seconds for a large concert hall, about 1.4 seconds for a chamber music hall, 0.5–1.0 seconds for a speech auditorium, and about 0.35 seconds for a listening test room. The acoustic parameters for concert halls and auditoria, and their measurements, are discussed further in Section 17.9.2.

2.4.3 Sound Pressure Level in a Room

The amplifying effect of reflections and room reverberation can be understood easily by considering the summation of direct and reverberant sound. The direct sound pressure is inversely proportional to the distance between source and receiver, as stated in Equation (2.21). The level of the reverberant field is approximately constant in the whole room, and it can be considered to be approximately incoherent with the direct sound. Based on the summing of two power levels, the total sound pressure level L_p in dB will be

$$L_p = L_W + 10 \log_{10} \left(\frac{Q}{4\pi r^2} + \frac{4}{S} \right), \qquad (2.33)$$

where L_W is the sound power level (Equation (2.17)) of the source in dB, Q is the directivity of the source, r is the distance of the source from the receiver in metres, and S is the absorption area in m^2 of room surfaces (see Equation (2.32)). The directivity Q of an omnidirectional source (a source that radiates equally in all directions) is 1.0 and for other sources it can have larger or smaller values, depending on the direction.

The first term inside the logarithm of Equation (2.33) corresponds to the intensity caused by the direct sound, it being proportional to directivity and inversely proportional to the area of a sphere of radius r. The second term corresponds to the reverberant field, which is inversely proportional to the absorption area of the room. Note that this expression says that the reverberant field is equal in the whole room. This formula is derived from a statistical formulation of reverberation and it may not be a valid approximation for complex room geometries.

Figure 2.22 characterizes the SPL dependency on the distance r separately for direct sound, reverberant sound, and the total sound level. The direct sound pressure level decreases by 6 dB for every doubling of the distance. At a certain distance, called the *reverberation distance* or the *radius of reverberation*, the direct and reverberant fields have the same level, and this is

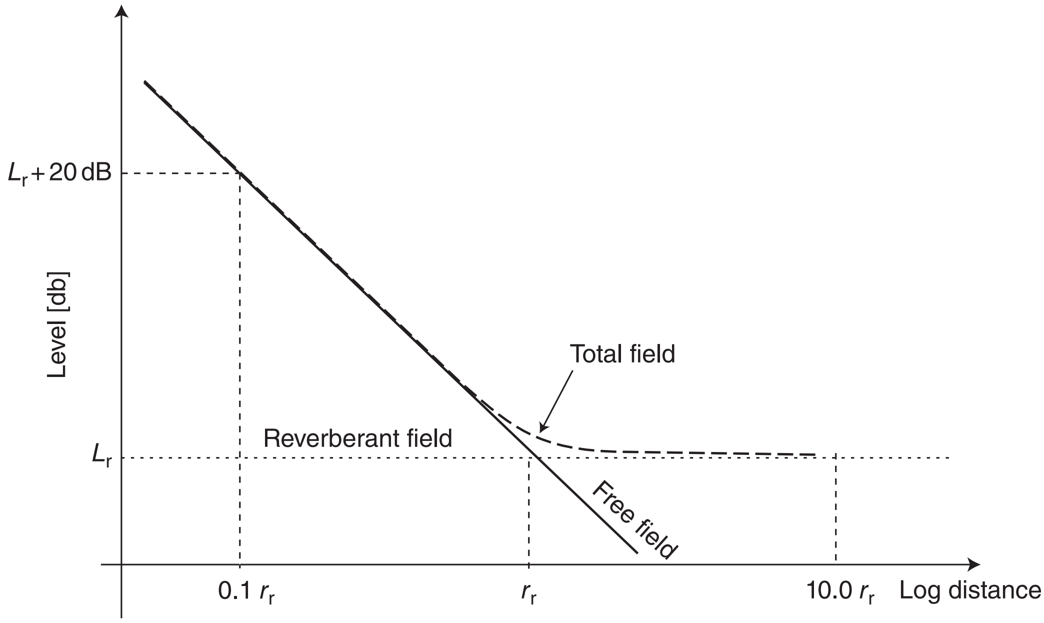

Figure 2.22 The theoretical behaviour of the sound pressure level in a room for a steady-state sound as a function of the source–receiver distance due to (a) a direct sound in a free field (solid line), (b) a reverberant field (dotted line), and (c) the total sound field (dashed line). The distance r_r is the reverberation distance, and L_r is the sound pressure level caused by the reverberant field.

obtained from Equation (2.33) by equating the two terms inside the logarithm and solving for r_r:

$$r_r = \frac{1}{4}\sqrt{\frac{QS}{\pi}}. \tag{2.34}$$

Beyond the reverberation distance the total field remains approximately constant, so that the perceived loudness does not essentially decrease, but the sound is perceived to be more reverberant since the direct sound level decreases.

2.4.4 Modal Behaviour of Sound in a Room

While Figures 2.19 and 2.21 characterize the temporal evolution of a sound field in a room, another picture is obtained by looking at sound transfer properties in the frequency domain. A hard-walled room can be understood as a three-dimensional resonator where a large number of modal frequencies can be found in the audible frequency range. For a rectangular room with hard walls, the frequencies of normal modes can be computed analytically from

$$f(n_x, n_y, n_z) = \frac{c}{2}\sqrt{\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2}, \tag{2.35}$$

where c is the sound velocity; L_x, L_y, and L_z are the dimensions of the room in the three directions; and integer variables n_x, n_y, and n_z are given all the combinations of the values 0, 1, 2, . . . The lowest modes correspond to plane-wave standing wave resonances between opposite walls, such as with $n_x = 1$, $n_y = 0$, $n_z = 0$, in Equation (2.35). Cross-modes, such as for $n_x = 1$, $n_y = 1$, and $n_z = 1$, are not as easy to conceive or visualize.

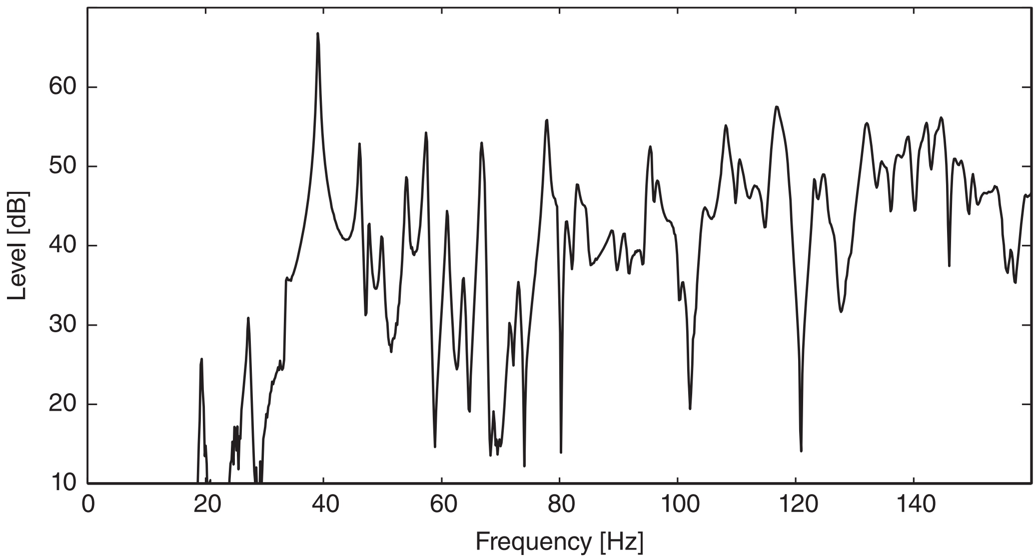

Figure 2.23 Magnitude response measured in a hard-walled room from a source to a receiver. (Below a strong mode at 39 Hz the level of modes is lower due to the roll-off of the loudspeaker response.) Towards higher frequencies the modes start to overlap more and the field is characterized as diffuse.

The density of modes increases rapidly as a function of frequency, and is proportional to the square of the frequency. Note that the density of modal frequencies d_f is approximately constant for one-dimensional acoustic systems, such as strings (Figure 2.13) and tubes (Figure 2.12), directly proportional to the frequency ($d_f \propto f$) for two-dimensional acoustic systems, such as a membrane (Figure 2.15), and proportional to the square of the frequency ($d_f \propto f^2$) for three-dimensional acoustic systems, such as rooms.

Figure 2.23 plots the magnitude response measured in a hard-walled room. The magnitude response (see Section 3.2.5) describes how each frequency component is emphasized or attenuated (in dB) when transferred from the source to the receiver. The lowest modes are quite separate peaks, but at higher frequencies the modes start to overlap and fuse. A transition frequency between these regions is called the *critical frequency* f_c (Schroeder, 1987), defined as

$$f_c = K\sqrt{T_{60}/V}, \tag{2.36}$$

where $K = 2000 \ldots 4000$, T_{60} is the reverberation time, and V is the volume of the room. Above the critical frequency the acoustic field is said to be *diffuse*. A diffuse field is composed of a large number of wavefronts travelling in all directions with equal probability, contributing to the high density of reflections and the high density of modes. Thus, a diffuse field can be modelled by statistical means, as the group behaviour of modes in the frequency domain and reflections in the time domain.

2.4.5 Computational Modelling of Closed Space Acoustics

The acoustic behaviour of real rooms, auditoria, and concert halls is very complex. This can be understood when estimating the possible number of degrees of freedom for vibration in the sound field. At the highest audible frequencies the wavelength is about 1 cm. Each such cube has three degrees of freedom to vibrate. Thus, a hall with dimensions $20 \times 40 \times 10$ metres has about $3 \cdot 2000 \cdot 4000 \cdot 1000 = 24,000,000,000$ degrees of freedom to vibrate! Only for very

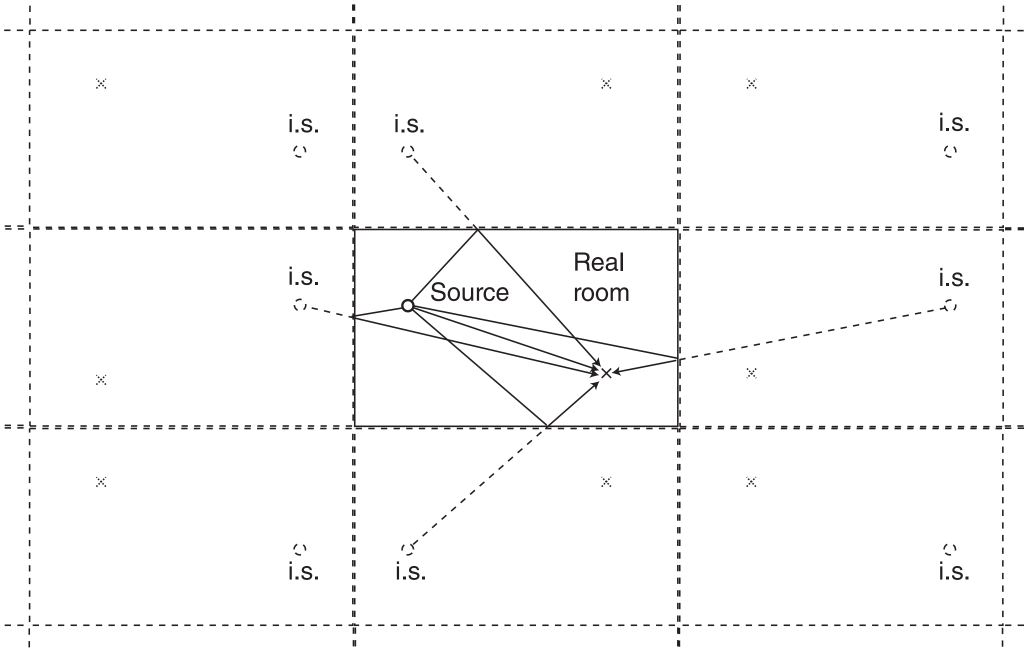

Figure 2.24 The principle of image source modelling. The mirrored rooms surrounding the real room and the related image sources (i.s.) are computed to find the paths of sound waves arriving at the receiver point in the real room. Only four first-order reflections are illustrated.

simple geometries, such as a shoebox with hard walls, can the field behaviour be computed by analytical means.

To meet the needs of computing and designing acoustics for complex spaces, a number of approximate numerical techniques have been developed. A conceptually simple technique is based on the *image source method*. Figure 2.24 illustrates a rectangular room with a source and a receiver. According to wave reflection laws, as illustrated in Figures 2.16 and 2.19, the reflections can be modelled as coming from image sources in image rooms. Each reflection in the real room corresponds to penetrating a wall between image rooms. Each path has an attenuation due to the $1/r$ distance law and damping for each reflection, and the wave will be delayed by the propagation time of the wave from the (image) source to the receiver. The frequency-domain room response in Figure 2.23 has been computed using the image source method as applied to a shoe-box-shaped room. If the room is not rectangular, the method can still be used, although the visibility of each image source has to be validated in each case (Kristiansen *et al.*, 1993).

Another commonly used numerical computational technique is the *ray-tracing* method. A large number of 'sound rays', analogous to light rays, are sent from the source in different directions. Each ray is traced through reflections from surfaces until the energy of the ray is below a certain threshold. The receiver can be modelled as a volume, such as a sphere or a cube, and each traced ray that hits it adds to the response record. The ray-tracing technique is statistical, but if the number of rays is sufficiently large and the receiver is small enough, the result may yield a good approximation of the room response.

In a room with a complex shape, both image-source and ray-tracing methods are problematic at low frequencies. Features like corners, object edges, and curved surfaces exhibit phenomena, such as diffraction, that are not easily simulated with these techniques. Different element-based techniques are more accurate in such cases. The *finite-element method* (FEM) (Thompson, 2006; Zienkiewicz and Taylor, 1977) assumes the space is a set of finite-sized elements having mass, spring, and damping properties. Given the geometry of a room and the sound source(s), the sound field at any point can be solved.

A variation of the element method is the *boundary-element method* (BEM) (Gumerov and Duraiswami, 2005), where only the boundaries of the space are taken into account in the first phase, and later the sound field at any point in the space can be solved. One more element-oriented technique is the *finite-difference time-domain method* (FDTD) (Botteldooren, 1995). The problem with all element-based methods is that the number of elements grows very rapidly when the wavelength of the highest frequency of interest decreases.

Summary

This chapter presented an overview of fundamental concepts in physical acoustics that are considered important in understanding communication by sound and voice, including the wave behaviour of sound in a free field, at material boundaries, and in closed spaces. References are given for further study on these issues, if needed.

While in physical acoustics such properties of sound as spatial distribution, two-way interaction between subsystems, energy behaviour, and specific physical variables are important, in the next chapter we make abstractions that simplify these issues. In a signal processing approach, we are more interested in looking at sound signals as abstract variables and their relationships as transfer functions and one-way interactions.

Further Reading

To better understand the general fundamentals of acoustics and wave equations, the reader is referred to such books as Beranek and Mellow (2012) and Morse and Ingard (1968).

Physical acoustics contains many subfields and requires concepts that are not discussed above. General *linear acoustics* deals with sound and vibration in fluids (gases and liquids) and solid materials with different geometries and structures, assuming that the condition of linearity (Section 3.2.1) is valid. If an acoustic system is non-linear, it is typically much more difficult to study. The subfield of *non-linear acoustics* deals with waves having high pressure levels, such as *shock waves* (Pierce, 1989; Raspet, 1997) and *cavitation* (Lauterborn, 1997).

Among other popular topics in acoustics research are *underwater acoustics*, also called *hydroacoustics* (Crocker, 1997, part IV); *ultrasound*, which studies sound above normally audible frequencies (Crocker, 1997, part V); *infrasound*, which concerns itself with sounds below normally audible frequencies (Gabrielson, 1997); *noise control* (Crocker, 1997, part VIII); *room*, *building*, and *architectural acoustics*, including *concert hall acoustics* (Ando, 2012; Barron, 2009; Beranek, 2004); *acoustics of musical instruments* (Fletcher and Rossing, 1998) and the *singing voice* (Sundberg, 1977); and *acoustic measurement techniques* (Beranek, 1988; Crocker, 1997, part XVII).

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