



Aalto University
School of Science

MS-E2114 Investment Science

Lecture II: Fixed income securities

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This lecture

- ▶ **Fixed income securities** are debt instruments which provide (i) returns in the form of regular, or fixed, interest payments and (ii) repayments of the principal when the security reaches maturity
- ▶ Refers especially to bonds that pay a periodic, fixed, non-random coupon and other similar instruments
 - ▶ E.g., bonds issued by corporations, sovereign states, municipalities, etc.
- ▶ The term also encompasses instruments whose interest depends on a reference rate such as EURIBOR
- ▶ In this lecture, we derive valuation formulas for several types of fixed income securities

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Terminology

- ▶ **Financial instruments** are monetary contracts between parties
 - ▶ Cash, evidence of ownership, contract to receive cash or other financial instruments
- ▶ **Security** = A readily transferable financial instrument which is traded in a well-developed market
 - ▶ Governed by laws on public market securities (e.g. regarding insider information)
- ▶ **Fixed income instrument** = A financial instrument which provides a preagreed stream of cash flows
 - ▶ Interest rate may be fixed or depend on a possibly changing reference rate such as EURIBOR
 - ▶ Yet the issuer may **default** (e.g., bankruptcy)
- ▶ **Fixed income security** = A fixed income instrument which is a security, too

Examples of fixed income instruments

- ▶ Money market instruments (securities)
 - ▶ Tradeable instruments that offer a guaranteed interest (possibly floating-rate) for a short period of time
 - ▶ Certificates of deposit offered by banks
 - ▶ Short term notes (1 yr or less) by corporations (= commercial paper)
- ▶ U.S. government securities
 - ▶ Treasury bills ("T-bills") < 1 year
 - ▶ Treasury notes 2-10 years
 - ▶ Treasury bonds > 10 years
- ▶ Other bonds (securities)
 - ▶ Municipal bonds (issued by e.g. by cities)
 - ▶ Corporate bonds

Further examples (these are not usually securities)

- ▶ **Mortgages** are used by purchasers of real property to raise funds to buy real estate, or alternatively by existing property owners to raise funds for any purpose, while putting a lien on the property being mortgaged
 - ▶ **Reverse mortgages** allow owners of home equity to borrow against the value of their home
- ▶ **Annuities** provide a series of payments at equal intervals (e.g., regular deposits to a savings account, monthly insurance payments, and pension payment)

Credit ratings

- ▶ Bonds offer a **fixed cash flow** *unless* the issuer defaults
- ▶ Ratings provided by credit rating agencies
 - ▶ The 'Big Three': Moody's, Standard & Poor's, Fitch
- ▶ U.S. Treasury securities have been considered the least risky historically
- ▶ Ratings are traditionally grouped into grades of *Investment Grade* (IG) and *High Yield* (HY)
 - ▶ Investment grade = Baa3 or better (Moody's), BBB- or better (S&P, Fitch)
 - ▶ High yield = Ba1 or worse (Moody's), BB+ or worse (S&P, Fitch)
 - ▶ High yield bonds are also known as *junk bonds*
 - ▶ https://en.wikipedia.org/wiki/Credit_rating

Market for future cash

- ▶ Fixed income securities define the time value of money
- ▶ Markets for many types of these securities are extremely well-developed
- ▶ Market prices for these securities reflect
 1. Time value of money
 2. Fundamental risk premium (= compensation for the risk that the issuer defaults)
 3. Market price risk premium (= compensation for the risk that the value of the security goes down)
 4. Supply and demand

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Perpetual annuity

- ▶ Receive $A \text{ €}$ annually forever starting in a year's time
 - ▶ $r =$ interest rate
- ▶ The net present value of this cash flow stream is ($r > 0$)

$$\begin{aligned}P &= \sum_{k=1}^{\infty} \frac{A}{(1+r)^k} = \frac{A}{1+r} + \frac{1}{1+r} \sum_{k=1}^{\infty} \frac{A}{(1+r)^k} \\&= \frac{A}{1+r} + \frac{P}{1+r} \\ \Rightarrow P &= \frac{A}{r}\end{aligned}$$

- ▶ If $A = 25000 \text{ €}$ and $r = 0.1 = 10\%$, then
 $P = 25\,000 \text{ €} / 0.1 = 250\,000 \text{ €}$

Cash flows of finite length

- ▶ Get A € annually for the next n years
 - ▶ Future cash flows discounted with interest rate r
- ▶ Present value can be computed from the perpetual annuity by subtracting the cash flows that occur after the n -th year

$$\begin{aligned} P &= \sum_{k=1}^n \frac{A}{(1+r)^k} \\ &= \underbrace{\sum_{k=1}^{\infty} \frac{A}{(1+r)^k}}_{\frac{A}{r}} - \underbrace{\sum_{k=n+1}^{\infty} \frac{A}{(1+r)^k}}_{\frac{1}{(1+r)^n} \sum_{k=1}^{\infty} \frac{A}{(1+r)^k} = \frac{1}{(1+r)^n} \frac{A}{r}} \\ \Rightarrow P &= \frac{A}{r} \left(1 - \frac{1}{(1+r)^n} \right) \end{aligned}$$

Examples of streams with finite life

- ▶ Get $A = 25\,000\text{ €}$ annually for 20 years
 - ▶ Interest rate $r = 0.1 = 10\%$

$$P = \frac{25\,000\text{ €}}{0.1} \left(1 - \frac{1}{(1 + 0.1)^{20}} \right) = 213\,000\text{ €}$$

- ▶ PV “only” 37 000 € less than that of the corresponding perpetual annuity on slide 11
- ▶ For high interest rates, the cash flows that occur in the more distant future do not matter that much

Example: Consumer credit

- ▶ Loan $P = 10\,000\text{€}$
 - ▶ Interest is paid monthly at the nominal rate of 12% p.a.
 $\Rightarrow r = 0.12/12 = 0.01$
 - ▶ Amortize (=pay back) in 3 years in **equal monthly payments**
($n = 36$)
- ▶ What are the monthly payments A ?
 - ▶ Finite life stream with unknown A
- ▶ Solving the pricing equation on slide 11 for A gives

$$A = \frac{r(1+r)^n P}{(1+r)^n - 1} = \frac{0.01(1+0.01)^{36} 10\,000\text{€}}{(1+0.01)^{36} - 1} \approx 332.14\text{€}$$

- ▶ Sum of payments $36 \times 332.14\text{€} = 11\,957.04\text{€}$
- ▶ Amortization table
 - ▶ (Loan at $n + 1$) = (Loan at n) - (Amortization at n)

Example: Consumer credit

n	Loan capital	Interest	Amort.	Payment	n	Loan ca.	Inter.	Amort.	Paym.
0	10000.0	100.0	232.1	332.1					
1	9767.9	97.7	234.5	332.1	19	5169.0	51.7	280.5	332.1
2	9533.4	95.3	236.8	332.1	20	4888.5	48.9	283.3	332.1
3	9296.6	93.0	239.2	332.1	21	4605.3	46.1	286.1	332.1
4	9057.4	90.6	241.6	332.1	22	4319.2	43.2	289.0	332.1
5	8815.9	88.2	244.0	332.1	23	4030.2	40.3	291.8	332.1
6	8571.9	85.7	246.4	332.1	24	3738.4	37.4	294.8	332.1
7	8325.5	83.3	248.9	332.1	25	3443.6	34.4	297.7	332.1
8	8076.6	80.8	251.4	332.1	26	3145.9	31.5	300.7	332.1
9	7825.2	78.3	253.9	332.1	27	2845.2	28.5	303.7	332.1
10	7571.3	75.7	256.4	332.1	28	2541.6	25.4	306.7	332.1
11	7314.9	73.2	259.0	332.1	29	2234.8	22.4	309.8	332.1
12	7055.9	70.6	261.6	332.1	30	1925.0	19.3	312.9	332.1
13	6794.3	67.9	264.2	332.1	31	1612.2	16.1	316.0	332.1
14	6530.1	65.3	266.8	332.1	32	1296.1	13.0	319.2	332.1
15	6263.3	62.6	269.5	332.1	33	977.0	9.8	322.4	332.1
16	5993.8	59.9	272.2	332.1	34	654.6	6.6	325.6	332.1
17	5721.6	57.2	274.9	332.1	35	329.0	3.3	328.9	332.1
18	5446.6	54.5	277.7	332.1	36	0.1	≈ 0		

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Bonds as examples of fixed income securities

- ▶ A bond pays its **principal** (also known as **face value** or **par value**) on its **maturity date**
 - ▶ E.g. pay a face value of 1000€ on 1.1.2029
- ▶ Most bonds pay periodic interest payments (also known as **coupon payments**)
 - ▶ **Coupon rate** defined as a percentage of face value
 - ▶ E.g., coupon rate 9 % p.a. \Rightarrow receive 90€ on every 1st of January until 1.1.2029
 - ▶ Coupon rate of a par bond (issue price = face value) is close to the prevailing interest rate when the bond is issued
 - ▶ If the bond is issued at a discount (issue price is less than face value), then the coupon rate can be less
 - ▶ Price appreciation to maturity generates further return to the investor
 - ▶ Originally, actual coupons were attached to printed bond certificates

Bond yield

- ▶ **Yield (To Maturity)(YTM)** = Internal rate of return (IRR) of a bond per annum (p.a.) at its current price
 - (i) Face value F
 - (ii) m coupon payments p.a., each payment C/m
 - (iii) n periods (payments) left to maturity
- ▶ If the current price is $P \Rightarrow$, then YTM is the rate λ such that

$$P = \frac{F}{[1 + (\lambda/m)]^n} + \underbrace{\sum_{k=1}^n \frac{C/m}{[1 + (\lambda/m)]^k}}_{\text{Finite life stream}}$$
$$\Rightarrow P = \frac{F}{[1 + (\lambda/m)]^n} + \frac{C}{\lambda} \left(1 - \frac{1}{[1 + (\lambda/m)]^n} \right)$$

The formula is best understood if you substitute $r = \lambda/m$ as the periodic (e.g. monthly) IRR, and then obtain an annualized rate (= YTM) by linear annualization $\lambda = rm$.

Bond yield

- ▶ As with IRR in general, YTM (λ) is computed numerically
- ▶ **Accrued interest** (AI) tells how much interest the bond has accrued since the last coupon payment
 - ▶ Linear interpolation:

$$AI = \frac{\text{Days since last coupon}}{\text{Days in period}} \times \text{Coupon payment}$$

- ▶ Consider a bond which has face value 1 000€, coupon rate 9 % with coupon payments every Feb 15 and Aug 15.
- ▶ If this bond is bought on May 5, there are 83 days since last coupon payment and 99 days until next payment
- ▶ $AI = \frac{83}{83+99} \times \frac{9\%}{2} \times 1\,000\text{€} = 20.52\text{€}$

Example of bond quotes

U.S. Treasury Quotes Wednesday, January 13, 2016					
Treasury Notes & Bonds					
Maturity	Coupon	Bid	Asked	Chg	Asked yield
1/31/2020	1.250	99.45	99.47	0.0469	1.385
1/31/2020	1.375	99.93	99.95	0.0234	1.389
2/15/2020	3.625	108.82	108.84	0.0391	1.393
2/15/2020	8.500	128.23	128.25	0.0547	1.369
2/29/2020	1.250	99.38	99.39	0.0703	1.402
2/29/2020	1.375	99.91	99.93	0.0938	1.393
3/31/2020	1.125	98.84	98.85	0.1250	1.407
3/31/2020	1.375	99.82	99.84	0.0859	1.415
4/30/2020	1.125	98.80	98.81	0.1094	1.411
4/30/2020	1.375	99.76	99.77	0.0703	1.429
5/15/2020	3.500	108.56	108.58	0.0703	1.451
5/15/2020	8.750	130.61	130.63	0.1094	1.437
5/31/2020	1.375	99.66	99.68	0.1172	1.451

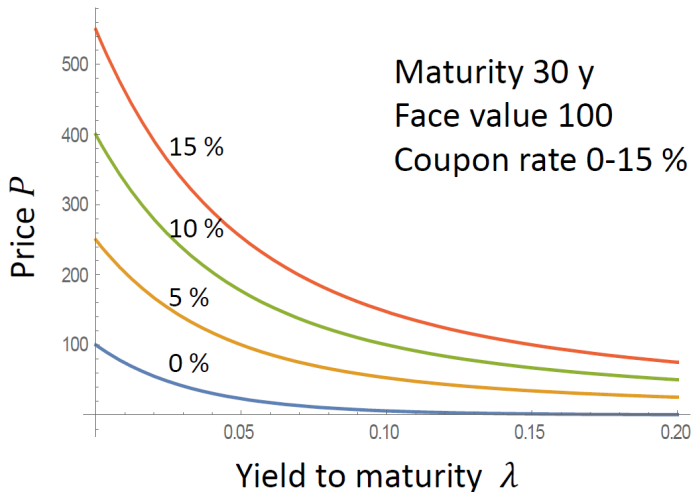
Coupon: Annual rate (%)

Ask and bid prices: % of face value

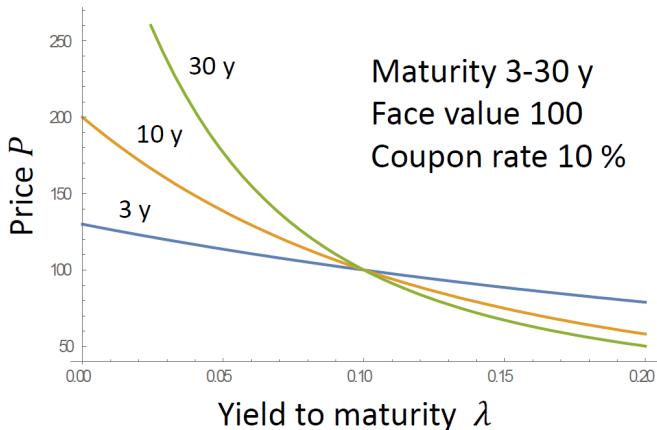
Chg: Daily change in asked price

Asked yield: yield to maturity at asked price

Price-yield curve



Price-yield curve



Price-yield curve

- ▶ Yield $\lambda = 0$ if and only if the price equals the total cash flow, that is, $P = F + nC/m$
- ▶ Yield λ equals the coupon rate if and only if price $P =$ face value F
- ▶ If the yield λ increases, the price P of bonds with small coupon rates decline more than the price of bonds with high coupon rates
 - ▶ The bigger early coupon payments are less affected by the rising interest rates
- ▶ For bonds of longer maturity, the price-yield curve is steeper
 - ▶ Steeper curve \iff bond prices are more sensitive to the interest rate

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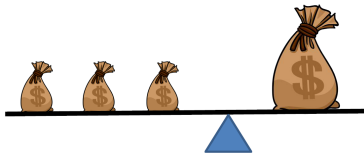
Duration

- ▶ Consider the cash flow stream (c_0, c_1, \dots, c_n) which gives c_i at time $t_i, i = 0, \dots, n$
- ▶ The **duration** of this cash flow stream is

$$D = \frac{PV(t_0)t_0 + PV(t_1)t_1 + \dots + PV(t_n)t_n}{PV},$$

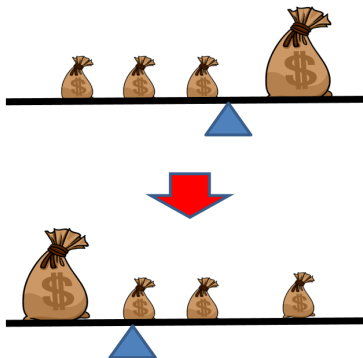
where $PV(t_i), i = 0, 1, \dots, n$ is the present value of c_i and $PV = PV(t_0) + \dots + PV(t_n)$

- ▶ This is the PV weighted average of the payment times t_0, t_1, \dots, t_n of the cash flow stream (c_0, c_1, \dots, c_n)
 - ▶ i -th weight = share of the $PV(t_i)$ out of the total PV
 - ▶ By definition, these weights add up to one



Duration

- ▶ Duration is a measure of the bond's 'average' maturity
 - ▶ If there are no coupons, then duration = maturity
 - ▶ If there are coupons, then duration < maturity
 - ▶ For two bonds with the same total cash flow (i.e., coupons + face value), the duration is shorter for the one with higher coupon rate



Macaulay duration

- ▶ What interest rate r should one use when computing duration?
- ▶ Macaulay duration: $r = \text{YTM}$

$$\Rightarrow D = \frac{\sum_{k=1}^n \frac{k}{m} \frac{c_k}{(1 + \frac{\lambda}{m})^k}}{PV}, \text{ where}$$

$$PV = \sum_{k=1}^n \frac{c_k}{(1 + \frac{\lambda}{m})^k}$$

- ▶ For bonds (derived in exercises)

$$D = \frac{1 + y}{my} - \frac{1 + y + n(c - y)}{mc[(1 + y)^n - 1] + my}, \text{ where}$$

m = periods per year

n = nr of periods left

$y = \lambda/m$ = yield per period

c = coupon rate

Modified duration

- ▶ The present value of cash flow c_k is

$$PV_k = \frac{c_k}{[1 + (\lambda/m)]^k}$$
$$\Rightarrow \frac{dPV_k}{d\lambda} = -\frac{(k/m)c_k}{[1 + (\lambda/m)]^{k+1}} = -\frac{(k/m)PV_k}{1 + (\lambda/m)}$$

- ▶ The price sensitivity of a bond is

$$P = \sum_{k=1}^n PV_k$$
$$\Rightarrow \frac{dP}{d\lambda} = \sum_{k=1}^n -\frac{(k/m)PV_k}{1 + (\lambda/m)} = -\frac{1}{1 + (\lambda/m)} \frac{\sum_{k=1}^n (k/m)PV_k}{P} P$$
$$\Rightarrow \frac{dP}{d\lambda} = -\frac{1}{1 + (\lambda/m)} DP = -D_M P,$$

where D_M is the **modified duration** $D_M = D/[1 + (\lambda/m)]$

Applying modified duration

- ▶ Consider a bond such that
 - ▶ Maturity 30 y, no coupons (i.e., coupon rate 0 %)
- ▶ Assume that interest rates rise from 10 % to 11%

$$\lambda \rightarrow \lambda + \Delta\lambda, \quad \lambda = 0.1, \Delta\lambda = 0.01$$

- ▶ No coupons $\Rightarrow D = \text{Maturity} \Rightarrow D_M = 30/[1 + 0.1] \approx 27.27$
- ▶ Linear approximation:

$$\begin{aligned}\Delta P &\approx -D_M P \Delta\lambda \\ \Rightarrow \frac{\Delta P}{P} &\approx -D_M \Delta\lambda = -27.27 \times 0.01 = -27.27\%\end{aligned}$$

- ▶ Price approx. sinks by 27 % if the interest rate rises by 1%

The actual price drop when yield changes from 10% to 11% is 23.78%. For smaller changes the approximation is more accurate: For 0.1% change, the drop is 2.689% and for 0.01% change, 0.2723%.

Application of modified duration

- ▶ Consider another bond with
 - ▶ Maturity 30 y, coupon rate 10 %, 2 coupons per year
 - ▶ Price = face value, i.e., YTM is 10 %

- ▶ Macaulay duration

$$D = 9.938 \Rightarrow D_M = 9.938 / [1 + (0.1/2)] \approx 9.47$$

- ▶ Price change

$$\frac{\Delta P}{P} \approx -D_M \Delta \lambda = -9.47 \times 0.01 = -9.47\%$$

- ▶ The relatively decline in price is now much less because of the coupon payments

The actual price drop when yield changes from 10% to 11% is 8.72%. For smaller changes the approximation is more accurate: For 0.1% change, the drop is 0.9386% and for 0.01% change, 0.09457%.

Duration of a portfolio

- ▶ **Portfolio** of bonds = a collection consisting of a set of m bonds
- ▶ Price $P = P_1 + P_2 + \dots + P_m$
 - ▶ P_i = price of bond $i = 1, 2, \dots, m$

Theorem

(Duration of a portfolio) Suppose there are m fixed-income securities with prices and durations of P_i and D_i , respectively, $i = 1, 2, \dots, m$, all computed using the same yield. Then the price P and duration D of the portfolio consisting of the aggregate of these securities are

$$P = P_1 + P_2 + \dots + P_m$$

$$D = w_1 D_1 + w_2 D_2 + \dots + w_m D_m,$$

where $w_i = P_i/P, i = 1, 2, \dots, m$

Duration of a portfolio

- **Proof.** Outline for the case of two securities A and B :

$$D^{A+B} = \sum_{k=0}^n \frac{PV_k^{A+B} t_k}{P^{A+B}}$$

$$D^A = \sum_{k=0}^n \frac{PV_k^A t_k}{P^A}$$

$$D^B = \sum_{k=0}^n \frac{PV_k^B t_k}{P^B}$$

$$\Rightarrow P^A D^A + P^B D^B = \sum_{k=0}^n t_k (PV_k^A + PV_k^B)$$

Duration of a portfolio

- ▶ Divide both sides of equation by $P = P^{A+B} = P^A + P^B$

$$\Rightarrow \frac{P^A D^A + P^B D^B}{P} = \frac{\sum_{k=0}^n t_k (PV_k^A + PV_k^B)}{P} = D$$

$$\Rightarrow D = \frac{P^A}{P} D^A + \frac{P^B}{P} D^B$$

- ▶ By definition, duration of a portfolio of A and B is

$$D^{A+B} = \sum_{k=0}^n \frac{t_k PV_k^{A+B}}{P^{A+B}} = \sum_{k=0}^n \frac{t_k PV_k^{A+B}}{P}$$

- ▶ Thus, $PV_k^{A+B} = PV_k^A + PV_k^B$ implies $D^{A+B} = D$
 - ▶ Holds when payments from A and B are discounted with the same rate for each period k , assuming the same yield
 - ▶ This assumption of identical interest rates does not hold for Macaulay duration which uses YTM for each bond

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Immunization

- ▶ **Immunization** is the development of an investment strategy when:
 - ▶ An investor has a liability stream (a cash flow stream the investor has to pay) that is sensitive to interest rates
 - ▶ He or she wants to construct an investment portfolio to match this liability stream both in terms of
 - ▶ present value and
 - ▶ interest rate sensitivity
- ▶ The combination of the portfolio and the liability stream is insensitive (immune) to (small) interest rate changes
- ▶ If an investor wants to immunize a bond portfolio, then he or she may have to short other bonds, which may be difficult or expensive in practice

Immunization

- ▶ **Principle:** Buy a portfolio of equal NPV whose interest rate sensitivity is the same as that of the liability stream being immunized
- ▶ If there are zero coupon bonds with many enough maturities, then perfect matching of cash flows is possible
 - ▶ Would match interest rate sensitivities exactly
 - ▶ This is difficult, however, because zero coupon bonds are rare and there may be no bonds whose maturities coincide with those of the cash flows of the portfolio
- ▶ The other method is to use **duration**
 - ▶ First-order (i.e., first derivative) approximation of interest rate sensitivity

Immunization

- ▶ **Task:** Immunize a liability stream with duration D and price P
- ▶ Bonds A and B available for immunization
- ▶ Buy A and B for total amount V_A and V_B (unit price times units bought) such that

$$P = V_A + V_B$$

$$D = w_A D_A + w_B D_B, \text{ where}$$

$$w_i = \frac{V_i}{P}, i = A, B$$

- ▶ In practice, more than two bonds would used
 - ▶ Helps diversify risk (of default)
 - ▶ Leads to more variables than equations \Rightarrow there can be many solutions

Example: Immunization

- ▶ A company is liable to pay 1 million € in 10 years
 - ▶ No coupons \Rightarrow Duration 10 y
- ▶ Immunize using the following three bonds whose face value is 100 € and which pay two coupons per year

Bond	Coupon rate	Maturity (y)	Price (€)	YTM	Duration (y)
1	6%	30	69	9%	11.44
2	11%	10	113	9%	6.54
3	9%	20	100	9%	9.61

- ▶ The PV of the liability at the prevailing rate is

$$P = \frac{1\,000\,000\text{€}}{[1 + (0.09/2)]^{20}} \approx 414\,634\text{€}$$

Example: Immunization

- ▶ If we use bonds 1 & 2

$$\begin{aligned} & \begin{cases} P &= V_1 + V_2 \\ D &= \frac{V_1}{P} D_1 + \frac{V_2}{P} D_2 = 10 \end{cases} \\ \Rightarrow & \begin{cases} V_1 &= P \frac{D-D_2}{D_1-D_2} \approx 292\,788 \\ V_2 &= P \frac{D_1-D}{D_1-D_2} \approx 121\,854 \end{cases} \\ \Rightarrow & \begin{cases} \frac{V_1}{P_1} &= \frac{292\,788}{69} = \mathbf{4241 \text{ units of bond \# 1}} \\ \frac{V_2}{P_2} &= \frac{121\,854}{113} = \mathbf{1078 \text{ units of bond \# 2}} \end{cases} \end{aligned}$$

Example: Immunization

- ▶ If we use bonds 2 & 3
 - ▶ No solution with positive amounts of bonds (weighted average of $D_2 = 6.54$ and $D_3 = 9.61$ less than $D = 10$ with all positive weights)

$$\begin{cases} V_2 &= P \frac{D_3 - D}{D_3 - D_2} \approx -52\,575 \\ V_3 &= P - V_2 \approx 467\,317 \end{cases}$$
$$\Rightarrow \begin{cases} \frac{V_2}{P_2} &= -\frac{52\,575}{113} = \text{sell short } \mathbf{465} \text{ units of bond \# 2} \\ \frac{V_3}{P_3} &= \frac{467\,317}{100} = \text{purchase } \mathbf{4673} \text{ units of bond \# 3} \end{cases}$$

Example: Immunization

		<i>Percent yield</i>		
		9.0	8.0	10.0
<i>Bond 1</i>	<i>Price</i>	69.04	77.38	62.14
	<i>Shares</i>	4241.00	4241.00	4241.00
	<i>Value</i>	292798.64	328168.58	263535.74
<i>Bond 2</i>	<i>Price</i>	113.01	120.39	106.23
	<i>Shares</i>	1078.00	1078.00	1078.00
	<i>Value</i>	121824.78	129780.42	114515.94
<i>Obligation</i>	<i>Value</i>	414642.86	456386.95	376889.48
Surplus		-19.44	1562.05	1162.20

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