MEC-E1004 Principles of Naval Architecture

Simpson's Integration of areas and volumes

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Calculating areas and volumes

Tips before going to calculations

- This tutorial is to explain how to use the excel sheet in area and volume calculations
- By this stage you should have your hull form lines ready.
- The excel sheet is based on Simpson's first rule integration (odd number of ordinates must be used)

Waterplane Area and LCF

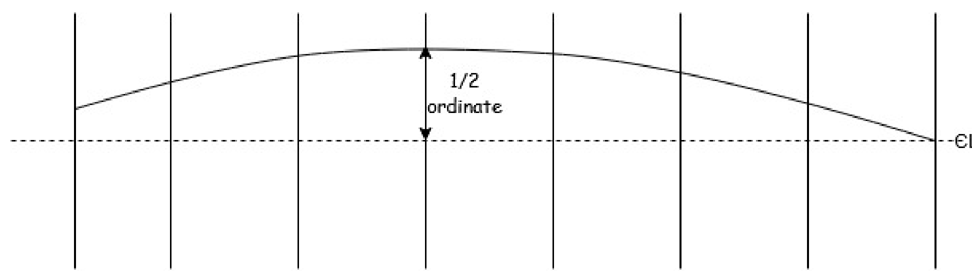
- The first step is to define the length of the waterplane area, and the number of intervals.

Length	150	m
Intervals:	10	-
Spacing, s:	**15**	**m**

Waterplane Area and LCF

- As the ship is symmetric about the centerline, only ½ ordinates will be used.
- Half-ordinates as shown in figure are measured at each frame and to be inserted in the table.

		1/2 ordinates	SM	Product for area	Lever @ Frame
Frame [-]	x-coordinate	y_n [m]	k_n [-]	$y_n * k_n$ [m^2]	R_n [m]
0	0	1,0	1	1,0	0
1	15	7,5	4	30,0	1
2	30	12,0	2	24,0	2
3	45	13,5	4	54,0	3
4	60	14,0	2	28,0	4
5	75	14,0	4	56,0	5
6	90	14,0	2	28,0	6
7	105	13,5	4	54,0	7
8	120	12,0	2	24,0	8
9	135	7,0	4	28,0	9
10	150	0,0	1	0,0	10
			Σ	327	m^2

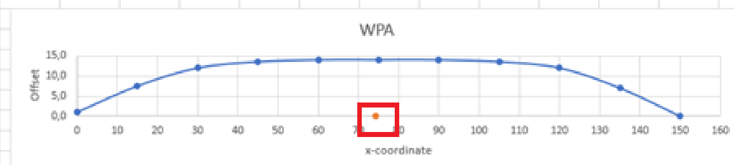


Waterplane Area and LCF

- The results are highlighted in red; the waterplane area of the waterline and the longitudinal center of floatation from frame zero.
- You can then repeat the same procedures for any WL you want.

		1/2 ordinates	SM	Product for area	Lever @ Frame 0	moment of area	
Frame [-]	x-coordinate	y_n [m]	k_n [-]	$y_n * k_n$ [m²]	R_n [m]	$y_n * k_n * R_n$ [m³]	
0	0	1,0	1	1,0	0	0,0	
1	15	7,5	4	30,0	1	30,0	
2	30	12,0	2	24,0	2	48,0	
3	45	13,5	4	54,0	3	162,0	
4	60	14,0	2	28,0	4	112,0	
5	75	14,0	4	56,0	5	280,0	
6	90	14,0	2	28,0	6	168,0	
7	105	13,5	4	54,0	7	378,0	
8	120	12,0	2	24,0	8	192,0	
9	135	7,0	4	28,0	9	252,0	
10	150	0,0	1	0,0	10	0,0	
			Σ	327	m²	1622	m³

WPA	3270	m2
LCF From fr0	74,404	m



Second moment of area I_T and I_L

- The second moment of area about the transverse axis equals.

$$I_L = \int x^2 y dx$$

- The second moment of area about centerline equals

$$I_T = 1/3 \int y^3 dx$$

- The excel calculates I_L through the center of flotation and I_T

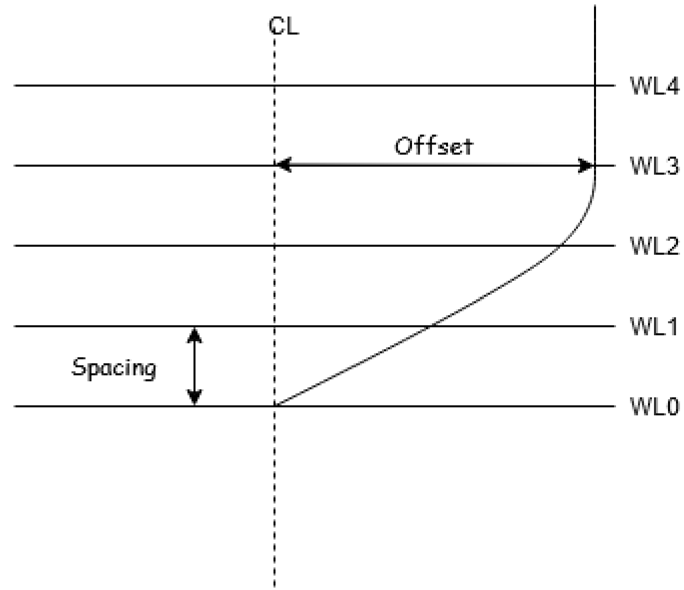
		1/2 ordinates	SM	Product for area	Lever @ Frame 0	moment of area	F(IL)	F(IT)
Frame [-]	x-coordinate	y_n [m]	k_n [-]	$y_n * k_n$ [m]	R_n [-]	$y_n * k_n * R_n$ [m]	yn * kn*Rn^2	yn^3* kn
0	0	1.0	1	1.0	0	0.0	0.0	1.0
1	18	7.5	4	30.0	1	30.0	30.0	1687.5
2	36	12.0	2	24.0	2	48.0	96.0	3456.0
3	54	13.5	4	54.0	3	162.0	486.0	9841.5
4	72	14.0	2	28.0	4	112.0	448.0	5488.0
5	90	14.0	4	56.0	5	280.0	1400.0	10976.0
6	108	14.0	2	28.0	6	168.0	1008.0	5488.0
7	126	13.5	4	54.0	7	378.0	2646.0	9841.5
8	144	12.0	2	24.0	8	192.0	1536.0	3456.0
9	162	7.0	4	28.0	9	252.0	2268.0	1372.0
10	180	0.0	1	0.0	10	0.0	0.0	0.0
			Σ	327	[m]	1622	9918	51608

WPA	3924	m2
LCF From fr0	89.284	m
LCF from amidship	-1	+ve forward
IL at CF	7280214.606	m4
IT	206430	m4

Cross Sectional Area and its Center

- **The same procedure is followed for calculating the cross sectional area of a station. Instead of using ½ ordinates among longitudinal intervals, they are measured at every waterline (vertical intervals).**

Maximum draft	5	m
Intervals:	10	-
Spacing, s:	0,5	m

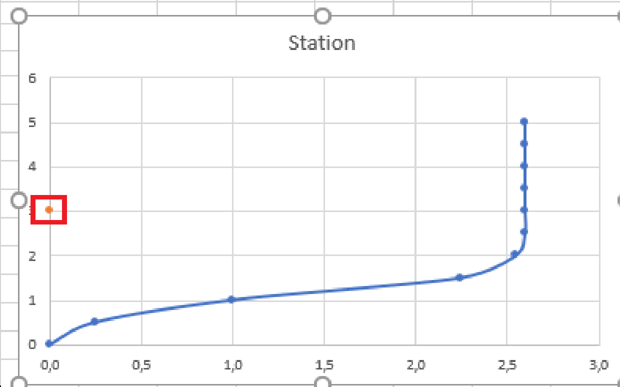


Cross Sectional Area and its Center

- The results in such a case are the cross sectional area of the station and its center.

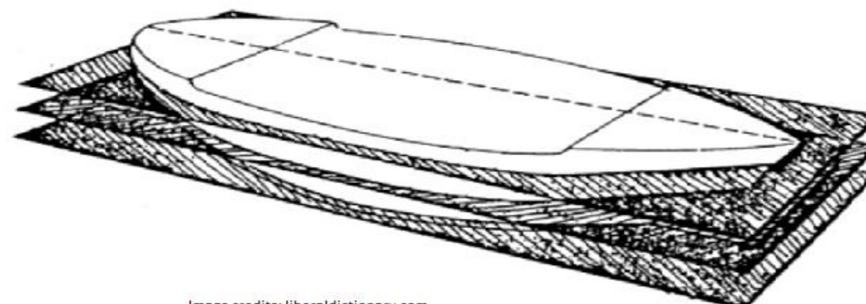
		1/2 ordinates	SM	Product for area	Lever @ Frame (	moment of area	
WL [-]	Z-coordinate	y_n [m]	k_n [-]	$y_n * k_n$ [m^2]	R_n [m]	$y_n * k_n * R_n$ [m^3]	
0	0	0,0	1	0,0	0	0,0	
1	0,5	0,3	4	1,0	1	1,0	
2	1	1,0	2	2,0	2	4,0	
3	1,5	2,3	4	9,0	3	27,0	
4	2	2,6	2	5,1	4	20,4	
5	2,5	2,6	4	10,4	5	52,0	
6	3	2,6	2	5,2	6	31,2	
7	3,5	2,6	4	10,4	7	72,8	
8	4	2,6	2	5,2	8	41,6	
9	4,5	2,6	4	10,4	9	93,6	
10	5	2,6	1	2,6	10	26,0	
			Σ	61,3	m^2	369,6	m^3

Cross sectional area	20,4	m2	
Center of CSA	3,015	m	from keel



Volumes and LCB/KB

- The numerical integration is just a tool and can be used in voulme integration as well. In this step cross sectional areas or waterplane areas are used to obtain the ship's volume of displacement.



Volumes and LCB

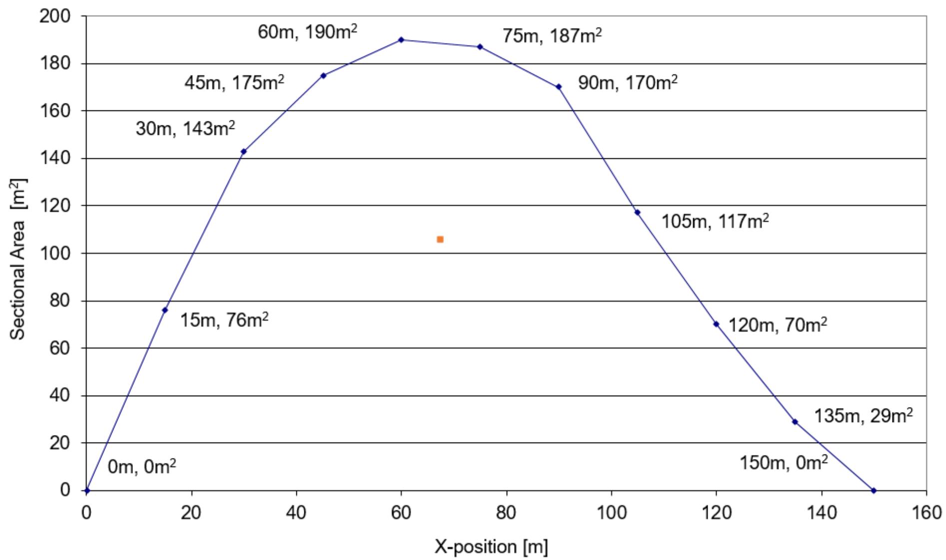
- **You should have the cross sectional area for every station ready at this point. You follow the same procedure discussed previously, and the results will be the volume of displacement, the longtiduinal center of buoyancy, and the displacement in tonnes.**

		Cross-sectional area	SM	Product for volume	Lever @ Frame 0	moment of volume
Frame [-]	x-coordinate	A_n [m^2]	k_n [-]	$A_n * k_n$ [m^2]	R_n [m]	$A_n * k_n * R_n$ [m^3]
0	0	0	1	0	0	0
1	15	76	4	304	1	304
2	30	143	2	286	2	572
3	45	175	4	700	3	2100
4	60	190	2	380	4	1520
5	75	187	4	748	5	3740
6	90	170	2	340	6	2040
7	105	117	4	468	7	3276
8	120	70	2	140	8	1120
9	135	29	4	116	9	1044
10	150	0	1	0	10	0
			Σ	3482	m^2	15716

Volume	17410	m3
Density of water	1,025	t/m3
Displacement	17845,25	t
LCB from fr0	67,702	m

Volumes and LCB

- **Provided in another sheet is the sectional area curve as a result.**



Volumes and KB

- **The final step is to obtain the volume of displacement using another way by integrating the waterplane areas in order to get the vertical center of buoyancy above the keel (KB).**
- **Make sure that the volume by the two ways is the same.**

		WPA	SM	Product for volume	Lever @ keel	moment of volume	
WL [-]	Z-coordinate	y_n [m^2]	k_n [-]	$y_n * k_n$ [m^3]	R_n [m]	$y_n * k_n * R_n$ [m^4]	
0	0	0,0	1	0,0	0	0,0	
1	0,5	300,0	4	1200,0	1	1200,0	
2	1	950,0	2	1900,0	2	3800,0	
3	1,5	1450,0	4	5800,0	3	17400,0	
4	2	1900,0	2	3800,0	4	15200,0	
5	2,5	2150,0	4	8600,0	5	43000,0	
6	3	2330,0	2	4660,0	6	27960,0	
7	3,5	2370,0	4	9480,0	7	66360,0	
8	4	2390,0	2	4780,0	8	38240,0	
9	4,5	2400,0	4	9600,0	9	86400,0	
10	5	2400,0	1	2400,0	10	24000,0	
			Σ	52220,0	m^3	323560,0	m^4

Volume	17406,7	m3	
KB	3,098	m	above keel

Thank you

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