Simpson's First Rule (application on areas)

1. Simpson's rule can be used to integrate the area under any curve; it can be used in waterplane area and cross-sectional area integration.
2. Make sure that the area has even number of divisions.
3. For obtaining the center of area, moments are calculated about whether the aft perpendicular or the amidships. When calculating moments about amidships, +ve and -ve signs should be considered for areas forward or abaft amidships.
4. Some assumptions must be made to calculate the new design data such as that CB and draught do not change.

½ ordinates	Simpson's multiplier	Product for area	Lever@ AP	Product for 1st moment	Lever@ AP	Product for 2nd moment
y_0	1	$1y_0$	$0h$	0	$0h$	0
y_1	4	$4y_1$	$1h$	y_1h	$1h$	y_1h^2
y_2	2	$2y_2$	$2h$	$2y_2h$	$2h$	$4y_2h^2$
y_3	4	$4y_3$	$3h$	$3y_3h$	$3h$	$9y_3h^2$
y_4	2	$2y_4$	$4h$	$4y_4h$	$4h$	$16y_4h^2$
y_5	4	$4y_5$	$5h$	$5y_5h$	$5h$	$25y_5h^2$
y_6	1	$1y_6$	$6h$	$6y_6h$	$6h$	$36y_6h^2$
		$\sum_A$		$\sum MA \times h$		$\sum i \times h^2$

The area according to Simpson's rule is given by $A = \frac{h}{3}(\sum_A)$,

and the first moment of area is $= \frac{h}{3}(\sum MA \times h) = \frac{h^2}{3}\sum MA$

Then the center of area is $= \dfrac{\frac{h^2}{3}\sum MA}{\frac{h}{3}\sum_A} = h\dfrac{\sum MA}{\sum_A}$,

The second moment of area about aft perpendicular $I_{AP} = \frac{h}{3}\sum i \times h^2 = \frac{h^3}{3}\sum i$

Finally, using parallel axis theorem, the second moment of area about the center of floatation is obtained as follows: $I_F = I_{AP} - Ax^2$

Where, x is the distance between the aft perpendicular and the center of floatation.

Example

A ship 180 m long has half-widths of waterplane of 1, 7.5, 12, 13.5, 14, 14, 14, 13.5, 12, 7 and 0 m respectively. Calculate: (a) Waterplane area (b) Waterplane area coefficient.

• Solution Procedure:

1. A simple way to calculate the waterplane area is to use the tabular format for integration:

½ ordinates	Simpson's multiplier	Product for area
1	1	1
7.5	4	30
12	2	24
13.5	4	54
14	2	28
14	4	56
14	2	28
14	4	56
14	2	28
13.5	4	54
12	2	24
7	4	28
0	1	0

$$\sum_A = 411$$

2. The longitudinal spacing $(h) = {}^{180}/_{10} = 18\ m.$
3. The waterplane area $WPA = 2 \times \frac{1}{3} \times 18 \times 411 = 4932\ m^2.$
4. Finally, the waterplane area coefficient $C_W = \frac{A_W}{L \times B} = \frac{4932}{180 \times 14 \times 2} = \ 0.978$

Simpson's First Rule (application on volumes)

Example

The immersed cross-sectional areas of a ship 120 m long, commencing from aft, are 2, 40, 79, 100, 103, 104, 104, 103, 97, 58, 0 m^2. Calculate: (a) volume of displacement (b) LCB.

Solution procedure:

1. The same procedure is followed in volume calculations:

Frame	CSA	Simpson's multiplier	Product for area	Lever @Fr0	Moment of volume
0	2	1	2	0	0
1	40	4	160	1	160
2	79	2	158	2	316
3	100	4	400	3	1200
4	103	2	206	4	824
5	104	4	416	5	2080
6	104	2	208	6	1248
7	103	4	412	7	2884
8	97	2	194	8	1552
9	58	4	232	9	2088
10	0	1	0	10	0
			$\sum_V = 2388$		$\sum \boldsymbol{MA} = \mathbf{12352}$

2. The volume of displacement$= \frac{h}{3} \times \sum_V \quad = \frac{120}{10\times 3} \times 2388 = 9553\ m^3$
3. The longitudinal center of volume (LCB) $= h \times \frac{\sum MA}{\sum V} = 12 \times \frac{12352}{2388} = 62.1\ m$