Sectional Modulus Tutorial

In the lecture notes, we explain how the sectional modulus is important in the hull girder analysis, here we recall the equations relate the hull bending to the sectional modulus:

$$\sigma = \frac{Mz}{I}$$

The sectional modulus: $Z_{deck} = \frac{I}{z_{deck}}$, and $Z_{keel} = \frac{I}{z_{keel}}$ where z_{deck} and z_{keel} are the distance from the neutral axis of the ship section considered to the deck and the bottom as they are the farthest from the neutral axis and they are likely to have the highest bending stresses. In order to calculate the sectional modulus, we shall calculate the first and second moment of area of each structural component (only those contribute in the longitudinal stiffness) about its centroid and the neutral axis of the section.

The second moment of area of such a structural component will be at first calculated at the component's centroid then at the neutral axis using the parallel axis theorem.

$$I_x = i_x + a_i \cdot h^2$$

Where I_x is the moment of inertia about the neutral axis of the section, i_x is the moment of inertia about the centroid of the component, a_i is the cross-sectional area of the structural component, and h is the distance from the centroid to the neutral axis.

The main procedure to be followed to calculate the sectional modulus is shown below:

1. Calculate the area of each component (a_i).
2. Take the baseline as your first reference line and calculate the height of each component's area centroid above the baseline (h_i).
3. Calculate the 1st moment of area about the baseline ($a_i h_i$).
4. Calculate the 2nd moment of area of each component about the baseline ($a_i h_i^2$).
5. Calculate the moment of inertia of each component about its own horizontal axis passing through the centroid (i_x).

   **The moment of inertia of the plate or any rectangular cross-section is equal to: $i_x = \frac{bd^3}{12}$

   where b is the dimension parallel to the neutral axis (breadth of deck plate or thickness of the vertical side plate) and d is the dimension perpendicular to the neutral axis.

6. Calculate the distance of the neutral axis above the baseline ($\frac{\sum a_i h_i}{\sum a_i}$).
7. Calculate the moment of inertia of the total section about the baseline ($I_{BL} = \sum a_i h_i^2 + \sum i_x$).
8. Calculate the moment of inertia of the total section about the NA ($I_{NA} = I_{BL} - Ah_{NA}^2$).
9. Calculate the section modulus for deck and keel.

Ship section idealization

The idealized ship section is based on using effective thickness concept for the decks, sides, and bottom structure. The effective thickness takes account of longitudinal stiffeners and girders and is calculated as follows:

$$t_e = t + \frac{\sum_{i=1}^{n} a_i}{l}$$

$$n = \frac{l}{s} - 1$$

Where:

t = plate thickness

t_e = effective (idealized) plate thickness

a_i = sectional area of longitudinal member

l = stiffened panel width through which n longitudinal members are distributed

s = distance between each two consecutive longitudinal members

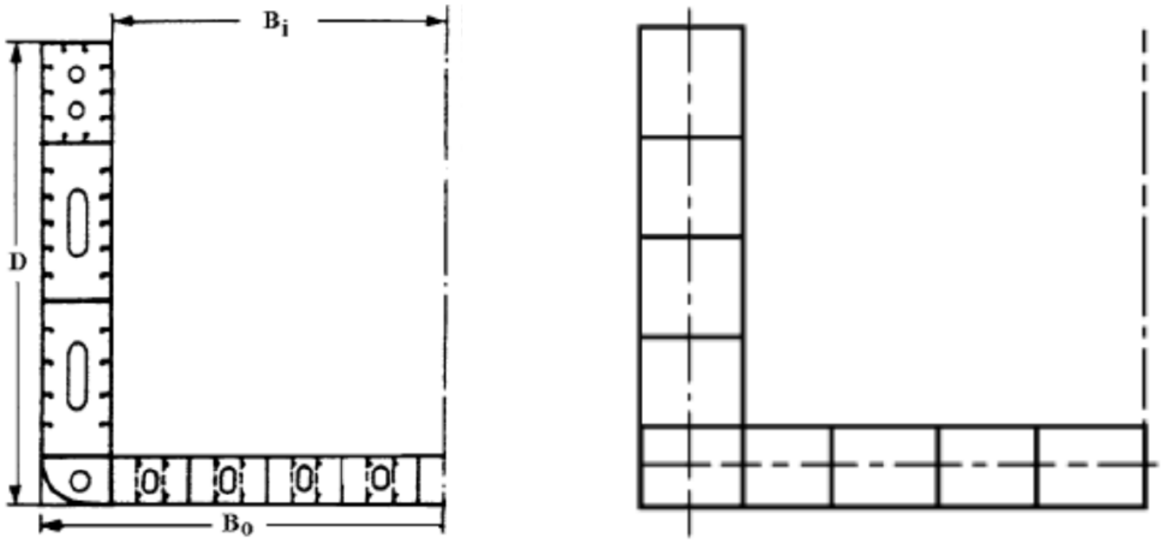


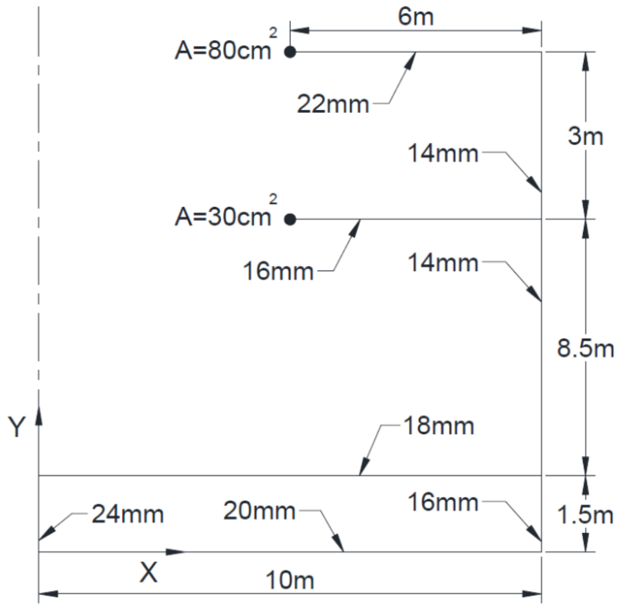
Figure 1 ship section of a container ship and corresponding idealization

It will be noted that the moment of inertia of what is called horizontal material (like deck plate) about its own neutral axis is sufficiently small to be negligible. The calculation is usually carried out for one side of the ship only, and therefore the results have to be multiplied by two, as will be illustrated in the following example.

Example

Consider the idealized ship section shown in the figure. Calculate:

- The sectional modulus at the deck and the bottom.
- The maximum bending stress ($M = 400\ MN \cdot m$).
- The factor of safety (FOS) ($\sigma_y = 235\ MPa$).



Solution:

- We can follow the same procedure discussed above but in a tabular format to avoid any mistakes:

Item	Number of parts	Horizontal dimension	Vertical dimension					
	n	b	d	h_i	a_i	$a_i h_i$	i_x	$a_i h_i^2$
[-]	[-]	[m]	[m]	[m]	[m²]	[m³]	[m⁴]	[m⁴]
Bottom plating	1	10.000	0.020	0.000	0.200	0.000	0.000	0.000
Inner bottom plating	1	10.000	0.018	1.500	0.180	0.270	0.000	0.405
Strength deck plating	1	6.000	0.022	13.000	0.132	1.716	0.000	22.308
2nd deck plating	1	6.000	0.016	10.000	0.096	0.960	0.000	9.600
Side plating	1	0.014	11.500	7.250	0.161	1.167	1.774	8.463
Bilge	1	0.016	1.500	0.750	0.024	0.018	0.005	0.014
Center girder (1/2)	1	0.012	1.500	0.750	0.018	0.014	0.003	0.010
Upper hatch side girder	1	-	-	13.000	0.008	0.104	0.000	1.352
Lower hatch side girder	1	-	-	10.000	0.003	0.030	0.000	0.300
				∑	0.822	4.279	1.782	42.451

- The height of the NA above the baseline: $NA = \frac{\sum a_i h_i}{a_i} = \frac{4.279}{0.822} = 5.205\ m$
- The total moment of inertia (both sides) about the NA:

$$I_{NA} = I_{BL} - A \cdot h_{NA}^2 = \sum i + \sum a_j h_j^2 - \sum a_j \cdot h_{NA}^2$$
$$= 2 \times \left\{ 1.782 + 42.451 - 0.822 \times 5.205^2) \right\} = 43.927\ m^4$$

- The distance from the neutral axis to the deck and the bottom:

$$z_{deck} = 7.795\ m,\ \ z_{bottom} = 5.205\ m$$

- The sectional modulus at the deck and the bottom respectively:

$$Z_{deck} = \frac{I}{z_{deck}} = \frac{43.927}{7.795} = 5.635\ m^3$$
$$Z_{bottom} = \frac{I}{z_{bottom}} = \frac{43.927}{5.205} = 8.439\ m^3$$

- Since $z_{deck} = z_{\max}$; the maximum bending stress is at the deck:

$$\sigma_{deck} = \frac{M}{Z_{deck}} = \frac{400}{5.635} = 71\ MPa$$

- Finally, the factor of safety is given by:

$$FOS = \frac{\sigma_y}{\sigma_{deck}} = \frac{235}{71} = 3.31$$