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ASSESSMENT OF SHIP'S TRANSVERSE STABILITY AT SEA

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Abstract—The main objective of this paper is to present the application of the random decrement technique to nonlinear ship roll motion. The validity, accuracy and reliability of the method are investigated using model experimental and full-scale ship test data. The predicted natural frequency obtained using either the random decrement or autocorrelation function method can be used to determine the value of the metacentric height for a ship rolling under the action of unknown random excitation.

Since the random decrement method does not require the measurements of wave height, and roll motion of a ship can be easily measured in a seaway, it is expected that the method would be particularly useful for the determination of the transverse stability of a ship sailing in a realistic sea.

1. INTRODUCTION

Ship's transverse stability at sea is one of the most important factors in determining its survivability. The metacentric height is usually used as a measure of initial transverse stability. The shape of the righting arm curve will determine the ability of the ship to come back to the upright position after she has been acted upon by an external moment. During roll motion, other factors affect the ability of the ship to return to the upright position: exciting, damping and inertia moments. The metacentric height and the righting arm curves are usually determined for every ship at the time of building. Ship's stability booklet gives enough information for the estimate of its metacentric height and GZ curves at different loading conditions. However, environmental and dynamic effects are not usually taken into consideration. The level of wave excitation may surpass that with which a ship can cope. Oblique waves are known to change the transverse stability characteristics of the ship. Beam wind can change the ship's righting moment characteristics through the introduction of steady list. The effect of ice accumulation on the bridge and deck on the vertical position of the centre of gravity is difficult to estimate. A method for obtaining an estimate for the ship's transverse stability at sea, which defines safe navigation conditions, can help in alerting the captain when a bad environmental condition is being met. Since wave measurement is difficult to obtain from a ship moving with forward speed, we have to depend only on the measured response of the ship.

Roberts *et al.* (1991) used the method of stochastic linearization to develop a method for the identification of the parameters in the roll differential equation using roll response only. The authors tested the method using digitally generated roll responses. This method is applicable for systems which have very weak nonlinearities. Good agreement was obtained for the cases when damping was very small and the restoring moment was linear. However, the agreement deteriorates when the damping or the nonlinearity in the restoration are large.

Haddara (1992) used the Fokker–Planck equation approach to extend the applicability of the random decrement technique to nonlinear systems. Assuming that the excitation satisfies certain conditions, one can show that the expected value of the roll angle satisfies approximately the free roll equation. Thus, using the stationary random roll response, one can determine all the parameters that can be obtained from a free roll decay curve. It was also shown [see Haddara (1992)] that the autocorrelation function for the roll angle satisfies a linearized roll equation. The method was tested using digitally generated and preliminary experimental data. Good agreement was obtained [see Haddara (1992) and Haddara and Wu (1993)].

The objective of the present work is to show the results obtained using this method when applied systematically to roll responses measured from three fishing vessel models in the towing tank of Memorial University. The roll responses obtained from a full-scale test using a 32.80 m fishing vessel are also analyzed and presented. Excellent agreement between predicted and actual values for the metacentric height was obtained in all cases. Good agreement was also obtained for the damping parameters. However, damping estimation was not as successful in some cases.

2. ANALYTICAL METHODS

The Random Decrement and Autocorrelation Function methods could be used in determining the natural frequency and roll damping of a ship subject to a randomly varying excitation. The two methods share one characteristic: no knowledge of the excitation is required as long as it is a zero mean, Gaussian, white noise random process. To determine all relevant parameters from the random decrement, autocorrelation function and free roll decay curves, an estimation method based on the theory of equivalent linearization is developed and presented in this paper.

2.1. *Random Decrement method*

The formulation of the random decrement existing in the literature is based on the assumption that the dynamic system is linear, time invariant and subjected to Gaussian, white noise excitation. Thus, one can use the principle of superposition in formulating the equation for the random decrement. For nonlinear systems the principle of superposition does not apply. However, we can use the Fokker–Planck equation to show that the random decrement is an approximation to the free decay curve [Haddara (1992)].

The nonlinear roll motion of a ship in random beam seas can be mathematically modelled by a second-order ordinary, nonlinear differential equation

$$\ddot{\phi} + N(\phi,\dot{\phi}) + D(\phi) = K(t) \tag{1}$$

where ϕ is the roll angle, $\dot{\phi}$ and $\ddot{\phi}$ are the first and second derivatives with respect to time, $N(\phi,\dot{\phi})$ and $D(\phi)$ are the nonlinear damping and restoring moments per unit virtual inertia and $K(t)$ is the wave exciting moment per unit virtual inertia.

It is assumed that the wave excitation $K(t)$ is a stationary Gaussian white noise random process which satisfies the following equations:

$$\langle K(t) \rangle = 0$$

$$\langle K(t_1)\, K(t_2) \rangle = \Psi_0\, \delta(t_1 - t_2)$$

where δ is the Dirac delta function, Ψ_0 is the variance of the excitation and the symbol $\langle\ \rangle$ means the ensemble average of the process.

Using the following change of variables:

$$\begin{matrix} y_1 = \phi \\ y_2 = \dot{\phi} \end{matrix} \text{ and let } Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}.$$

Then, Equation (1) can be expressed as

$$\dot{y}_1 = y_2$$

$$\dot{y}_2 = -N(y_1, y_2) - D(y_1) + K(t).$$

The stochastic process $Y(t)$ is assumed to be a Markov process. A Markov process satisfies the following conditional probability equation:

$$P_n(x_n t_n \mid x_1 t_1, x_2 t_2, \ldots, x_{n-1} t_{n-1}) = P_2(x_n t_n \mid x_{n-1} t_{n-1}).$$

Thus the process $Y(t)$ may be described by the conditional probability density function, $P_2(y_1, y_2, t \mid y_{10}, y_{20})$, where y_{10} and y_{20} are the initial values of the angle and velocity of roll motion. One can show that P_2 satisfies the following partial differential equation:

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial y_1}(y_2 P) + \frac{\partial}{\partial y_2}(N + D)P + \frac{\Psi_0}{2}\frac{\partial^2 P}{\partial y_2^2} \tag{2}$$

where P represents $P_2(y_1, y_2, t \mid y_{10}, y_{20})$. The solution of Equation (2) subject to the initial condition:

$$P(y_1, y_2, t \mid y_{10}, y_{20}) = \delta(y_1 - y_{10})\, \delta(y_2 - y_{20})$$

as $t \to 0$, yields the conditional probability density function which describes the process $Y(t)$ completely.

In order to obtain an equation describing the expected value propagation, we rewrite Equation (2) as

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} P(y_1, y_2, t \mid y_{10}, y_{20}) &= \frac{P(y_1, y_2, t + \mathrm{d}t \mid y_{10}, y_{20}) - P(y_1, y_2, t \mid y_{10}, y_{20})}{\mathrm{d}t} \\ &= -\frac{\partial}{\partial y_1}(y_2 P) + \frac{\partial}{\partial y_2}(N + D)P + \frac{\Psi_0}{2}\frac{\partial^2 P}{\partial y_2^2}. \end{aligned} \tag{3}$$

Multiplying Equation (3) by y_1 and integrating the equation with respect to y_1 and y_2 from $-\infty$ to ∞ gives

$$\dot{\mu}_1 = \mu_2 \tag{4}$$

where μ_1 and μ_2 are the expected values of y_1 and y_2, respectively. We then repeat the same process using y_2 and get

$$\dot{\mu}_2 = -\langle N(y_1, y_2) + D(y_1) \rangle.$$

This equation can be expanded in its Taylor series about μ_1 and μ_2. Retaining the first-order terms only gives

$$\dot{\mu}_2 = -\langle N(\mu_1,\mu_2) + D(\mu_1) + \left[(y_1-\mu_1)\frac{\partial}{\partial y_1} + (y_2-\mu_2)\frac{\partial}{\partial y_2}\right][N(\mu_1,\mu_2) + D(\mu_1)]\rangle$$
$$= -N(\mu_1,\mu_2) - D(\mu_1). \tag{5}$$

Substituting Equation (4) into Equation (5), we get

$$\ddot{\mu}_1 + N(\mu_1\dot{\mu}_1) + D(\mu_1) = 0. \tag{6}$$

We see from this equation that the expected value of the nonlinear random roll motion satisfies, to a first-order approximation, the differential equation of its free roll motion.

2.2. *Autocorrelation Function*

It has been shown by Vandiver *et al.* (1982) that the autocorrelation function of the response of a linear time invariant system excited by a zero mean, stationary, Gaussian random process is linearly related to its random decrement. For nonlinear systems, the autocorrelation function of the random roll response satisfies an equivalent linear differential equation governing the free roll motion [Haddara (1983)].

The elements of the autocorrelation matrix of the process $Y(t)$ are defined by

$$C_{ik}(\tau) = \int\int y_{i0}\, y_k\, P(Y,\tau \mid Y_0)\, P_s(Y_0)\, \mathrm{d}Y\, \mathrm{d}Y_0$$

where $P_s(Y)$ is the steady state probability function and it is independent of t and Y_0.

If we multiply both sides of Equation (3) by $y_{i0}\, y_k P_s(Y_0)$ and integrate the two sides of the equation from $-\infty$ to ∞, we get the following differential equations describing the elements of the correlation matrix:

$$\dot{C}_{11}(\tau) = C_{12}(\tau)$$
$$\dot{C}_{21}(\tau) = C_{22}(\tau)$$
$$\dot{C}_{12}(\tau) = -\langle y_{10}(N+D)\rangle$$
$$\dot{C}_{22}(\tau) = -\langle y_{20}(N+D)\rangle.$$

Using a Gaussian closure technique, the four equations can be combined into one approximate differential equation for the autocorrelation function C_{11} of the roll angle:

$$\ddot{C}_{11} + b_e\dot{C}_{11} + \omega_e^2 C_{11} = 0 \tag{7}$$

where b_e and ω_e are the equivalent linear damping and equivalent natural frequency, respectively.

If the damping and restoring moment take the form

$$N(y_1,y_2) = 2\zeta\omega_\phi(y_2 + \epsilon y_2^3)$$
$$D(y_1) = \omega_\phi^2(y_1 + \alpha_1 y_1^3 + \alpha_2 y_1^5)$$

then the equivalent linear damping and natural frequency are given by

$$b_e = 2\zeta\omega_\phi (1 + 3\epsilon\omega_\phi^2\sigma^2)$$

$$\omega_e^2 = \omega_\phi^2 (1 + 3\alpha_1\sigma^2 + 15\alpha_2\sigma^4)$$

where σ^2 is the variance of linear rolling motion.

Equation (7) shows that the autocorrelation function of a nonlinear, time invariant system excited by a zero mean, stationary, Gaussian, white noise random process satisfies the differential equation describing the free motion of an equivalent linear system.

2.3. *The proposed method for parameter estimation*

In this paper, the following nonlinear roll equation is used to describe the free roll decay, the random decrement and the autocorrelation function curves:

$$\ddot{\phi} + b_1\dot{\phi} + b_3\dot{\phi}^3 + \omega_\phi^2(\phi + \alpha_1\phi^3 + \alpha_2\phi^5) = 0 \tag{8}$$

where b_1 and b_3 are linear and nonlinear damping coefficients, respectively; ω_ϕ is the natural frequency; α_1 and α_2 are the nonlinear restoring coefficients.

For a specified center of gravity, the stability (GZ) curve can be approximated by

$$GZ(\phi) = GM(\phi + \alpha_1\phi^3 + \alpha_2\phi^5) \tag{9}$$

where GM is the transverse metacentric height. The values of α_1 and α_2 are determined using a least-squares technique to fit Equation (9).

To evaluate the natural frequency and the damping parameters, the method of equivalent linearization given by Krylov and Bogoliubov (Hagedorn, 1988) is applied here. This technique assumes that a given nonlinear differential equation can be replaced by an equivalent linear differential equation such that the solution of the two equations can be made to differ from each other by an error of the order ϵ^2.

The nonlinear differential Equation (8) can be rewritten as the form

$$\ddot{\phi} + \omega_\phi^2\phi = \epsilon f(\phi,\dot{\phi}) \tag{10}$$

where

$$\epsilon f(\phi,\dot{\phi}) = -(b_1\dot{\phi} + b_3\dot{\phi}^3 + \alpha_1\omega_\phi^2\phi^3 + \alpha_2\omega_\phi^2\phi^5). \tag{11}$$

The equivalent linear equation is given by

$$\ddot{\phi} + b_e\dot{\phi} + \omega_e^2\phi = 0. \tag{12}$$

Following the theory of the first approximation, one starts with a solution of the form

$$\phi = R\cos\psi = R\cos(\omega_\phi t + \theta).$$

The linearized parameters b_e and ω_e are defined by the following functions:

$$b_e = \frac{\epsilon}{\pi R\omega_\phi}\int_0^{2\pi} f(R\cos\psi, -R\omega_\phi\sin\psi)\sin\psi d\psi \tag{13}$$

$$\omega_e^2 = \omega_\phi^2 - \frac{\epsilon}{\pi R}\int_0^{2\pi} f(R\cos\psi, -R\omega_\phi \sin\psi)\cos\psi d\psi. \tag{14}$$

Substituting Equation (11) into Equations (13) and (14), respectively, and using

$$\phi = R\cos\psi \qquad \dot{\phi} = -R\omega_\phi \sin\psi,$$

we obtain

$$b_e = b_1 + \frac{3}{4}R^2\omega_\phi^2 b_3 \tag{15}$$

$$\omega_e^2 = \omega_\phi^2\left(1 + \frac{3}{4}\alpha_1 R^2 + \frac{5}{8}\alpha_2 R^4\right). \tag{16}$$

The relationship between the damped natural frequency and the equivalent natural frequency can be found from the equivalent linear Equation (12). That is,

$$\omega_d = \sqrt{\omega_e^2 - b_e^2/4}. \tag{17}$$

By combining Equations (16) and (17), a new formula for predicting the natural frequency of a nonlinear system from its decay curve is developed. It is expressed as

$$\omega_\phi = \sqrt{\frac{\omega_d^2 + b_e^2/4}{1 + \frac{3}{4}\alpha_1 R^2 + \frac{5}{8}\alpha_2 R^4}}. \tag{18}$$

The equivalent damping coefficient b_e can be calculated from a decay curve using the formula

$$b_e = \frac{4}{T}\ln\left(\frac{\phi_k}{\phi_{k+1}}\right)$$

where T is the period of a cycle, and ϕ_k and ϕ_{k+1} are two consecutive peak amplitudes.

The values of b_e and R vary from cycle to cycle in a decay curve. Using the mean values of b_e and R in Equation (18), we have

$$\omega_\phi = \sqrt{\frac{\omega_d^2 + \bar{b}_e^2/4}{1 + \frac{3}{4}\alpha_1 \bar{R}^2 + \frac{5}{8}\alpha_2 \bar{R}^4}}. \tag{19}$$

This formula provides a practical tool for predicting the natural frequency of a nonlinear system using the information available from a decay curve.

The nonlinear damping coefficients are determined using a least-squares method to fit Equation (15) in which roll angle amplitude R takes the average of two consecutive peak amplitudes. That is,

$$R = \frac{\phi_k + \phi_{k+1}}{2}.$$

The method described above is capable of dealing with nonlinear damping and nonlinear restoring moments. The results obtained from the analysis of both generated data and real roll data showed that the method provides considerably accurate estimation for the natural frequency and damping coefficients.

3. EXPERIMENTAL STUDY

3.1. *Experiments*

The roll experiments in random waves were performed on three fishing vessel models—M363, M365 and M366—in the towing tank of Memorial University of Newfoundland. These models belong to the "less than 25 m" class and have similar dimensions but varying hull forms. The body plans of the three models are shown in Figs 1–3.

Each model was positioned for beam wave encounter in the wave tank with no forward speed. The unidirectional JONSWAP sea-spectrum was used for generating random waves, as it is a reasonable representation of a North Atlantic wave energy distribution. This spectrum, expressed as a function of frequency, is given by

$$S(f) = \frac{A}{f^5}\exp(-B/f^4)\,\gamma^a$$

where

$$A = \frac{5}{16}\frac{H_s^2 f_m^4}{\gamma^{1/3}},\ B = \frac{5}{4}f_m^4,\ a = \exp -\left[\frac{(f-f_m)^2}{(2\sigma^2 f_m^2)}\right].$$

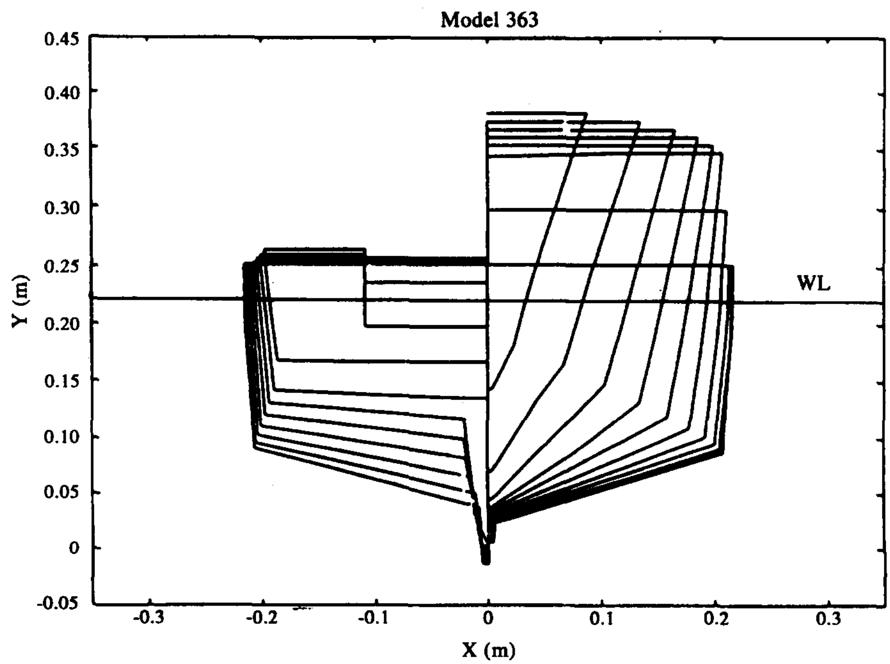

FIG. 1. Body plan of M363.

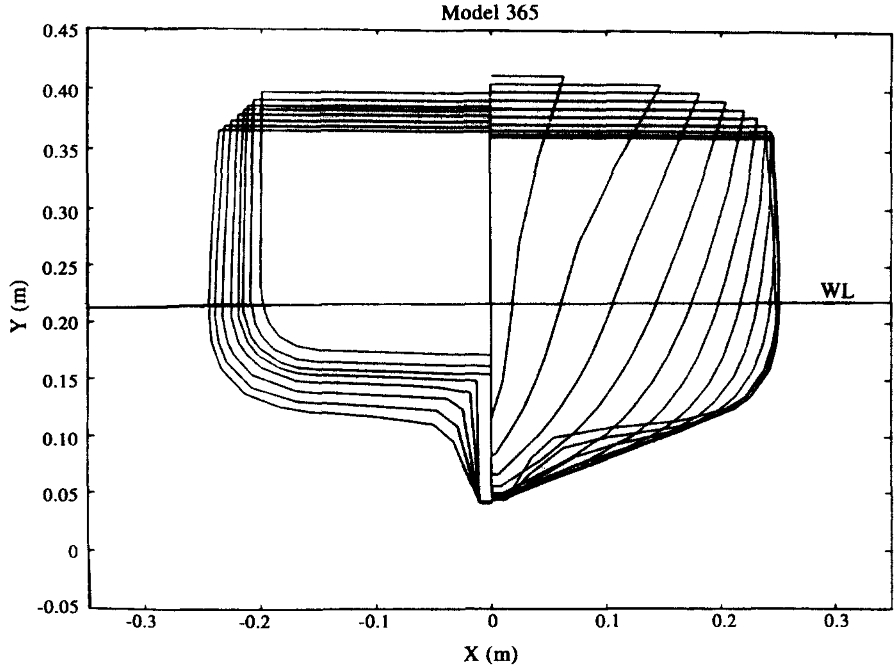

FIG. 2. Body plan of M365.

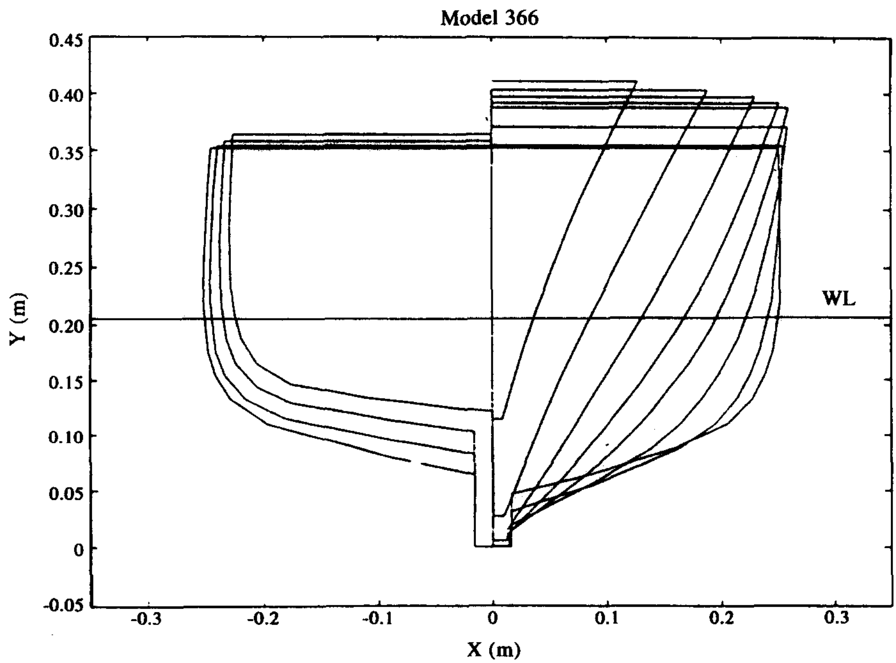

FIG. 3. Body plan of M366.

H_s is significant wave height and f_m is wave modal frequency. Peak enhancement factor γ took the value of 3.3 in the experiments and shape parameter $\sigma = 0.07$ for $f \leq f_m$ and $\sigma = 0.09$ for $f > f_m$.

For model M363, a dynamometer was attached to the model to measure the roll angle. The dynamometer prevented the model from motions other than rolling and heaving. For models M365 and M366, a gyroscope was installed on the model for measuring roll motion. The model was tethered from its bow and stern by two strings running through a pair of pulleys. Two small weights were fastened at the end of the two strings, which provided the model small adjustments for flexibility. This set-up allowed the model to move freely in six degrees of freedom.

The free roll tests and the roll tests in random beam waves were performed for each model at a series of GM values. To see the effect of different wave height on the random decrement, each model was subjected to two wave spectra with the same modal frequency but different significant wave heights. The sample duration of each record is 400 sec in order to meet stationarity requirements and provide enough data for analysis.

3.2. *Results*

The random decrement and the autocorrelation function have been calculated from the records of random rolling tests. For comparison purposes, the random decrement and the autocorrelation function curves are plotted with the corresponding free decay curve with the same initial angle and the same *GM* value. Figures 4–6 show the comparison of the random decrement and the free roll decay curves for the three different models. Figures 7–9 compare the autocorrelation function curve and the free roll decay curve for the same cases.

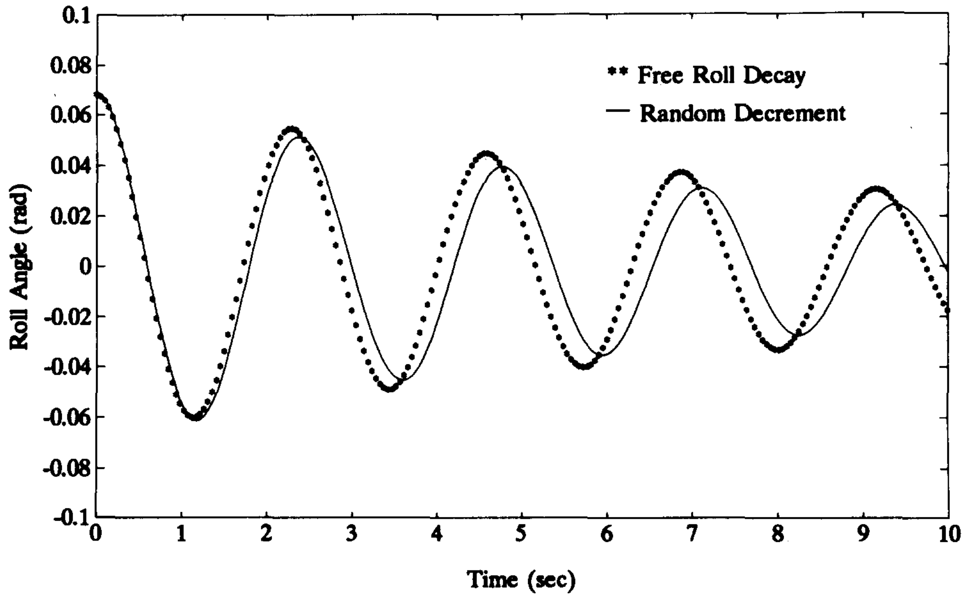

FIG. 4. Comparison of random decrement and free decay (M363).

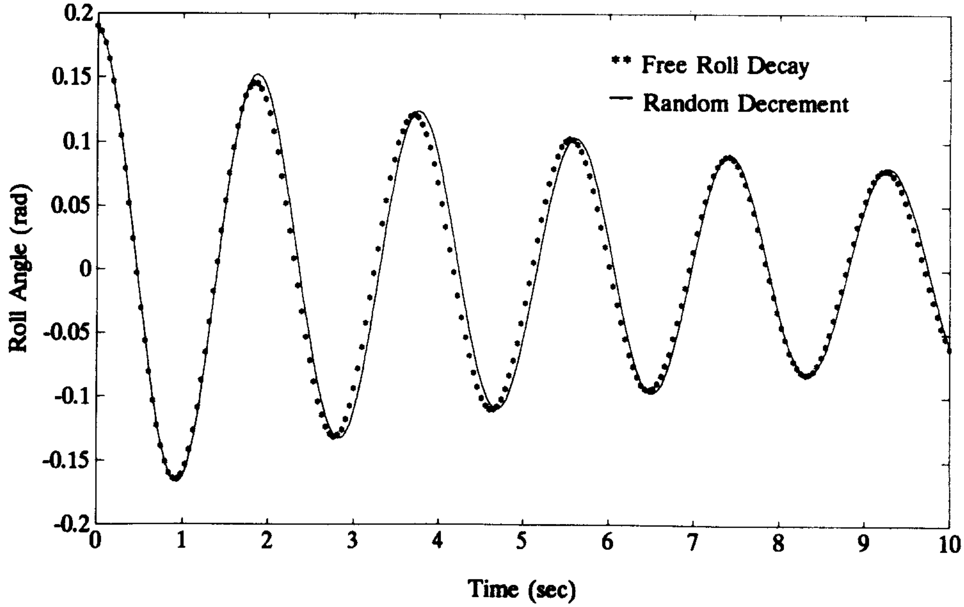

FIG. 5. Comparison of random decrement and free decay (M365).

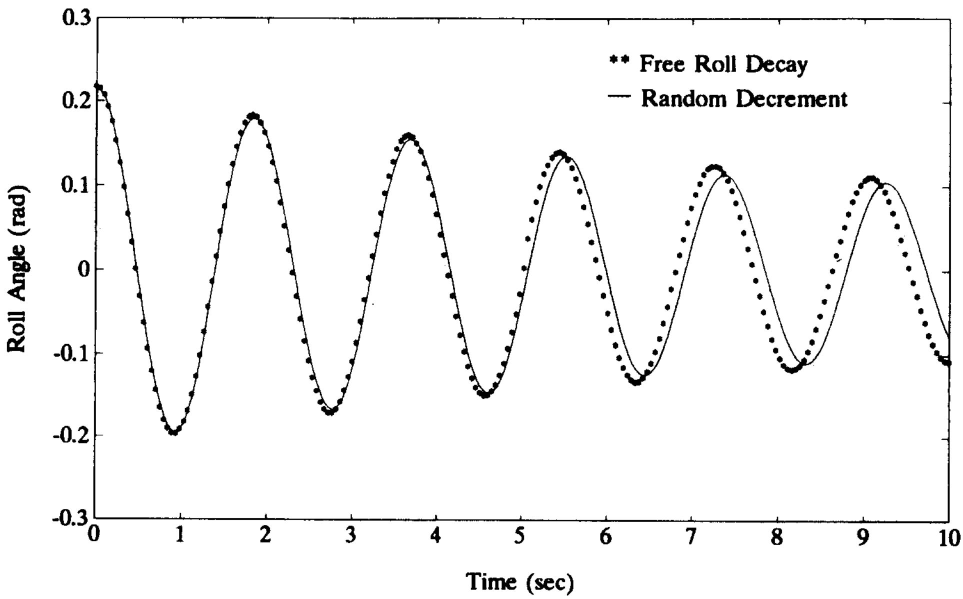

FIG. 6. Comparison of random decrement and free decay (M366).

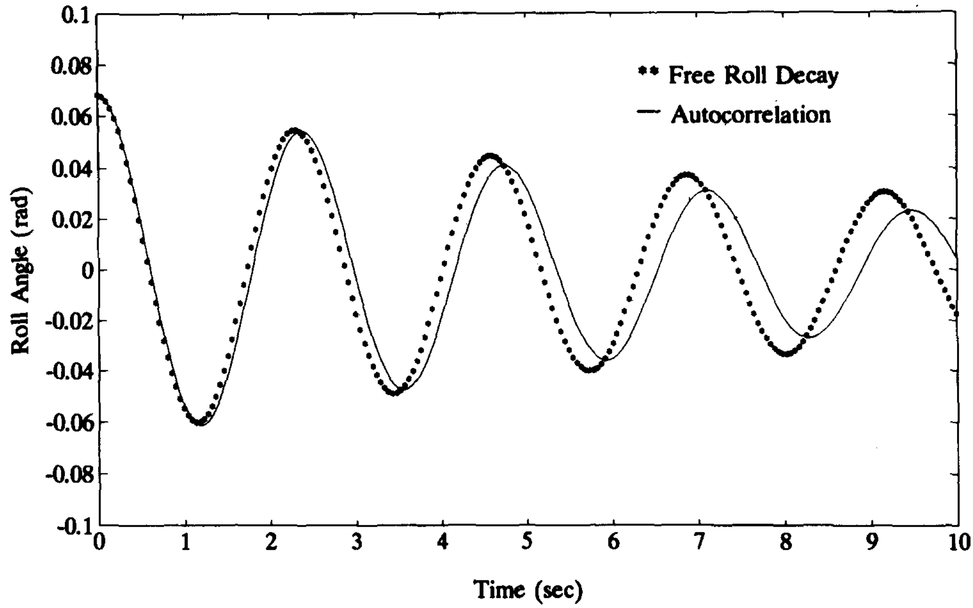

FIG. 7. Comparison of autocorrelation and free decay (M363).

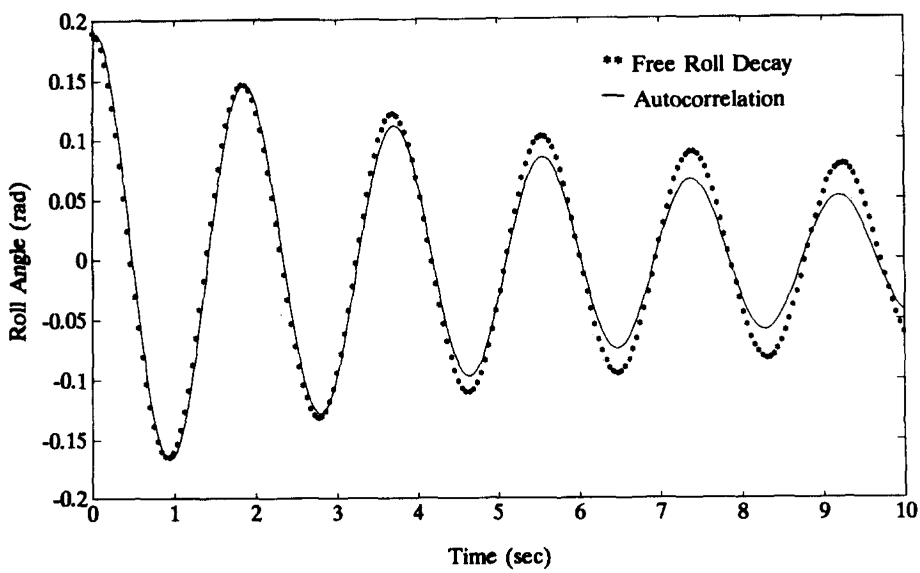

FIG. 8. Comparison of autocorrelation and free decay (M365).

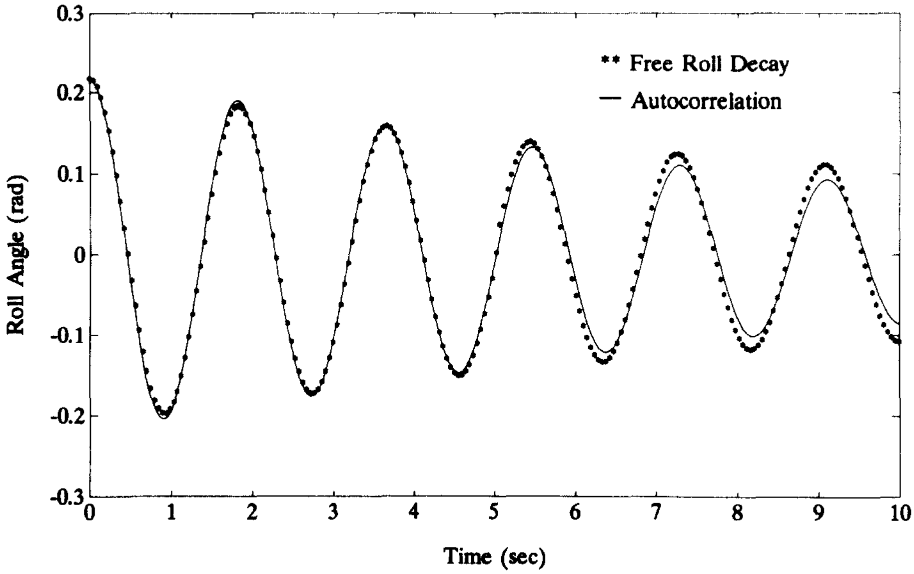

FIG. 9. Comparison of autocorrelation and free decay (M366).

It is seen that for M365 and M366, both the random decrement and the autocorrelation function curves closely resemble the actual free roll decay curves. Such resemblance for M363 is reasonable but not as good as in the cases of M365 and M366 due to the large nonlinear damping of M363. These results are expected because of the first-order approximation used in the derivation of the random decrement. In the autocorrelation function, the nonlinearities in the damping and restoration are replaced with equivalent linear quantities. As a consequence, the system with larger nonlinearity would produce larger errors in calculation of the random decrement and the autocorrelation functions. Also, M363 experiments were conducted with the model constrained against sway motion while models M365 and M366 were allowed to sway. Restraining sway seems to influence the accuracy of the parametric identification.

To determine the roll parameters from free roll response and the random decrement, the proposed estimation method described previously was applied. The values of the damping coefficients and the natural frequency were estimated using the computer program written on the basis of Equations (15) and (19) in which α_1 and α_2 took the values obtained from GZ curve fit. Table 1 shows the parameters estimated from the

TABLE 1. PARAMETERS FROM RANDOM DECAY AND FREE DECAY

Model	Comparison	b_1	b_3	ω_ϕ	α_1	α_2
M363	Free decay	0.1509	2.0270	2.7453	1.2832	−1.3293
	Random decay	0.2041	1.7636	2.6737	1.2832	−1.3293
M365	Free decay	0.0894	0.7987	3.4052	−0.3640	−2.2695
	Random decay	0.1079	0.5702	3.4053	−0.3640	−2.2695
M366	Free decay	0.0933	0.2412	3.4643	0.1480	−1.5676
	Random decay	0.0971	0.2897	3.3942	0.1480	−1.5676

random decrement curves and those from the free roll decay curves. We see in Table 1 that M363 has the highest linear and nonlinear damping among the three models because of the effects of its long hard chine, deep skeg and low rise of floor. M366 has the lowest nonlinear damping coefficient.

The natural frequencies predicted from the random decrement and the autocorrelation function are compared with those estimated from the free roll decay curves. The plots of the square of the natural frequency vs *GM* values are shown in Figs 10–12 for the three models. It seems that M366 gives the best results and M363 gives larger errors. It is seen that both the random decrement method and the autocorrelation function method provide very good estimates for the natural frequency obtained from free roll decay. The results obtained using the two methods are very close to each other. From a linear regression, one can find that the natural frequency predicted using the autocorrelation function curve is slightly more accurate than those obtained from the random decrement.

The relationship between the square of the natural frequency and the *GM* value is demonstrated by the straight lines obtained using a least-squares fit. Thus, the natural frequency predicted from either the random decrement or the autocorrelation function curve can be used to predict instantaneous value of the metacentric height of the ship sailing under varied loading conditions in a realistic sea.

It is seen that the random decrement can be used to approximately estimate the nonlinear damping coefficients. The damping parameters estimated from the random decrement produce a free decay curve which compares well with the actual free decay curve. However, it is also found that not all of the random roll tests have been successfully used as a substitute for actual free roll decay in the prediction of nonlinear damping coefficients. One explanation to this problem is that the JONSWAP spectrum,

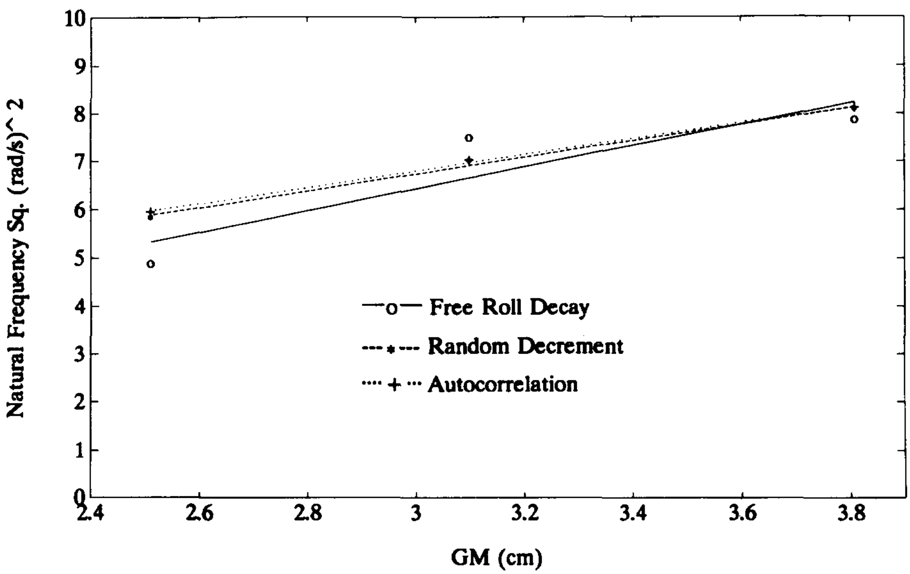

FIG. 10. ω_ϕ predicted from three methods (M363).

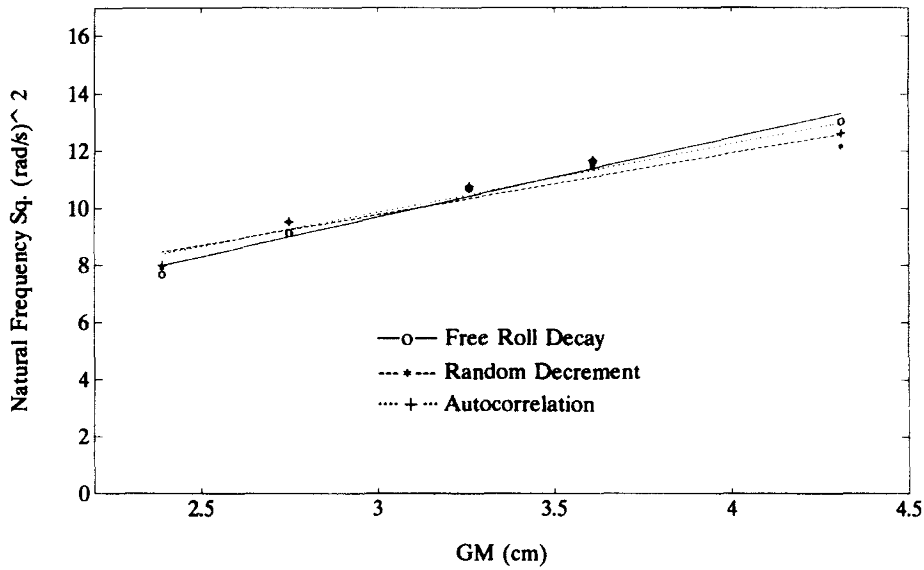

FIG. 11. ω_ϕ predicted from three methods (M365).

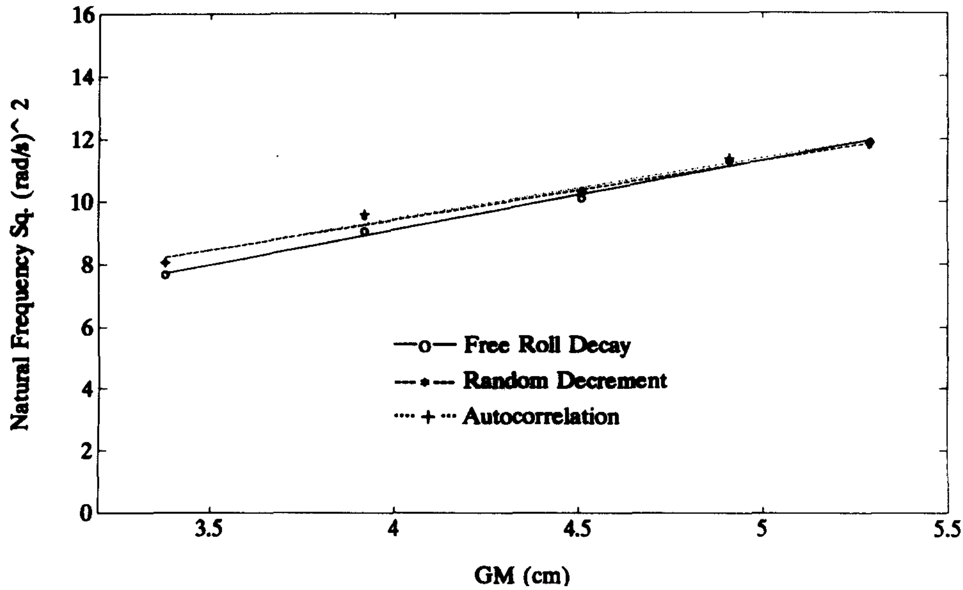

FIG. 12. ω_ϕ predicted from three methods (M366).

which was used for generating random waves, is different from the white noise spectrum that was assumed in the derivation of the random decrement. When the modal frequency of excitation is away from the natural frequency of the system, the wave spectrum is not sufficiently broader than the response band-width. The problem may also arise from the calculation of the expected values. When the number of segments is limited by the length of a record, the ensemble average of the segments may not represent the true random decrement. The first-order approximation used in the derivation of the random decrement would also produce some errors.

3.3. *Reliability of the Random Decrement method*

For linear systems, if the wave excitation is a stationary, zero mean, Gaussian, white noise random process, the random decrement should be dependent only on the system characteristics and independent of the ambient excitation. So, neither the type nor the intensity of the input should affect the scale and the form of the random decrement curve. This point is investigated here for nonlinear systems.

Figure 13 shows two random decrement curves extracted from the repeated tests conducted in random waves with the same wave spectrum. Although the time series of both wave excitation and roll response were different in the two tests, the random decrement curves extracted from two records of random roll response are almost identical. The excellent agreement obtained for the three models show that the random decrement method can produce repeatable decrement curves from the nonlinear roll response subject to the wave excitation with the same spectrum. This is important for ensuring a reliable and unique solution of the roll parameters to be obtained.

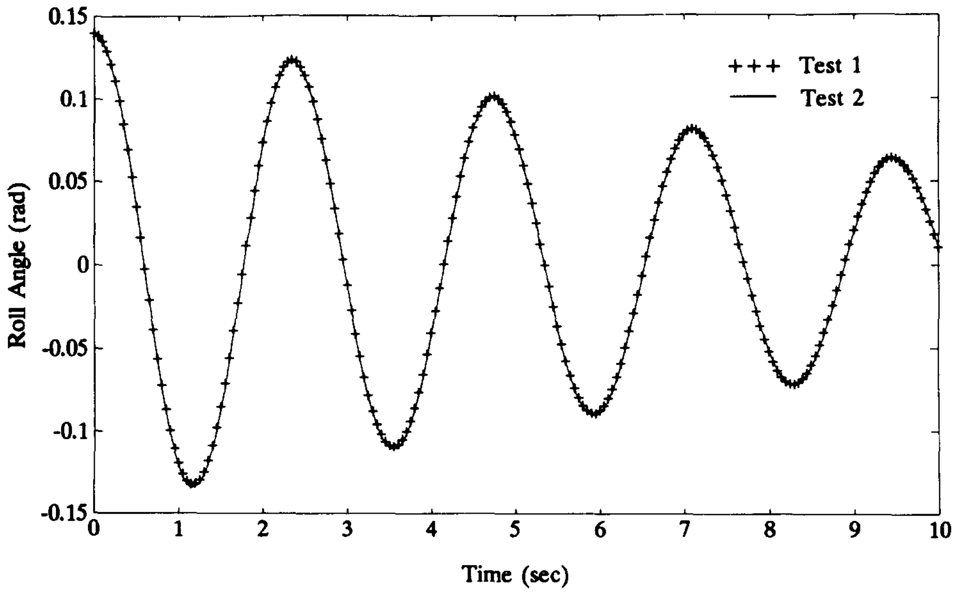

FIG. 13. Random decrement from repeated tests (M363).

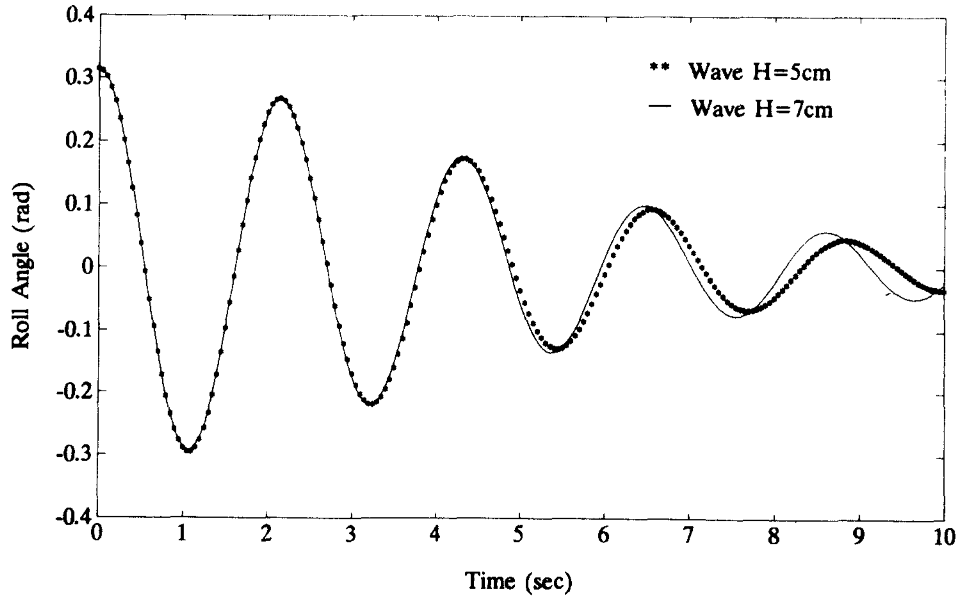

FIG. 14. Effect of wave height on random decrement (M363).

Figure 14 compares the random curves obtained from the random roll response to two wave spectra with the different significant wave heights and the same modal frequency. Although the significant amplitudes of the two random roll records are much different, their random decrement curves closely resemble each other. The good results obtained for the three models lead to the conclusion that the effect of varying significant wave height on the random decrement is weak and can be neglected.

4. FULL-SCALE SHIP TESTS

The real ship tests were carried out on the fishing vessel "*Newfoundland Alert*" during its fishing trip at sea in September 1992. The particulars of the ship are given in Table 2 and the ship layout is shown in Fig. 15. The instruments used for measuring roll motion of the ship are the same as those used in the experiments for M365 and M366.

TABLE 2. GENERAL PARTICULARS OF THE SHIP

Vessel's name	*Newfoundland Alert*
Type of vessel	Fishing vessel
Length between perpendicular	32.80 m
Breadth moulded	10.00 m
Depth moulded	6.80 m
Summer load draft	4.011 m
Displacement at SLWL	673 t.
Lightship weight	406 t.
Date keel laid	25 February 1988
Builder	Marystown Shipyard Ltd

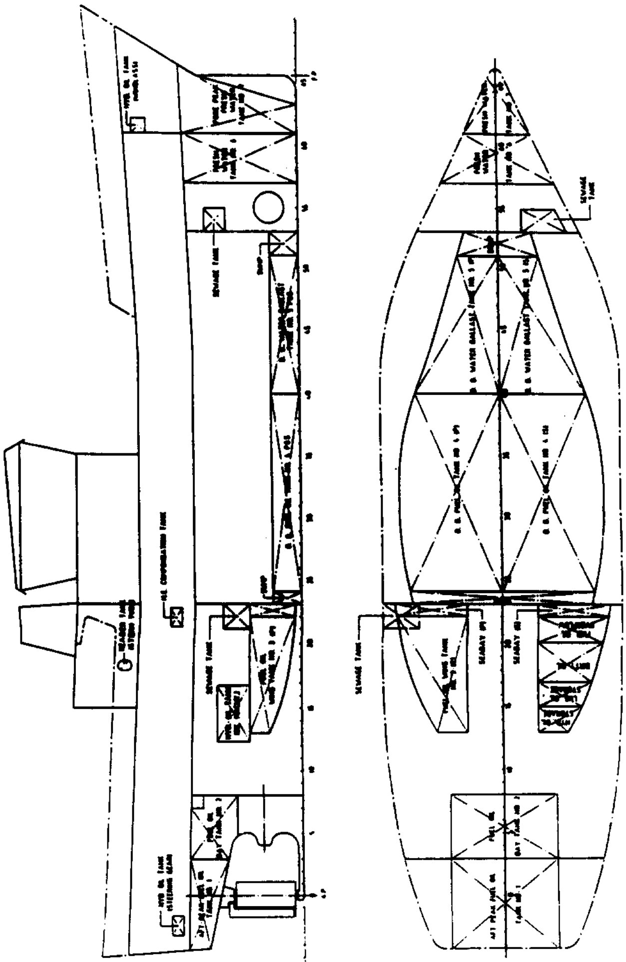

FIG. 15. Full-scale ship layout.

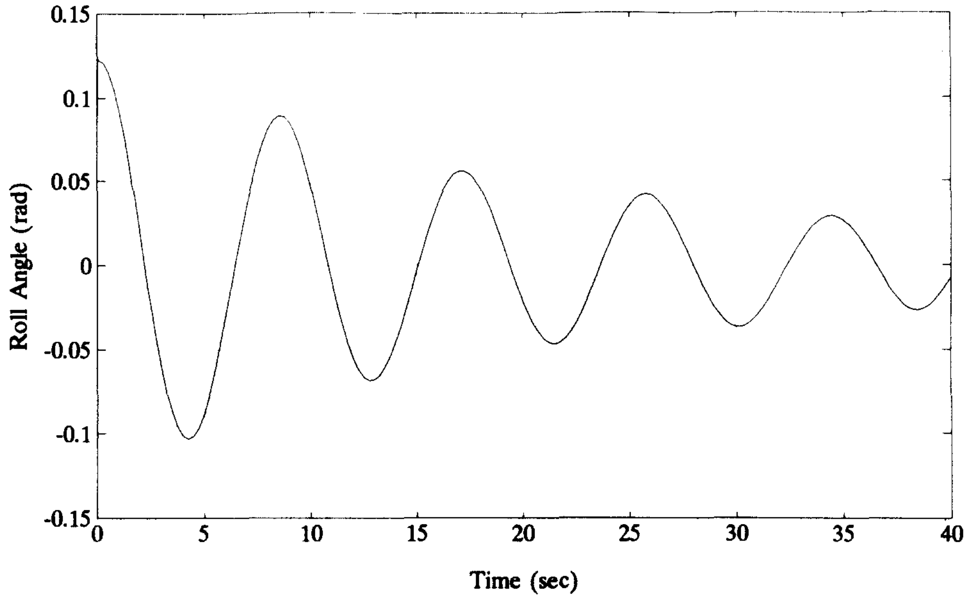

FIG. 16. Random decrement (ship: GM = 0.6989 m).

The loading condition of the ship changed from time to time during the trip because of the change in weight of the catch, the consumption of fuel and fresh water. To find the instantaneous GZ curve and GM value of the vessel, we need to calculate the weight and the center of gravity of each item causing the changes of mass and mass distribution of the vessel. In addition, there are several tanks located in the ship. When the tanks are partially filled with liquid, the stability of the ship is adversely affected by free surface effect, which causes a loss in GM. For a real ship sailing in a realistic sea, six degrees of freedom of motion and forward speed are involved. In addition, the ship is subject to varied sea states. Therefore, the situation of a ship at sea is more complicated than in numerical simulation or in experiments.

Figure 16 shows the random decrement curve extracted from the record of roll motion measured with the ship in a seaway. The estimated roll parameters obtained from the random decrement method are given in Table 3 for three loading conditions. The results show that the real ship tests used smaller sample size or less segments than those used in the simulation and the experiments to form an equally accurate random decrement curve. In addition, the damping coefficients obtained from the real ship tests show smaller variance than either the numerical simulation (Haddara and Wu,

TABLE 3. PARAMETERS ESTIMATED FROM RANDOM DECREMENT

Case	GM (m)	b_1	b_3	ω_ϕ	α_1	α_2
1	0.6989	0.0733	3.2782	0.7350	0.4025	−0.4818
2	0.7091	0.0775	4.5473	0.7401	0.3910	−0.4747
3	0.7287	0.0868	3.5923	0.7653	0.3671	−0.4611

1993) or the model experiment. One explanation is that the frequency band of the sea waves is probably broader than those used in the simulation and the experiments. Another reason is that the wave excitation of the actual sea is a true random process.

Figure 17 shows the plot of the square of natural frequency vs *GM* value obtained from the random decrement method and the autocorrelation function method. The two methods not only succeeded in distinguishing the small changes in the natural frequency with different loading conditions but also gave the results satisfying the linear relationship between the square of the natural frequency and the *GM* value. It has been found that if the random decrement method fails to provide good estimation from noisy data in the case of small roll motion, the autocorrelation function method correspondingly also provides bad results.

5. CONCLUSIONS

The feasibility of extending the random decrement technique to the case of nonlinear systems has been investigated. The method was applied to the nonlinear ship roll motion in random seas. Roll responses obtained from model experiments as well as full-scale tests were used. The theory behind the method assumes that roll motion is excited by zero-mean, white Gaussian noise. However, the method gave good results in cases of experimental roll responses excited by JONSWAP wave spectrum as well as realistic sea waves.

The validation of the method covered various loading conditions, different hull shapes and different significant wave heights. In all cases, free roll response could be derived from the stationary random roll response by either calculating the expected value of the roll angle or its correlation function.

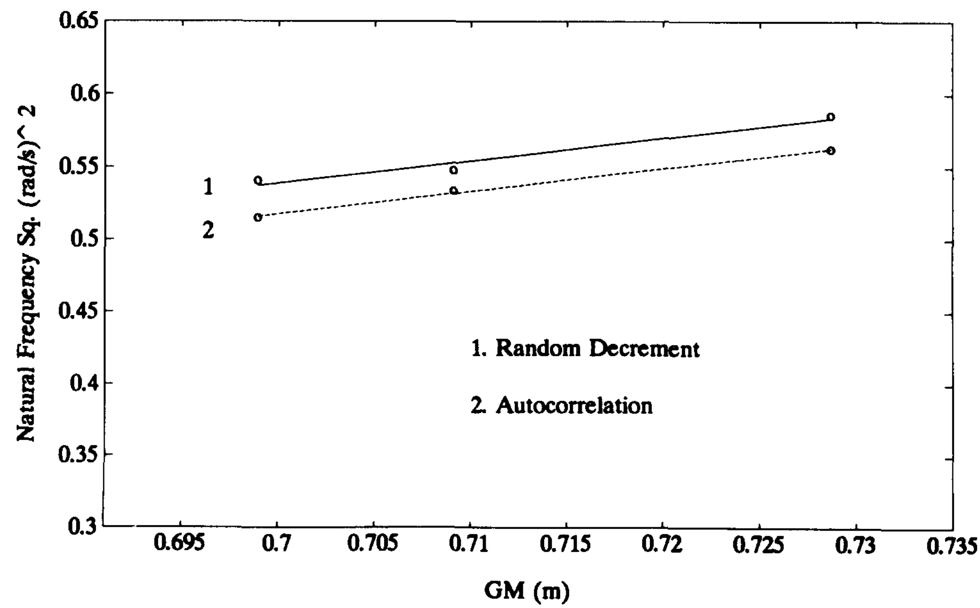

FIG. 17. Natural frequency predicted by two methods.

The values of the natural frequency predicted using the random decrement or the correlation function showed good agreement with those obtained from the free decay curves obtained from model tests and with those estimated from the stability booklet for the real ship. The linear relationship between the square of the natural frequency and the value of the metacentric height obtained can be used for the prediction of the latter while the ship is at sea.

Damping parameters can be predicted as well from the derived free decay curves. However, because of the approximations used, some of the predictions are not in good agreement. Further work is still required in this area.

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